Avalanche Dynamics on an Inclined Plane

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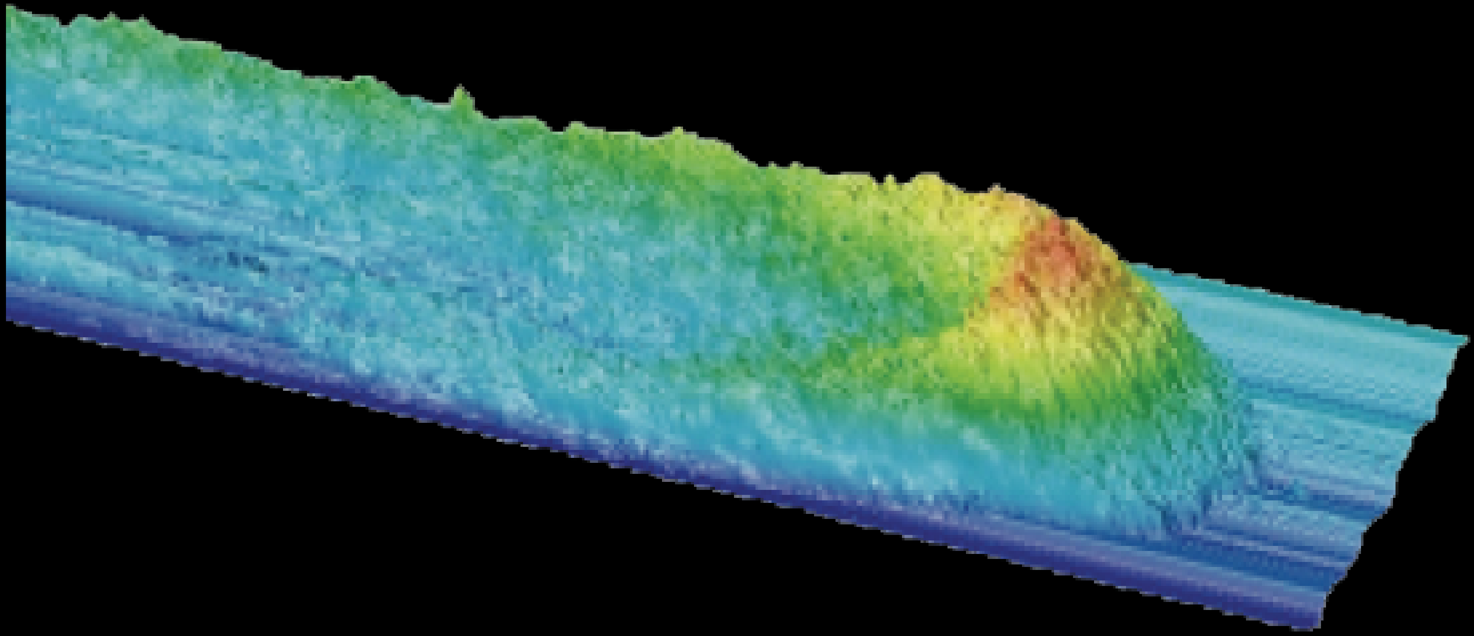

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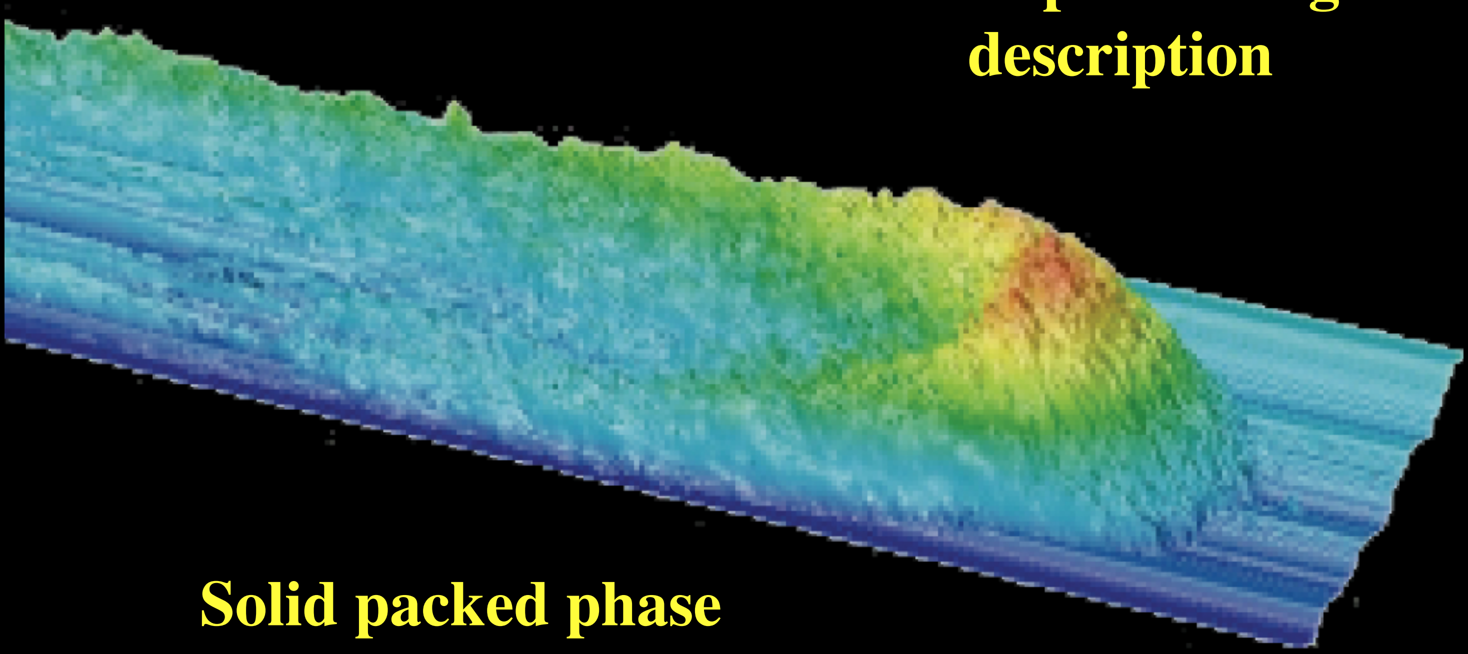
Motivation
Co-existing solid & fluid phases



Flowing fluid phase
Depth-averaged description

Solid packed phase
Material failure

Outline

The qualitative and quantitative dynamics of continuous avalanches on an inclined plane depends critically on materials properties.

A little background

The experiment: High speed imaging & Laser deflection

Results: ***The avalanche dynamics of smooth spherical particles are very different from the dynamics of avalanching sand (or other irregular grains)***

Some theory (Thomas Halsey)

Conclusions

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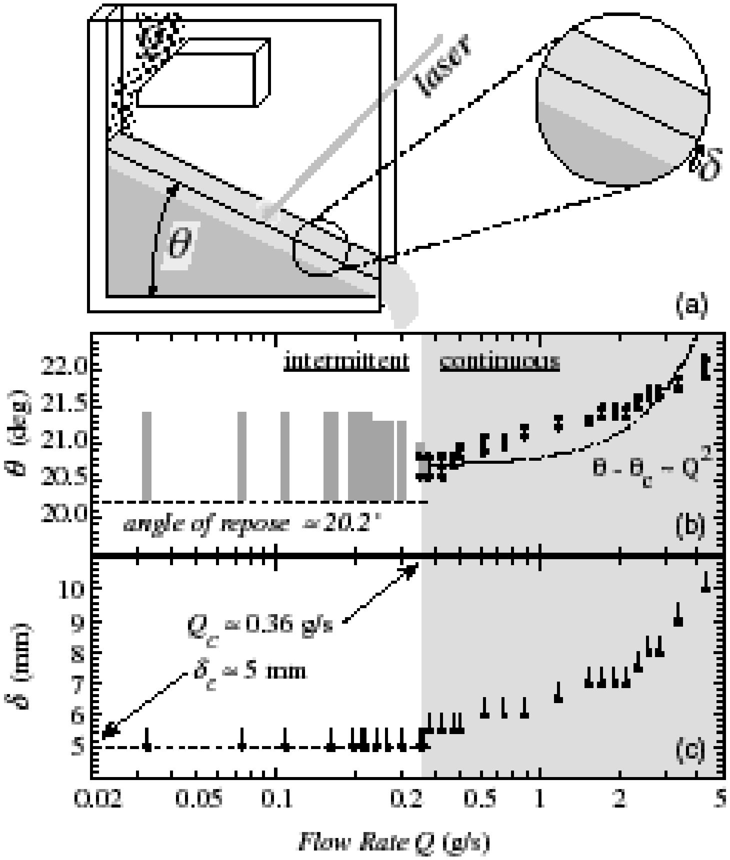
Avalanche Behavior in Bulk Systems

