Non–Equilibrium Dynamics of Interacting Tunneling States in Glasses

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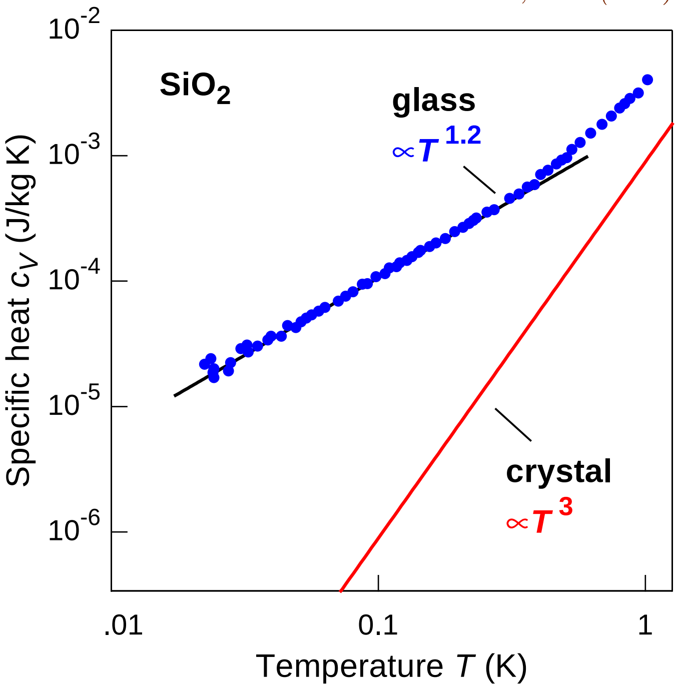
measurements in equilibrium:

- tunneling model $\longrightarrow$ neglects interactions between TSs
- recent experiments $\longrightarrow$ mutual interactions exist

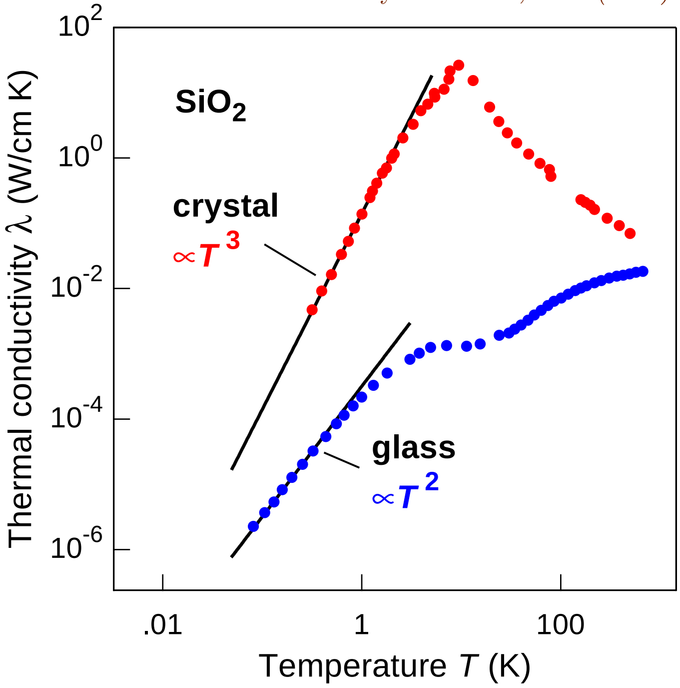
non–equilibrium measurements:

- decreased linear response in equilibrium (dipole gap)
  - $\hookrightarrow$ strongly coupled pairs of TSs (*A. Burin*)
- dynamics of (coupled) TSs
  - $\hookrightarrow$ depends on the sweep rate of an external field
  - $\hookrightarrow$ adiabatic approximation fails
  - $\hookrightarrow$ energy relaxation via mutual interaction

L.C. Lasjaunias et al.
Sol. State Com. **17**, *1045 (1975)*

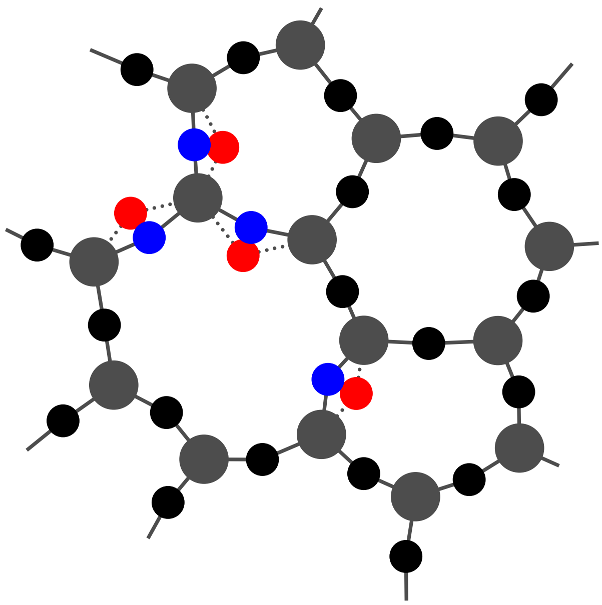


R.C. Zeller, R.O. Pohl
Phys. Rev. **B4**, *2029 (1971)*



↪ broad distribution
of low-energy states

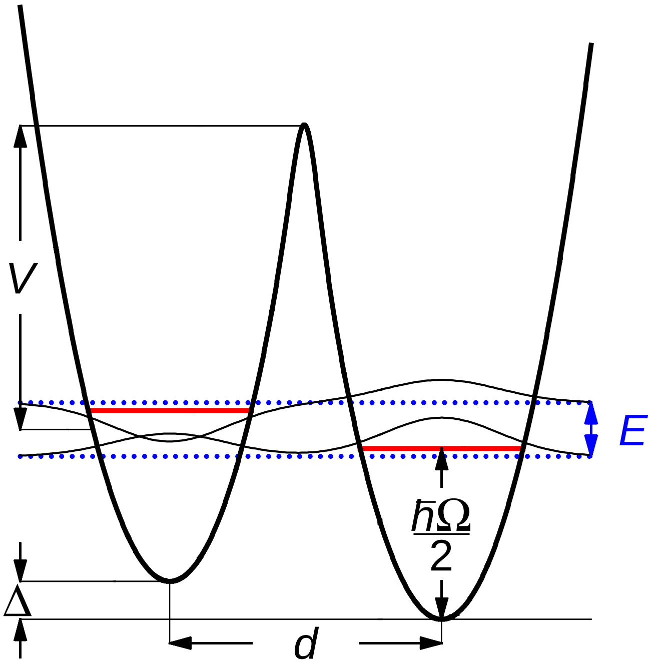
↪ localized defects
↪ strong coupling to phonons



Tunneling Model

- W.A. Phillips, J. Low. Temp. Phys. 7, 351 (1972)
- P.W. Anderson et al., Philos. Mag. 25, 1 (1972)

1.)



$$\widehat{H_0} = \frac{1}{2}\left(\Delta_0\ \widehat{\sigma_x} + \Delta\ \widehat{\sigma_z}\right)$$

$$E = \sqrt{\Delta_0^2 + \Delta^2}$$

$$\Delta_0 \simeq \hbar\Omega \mathrm{e}^{-\lambda}$$

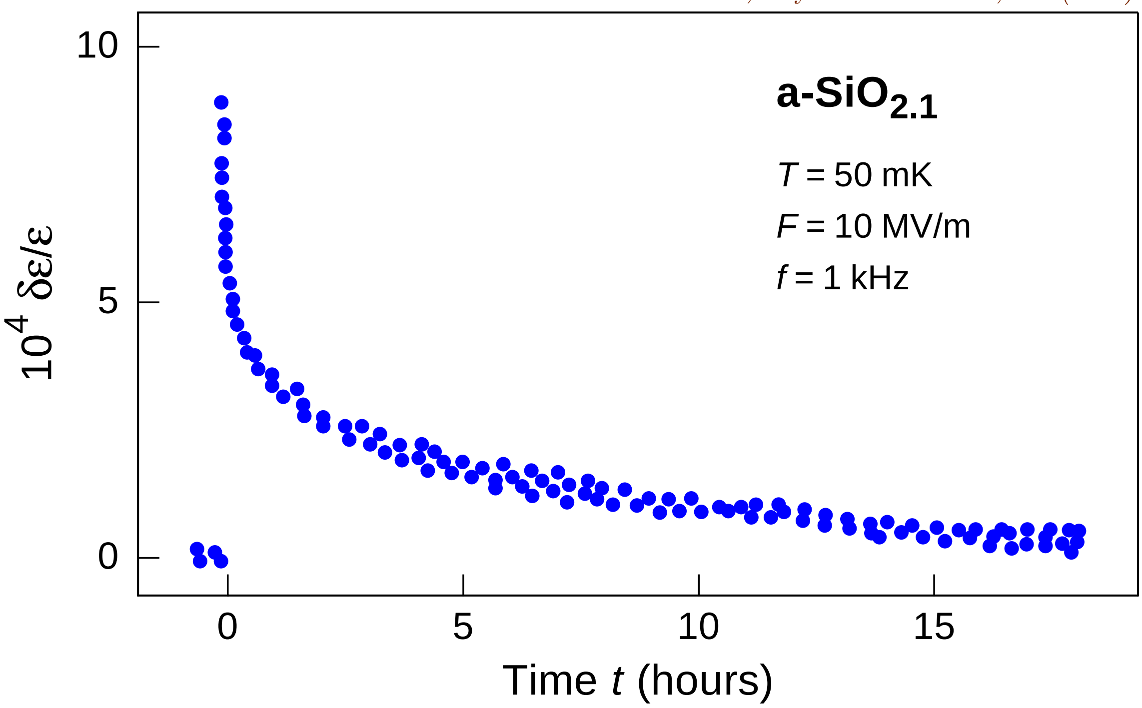
$$\lambda = \frac{d}{2\hbar}\sqrt{2mV}$$

