Decoherence of quantum transport in disordered metals by dynamical defects

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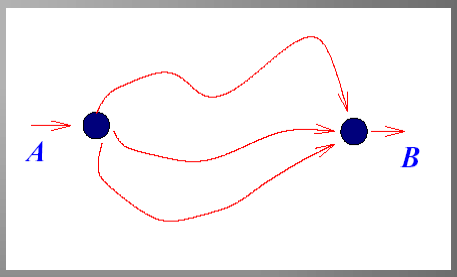
Outline

- Decoherence in weak localization: Overview
- Dephasing due to dynamic degrees of freedom:
  - Qualitative considerations
  - Results for identical dynamic defects
  - Average over different dynamic defects
- Discussion and Conclusions
- What is not understood yet?
- Appendix: Calculations

Noninteracting electrons with $p_F \ell \gg \hbar$ passing through scattering medium.



The probability is

$$W = \left| \sum_i A_i \right|^2 = \sum_i |A_i|^2 + \sum_{i \neq j} A_i A_j^* .$$

A_i is the propagation ***amplitude*** along the path i.

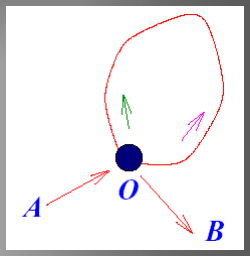
The 1st item – ***classical probability***, the 2nd one – ***interference term.***

Constructive interference

For the majority of the trajectories the phase gain,

$$\Delta\varphi = \hbar^{-1} \int_A^B \vec{p} \cdot d\vec{l} \gg 1 ,$$

and interference term vanishes.



Special case - trajectories with self-crossings. For these parts, the phase gains are ***the same***, and

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2A_1 A_2^* = 4|A_1|^2 .$$

Thus quantum effects double the result. As a result, the total scattering probability at the scatterer at the site O ***increases***.

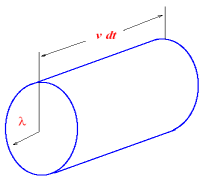
Estimate for 2D case

Interference volume : $v\lambda^2 dt,\ \lambda = \mathrm{h}/p_F$

"Trajectory" volume:

distance $\sqrt{Dt}$

volume : bDt b - thickness



Probability $\dfrac{v\lambda^2\, dt}{bDt}$.

Thus the relative correction is

$$\frac{\Delta\sigma}{\sigma} \sim -\frac{v\lambda^2}{bD}\int_{\tau}^{\tau_\varphi}\frac{dt}{t} = \frac{\Delta\sigma}{\sigma} \sim -\frac{v\lambda^2}{bD}\ln\frac{\tau_\varphi}{\tau}.$$

The lower limit is the elastic time, while τ_φ is the ***dephasing time***.

In a magnetic field $\vec{p} \to \vec{p} + (e/c)\vec{A}, \quad \vec{H} = curl\ \vec{A}$

additional phase $\Delta\varphi_H = \dfrac{2e}{c\hbar}\oint \vec{A}\cdot d\vec{l} = \dfrac{2e}{c\hbar}\oint \left(curl\ \vec{A}\right)\cdot d\vec{S} = 4\pi\dfrac{\Phi}{\Phi_0}$

Φ is the magnetic flux $\Phi_0 = 2\pi\hbar c/e$

The role of magnetic field is important at

$$t_H \le \tau_\varphi \quad \to \quad H \ge H_0 \sim \Phi_0/(D\tau_\varphi) \approx \hbar c/L_\varphi^2.$$

Quantum effects manifest themselves in extremely weak magnetic fields.

