Transport in 1D Granular Arrays

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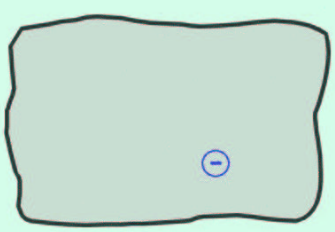
Santa Barbara, May 2003.

the Coulomb blockade



- a quantum dot

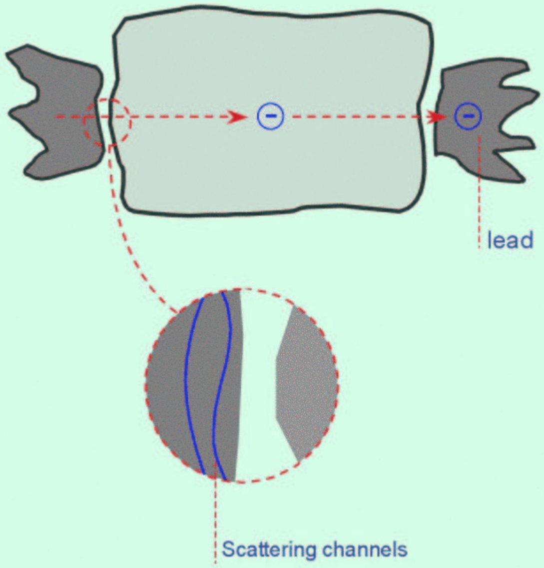
- accommodation of a single electron costs energy

$$E_c = \frac{e^2}{2C} >> T$$

- capacitance (↑ C)

the Coulomb blockade

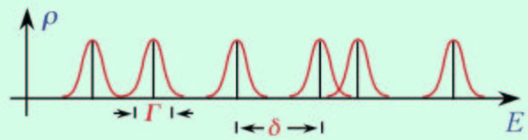
- coupling to external device



- dot coupled to leads by M channels with transmission coefficients: $0 < t_s < 1$.

dimensionless conductance: $g = \sum_{s=1}^{M} |t_s|^2$

- alternative interpretation: $g = \Gamma/\delta$

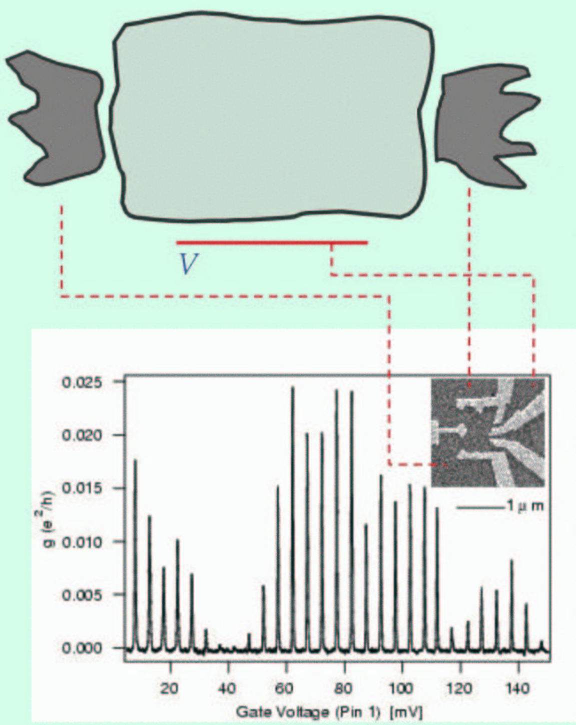


- for $g << 1$, the dot is in a state of 'Coulomb blockade': total conductance

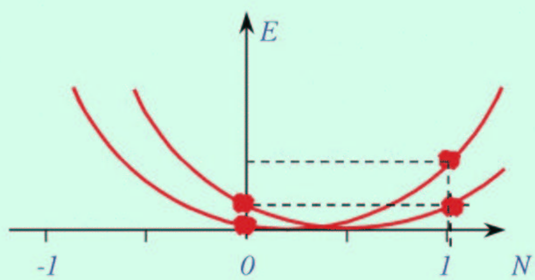
$$g_{tot} \sim \exp(-E_c/T)$$

the Coulomb blockade

- gate voltage as external probe



- gate electrode controls electrostatically preferred charge on the dot.