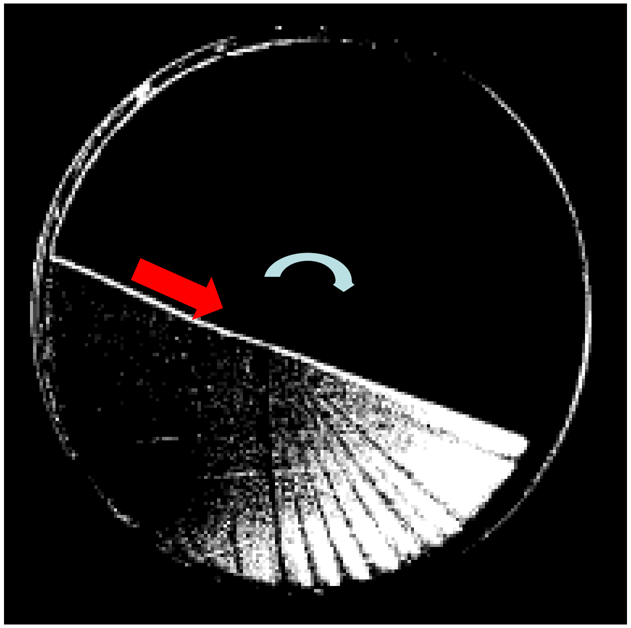
Avalanches on a heap

Lemieux & Durian
PRL **85**, 2473 (2000)



Avalanches in a rotating drum

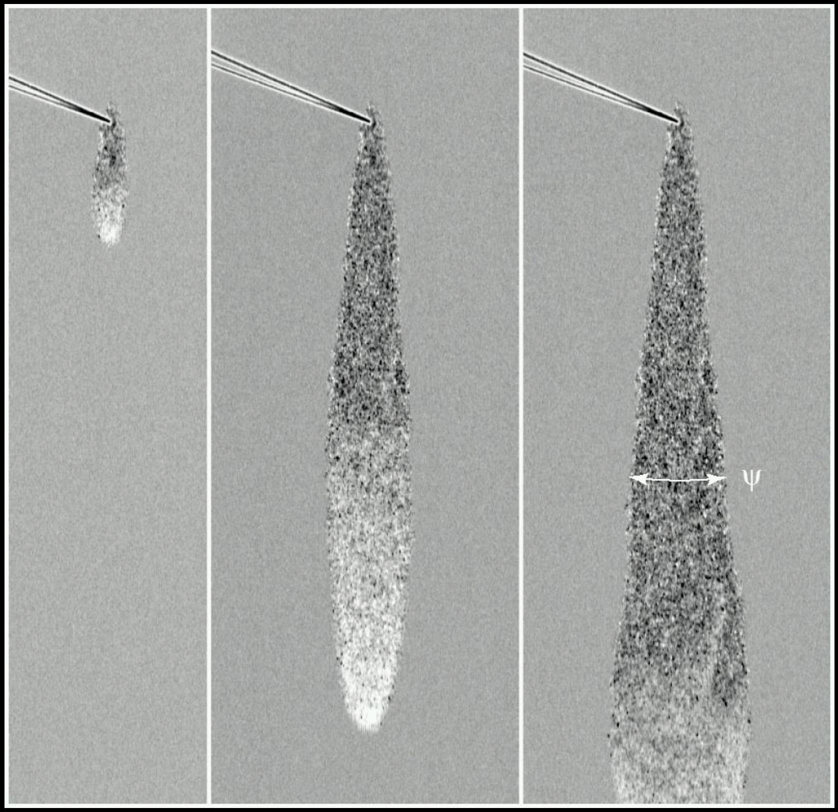
Rajchenbach, Adv. Phys.
49, 229 (2000)

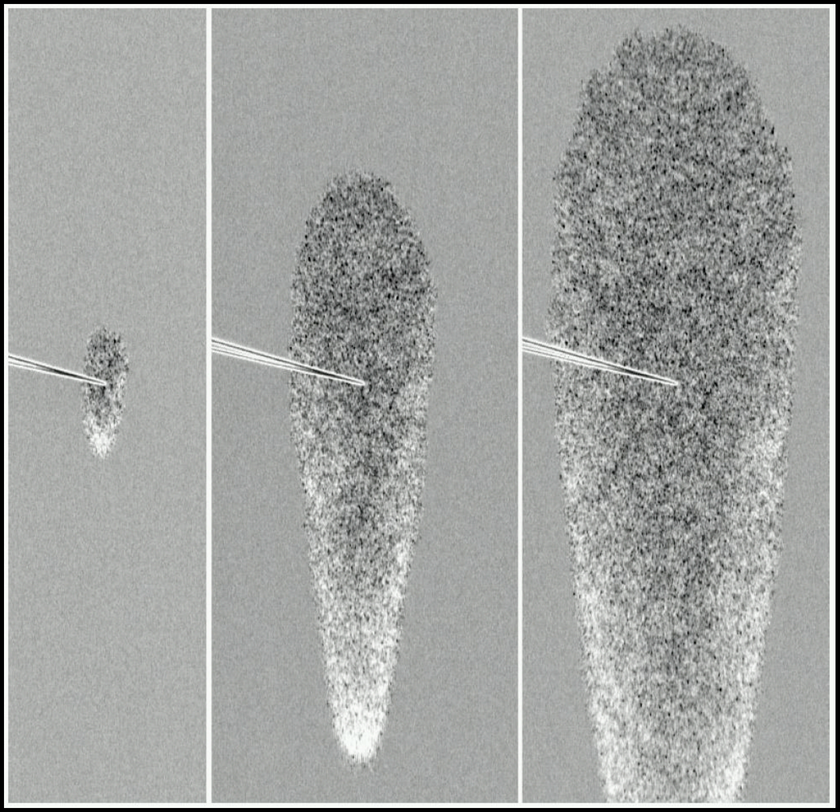


Emphasis on
Frequency & Size distribution

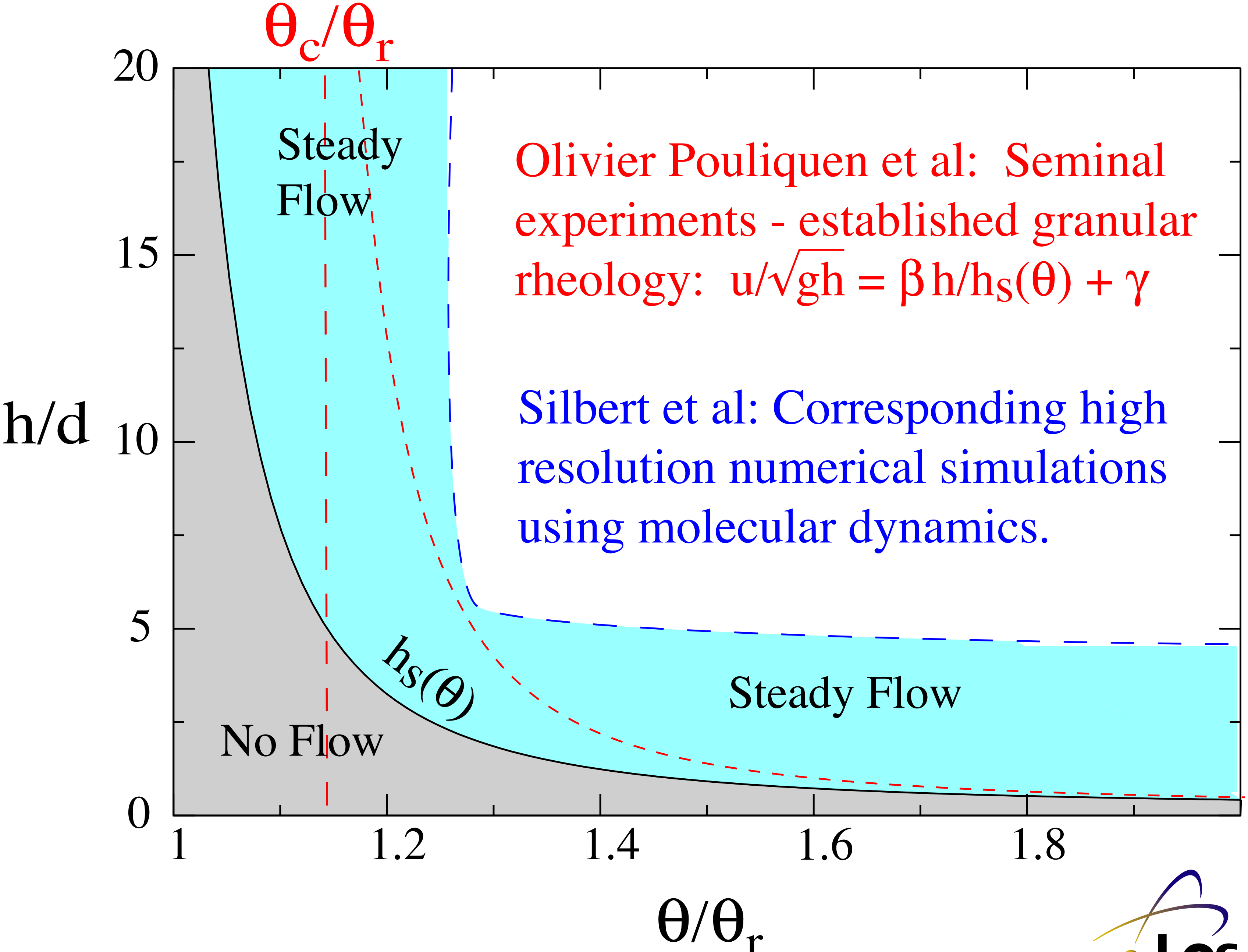
Avalanches on a metastable layer

Daerr and S. Douady, Nature **399**, 243 (1999)