2.) $P(\lambda, \Delta)\,\mathrm{d}\lambda \mathrm{d}\Delta = P_0\,\mathrm{d}\lambda \mathrm{d}\Delta \quad \Rightarrow \quad P(E) \simeq P_0\ ; \quad E > \Delta_{0\mathrm{min}}$

3.) neglects interaction between tunneling states (TSs)

Sudden Application of a Bias Field

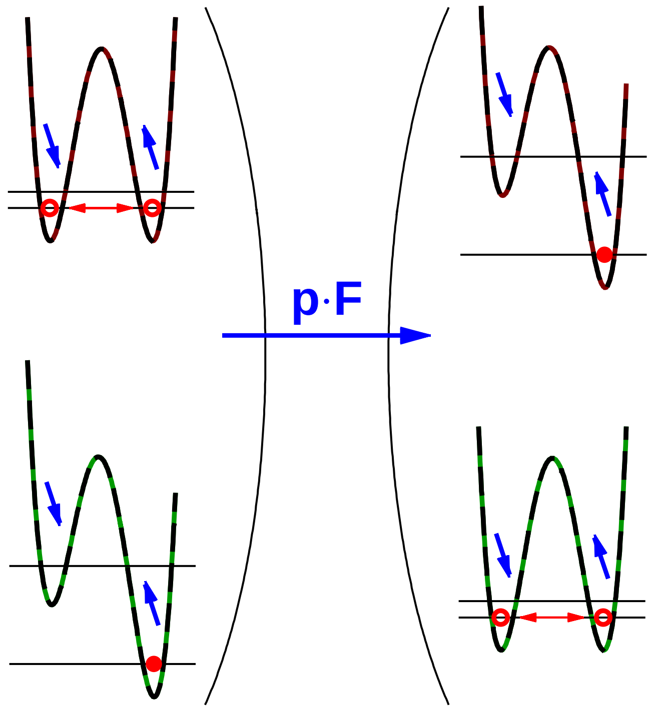
D.J. Salvino et al, Phys. Rev. Lett. **73**, *268 (1994)*



$$E = \sqrt{\Delta_0^2 + (\Delta + \mathbf{p}\cdot\mathbf{F})^2}$$

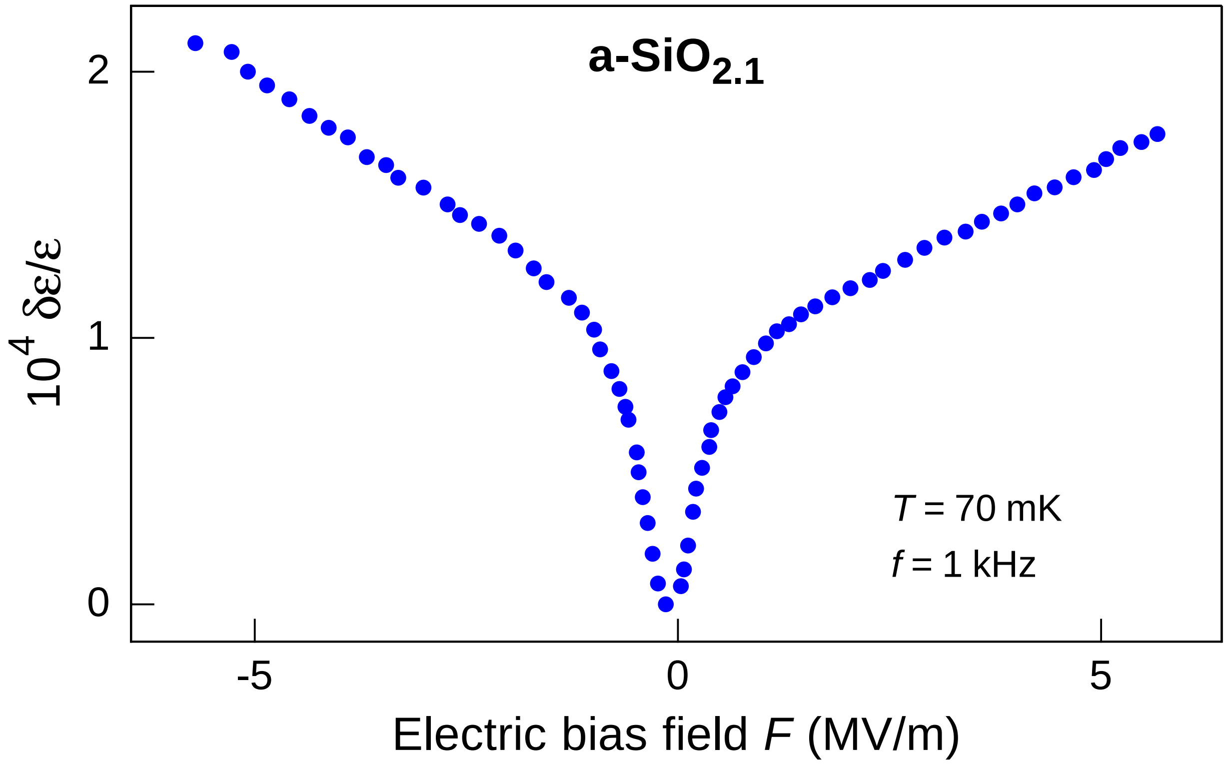
relevant TSs: $E \sim k_\mathrm{B}T$

large bias field: $\mathbf{p}\cdot\mathbf{F}_\mathrm{dc} \gg k_\mathrm{B}T$



(Fast) Bias Field Sweep

D.J. Salvino et al, Phys. Rev. Lett. **73**, *268 (1994)*

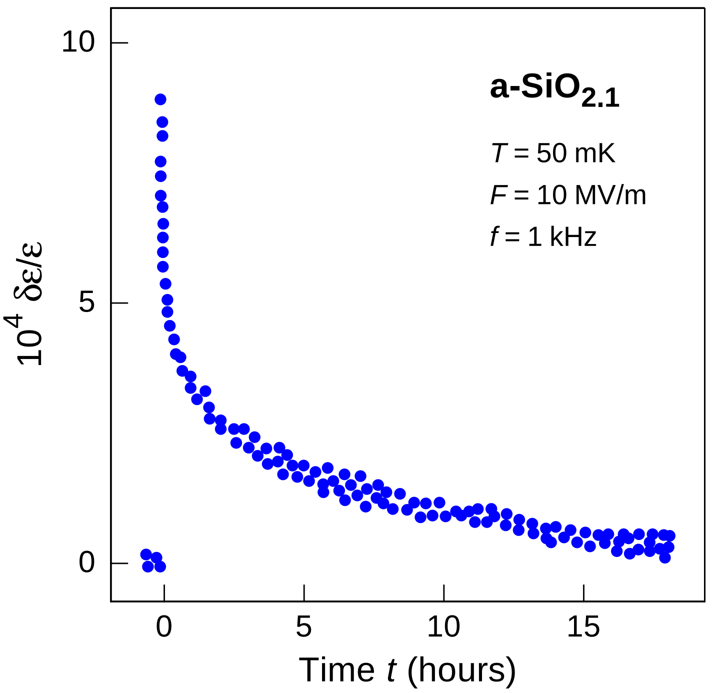


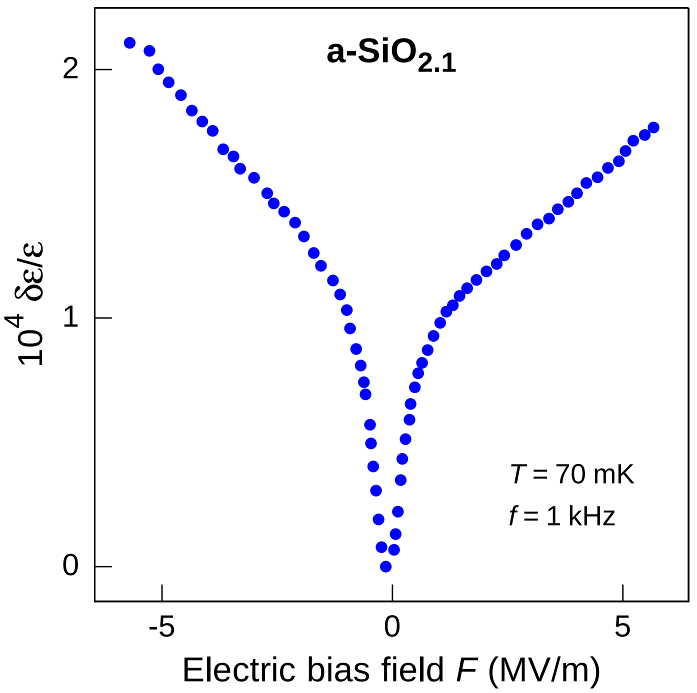
- strongly coupled TSs $\Rightarrow$ "**dipole gap**" in thermal equilibrium