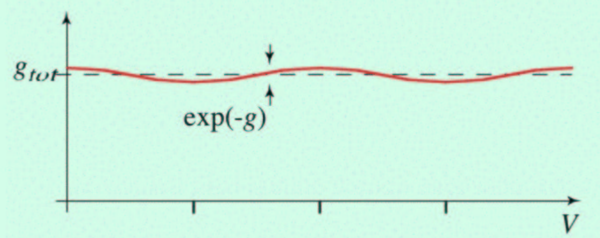
- generic values of V: transport blocked

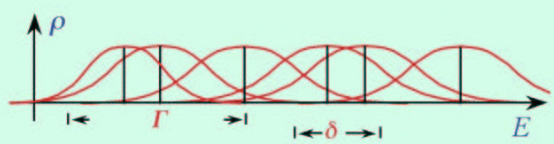
- $V = E_c/2$

  free current flow (Coulomb blockade 'peak')

... however,

- the phenomenon is extremely susceptible to changes in the value of the tunneling conductance, g.
  - for $g<<1$, exponential suppression of the conductance.
  - however, for $g>>1$, the Coulomb blockade diminishes down to a small correction: $\delta g \sim exp(-g)$

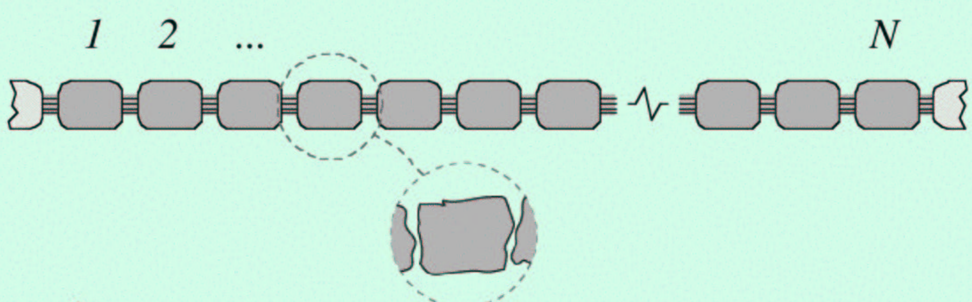




This contribution exists in parallel all sorts of other quantum corrections (Altshuler Aronov, weak localization ...) and is, therefore, nearly invisible.

question addressed in this talk:

- what happens if we consider an array of many strongly coupled $(g>>1)$ dots ?



main results:

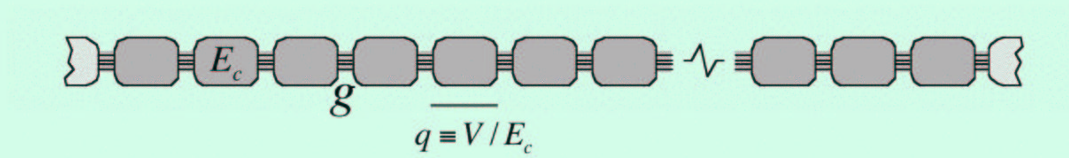
- the Coulomb blockade drives the system into an insulating phase.
- the corresponding charge gap is given by

$$\Delta \equiv E_c \exp(-g/4)$$

- at temperatures $T<\Delta$, both the conductance, and diff. capacitance show activated behavior:

