Quantum Ising and dimer models

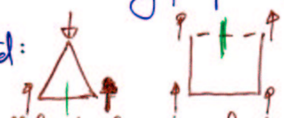
R. Moessner
ENS Paris

P. Chandra
E. Fradkin
S. Isakov
• S. Sondhi
O. Tchernyshyov

Hamiltonian

$$\mathcal{H} = \underbrace{\sum_{\langle ij\rangle} J_{ij} S_i^z S_j^z}_{\hat{J}} + \underbrace{\Gamma \sum_i S_i^x}_{\hat{\Gamma}} + \underbrace{h \sum_i S_i^z}_{\hat{h}}$$

- $\hat{J}$ is fully frustrated: in each elementary plaquette, one bond has to be frustrated:

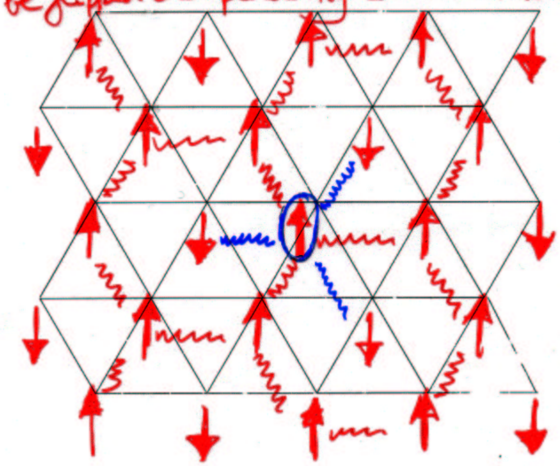


$\boxed{\text{Extensive } T=0 \text{ entropy}}$ all bonds afm — one bond afm (Villain)

- nearest-neighbour exchange only, with $|J_{\langle ij\rangle}| = J \ \forall \langle ij\rangle$
- Ising exchange (no XY term); diagonal in S^z basis ("classical model")
- transverse field generates quantum fluctuations: $[\hat{J}, \hat{\Gamma}] \neq 0$ as $[S^x, S^z] \neq$
- simple single-site term: spin-flip $S^x|\uparrow\rangle = |\downarrow\rangle$; $S^x|\downarrow\rangle = |\uparrow\rangle$
- classical field $[\hat{J}, \hat{h}] = 0$; useful to compare $\hat{\Gamma}$ and $\hat{h}$

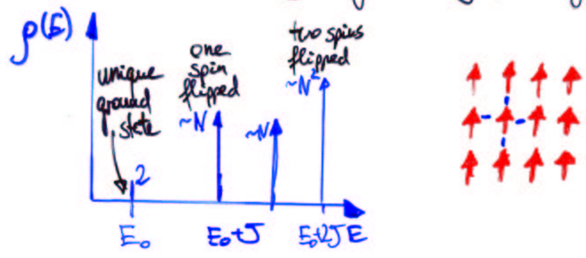
$\mathcal{H}$ is periodic: frustration is geometric, not due to quenched disorder

spins experiencing 0 net exchange field can be flipped
⇒ EXTENSIVE entropy @ $T=0$ since $\frac{1}{3}$ of spins can be flipped independently [Wannier, Houtappel, Anderson].

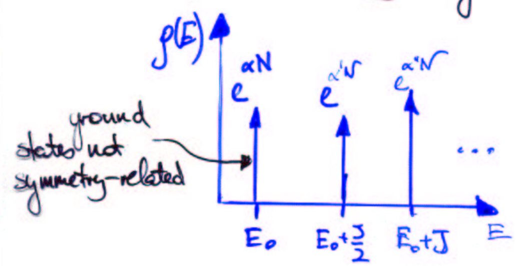


Density of states (classical)

- unfrustrated Ising magnet (e.g. ferromagnet)



- frustrated Ising magnet (e.g. triangular antiferromagnet)



reminiscent of Landau levels (quantum Hall effect)

→ at $T \ll J$, can project onto ground state
⇒ any perturbation is strong
⇒ many competing instabilities (of unknown universality class) – no obvious ordering pattern
⇒ huge low-energy density of states

Geometric frustration

degeneracy
+
quantum fluctuations

transverse field

what happens?

frustrated transverse-field Ising models

wide range of potential realizations

real anisotropic spin systems placed in a field

effective pseudospin models (e.g. in high-T_c)

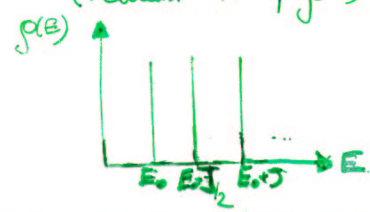
comparatively simple approach to quantum spin systems