Theory: A.L. Burin, J. Low Temp. Phys. **100**, *309 (1995)*

Relevant Pairs of TSs

D.J. Salvino et al, Phys. Rev. Lett. **73**, *268 (1994)*





relevant TSs

$E \sim k_\mathrm{B}T$

↙ ↘

contribute to

$$\frac{\delta\epsilon}{\epsilon} \propto \Delta_0^2/E^3 \tanh\left(\frac{E}{2k_\mathrm{B}T}\right)$$

if $\Delta_0 \sim k_\mathrm{B}T$

have very long

$$\tau_1 \propto \Delta_0^{-2}$$

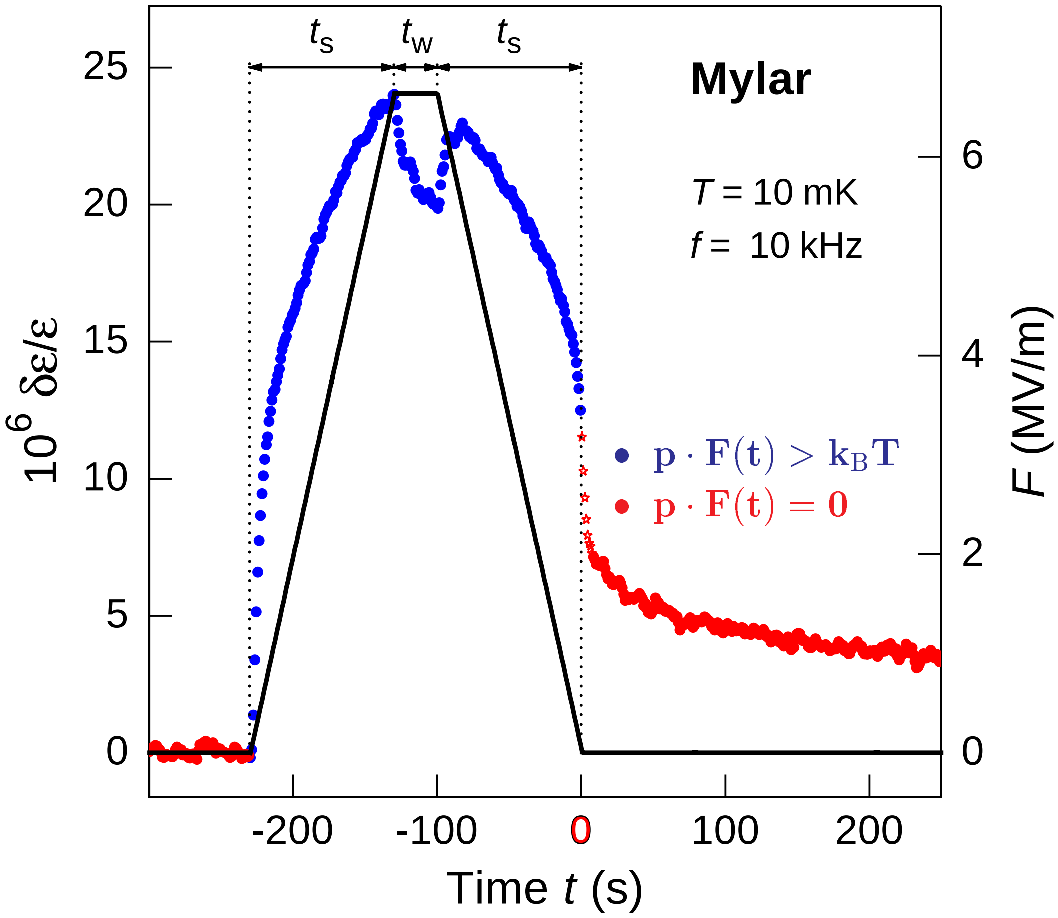
if $\Delta_0 \ll k_\mathrm{B}T$

↘ ↙

pair

Dynamics of Coupled TSs

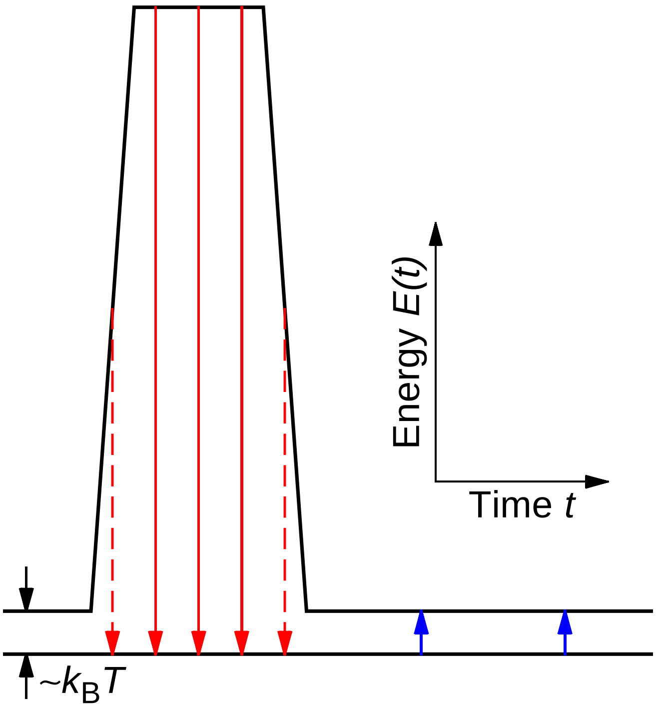
$$E(t) = \sqrt{\Delta_0^2 + (\Delta + \mathbf{p}\cdot\mathbf{F}(\mathbf{t}))^2}\ ; \qquad E \sim k_B T \quad \longrightarrow \quad \frac{\delta\epsilon}{\epsilon}$$



$$\left.\frac{\delta\epsilon}{\epsilon}\right|_{\text{res}} \simeq \underbrace{\frac{4\pi p^2}{9\epsilon_0\epsilon} P_0^2 U_0 \ln\left(\frac{\mathbf{p}\cdot\mathbf{F}}{T}\right)^2}_{A(F,T)} \ f\,(t)\ ; \qquad T < T_{\text{min}}$$

decay function

Pair Breaking by Relaxation



decay time

(longest relaxation time):

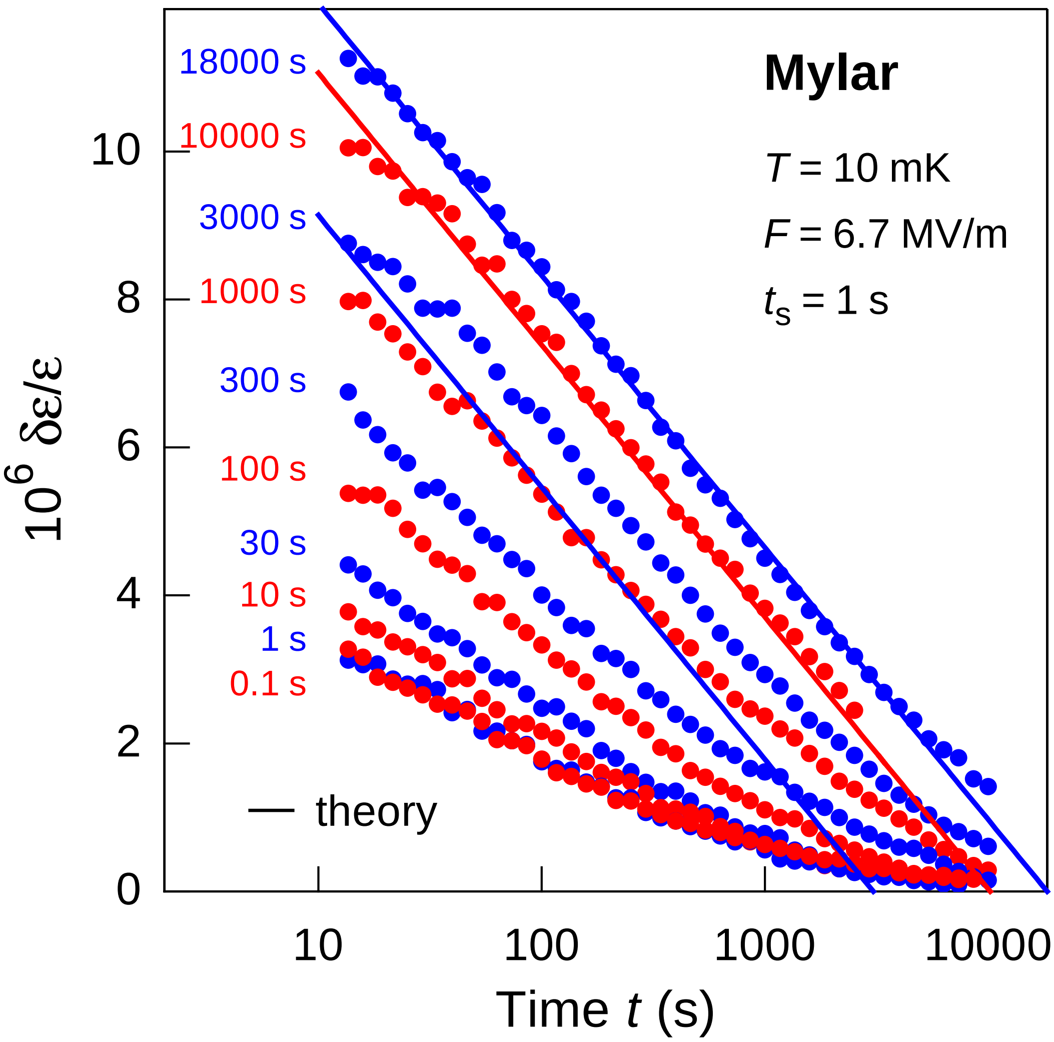
$$\tau_0 = t_{\mathrm{w}} \frac{\tau_1(E \approx k_{\mathrm{B}}T)}{\tau_1(E \approx \mathbf{p}\cdot\mathbf{F})}$$

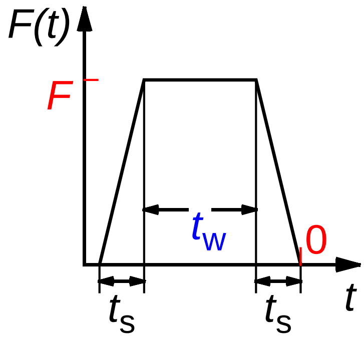
- 1–phonon process: $\tau_0 \sim t_{\mathrm{w}} \dfrac{\mathbf{p}\cdot\mathbf{F}}{2k_{\mathrm{B}}T} \gg t_{\mathrm{w}}$