$$g_{tot}, \partial_\mu N \sim \exp(-\Delta/T)$$

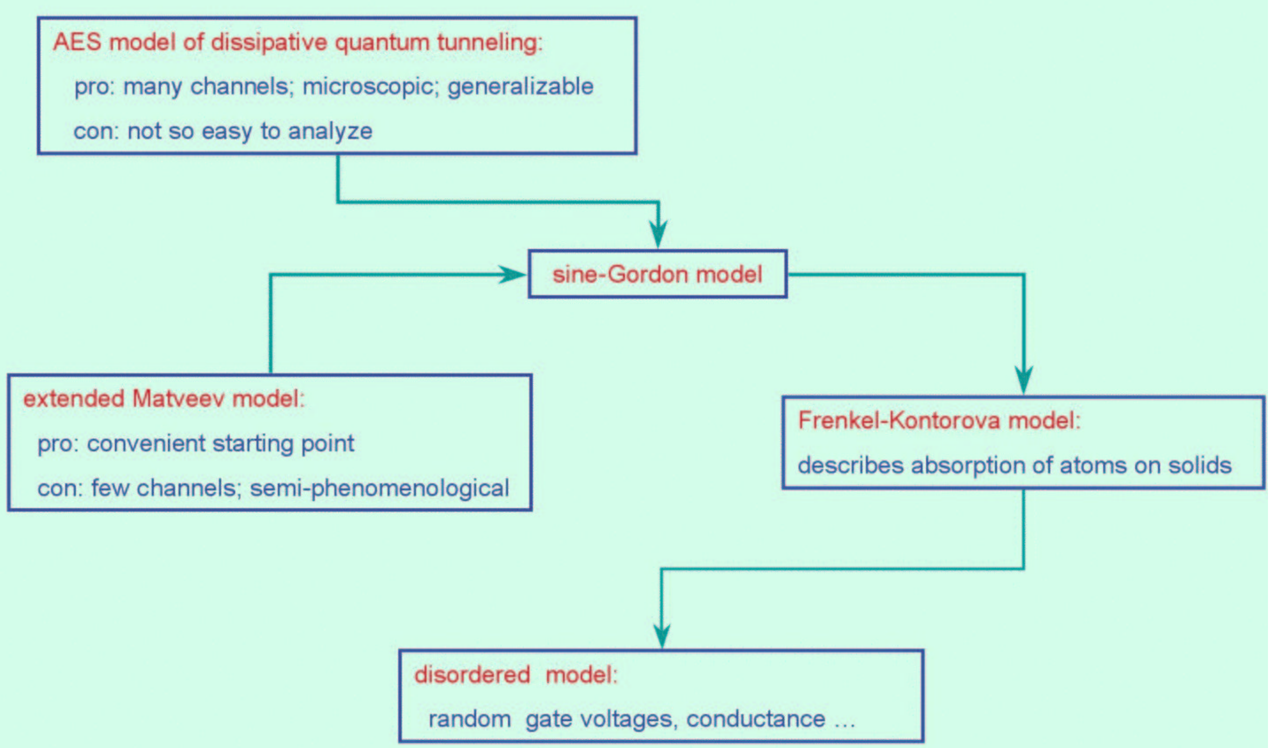
the system



- tunneling incoherent (effects of quantum interference negligible) for $T > g\,\delta$.

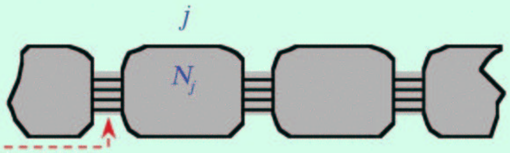
- mechanisms relevant to the physics of the system:
  - charging: E_c; and gate voltage: $q = V / E_c$
  - interface scattering: g

strategy



extended Matveev model

- Flensberg 93, Matveev 94: semi-phenomenological model of the Coulomb blockade in few channel quantum dots.
- generalization to an array:



- charge displacement field: $\Theta_j(\tau)$. Physical meaning: $\Theta_{j+1}(\tau) - \Theta_j(\tau) = N_j(\tau)$ = charge sitting on grain no. j.

$$S[\theta] = S_c[\theta] + S_{scatt}[\theta]$$

$$S_c[\theta] = \frac{1}{T}\sum_{j=1}^{N}\sum_{m}\left[E_c(\theta_{j+1,m} - \theta_{j,m} - q)^2 + |\omega_m||\theta_{j,m}|^2\right]$$

$$S_{scatt}[\theta] = D\, r \sum_{j=1}^{N}\int_0^{\beta} d\tau\ \cos(\theta_j(\tau))$$

- reflection coefficient;
- high energy cutoff
- for many channels: $r \longrightarrow \prod_{s=1}^{M} r_s << 1$

analysis of Matveev model

- a major simplification: physics controlled by temporal zero mode $\Theta_{m=0}$. Dynamic modes give rise to inessential renormalization factors :

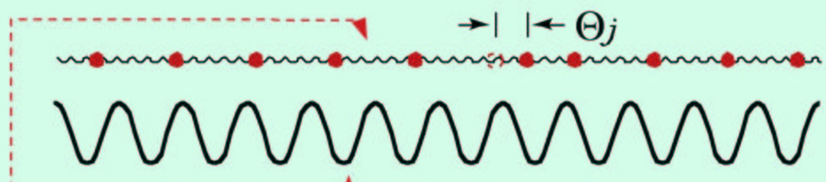
$$\langle\theta_j^2(\tau)\rangle = \frac{1}{N}\sum_k \sum_{m\neq 0}^{E_c/T} \frac{1}{E_C k^2 + |\omega_m|} = O(1)$$