Fazekas-Anderson strategy

connections to other Stat. Mech problems

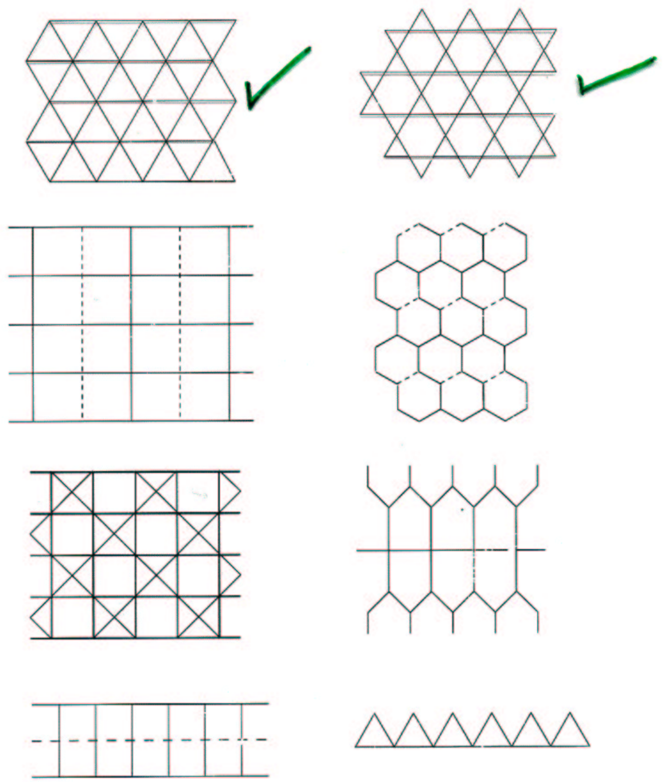
- height/dimer/loop/vertex models
- hopping
- stacked magnets
- gauge theories

degeneracy makes transv. field perturbation singular (→ Quantum Hall physics)



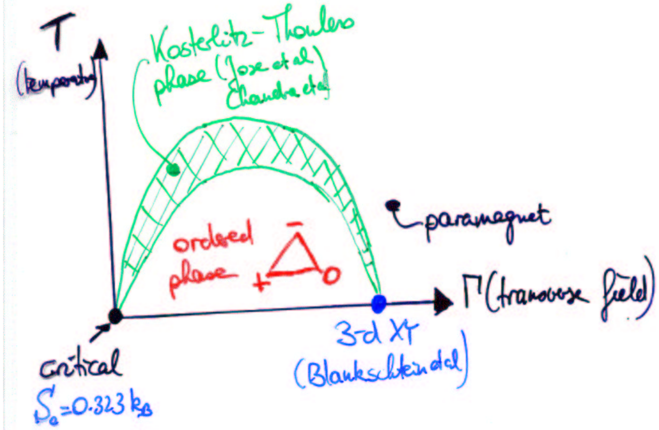
systematic study of lattices

new phases
new phase transitions



I) "order by disorder": triangular lattice

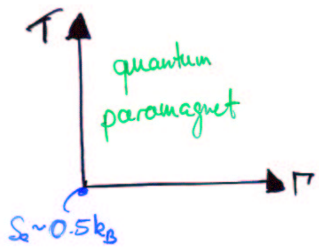
- quantum fluctuations favour configurations with maximal number of flippable spins
  ($|\uparrow\rangle + |\downarrow\rangle$ = aligned along field)
- such configurations tend to be ordered;



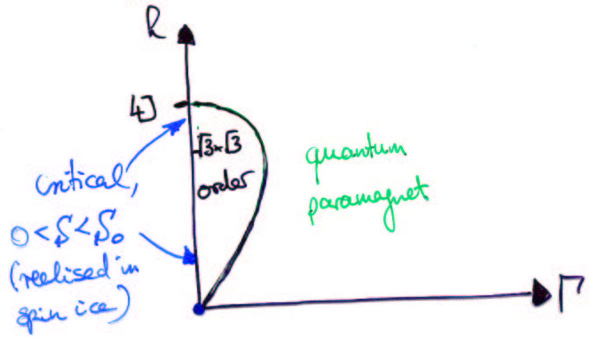
- square lattice frustrated Ising model also orders

II "disorder by disorder" : kagome lattice

- quantum fluctuations do not generate an order
  'The counterintuitive does not happen'
- $\Gamma = 0^+$ and $\Gamma = \infty$ states continuously connected