- distribution of relaxation times within tunneling model

  $\Rightarrow$ decay function: $f(t) = \ln\left(\dfrac{\tau_0}{t}\right)$

Waiting Time Dependence





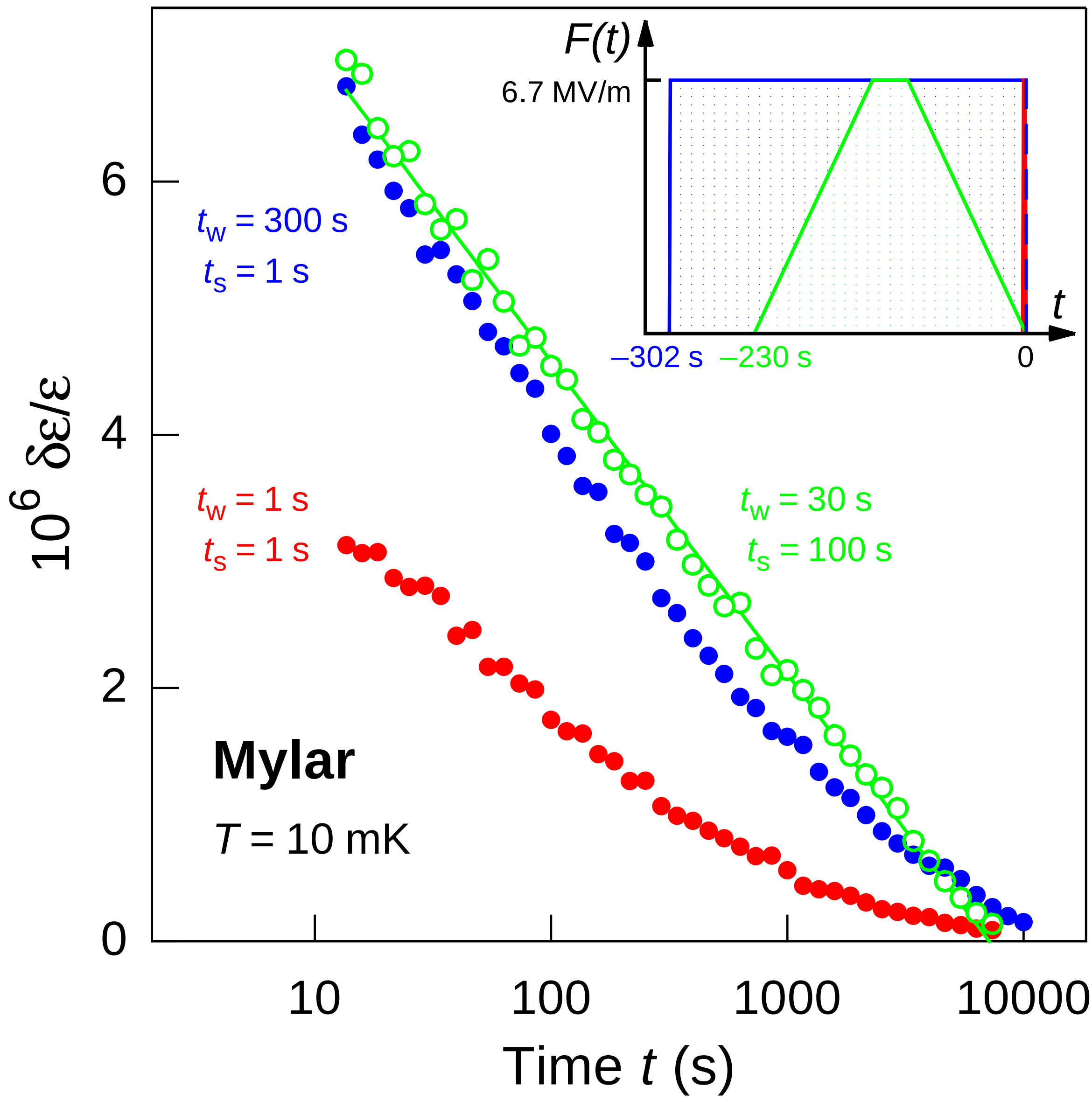
- $t_w > 3000$ s $\rightarrow$ $\tau_0 \sim t_w$

  $\hookrightarrow$ **relaxation enhanced by interaction ?**

- $t_w < 3000$ s $\rightarrow$ $\tau_0 \gg t_w$

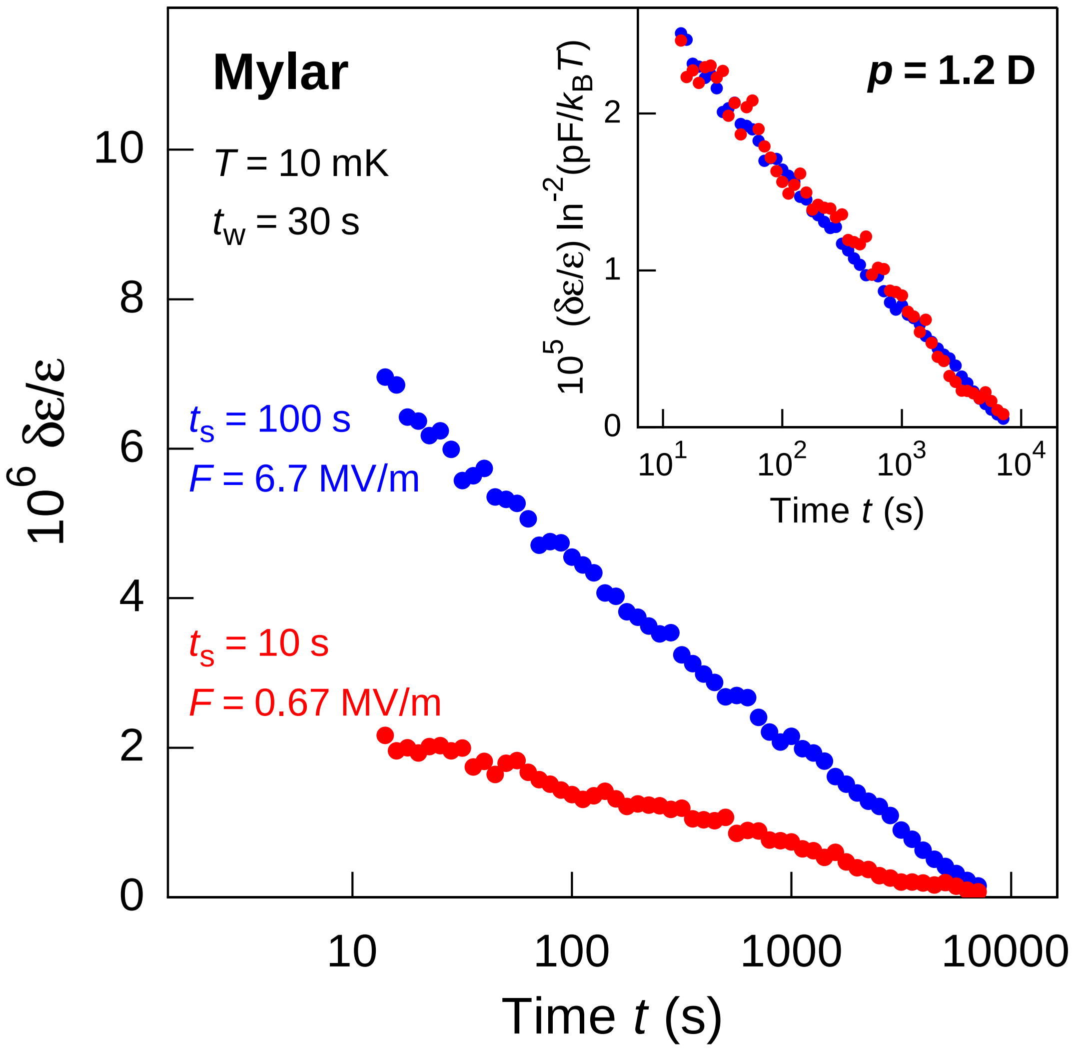
  $\rightarrow$ non–logarithmic decays

  $\hookrightarrow$ **additional pair breaking mechanism ?**



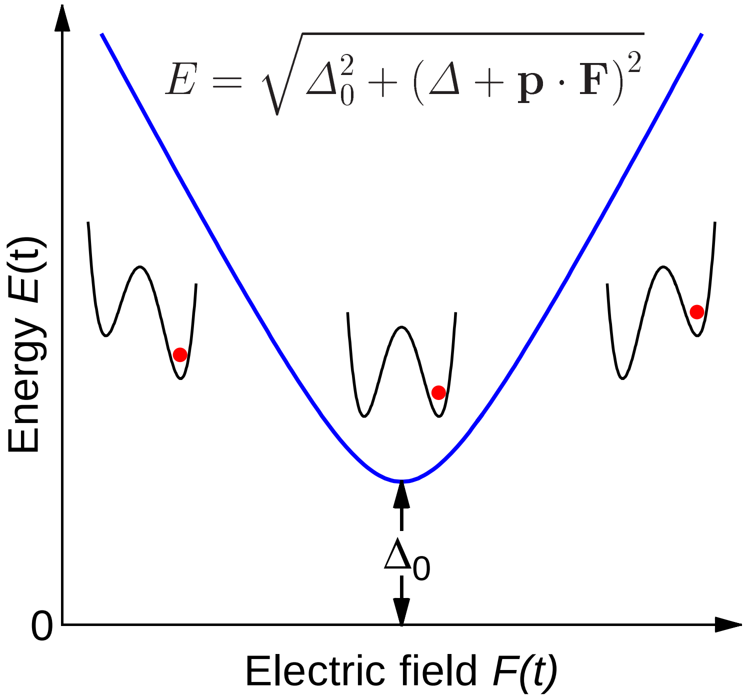
F(t)
6.7 MV/m
t
–302 s
–230 s
0
6
4
2
0
$10^6\ \delta\varepsilon/\varepsilon$
t_w = 300 s
t_s = 1 s
t_w = 1 s
t_s = 1 s
t_w = 30 s
t_s = 100 s
Mylar
T = 10 mK
10
100
1000
10000
Time t (s)

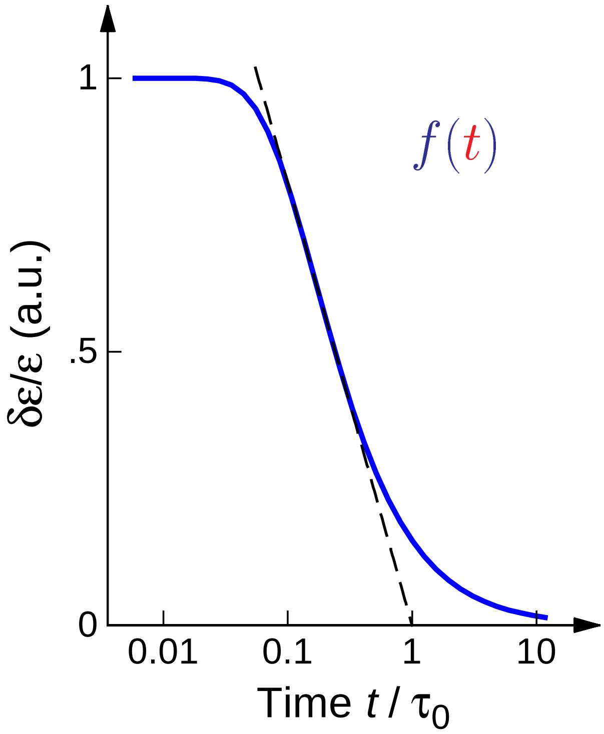
Scaling Behavior $F/t_s = \mathrm{const}$



- $\tau_0 = \tau_0\left(\frac{F}{t_s}\right)$!