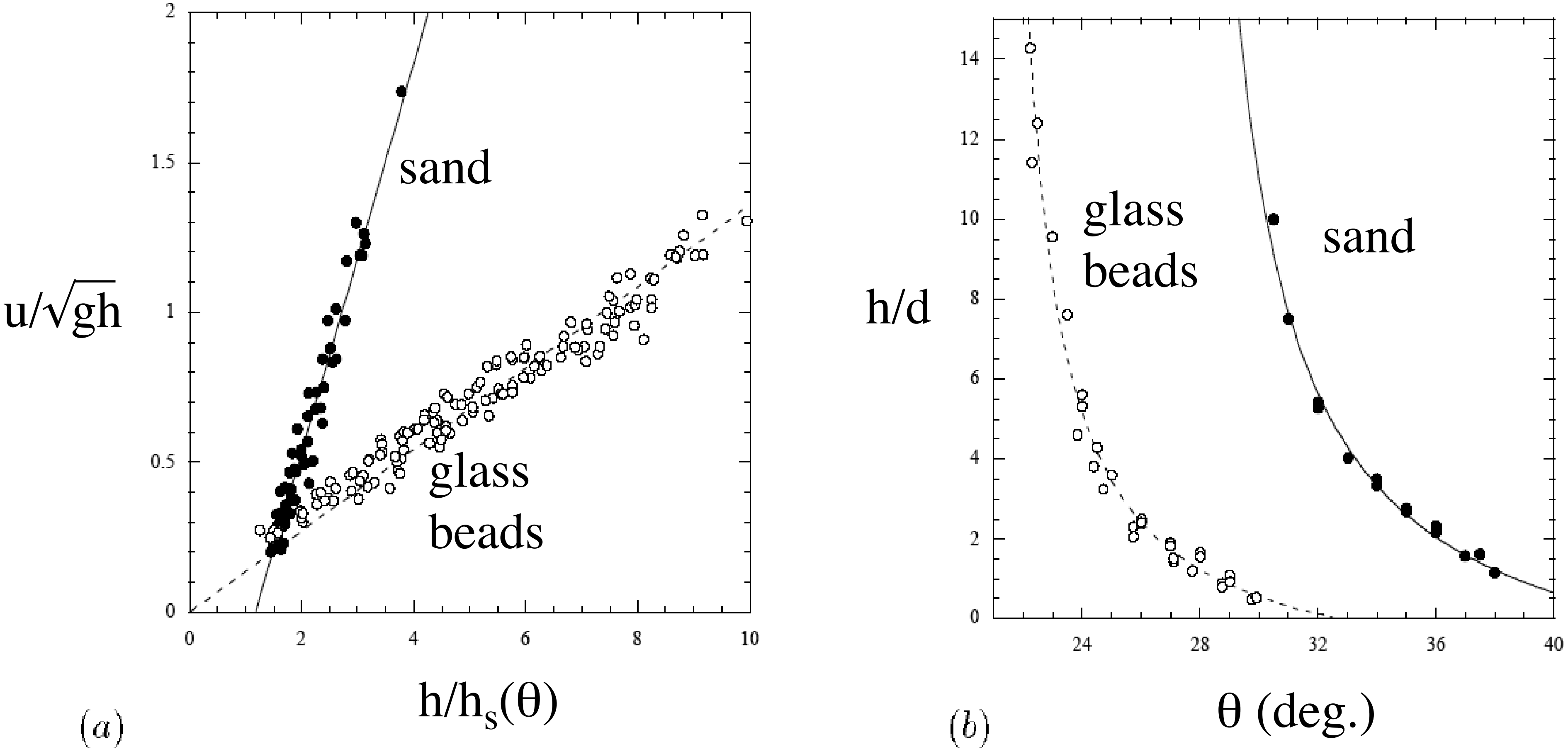
Phase Diagram for Granular Inclined Layer Flows



Olivier Pouliquen et al: Seminal experiments - established granular rheology: $u/\sqrt{gh} = \beta\, h/h_S(\theta) + \gamma$

Silbert et al: Corresponding high resolution numerical simulations using molecular dynamics.

Granular Rheology



Forterre & Pouliquen, J. Fluid Mech. **486**, 21 (2003)

Given that one understands uniform steady flows, what can one say about avalanche behavior where there are co-existing phases (solid & fluid/gas)

Material failure

Rheology of different granular materials

Fascinating Dynamics

Nonlinear wave equation - Burgers shock & breaking wave motion

BHE Physical Review Letters 94, 208001 (2005)

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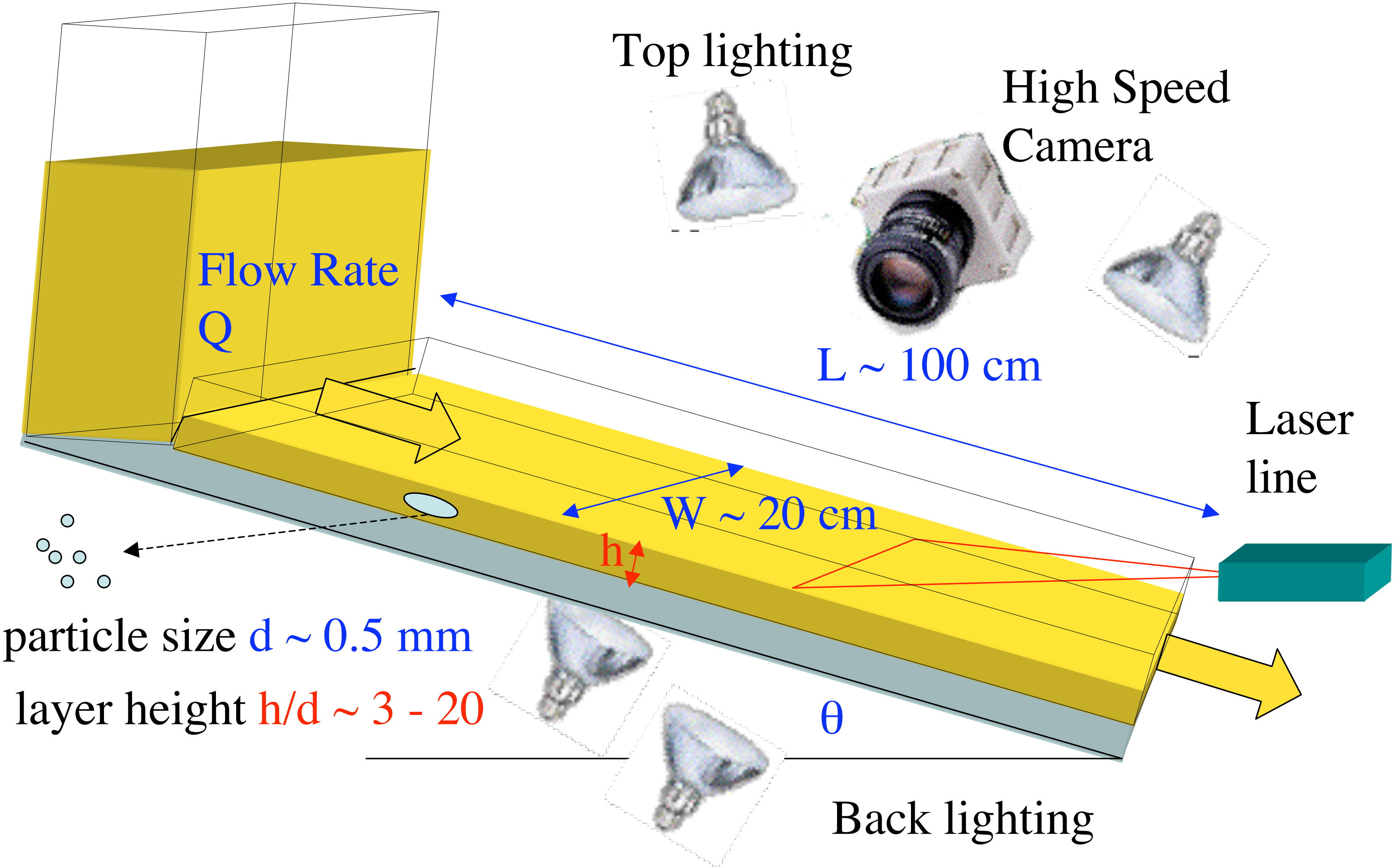
Experimental Setup