- More complex phase diagram in presence of additional longitudinal field h

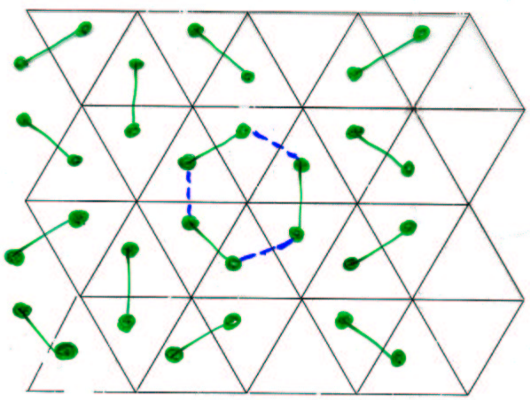


Methods

- variational ("flippability analysis") (Fazekas + Anderson)
- large-Γ / Landau-Ginzburg Wilson (Blankschtein et al)
- mappings (QDM, height model, ...) {Rokhsar-Kivelson, Hilhorst et. al.}
- numerics (QMC, exact diagonalisation)

Results

- order by disorder (triangular, square, planar pyrochlore, [hexagonal: 48-spin unit cell])
- "disorder by disorder" (kagome → quantum paramagnet)
- extended critical phases (triangular, kagome, planar pyrochlore)
- 'unusual' phase transitions (O(4) in hexagonal, floating KT in triangular)
- sliding phase (planar pyrochlore) → dDW (Chakravarty)
- topologically ordered phase (triangular QDM) → SR-RVB, quantum computing



$$H_{Ising} = J \sum S_i^z S_j^z + \Gamma \sum_i S_i^x$$ ← quantum dynamics

$$H_{QDM} = (\text{hardcore constraint for } J \to \infty) + \Gamma \sum_{\square} \{ |\,\cdot\rangle\langle\cdot\,| + h.c. \}$$

$$+\, v \sum_{\square} \{ |\,\cdot\rangle\langle\cdot\,| + |\,\cdot\rangle\langle\cdot\,| \}$$ diagonal term (RK potential)

- $\frac{v}{\Gamma} = 1$: exactly soluble ("RK") point ($T=0$)
- $\frac{v}{\Gamma} = 0$: transverse-field point ($T = 0^+$)
- mapping is a duality

Rokhsar-Kivelson quantum dimer model

$$H = -t \sum_{\square} \{ |\,\|\,\rangle\langle = | + h.c. \} + v \sum_{\square} \{ |\,\|\,\rangle\langle\,\|\,| + | = \rangle\langle = | \}$$

- dimers live on links of lattice
- H is sum over plaquettes: resonance + potential terms

Easily generalised to other lattices

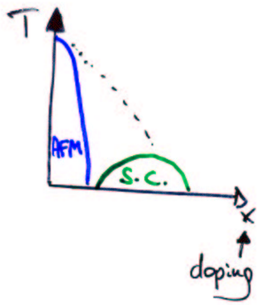
$$H_\triangle = -t \sum_{\diamond} \{ |\,=\,\rangle\langle\, /\!/ \,| + h.c. \} + v \sum_{\diamond} \{ | = \rangle\langle = | + | /\!/ \rangle\langle /\!/ | \}$$

+ symmetry-equivalent terms

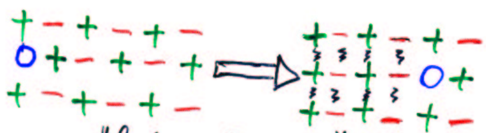
Note: plaquettes have perimeter of even length, e.g. $\diamond$ for triangular lattice

- Two paths to QDM:
  - dimer = SU(2) singlet
  - duality: dimer = frustrated bond

Digression: short-range RVB physics and the quantum dimer model

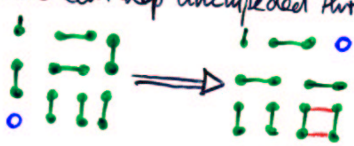


High-T_c problem: doped Mott insulator
How do holes hop through AFM?



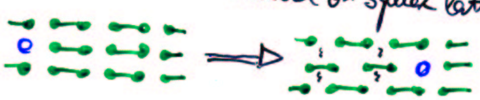
"frustrated hole motion" in Néel background

Alternative: spins form valence bonds (dimers) and holes can hop unimpeded through dimer liquid