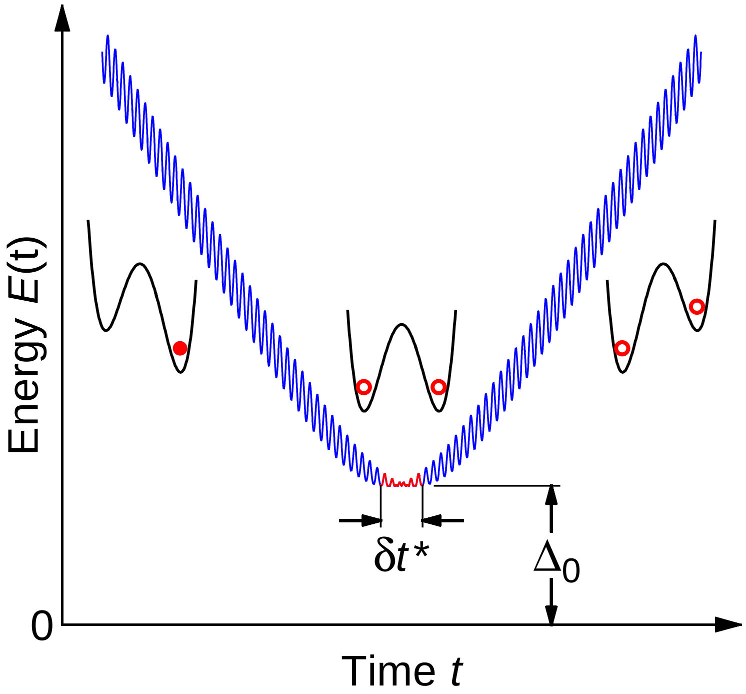
Pair Breaking by Non–Adiabatic Driving

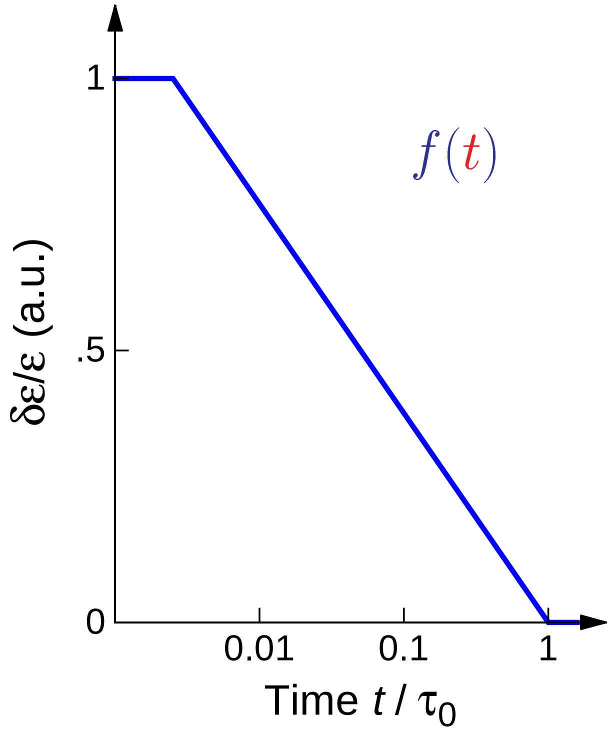




$$\Delta_0 \sim \Delta_{0c} \propto \sqrt{\frac{F}{t_s}} \quad \Longrightarrow \quad \tau_0 \propto \frac{t_s}{F}$$

