How laziness Can Lead to a Big Ego: Algorithms and Quenched Disorder

Alan Middleton
May 1, 2003

[Much of this work done in collaboration with Chen Zeng, David McNamara, Daniel Fisher, Jennifer Schwarz, Jan Meinke.]
Optimizing Routes
Why should a physicist care?

- Shortest path is the lowest energy state of an extended object in a disordered background. E.g., a vortex line in type-II superconductors.
- (Dual problem of breaking a rock: surface of least fracture cost.)

- \( \zeta = 2/3 \) is an exact result.
  - Inspired by simulation.
  - Information about optimal paths (power law) that wasn't obvious to computer scientists and mathematicians.
Algorithms for Disordered Materials

- Find mappings to graph problems.
- Then, when possible, apply fast, exact ground state algorithms.
  - Avoid minima by using nonphysical approximate solutions.
  - Improve solutions by finding paths.

What is tractable?
Dimensionality, max/min, kinetics/statics, Z (counting)

- Domain walls, random bond magnet
- Partition fn. Z, 2D EA spin glass
- Partition fn. Z, 2D elastic (per.)
- Percolation
- Shortest paths
- Any $d$: many non-intersecting lines in random potential
- Highest energy state or partition fn. Z, random bond magnet
- Coulomb glass ground state
- Ground state, two 2D layers Ising spin glass
- Barrier to motion of loop in plane.

In classification, polynomial could be of large degree, but worst case bounds rarely exceed $N^3$ and practically as fast as $-N^{1.2}$.

Can do better, typically? Good heuristics? Able to approximate well? Physical importance?
**P: what can we learn?**

- *Check quantitative predictions*, e.g., Le Doussal, Wiese, Chauve (cond-mat/0304164) \( d = 4 - \epsilon \)

\[ \zeta = 0.20829804\epsilon + 0.006858\epsilon^2 \]  

[Non-per. pins]

\[ \langle (u(x) - u(0))^2 \rangle = \left[ \frac{\epsilon}{18} + \frac{\epsilon^2}{108} \right] \ln(|x|) \]  

[Per. Pins, \( 2 < d < 4 \)]

- *More “qualitative” concepts*
  - Number of thermodynamic states
  - *Performance* of the algorithm.
  - Length scales and phase transitions.

---

Map interfaces to matching

Membrane energies: elast. & periodic pins

(111) cubic faces = Rhombus tiling

Very fast algorithms exist for the assignment (or marriage) problem.

(Old PC: \( 10^6 \) sites in 40 s)

Line segments on long axis

Matching/dimer covering (honeycomb lattice)

Multiple lines in disordered background
Does laziness lead to a big ego?

- How many states are there, at T=0, in the thermodynamic limit?
  - A ground state in the thermodynamic limit is stable to “flips” of finite clusters.
  - Are there ground states in the thermodynamic limit that are not related by global symmetries?

- One might actually take the limit of large $L$, watching the correlation functions in a finite volume or, equiv., look at changes in interior as change boundary conditions in a large sample.

[C. Newman and D. Stein].

---

**Sample expansion**

Start with $L^d$ sample.

Find ground state config.

Expand by adding spins to volume $(L')^d$.

Find ground state config.

*Compare bonds* (for SG) in common area of size $w^d$

What is probability $P(L,L',w)$ of any change?
3D spin glasses

- Pallassini, Liets, Junger, Young (cond-mat/0212551):

  "The ground states are determined exactly for systems up to size $L^3$ spins. ... The data are consistent with a picture where the surface of the excitations is not space-filling, such as the droplet or "TNT" picture. When allowing for large finite size corrections, the data are also consistent with a picture with space-filling surface, such as replica symmetry breaking."
Random Field Ising Magnet (RFIM)

- Ferromagnetically coupled magnetic spins $s_i$ (up/down) subject to random fields $h_i$.

$$H = -J \sum_{\langle ij \rangle} s_i s_j + \sum_i h_i s_i$$

- As vary $\Delta = \langle h^2 \rangle / J$, get a phase transition between FM and PM phases.

3D RFIM (w / D. Fisher)

Up to $256^3 \sim 1$ hour.

Find $\beta, \theta, \alpha, \text{“} d_f \text{“}$.

(continuous transition.)

Nested domain walls.

[Part of a ground state for a $64^3$ sample, near $\Delta_c$]
4D RFIM magnetization

4D RFIM: more than 2 states?

cf. Periodic with both +, BC's in window of size 2^4.
Running times – RFIM

- Can understand by studying how algorithm grows domains (much faster than physically.)
- Solve RFIM using max-flow approach
  - Based on max-flow / min-cut approach
  - “Push-relabel” (Goldberg & Tarjan)
Running times

- In the best case, at the worst location
  (i.e., when $\Delta \approx \Delta_c$, $\xi \approx L$), the height field in a
correlation volume has constant slope.
- Average number of relabels is then $\sim c L$. 

4D RFIM running time per site

![Graph showing the relationship between peak relabels per site and volume for 4D RFIM. The graph includes a linear fit.](chart.png)
P, NP & Physics

“The three problems discussed so far in this chapter (REACHABILITY, MAX FLOW, and MATCHING) represent some of the happiest moments in the theory of algorithms.”

- C. H. Papadimitriou, Computational Complexity

Algorithms & Physics

- What is P or NP-hard can be subtle.
- Find exponents, etc., to “high” precision.
- Study qualitative issues, also subtle.
- Performance of algorithms depends on phase in explicable fashion (?)?
- Other distinctions from CS, e.g., (Machta, et al); can growth processes be parallelized?
- Open issues .........................