- IR convergent !
- action of static sector:

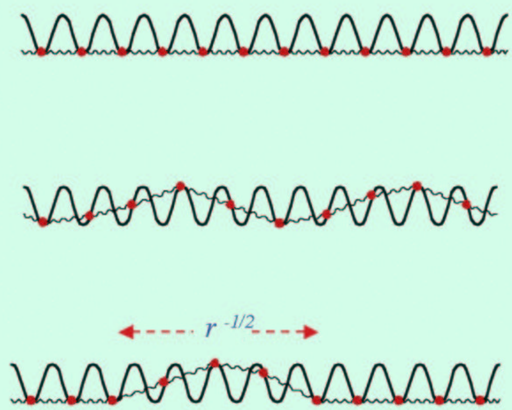
$$S[\theta] = \frac{E_c}{T}\sum_{j=1}^{N}\left[(\theta_{j+1} - \theta_j - q)^2 + r\cos(\theta_j)\right]$$

- interpretation I: lattice version of the classical sin-Gordon model
- interpetation II: action of Frenkel-Kontorova (1932) model of atomic absorption on substrates

Frenkel-Kontorova model



$$S[\theta] = \frac{E_c}{T}\sum_{j=1}^{N}\left[(\theta_{j+1}-\theta_j)^2 + r\cos(\theta_j + qj)\right]$$



- atoms follow substrate, $\Theta_j = -qj$, energy:

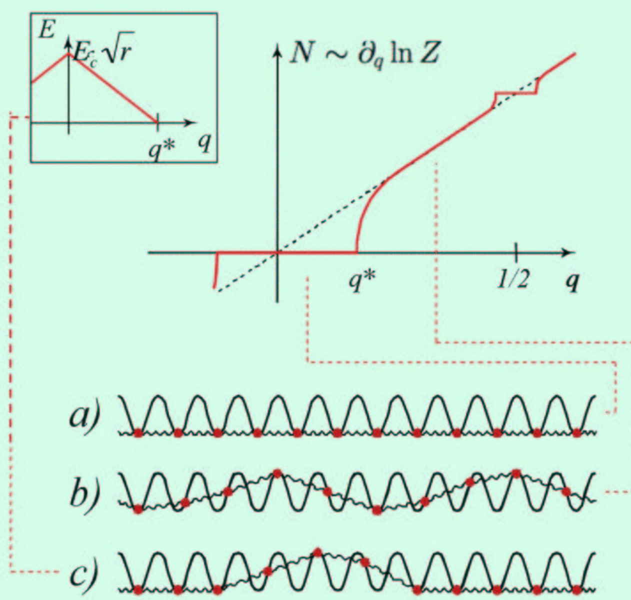
$$F[\theta] = \frac{NE_c}{T}\left[q^2 - r\right]$$

- ground state of the chain, $\Theta_j = 0$, energy:

$$F[\theta] = 0$$

- phase transition at critical value: $q^* \sim r^{1/2}$.
- excitations of the system: long solitons.

implications



- in an interval of width ~q^*, centered around $q=0$, the ground state of the system is q-independent (a)).
- for $q>q^*$, reentrance into q-dependent state (b)). However, plateau formation around other rational values of q.
- thermal fluctuations create q-dependent excited states (c)) that cost energy $\Delta \sim E_c r^{1/2}$

translation to the metallic context

- for zero gate voltage (and other rational values of q), the system is in an insulating state.
- the charge gap is given by: $\Delta \sim E_c r^{1/2}$
- one can show that the insulating state survives generalization to random values of q, however, with a lower gap: $\Delta \sim E_c r$.

the real thing

- shortcomings of the previous discussion
  - limitation to few channels.
  - unclear how quantum interference (localization, dephasing, etc.) can be built in.
  - connection to Coulomb blockade literature unclear.

- alternative approach: for $g>>1$, large charge fluctuations. Description in terms of the phase ϕ_i conjugate to the charge N_i ($[\phi_i, N_j] = -i\,\delta_{ij}$) is favorable.