Non–Adiabatic AC–Field Driving

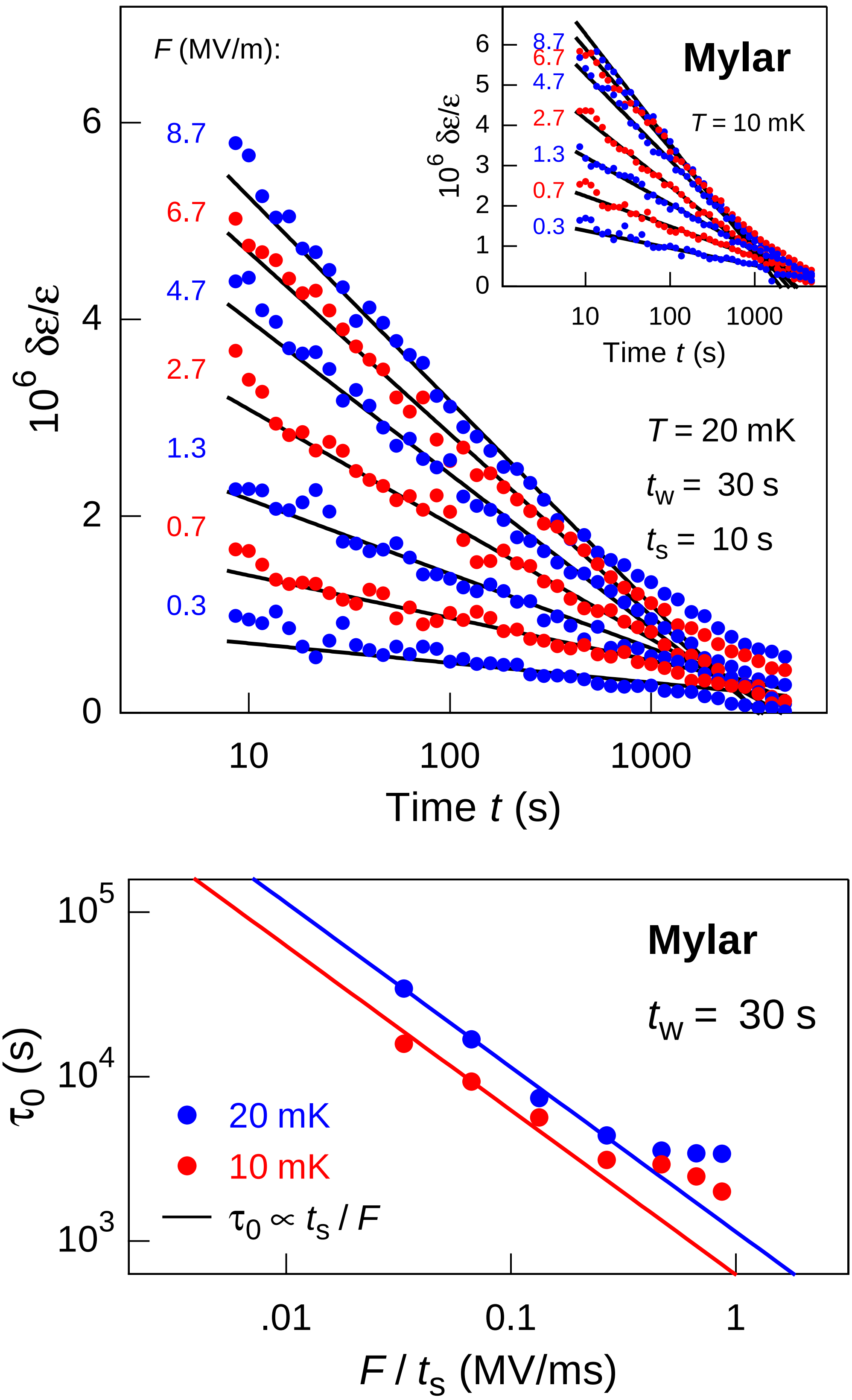




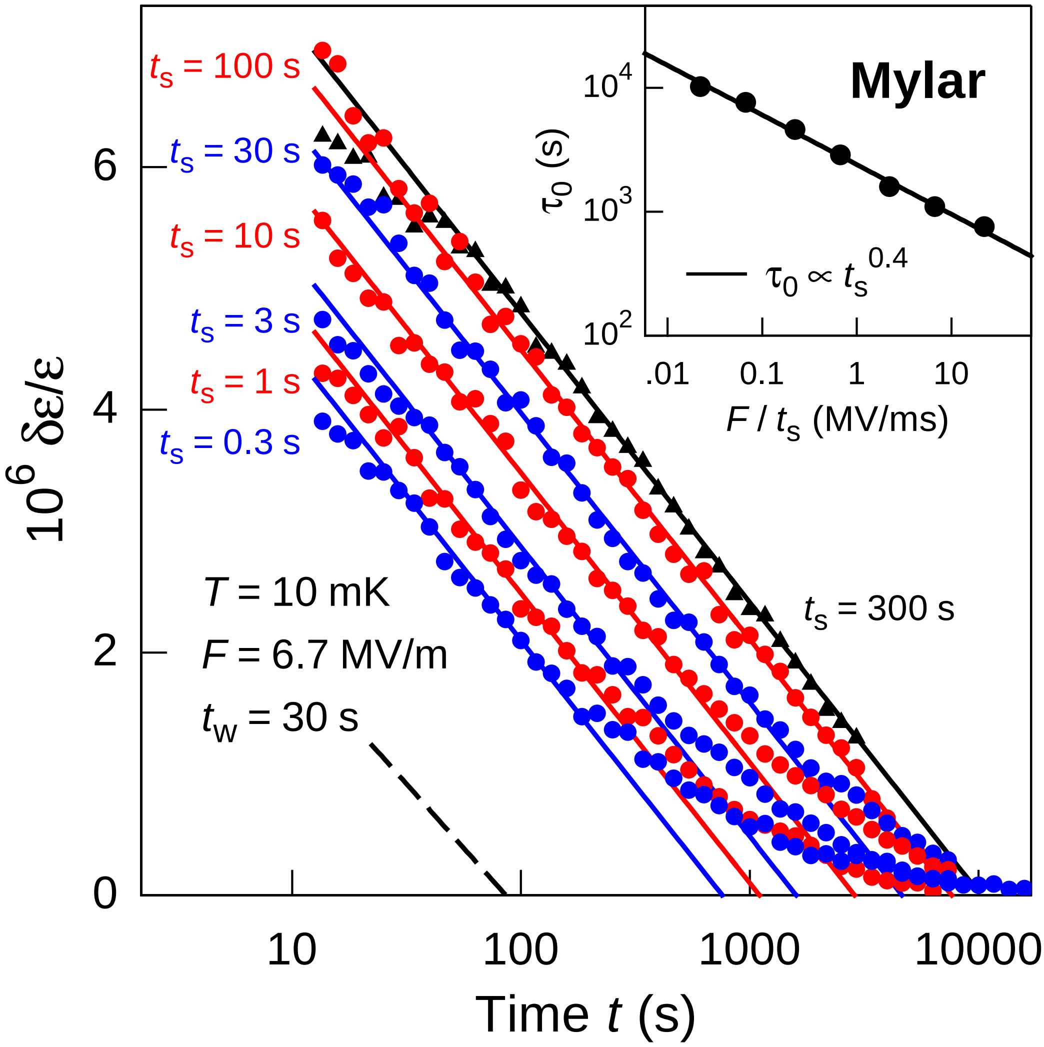
$$f(t) = C \ln\left(\frac{\tau_0}{t}\right) \quad ; \quad \tau_0 \propto \frac{t_\mathrm{s}}{F} \;?$$

$$\delta t^\star = 2F_\mathrm{ac}\,\frac{t_\mathrm{s}}{F}$$

Bias Field Dependence



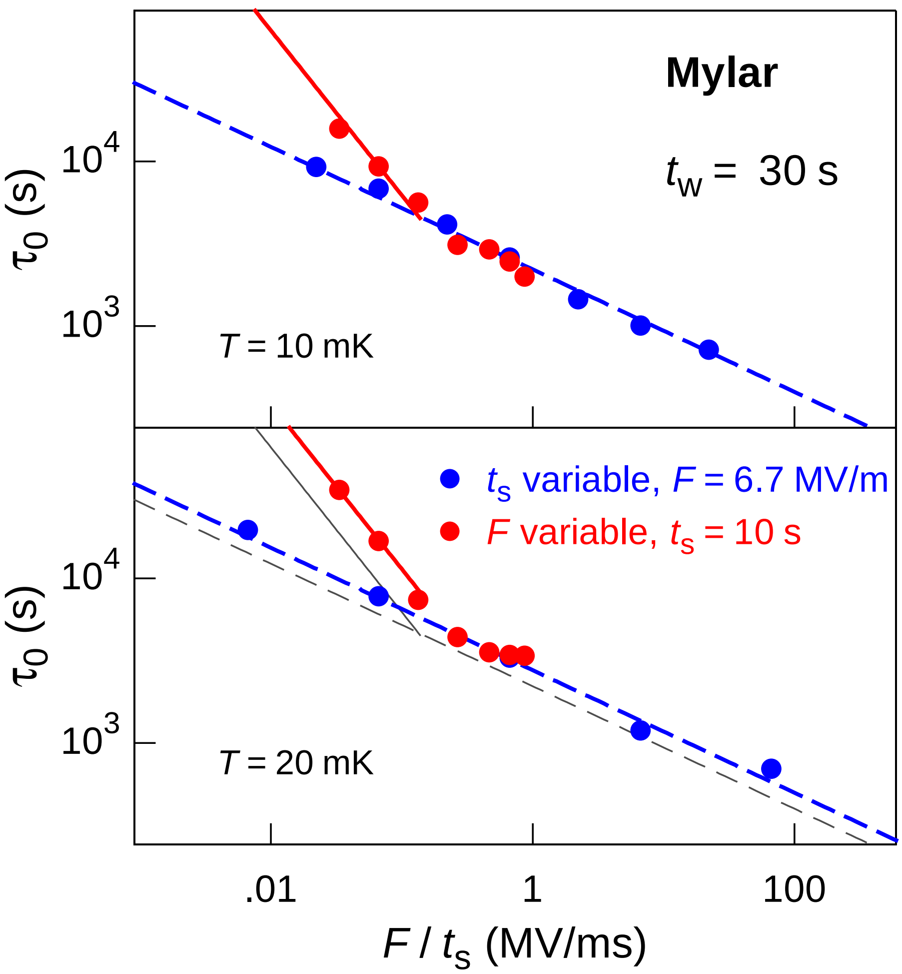
Sweep Time Dependence



expectation:

$$\tau_0 \propto \frac{t_s}{F}$$

Evidence for an additional Pair Breaking Process



$$\frac{\delta\epsilon}{\epsilon} \propto (1-\alpha)\ln\left(\frac{\tau_{0ac}}{t}\right) + \alpha\ln\left(\frac{\widetilde{\tau}_0}{t}\right)$$

$$\tau_{0ac} = \xi(T)\frac{t_s}{F}$$

$$\widetilde{\tau}_0 = 4700\,\mathrm{s}$$

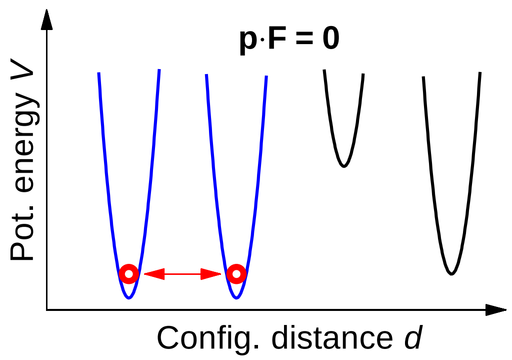
$$\log(\tau_0) = (1-\alpha)\log(\tau_{0ac}) + \alpha\log(\widetilde{\tau}_0)$$

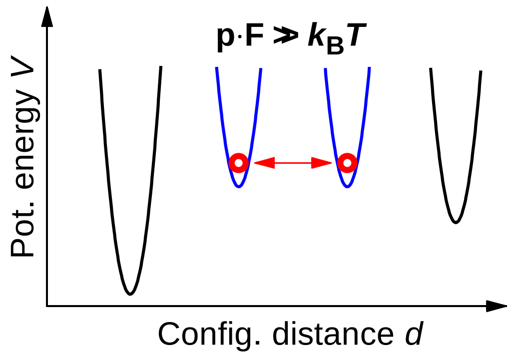
$$\alpha = \begin{cases} 0.6\,; & F > 1.5\,\mathrm{MV/m} \\ 0\,; & F < 1.5\,\mathrm{MV/m} \end{cases}$$

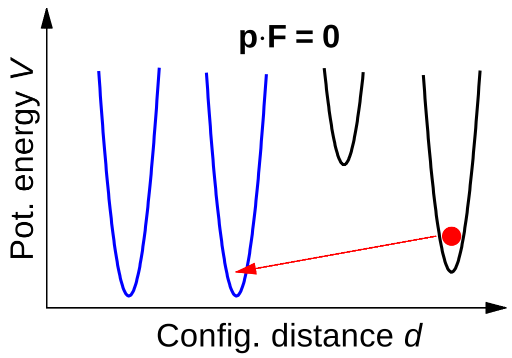
What is the extra Pair Breaking Process ?