Top lighting

High Speed Camera

Flow Rate Q

L ~ 100 cm

Laser line

W ~ 20 cm

h

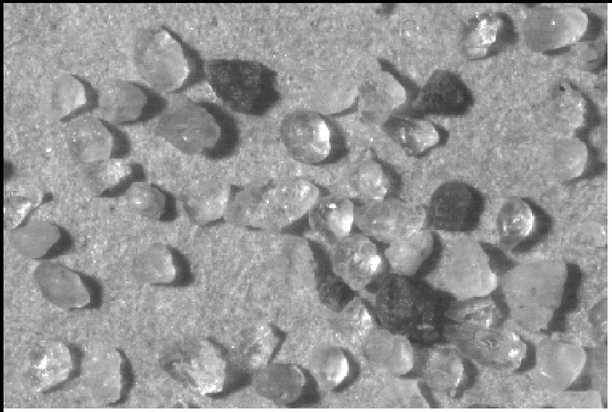
particle size d ~ 0.5 mm

layer height h/d ~ 3 - 20

θ

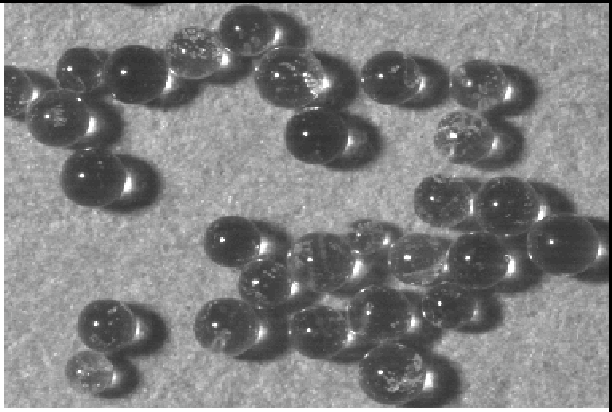
Back lighting

Characterization of the material



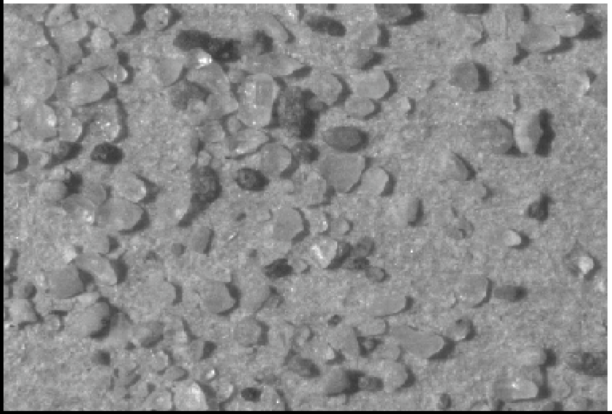
Sand

d=400 μm



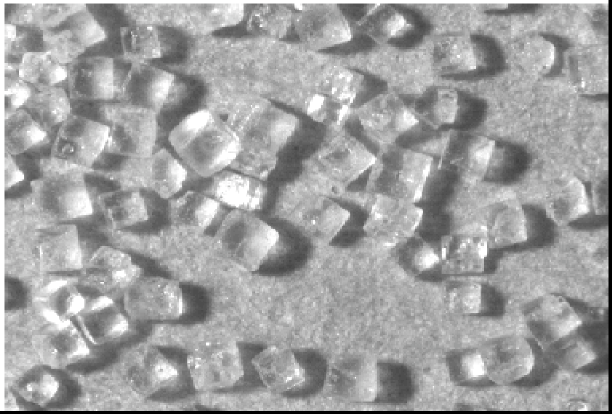
Glass beads

d=500 μm



Sand

d=150 μm



Salt

d=400 μm

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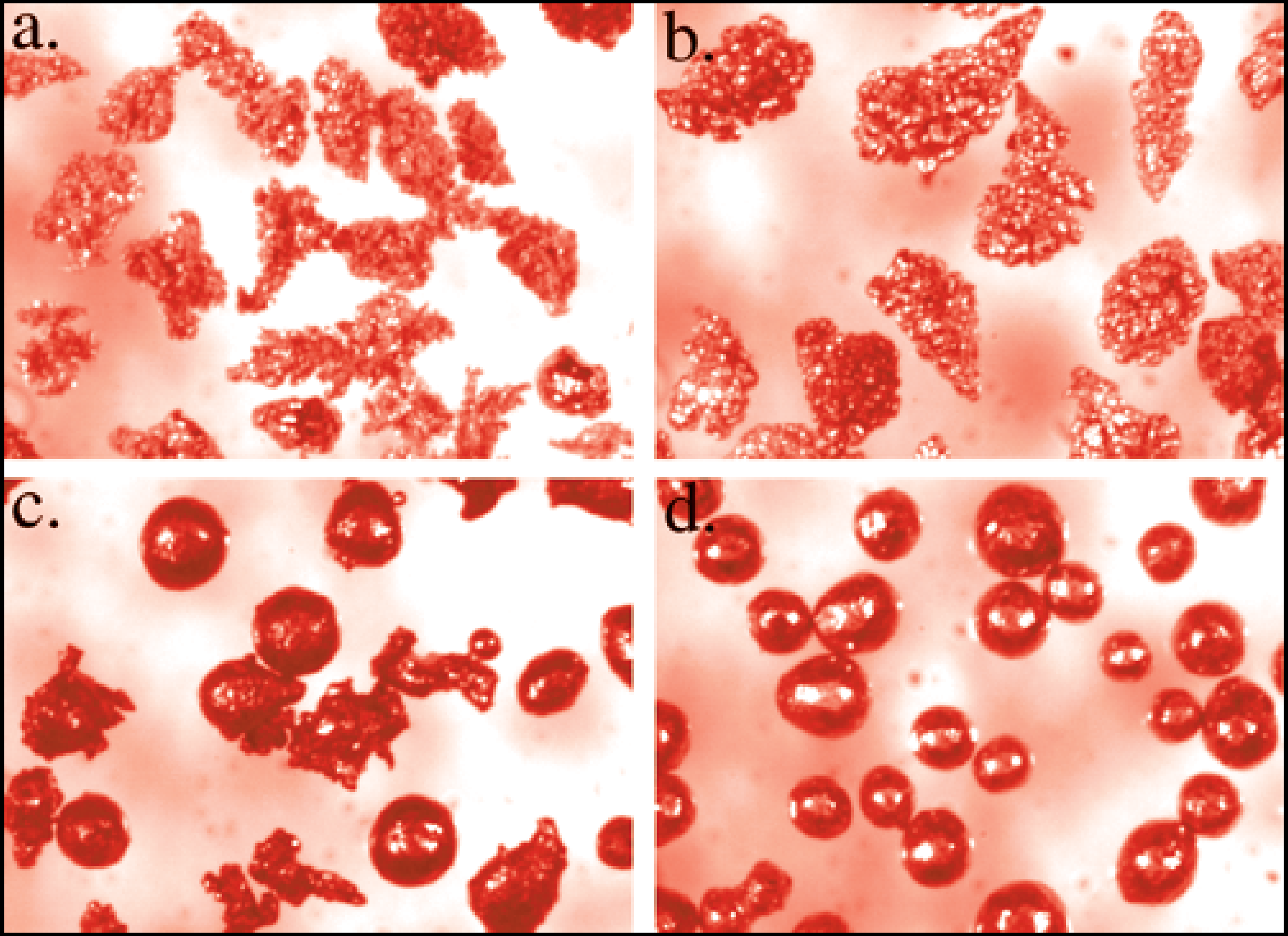
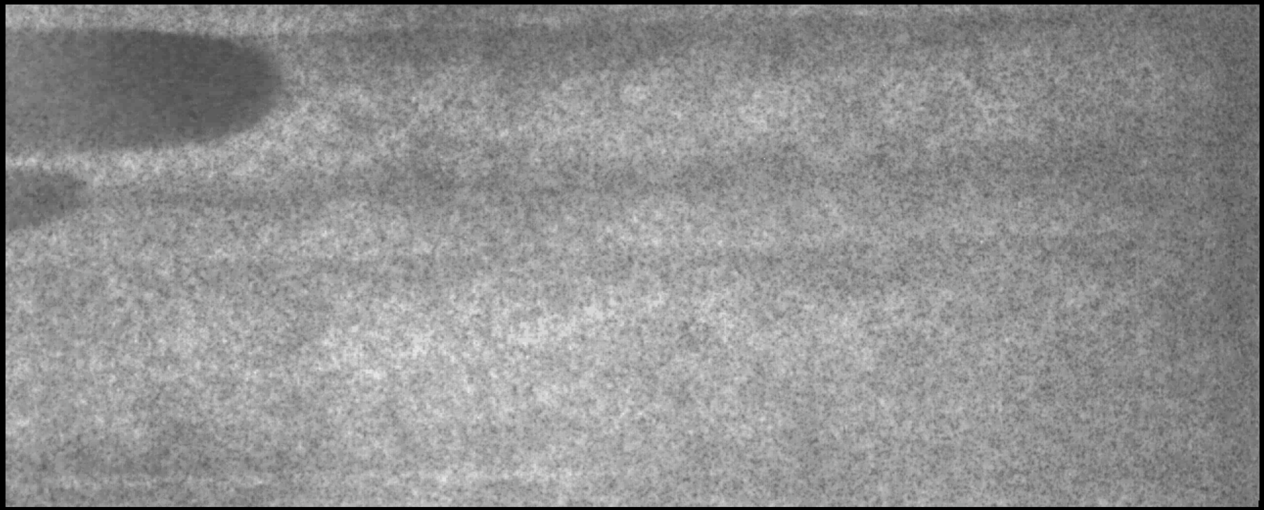
Charging Effects? Copper particles