PWA, '87
RK, '88

BUT: Quantum dimer model on square lattice has only solid phases



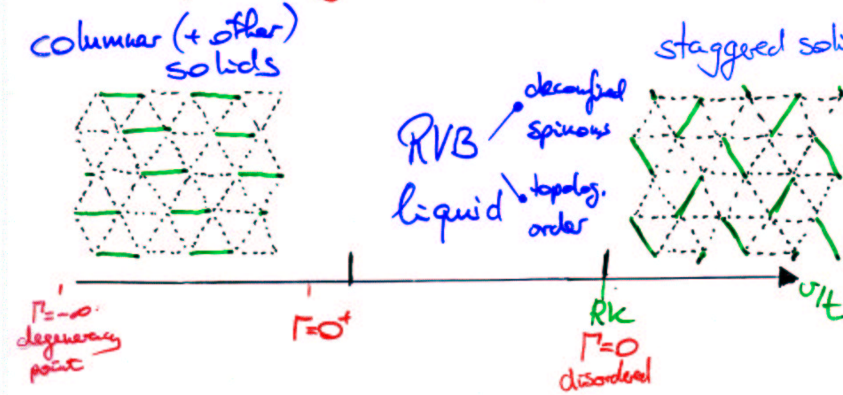
RK, Sachdev et al, Leung et al

∴ hole motion still frustrated!

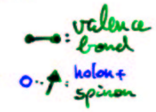
Q: RVB liquid impossible in principle?

An RVB liquid in the triangular lattice RK-QDM

fully-frustrated hexagonal transv. field Ising model $\xrightarrow{\text{dual}}$ triangular lattice quantum dimer model

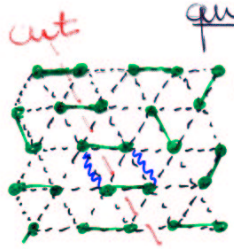


- Triangular RK-QDM realises short-range resonating valence bond liquid phase



- Topological order of interest in quantum computing (Kitaev, Ioffe et al., Troyer) Ivanov et al; Shtengel + Nayak
- connection to triangular spin liquids? (Sachdev, Lhuillier et al.)

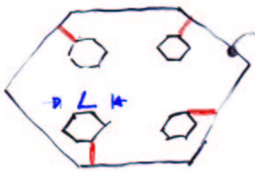
Topological order and quantum computing



- # of dimers crossing cut is even or odd (winding parity)
- winding parity conserved under action of any local $\mathcal{H}$.

⇒ on surface of non-trivial topology, get topologically distinct winding parity sectors, $|e\rangle$ or $|o\rangle$

- liquids in different sectors degenerate: "topological order" (Wen)



"chip" of triangular QDM.
Each cut is one q-bit with
$\langle e|o\rangle \sim e^{-L}$
⇒ immune to local sources of decoherence (Kitaev)

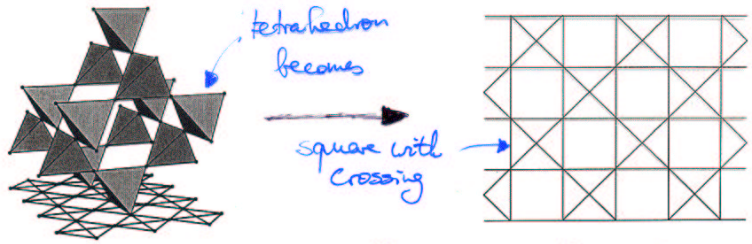
Proposal: compact quantum computer via triangular RVB liquid realised by Josephson Junction Array (Ioffe et al.)

Problems: switching
read in/out
'details'

Conclusions

- Frustrated quantum magnets display rich and interesting behaviour
  - ⇒ quantum disordered phases
  - ⇒ unusual critical and ordered phases
  - "strongly correlated physics"
- wide range of connections and potential realisations
  - ⇒ RVB theory of high-T_c : RVB liquid
  - ⇒ gauge theories } topological order
  - ⇒ quantum computing } topological order
- currently, large materials effort
  - ⇒ e.g. pyrochlore/spinel
- further directions
  - ⇒ ring-exchange models (RK point)
  - ⇒ non-Abelian/supersymmetric generalisations
  - ⇒ more general applicability of these concepts?

Sliding ice: two–dimensional pyrochlore



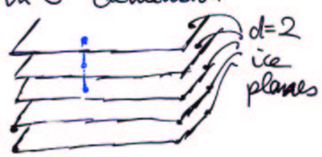
6 Ising ground states of tetrahedron = states of 6-vertex model = (square) ice H_2O configurations

• square ice is critical, with $S_0 = \frac{3}{4} \ln \frac{4}{3}$ (Lieb)

• Consider ferromagnetic stacking in 3^{rd} dimension

$$H_3 = -J_3 \sum_{i,n} S^z_{i,n} S^z_{i,n+1}$$

($J_3 \to \infty$ corresponds to "quantum ice")



⇒ critical phase persists for finite range $0 \le J_3 < J_{cr}$

"sliding ice"

• Chakravarty: use six-vertex model to describe conserved current in dDW.