3.) $$f(t, \tau_0) \propto \ln\left(\frac{\widetilde{\tau}_0}{t}\right) \; ;$$

$\widetilde{\tau}_0 =$ constant $\Longrightarrow$ quantum mechanical tunneling

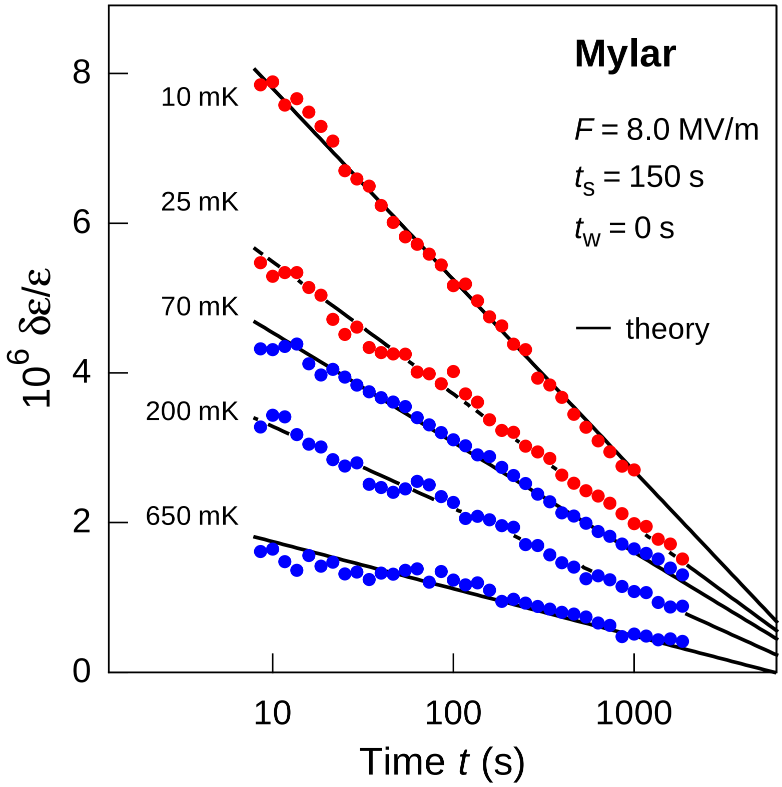


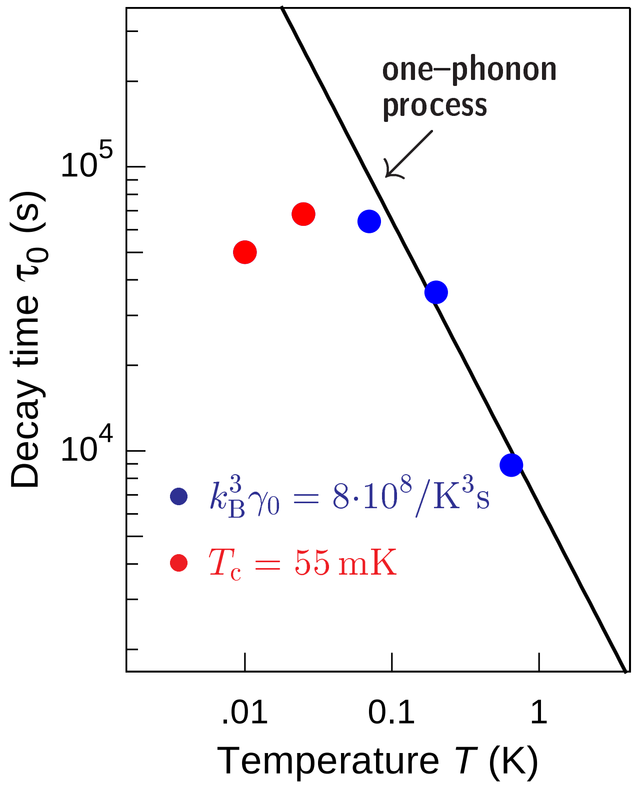




$\Longrightarrow$ field induced structural rearrangement

Interaction Mediated Relaxation





$$\tau_1^{-1} = \gamma_0 \Delta_0^2 E \coth\left(\frac{E}{2k_B T}\right) + \alpha_0 \left(\frac{\Delta_0}{E}\right)^2 k_B T$$

one–phonon process — *J. Jäckle, Z. Physik* ***257****, 212 (1972)*

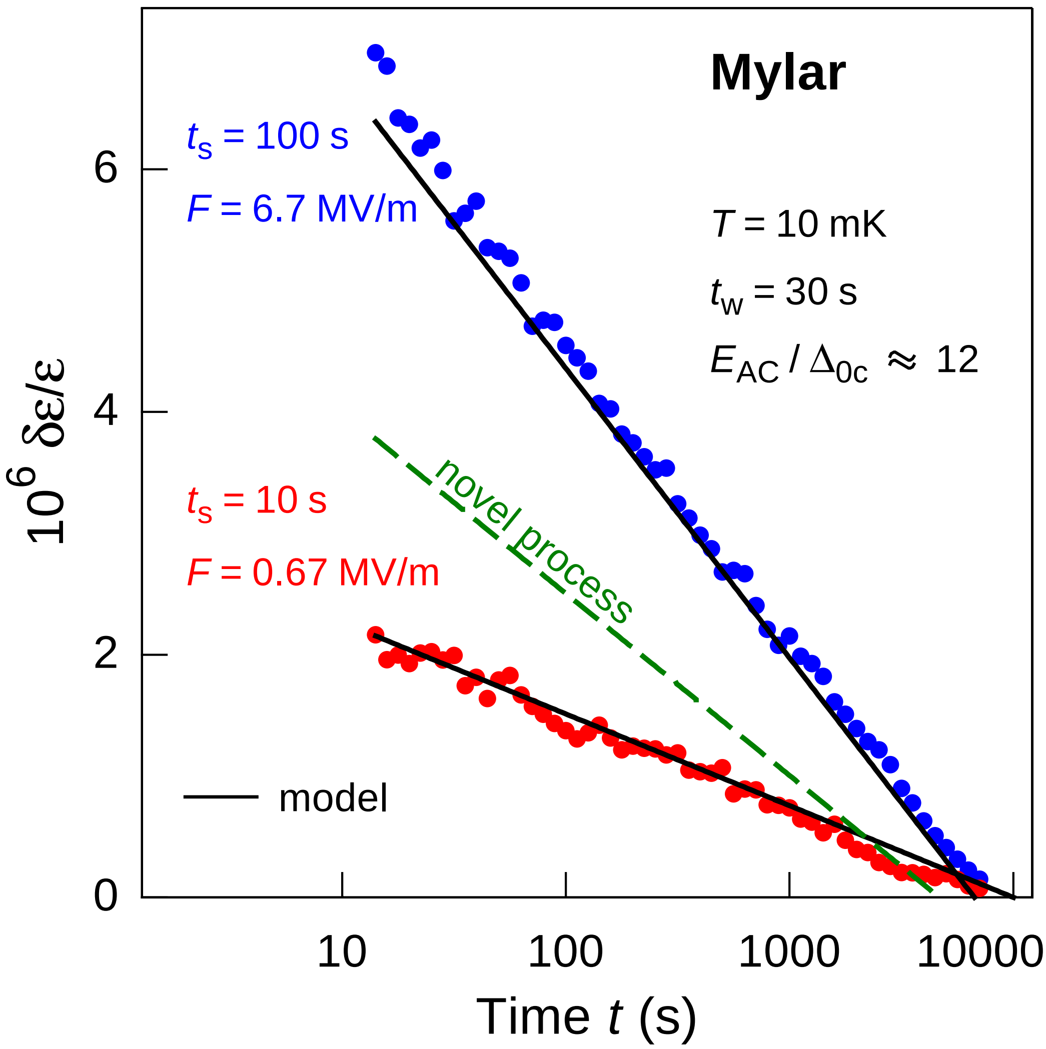
resonant pairs & spectral diffusion — *A.L. Burin et al, Phys. Rev. Lett.* ***80****, 2945 (1998)*

$$\hookrightarrow \quad k_B T_c = \sqrt{\alpha_0/\gamma_0 \ \tanh(0.5)}$$

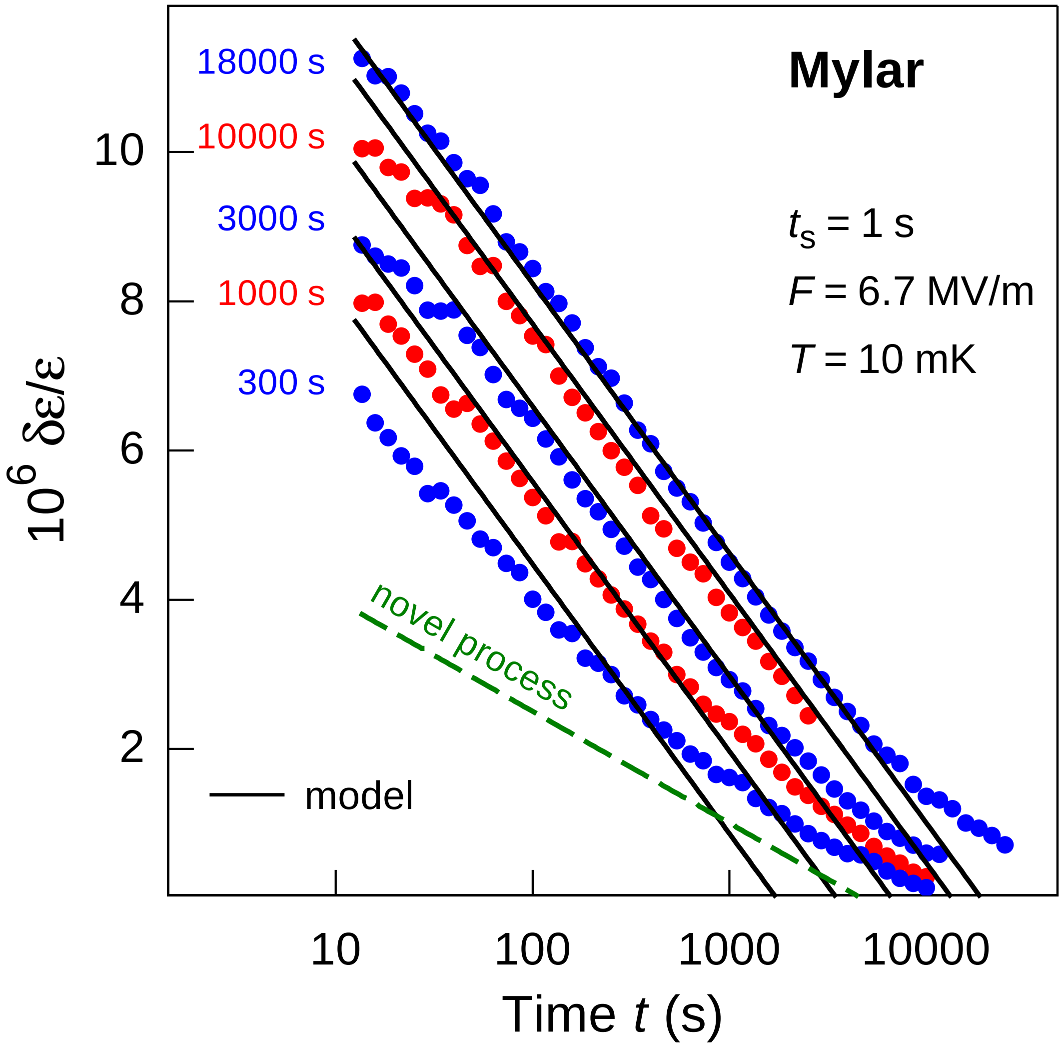
$$\hookrightarrow \quad \tau_0(T \leq 25\,\mathrm{mK}) \quad \Longrightarrow \quad T_c \simeq 55\,\mathrm{mK}$$

$$\frac{4}{3}\frac{p^2 P_0}{\epsilon_0 \epsilon} \simeq 1.35\,10^{-4} \qquad \Longrightarrow \qquad \begin{cases} P_0 U_0 \simeq 6.3\,10^{-4} \\ p \simeq 1.2\,\mathrm{D} \end{cases}$$