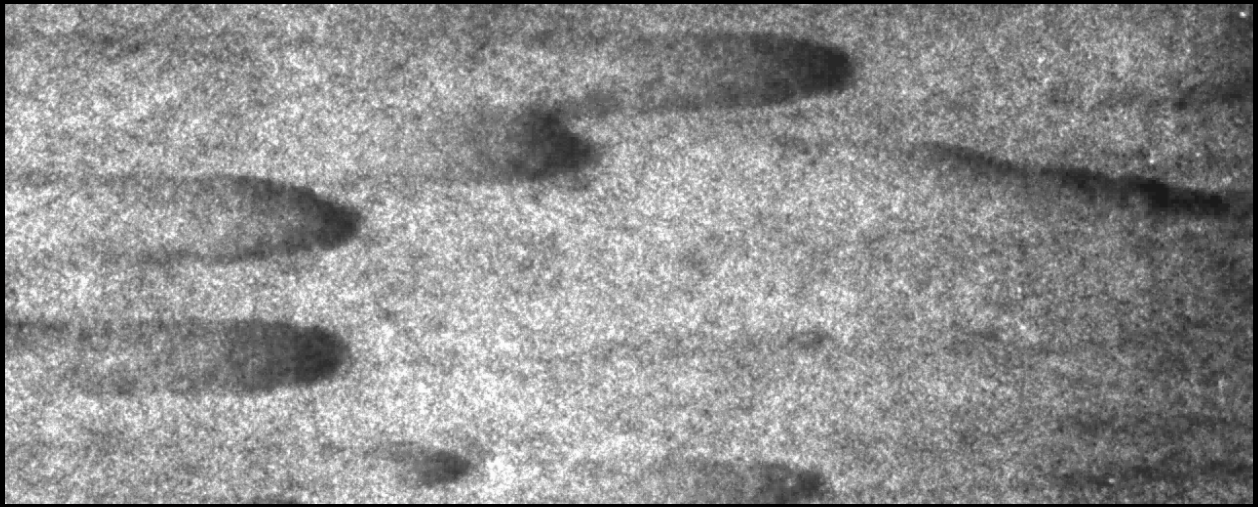


Sand

Small flow rate (0.17g/s-cm): static layer+avalanches



$\theta=33^{\circ}$



$\theta=38^{\circ}$

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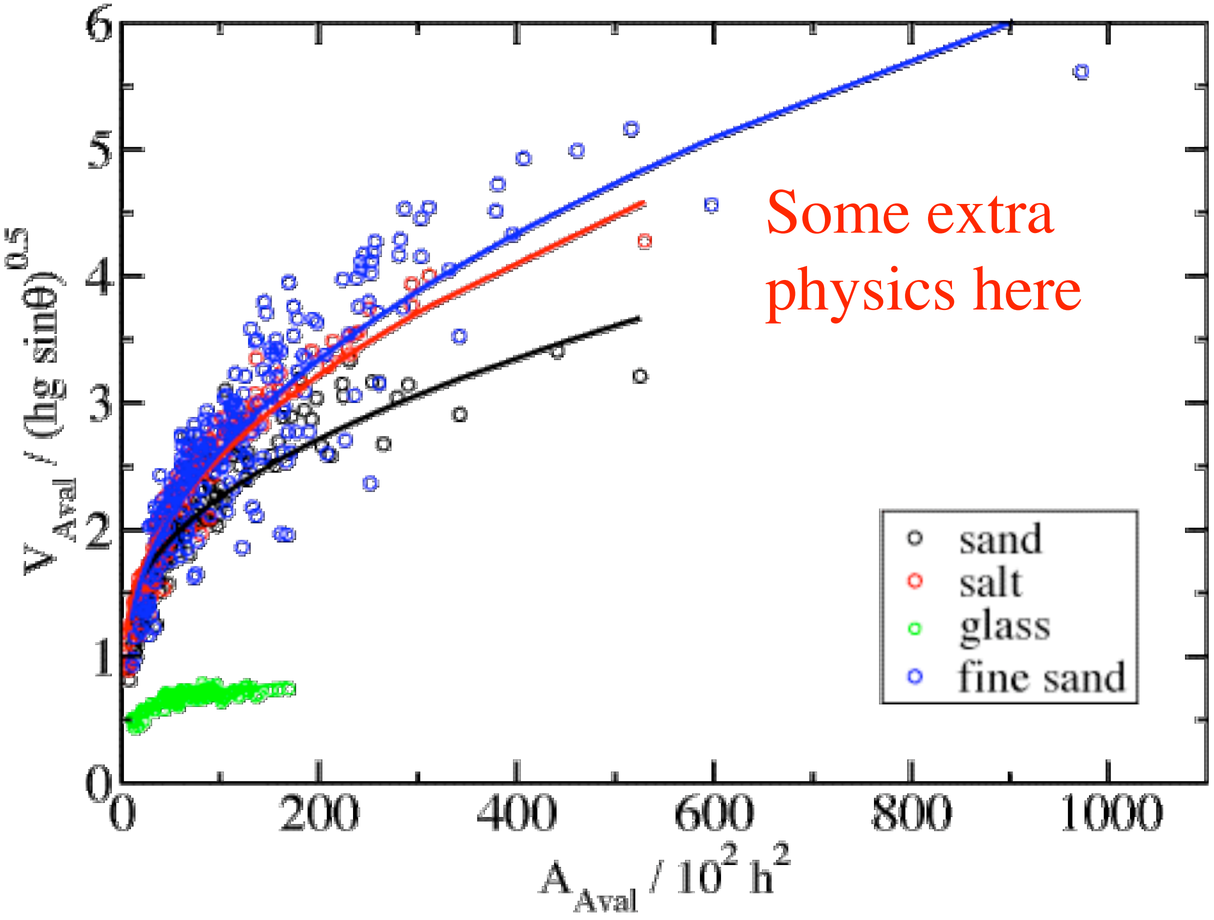
Glass beads (0.05 g/s-cm): avalanches

o

θ=25.2

Non-Dimensionalized Avalanche Velocity



Some extra physics here

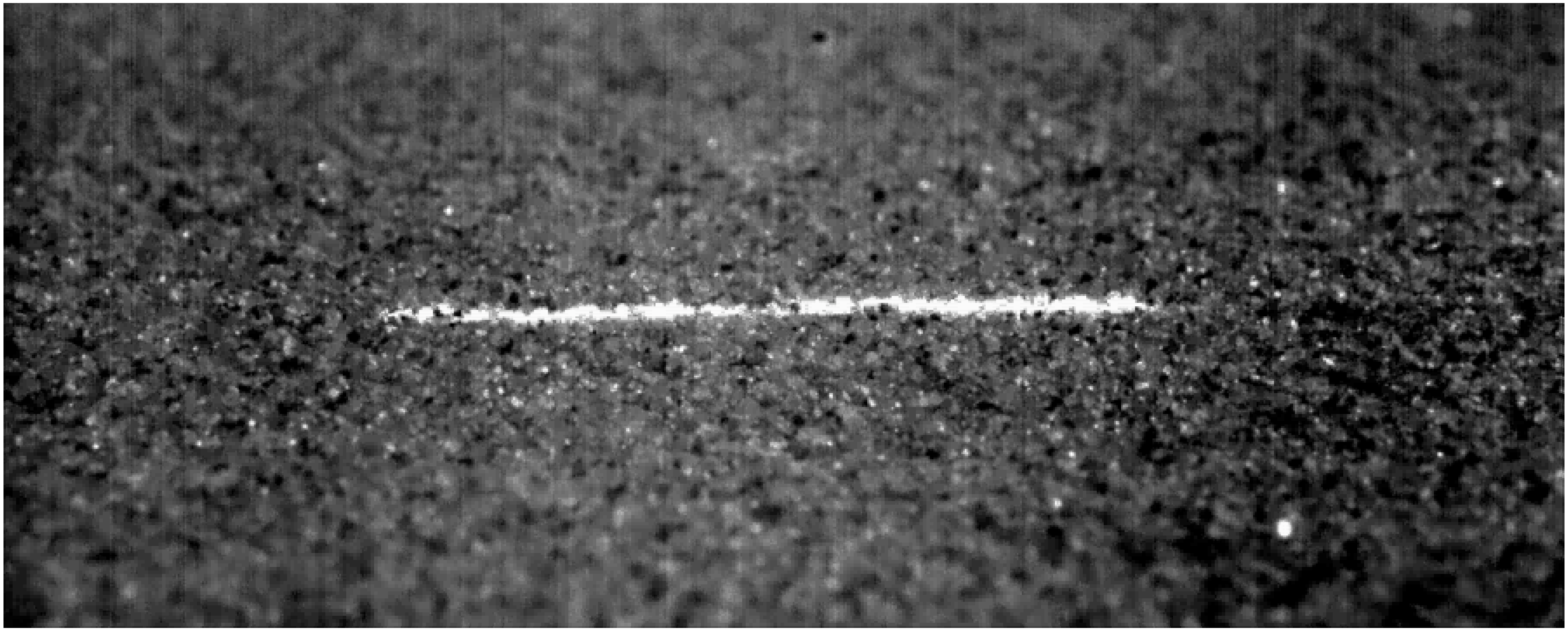
Rescale $v = v/(g_\theta h)^{1/2}$

$A = A/h^2$

→ Data Collapse - 2 groups

Measurement of the avalanche profile (side view)