- effective action: Ambegaokar, Eckern, Shoen 1984

$$S[\phi] = S_c[\phi] + S_{scatt}[\phi]$$

gate voltage

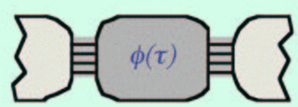
$$S_c[\phi] = \sum_j \int_0^\beta d\tau \left[\frac{\dot{\phi}_j^2}{4E_c} - iq\dot{\phi}_j \right]$$

conductance

$$S_{scatt}[\phi] = \frac{gT^2}{2} \sum_j \int_0^\beta d\tau d\tau' \frac{\sin^2(\delta\phi_j(\tau) - \delta\phi_j(\tau'))}{\sin^2(\pi T(\tau - \tau'))}$$

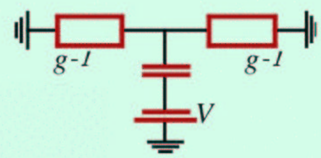
$$\delta\phi_j \equiv (\phi_{j+1} - \phi_j)/2$$

warmup: single grain



$$S_c[\phi] = \frac{1}{4E_c} \int_0^\beta d\tau \left[\dot{\phi}^2 + 4iE_c q\dot{\phi} \right]$$

$$S_{scatt}[\phi] = g\,T^2 \int_0^\beta d\tau d\tau' \frac{\sin^2((\phi(\tau) - \phi(\tau'))/2)}{\sin^2(\pi T(\tau - \tau'))}$$



- for $g>>1$, quadratic expansion:

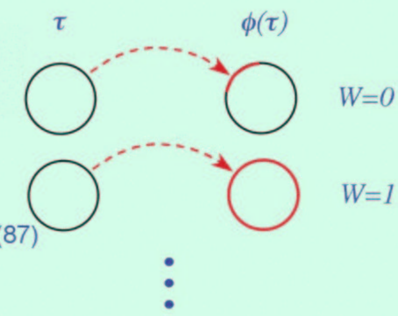
$$S[\phi] \approx \frac{1}{T} \sum_m \phi_m \left(\frac{\omega_m^2}{4E_c} + g|\omega_m| \right) \phi_{-m} - \frac{4E_c}{T} q^2$$

- anharmonic fluctuations lead to logarithmic corrections: $g_{tot} = \frac{g}{2}\left[1 - \frac{1}{g}\ln\frac{E_c}{T}\right]$

(Fazio & Schoen, 91, Golubev & Zaikin, 96, Efetov & Tschersich, 02, ...) small for $T > E_c exp(-g)$.

instantons

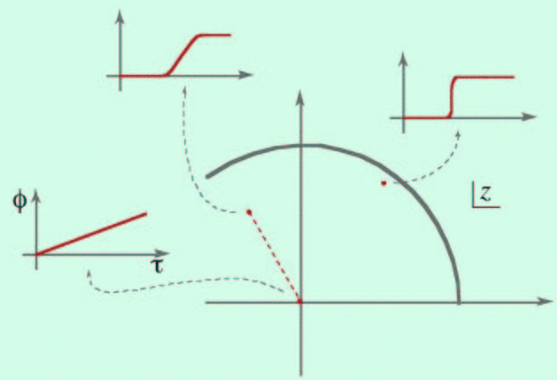
- mathematically: $\phi : S^1 \to S^1, \quad \tau \mapsto \phi(\tau)$



- topologically non-trivial excitations - instantons: Korshunov (87)

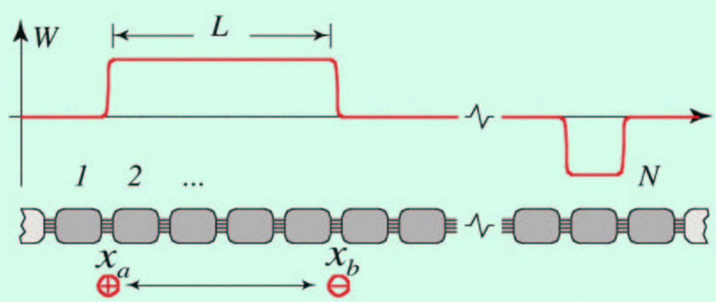
$$e^{i\phi(\tau)} = \frac{e^{2\pi i\tau T} - z}{1 - \bar{z}e^{2\pi i\tau T}}, \qquad |z| \le 1$$