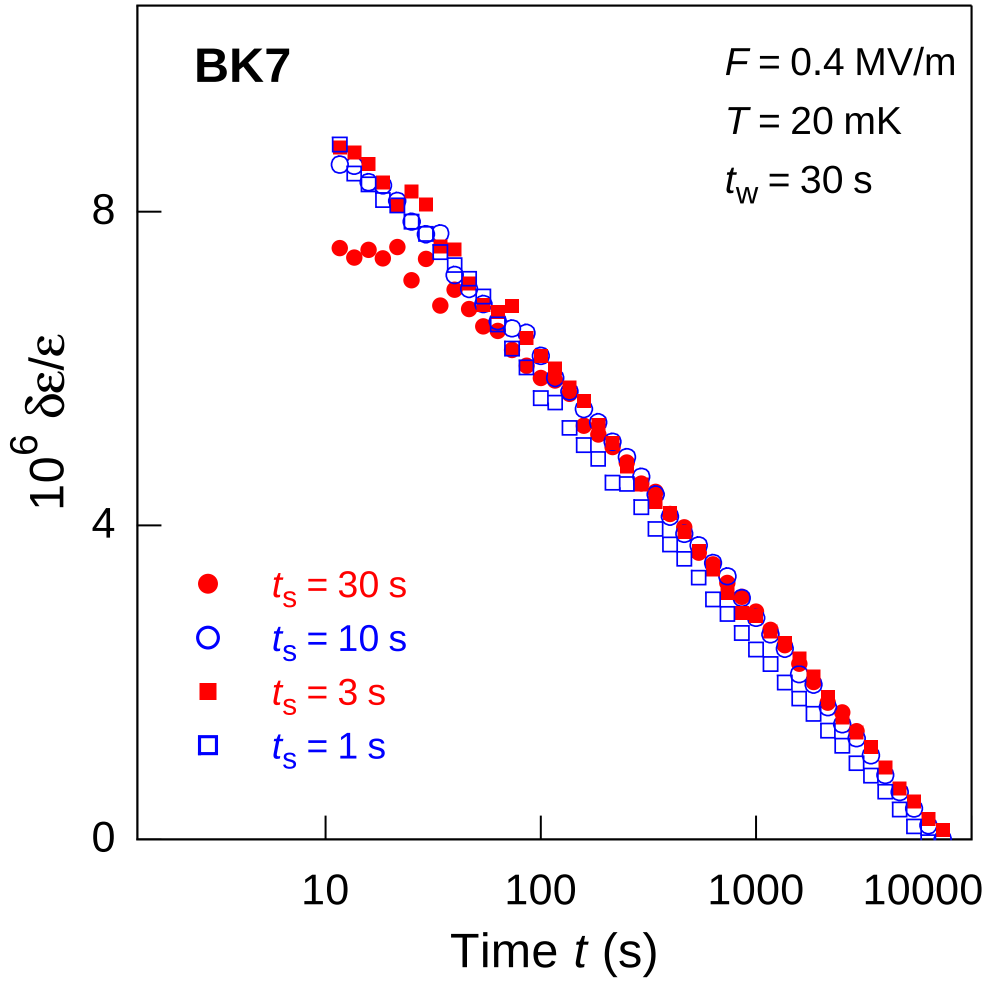
Non–Adiabatic Driving & Novel Process



Energy Relaxation & Novel Process



Non–Equilibrium Measurements on BK7

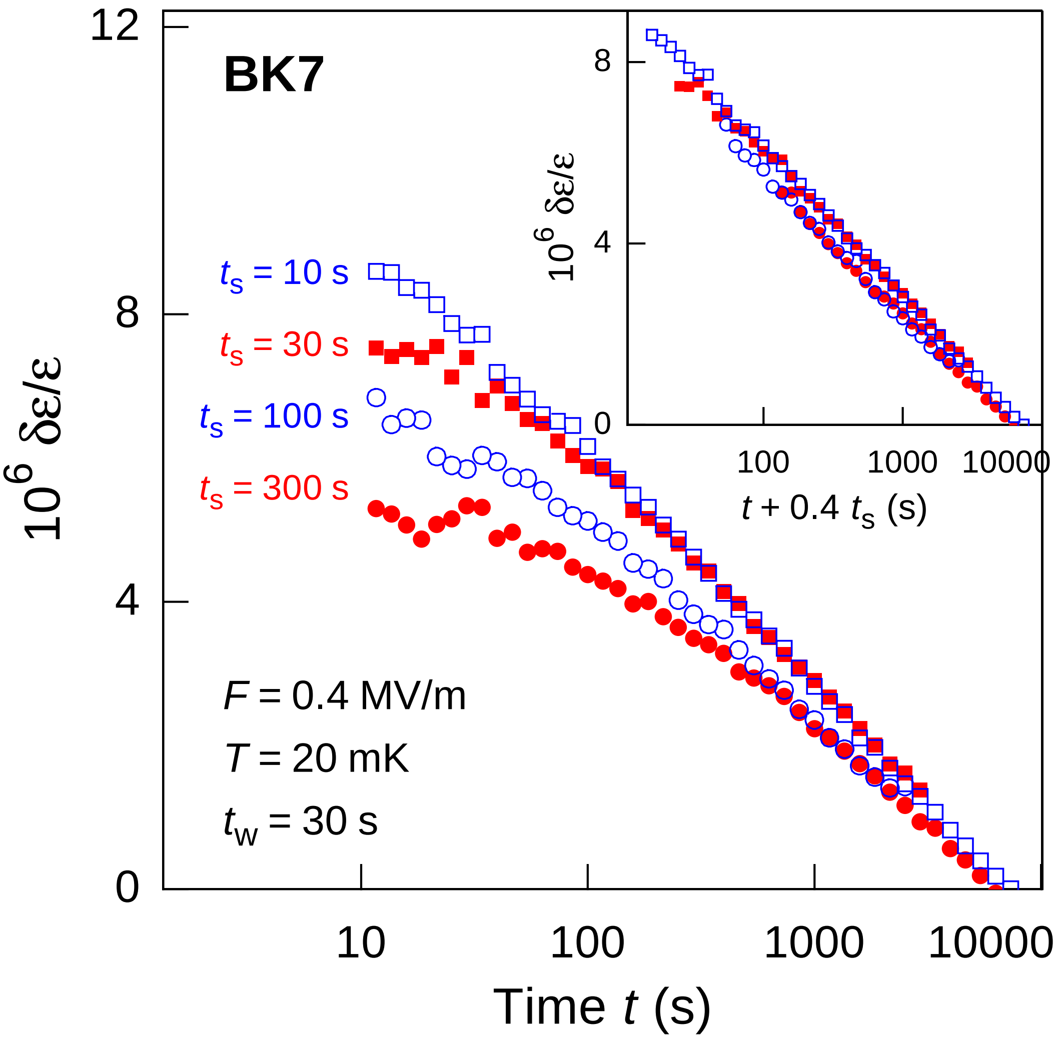


- no sweep time dependence

  & very little waiting time dependence $\Bigg\} \Longrightarrow \quad \dfrac{\Delta_{0\mathrm{min}}}{k_\mathrm{B}} \geq 1.6\,\mathrm{mK}$

- $\tau_0 \gg t_\mathrm{w} \dfrac{\mathbf{p}\cdot\mathbf{F}}{2k_\mathrm{B}T} \sim 150\,\mathrm{s}$
- τ_0 is temperature independent $\Bigg\} \Longrightarrow$ structural rearrangements ?

Non–Equilibrium Measurements on BK7



Conclusion

- Interaction between TSs matters

- Strongly coupled pairs govern non–equilibrium dynamics

- Field enhanced pair breaking processes

  $\hookrightarrow$ relaxation

  $\hookrightarrow$ non–adiabatic driving

  $\hookrightarrow$ add. process with constant $\widetilde{\tau}_0 \Rightarrow$ structural rearrangement ?

- $T < 55$ mK $\quad\rightarrow\quad$ interaction mediated relaxation in Mylar

Thanks to

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Danna Rosenberg

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U. S. Department of Energy

energy splitting:

$$E = \sqrt{\Delta_0^2 + (\Delta + \mathbf{p}\cdot\mathbf{F})^2}$$

relevant TSs: $E \sim k_{\mathrm{B}}T$

typically: $\mathbf{p}\cdot\mathbf{F} \ll k_{\mathrm{B}}T$

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dielectric excess response:

$$\left.\frac{\delta\epsilon}{\epsilon}\right|_{\mathrm{res}} \simeq A(F,T)\ f\,(t)$$

$$A(F,T) = \frac{4\pi p^2}{9\epsilon_0\epsilon} P_0^2 U_0 \ln\left(\frac{\mathbf{p}\cdot\mathbf{F}}{T}\right)^2$$

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relaxation rate for $E \sim k_{\mathrm{B}}T$**:**

$$\tau_1^{-1} \simeq 2.2\gamma_0 \Delta_0^2 k_{\mathrm{B}}T \ + \ \alpha_0 \frac{(\Delta_0)^2}{k_{\mathrm{B}}T}$$

$$\left[= \tau_1^{-1}\big|_{\mathrm{phonon}} \ + \ \tau_1^{-1}\big|_{\mathrm{TSs}}\right]$$

$$\hookrightarrow \quad k_{\mathrm{B}}T_{\mathrm{c}} \simeq \sqrt{\alpha_0/2.2\gamma_0}$$