Sand particles



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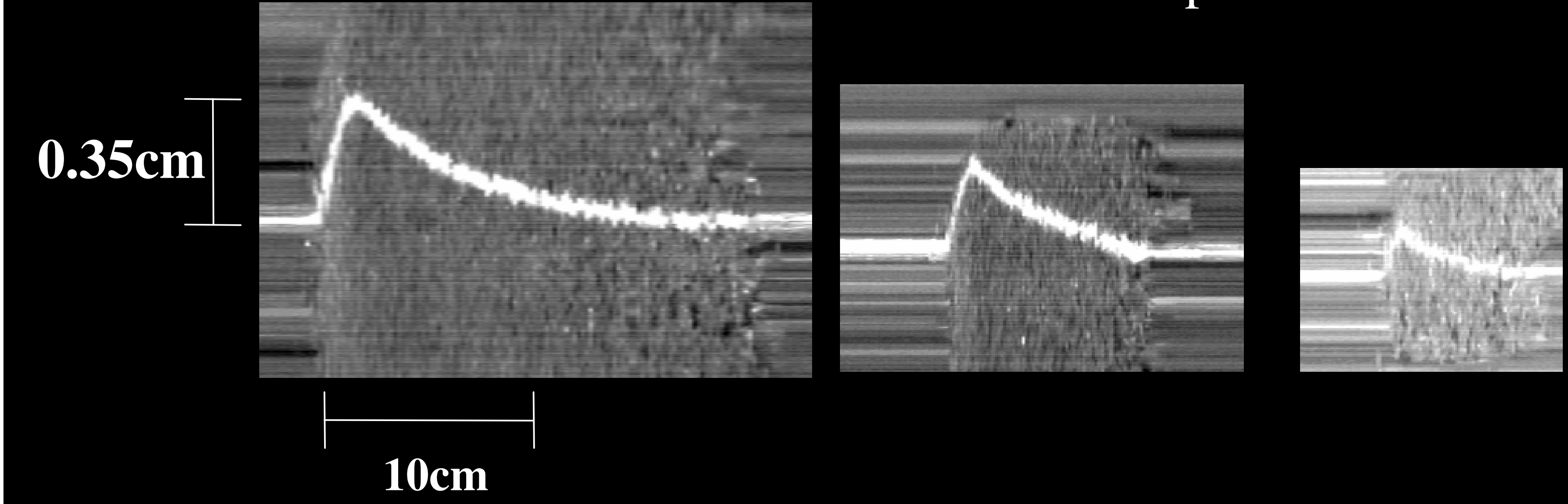
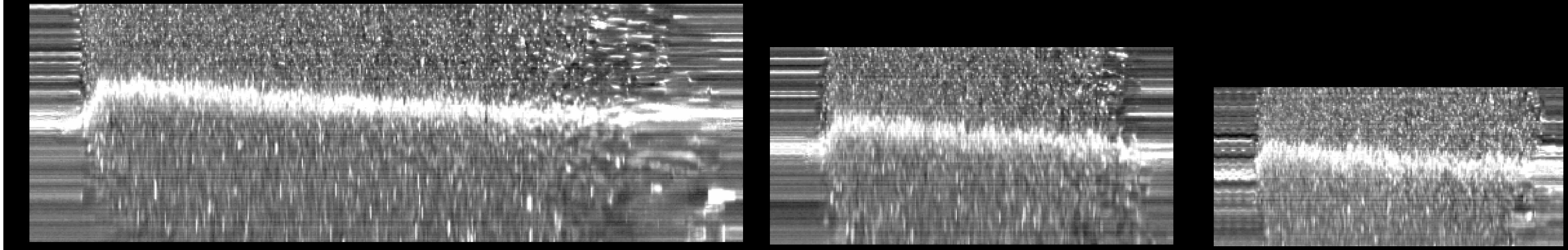
Measurement of the avalanche profile (side view)

Sand particles



Glass beads

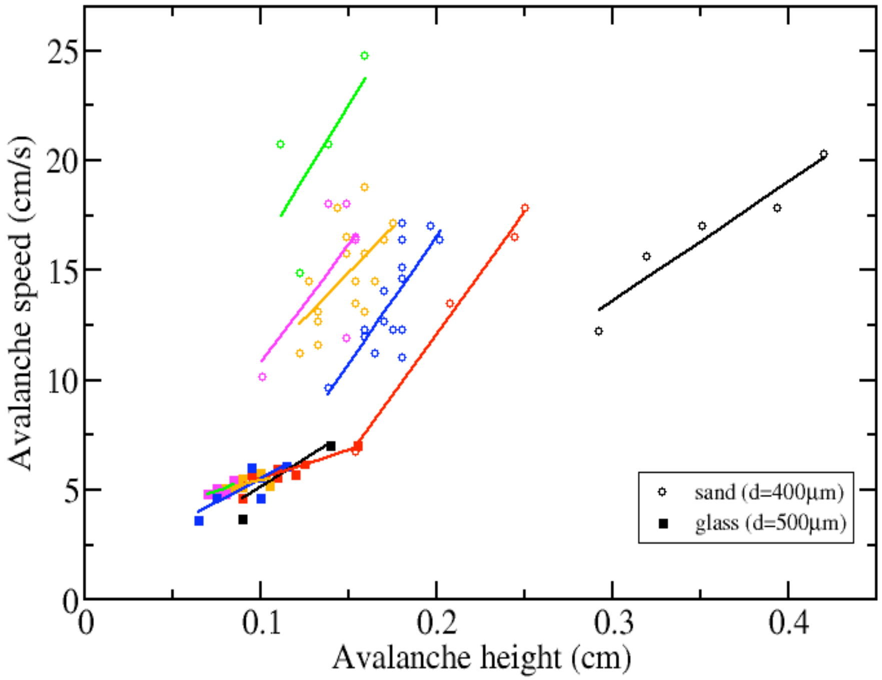


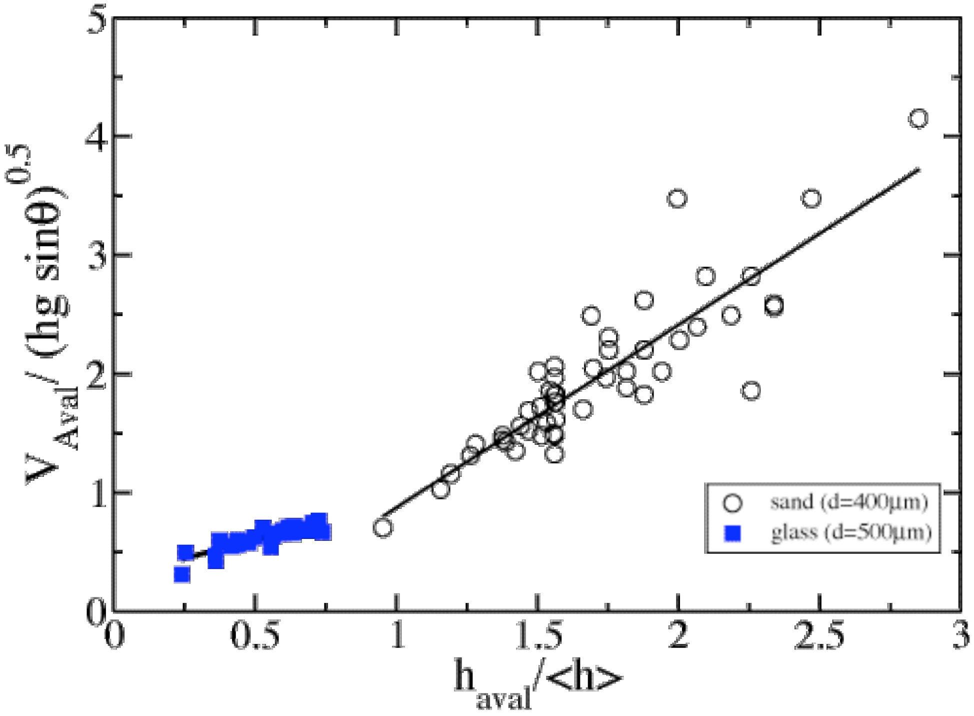
Avalanche speed vs height

Rescale $v = v/(g_\theta h)^{1/2}$

$h_{aval} = h_{aval}/h$

Data Collapse - 2 groups





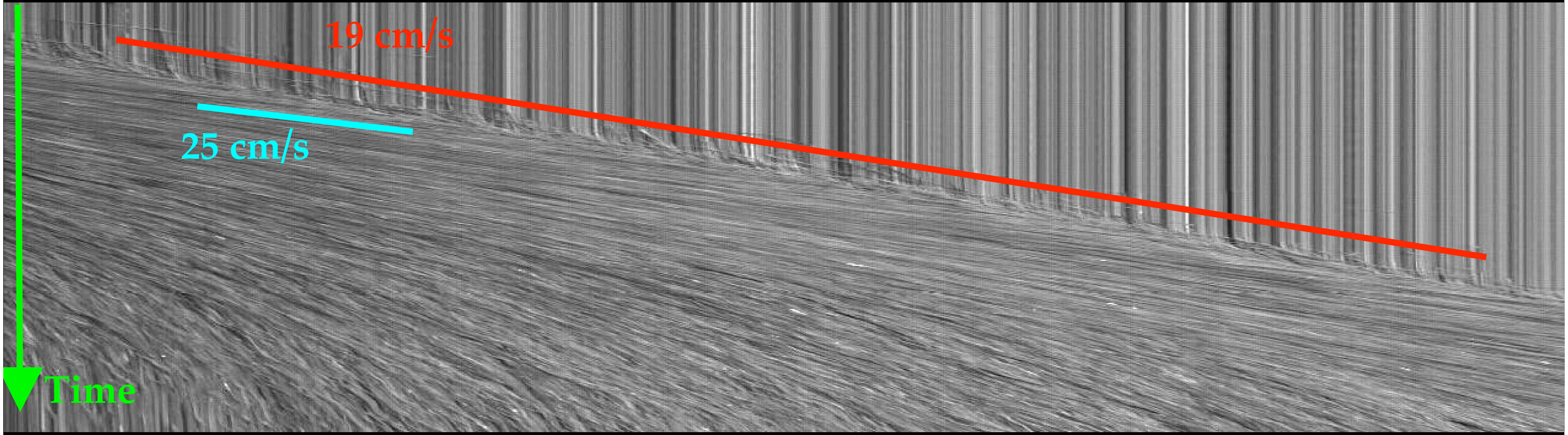
Avalanche height is relative to static layer, i.e., $h_{aval} = h_{max} - \langle h\rangle$