- instantons extremize scattering action
- responsible for gate voltage dependence
- action: $S = g + \frac{T}{E_c} + 2\pi i q$



instanton formation in the array

- consider phase fluctuation of the type:



- action: $S = g + \frac{T}{E_c} | L | + 2\pi i q L$
- re-interpret instanton configuration as a dipole of two opposite charges:
  - with the fugacity (core energy): $exp(-g/2)$,
  - interacting by one-dimensional Coulomb interaction: $(T/E_c)\,|x_a - x_b|$;
  - in a uniform external field: $2\pi i q$

$$\frac{Z}{Z_0} = \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(e^{-g/2} \frac{E_c}{T} \right)^{2k} \sum_{x_1 \ldots x_{2k}}^{N} e^{-\frac{T}{E_c}\sum_{a,b}^{2k}(-)^{a+b}|x_a - x_b| - 2\pi i q \sum_{a}^{2k}(-)^a x_a}$$

- fluctuation determinant

instantons in the array cont'd

- key to solving the problem is equivalence of the Coulomb gas to the sine-Gordon model:

$$S[\theta] = \frac{E_c}{T}\sum_{j=1}^{N}\left[(\theta_{j+1}-\theta_j-q)^2 - e^{-g/2}\cos(\theta_j)\right]$$

- fugacity (= pinning strength):

$$\prod_{s=1}^{M} r_s = e^{\frac{1}{2}\sum_{s=1}^{M}\ln(1-|t_s|^2)} \approx e^{-\frac{1}{2}\sum_{s=1}^{M}|t_s|^2} = e^{-g/2}$$

- cf. with the previous approach ! $\Delta \sim E_c e^{-g/4}$

dynamics and conductivity

- real time classical Langevin dynamics, $\theta_j \dashrightarrow \theta_j(t)$:

$$\frac{1}{g}\frac{d\theta_j}{dt} = E_c\left[\theta_{j+1}-2\theta_j+\theta_{j-1} - e^{-g/2}\sin(\theta_j + jq)\right] + E + \xi_j(t)$$

(E: external field)

the noise correlator: $\langle \xi_j(t)\xi_{j'}(t')\rangle = \frac{T}{g}\delta_{j,j'}\delta(t-t')$

- soliton - antisloliton creation as an "under-barrier" process: $n_s = l_s^{-1} exp(-\Delta/T)$,

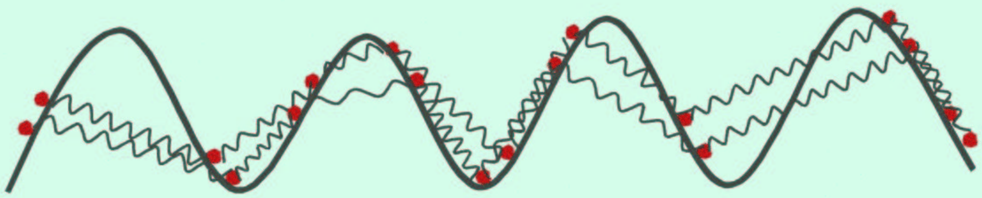
where $l_s^{-1} = exp(+g/4)$ is the solitin length and $\Delta = E_c\, exp(-g/4)$ is the charge gap,

- moving solitons, $\theta_j(t) = \theta(j - v_s t)$, where the soliton velocity: $v_s = l_s g E$

- current density: $J = e\, n_s v_s$; conductivity: $\sigma = g\, exp\{-\Delta/T\}$.

disorder

- random gate voltages: $q \dashrightarrow q_j$



- pinning energy: $E_{pin} = E_c\left(e^{-g/2}\right)^2$
- charge gap: $\Delta_{random} = \sqrt{E_c E_{pin}} \approx E_c e^{-g/2}$
- role of rare events ?

conclusions:

- 1D array (inelastic) is equivalent to a **classical** pinned charge density wave.
- **activation** behavior with the charge gap $\Delta \sim exp(-g/4)$.
- partition function is dominated by the **instanton** configurations.

open questions:

- generalization to D > 1.
- inclusion of quantum interference, i.e. how does this mechanism compete/cooperate with effects of localization ?
- role of disorder and rare events.