For glass avalanches, avalanche height is less than layer thickness

Space-time diagram

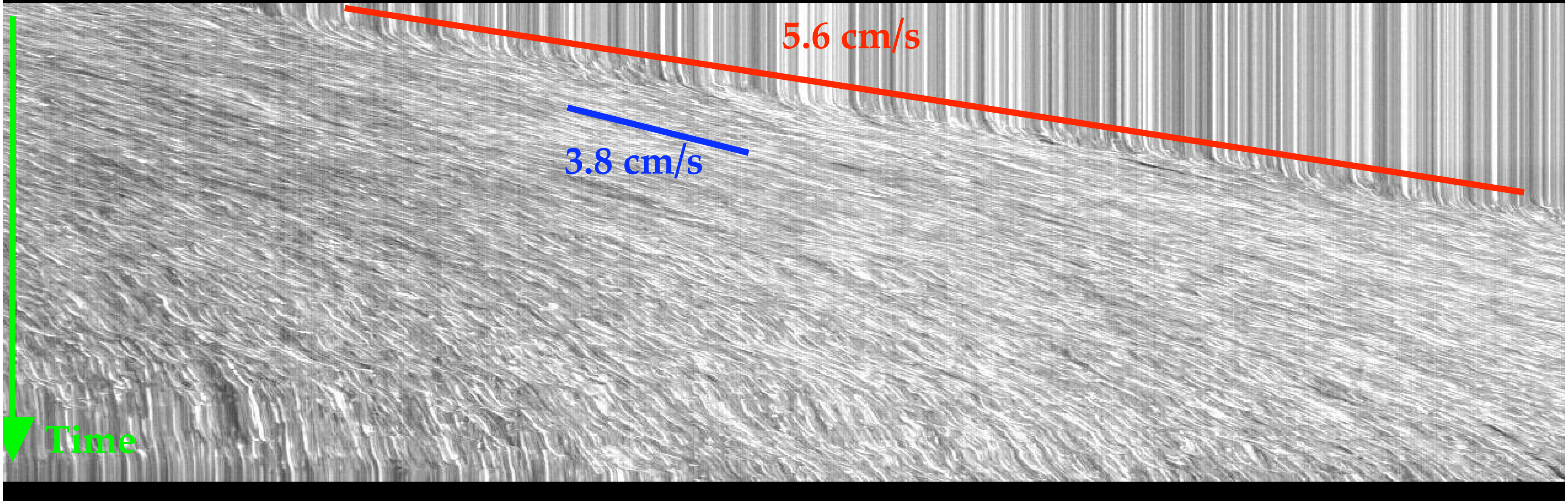
Sand particles, d=400 μm

+30%

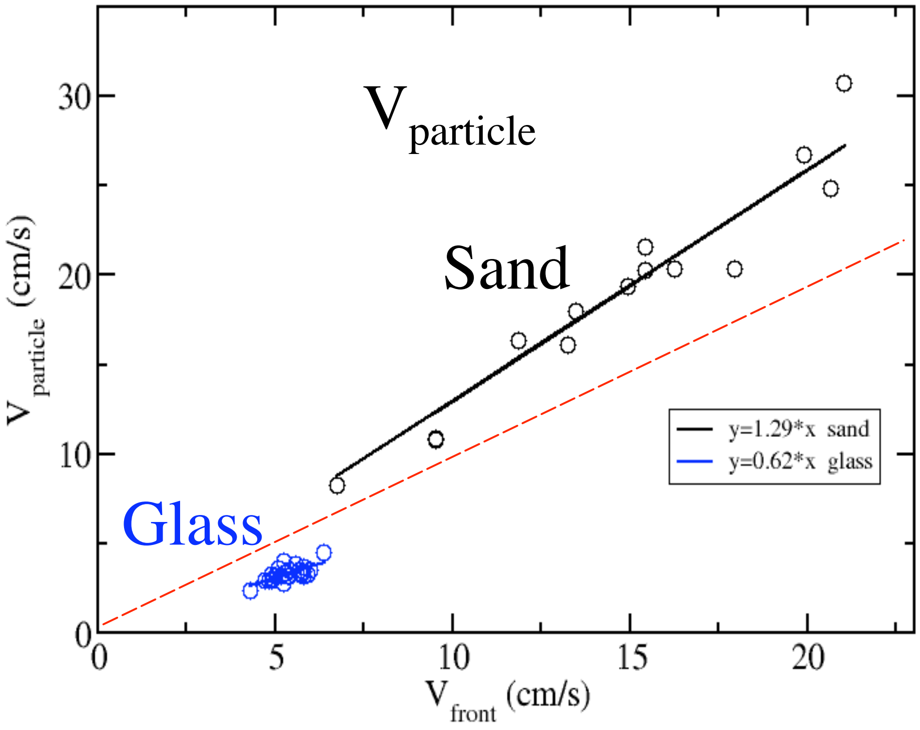


Glass beads, d=500 μm

-33%



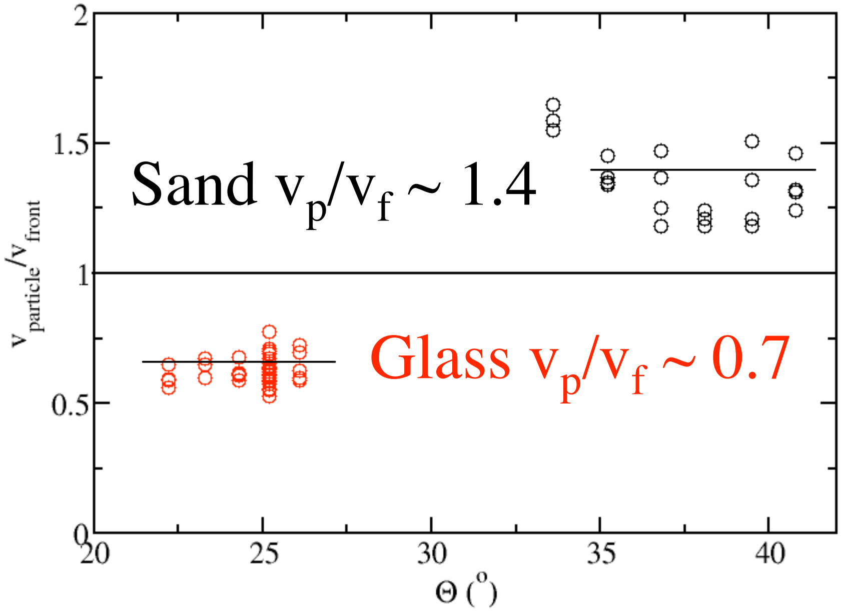
Particle velocity vs. front velocity



Two regimes of behavior: particle velocity faster (sand) or slower (glass beads) than front velocity

$V_{particle}$ taken just behind front

$V_{particle}/V_{front}$



High-Speed Visualization of Granular Avalanches
Side-View

Glass Beads

Sand

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Depth Averaged Equations

Savage-Hutter (1989)

mass conservation $\partial h/\partial t + \partial hu/\partial x = 0$

momentum conservation

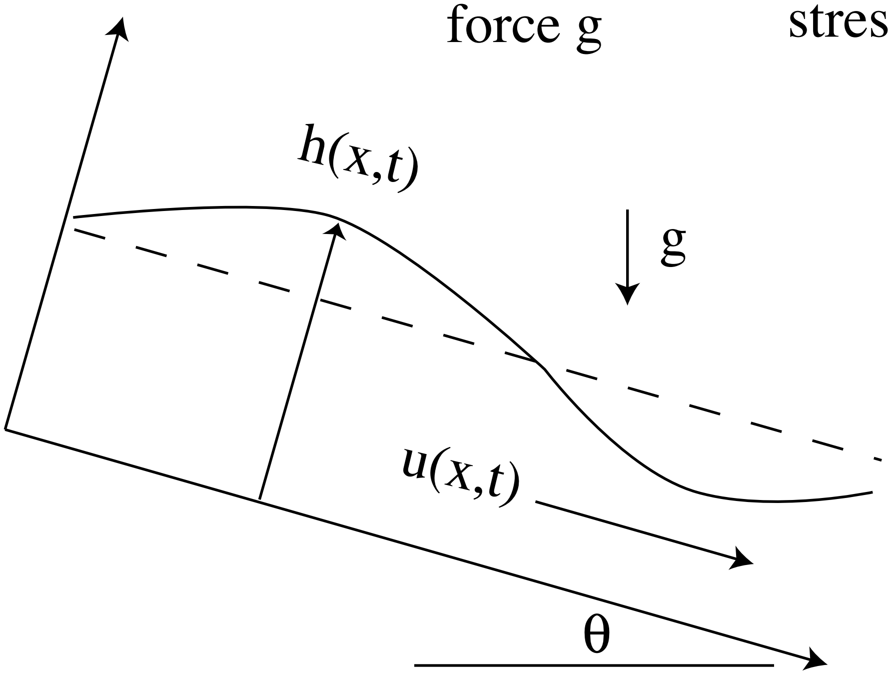
$$\rho(\partial hu/\partial t + \alpha \partial hu^2/\partial x) = (\tan\theta - \mu(u,h) - K\partial h/\partial x)\rho g h \cos\theta$$

tangential force g

tangential stress

pressure force

$K=\sigma_{xx}/\sigma_{zz} \sim 1$



Pouliquen Flow Rule

$u/\sqrt{gh} = \beta h/h_s(\theta) - \gamma$

$h_s(\theta)$ is thickness of layer where flow stops for θ

Non-dimensionalize

$x \rightarrow x/h_s \quad u \rightarrow u/\sqrt{gh_s} \quad t \rightarrow \sqrt{h_s/g}$

$$\partial h/\partial t + \partial hu/\partial x = 0$$

$$Fr^2(\partial hu/\partial t + \alpha\partial hu^2/\partial x) = (\tan\theta - \mu(u,h) - K\partial h/\partial x) \approx 0$$

$$Fr = U/(gh_s\cos\theta)^{1/2} \qquad \mu(u,h) = \tan\theta \text{ <--> } u/\sqrt{h} - \beta h + \gamma$$

$$\partial h/\partial t + a(h)\partial h/\partial x = \nu(h)\partial^2 h/\partial x^2$$

where $a(h) = (5\beta h^{3/2} - 3\gamma h^{1/2})/2$

and $\nu(h) = Kh^{3/2}$ (or $dh^{1/2}$ or $h_s^{3/2}$)

Generalized viscous Burgers equation => development of shocks

For glass $Fr < 0.5$ with $\beta \approx 0.14$, $\gamma = 0$ and $K=1$

$$\boxed{\partial h/\partial t + (5/4)h^{3/2}\partial h/\partial x = h^{3/2}\partial^2 h/\partial x^2}$$

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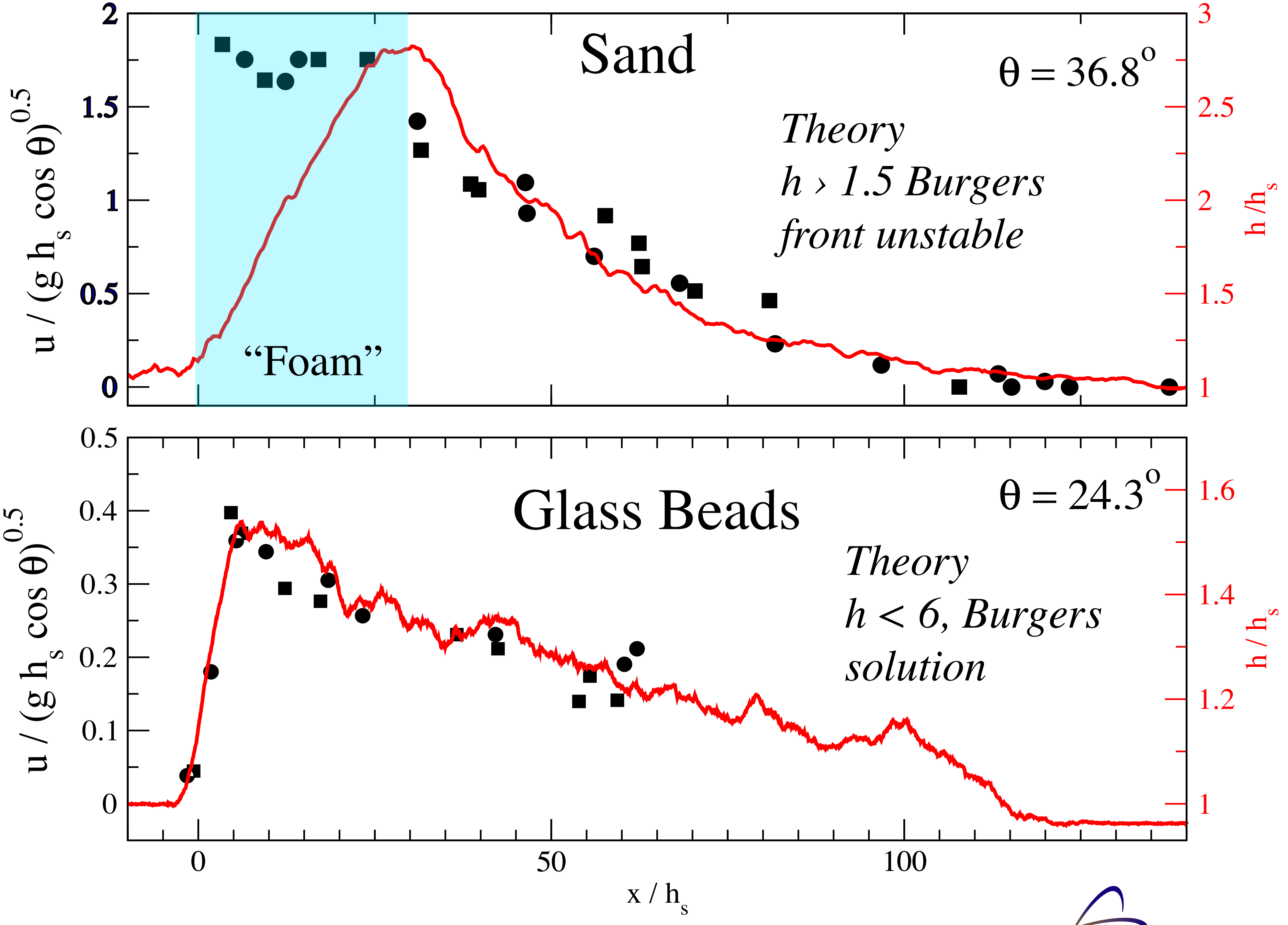
Numerical simulation of “avalanche”

Initial Condition $h(x,0) = 1 + 2\,e^{-(x/a)^2}$

Periodic BC

Superimposed Height & Velocity Profiles



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Conclusions & Opportunities

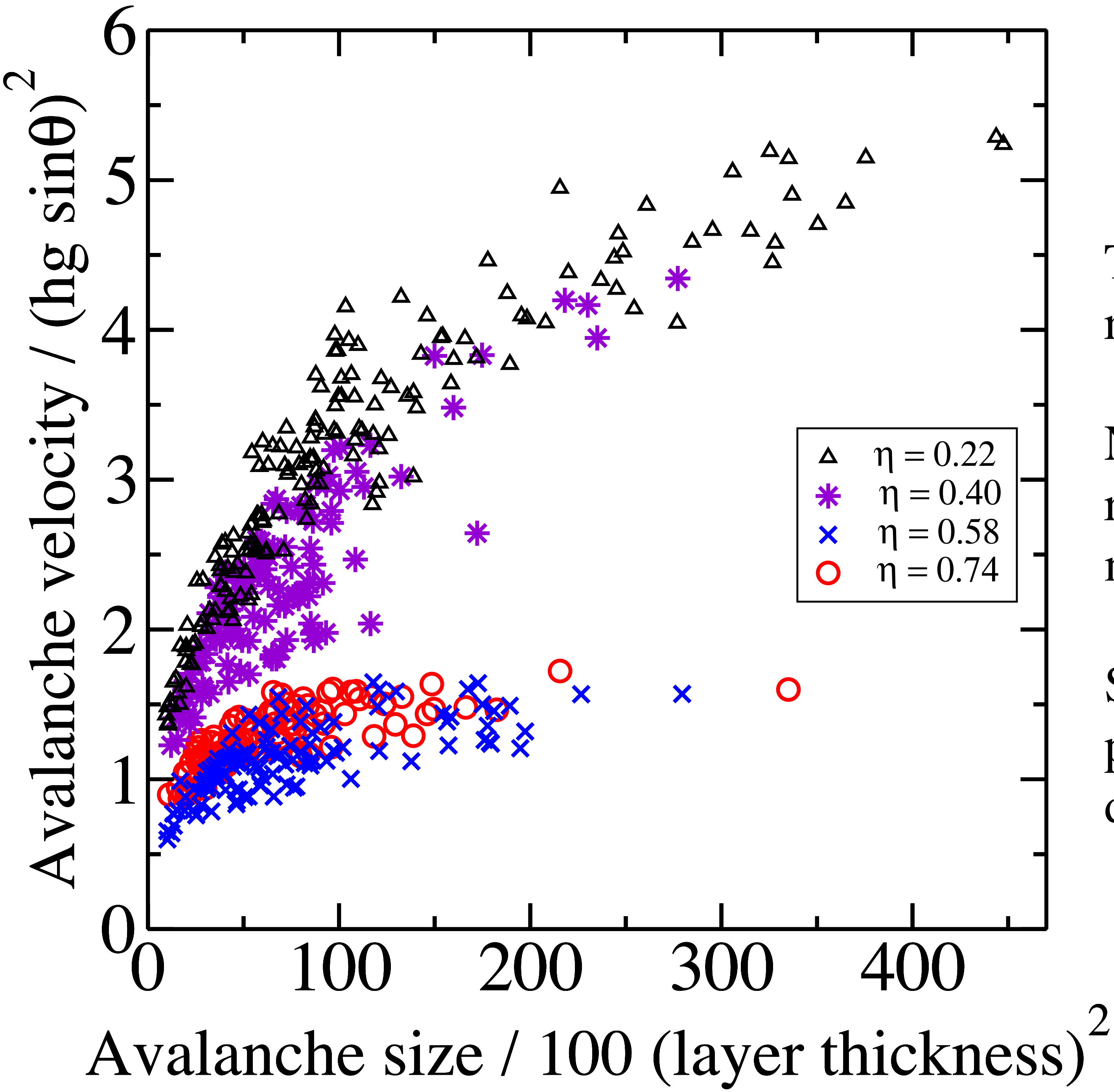
The dynamics of avalanches on an inclined plane depend in a qualitative and quantitative manner on the properties of the granular material.

Smooth glass beads avalanche in a progressive and smooth fashion whereas rough or irregular grains develop overturning fronts.

Depth-averaged equations provide a starting point for understanding the dynamics of avalanches but there are limitations that derive from the complicated flow rule for layer thicknesses near $h_{stop}(\theta)$.

The co-existence of solid and fluid/gas phases make avalanches on an inclined plane a rich and interesting system for understanding granular constitutive relations.

Copper Data



Two main groups remain:

No detailed flow rheology for these materials

Some different phenomena for dendritic copper

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High-Speed Visualization of Granular Avalanches
Top-View

Sand

Copper
Dendrites

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