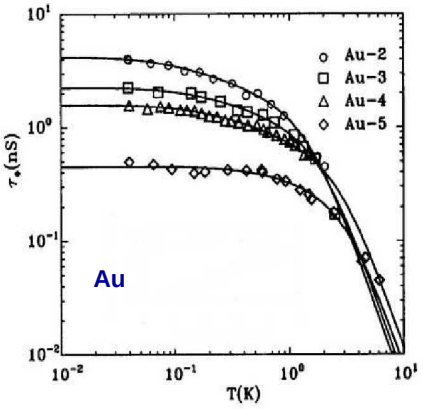
Conventional results

In the simplest situation $\tau_\varphi = \tau_{\mathrm{in}}$. In *clean* materials τ_{in} behaves as

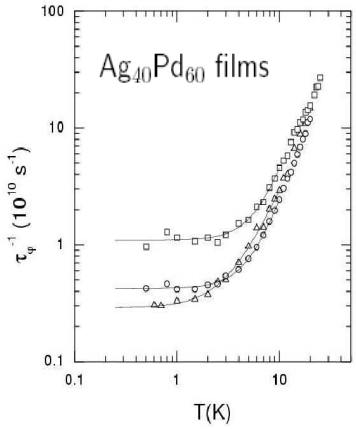
$$\tau_{\mathrm{ph}}^{-1} \propto T^3, \quad \tau_{ee}^{-1} \propto T^2.$$

In disordered materials $\longrightarrow$ quasi-elastic $e-e$ $\longrightarrow$ $\tau_\varphi^{-1} \propto T$.

Some experiments → saturations of dephasing time



Mohanty et al., 1997



Lin et al., 2000

Big activity, hot discussions …

Electromagnetic fields? (Altshuler&Kravtsov)

Probably not. Different materials give different results in the same configuration

Zero-point vibrations!? (Golubev&Zaikin)

Against general principles (Aleiner *et al.*, Imry, v. Delft)

Magnetic impurities?

Probably the case in good Ag, Au, and Cu wires, still some problems remain (Pierre *et al., 2003)*

Structural dynamic defects, two-level systems?

Previous studies of TLS-induced dephasing:

A. Zawadowski, Jan von Delft and D. C. Ralph, Phys. Rev. Lett. **83**, 2632 (1999). 2-channel Kondo model

I. L. Aleiner, B. L. Altshuler, Y. M. Galperin, and T. A. Shutenko Phys. Rev. Lett. **86**, 2629 (2000).

Y. Imry, H. Fukyama, and P. Schwab, Europhys. Letters, **47**, 608 (1999).
Kagn-Hun Ahn, P. Mohanty, Phys. Rev. B **63**, 195301 (2001) Structural TLS, AHV model, resonant processes

I.L. Aleiner, B.L. Altshuler, and Y.M. Galperin, Phys. Rev. B, **63**, 201401, (2001). Too many TLS required

New features:

Two mechanisms of dephasing:

Phase jumps versus phase wandering (diffusion)

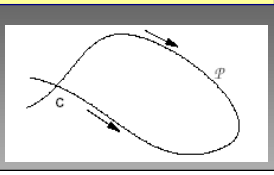
Role of the average procedure over different TLSs:

model of "tunneling states" with diagonal disorder

correlation of dephasing time with conductance

Dephasing due to "slow" degrees of freedom

Qualitative considerations



a slowly varying potential field $U(\vec{r},t)$ action $S=\int ds\sqrt{2m(\mathcal{E}-U)}$

variation ΔS $$\Delta S=-\int\frac{ds}{v}U(s,t)=-\int dt\,U(\vec{s}_t,t).$$

phase difference $$\Delta\varphi=[(\Delta\varphi)_+-(\Delta\varphi)_-]\propto\int_0^{t_0}dt\,[U(s_t,t)-U(s_{t_0-t},t_0-t)]$$

no spatial correlation between the scattering centers. $\overline{U(s_t,t)U(s_{t'},t')}\propto\delta(s_t-s_{t'})$

single-point correlation function

$$\overline{U(s,t)U(s,t')}\equiv\overline{U^2}f(t-t')\,,\ \overline{U^2}\equiv\overline{U^2(s,t)}\,,\ f(0)=1$$

variance $$\overline{\Delta\varphi^2}\propto\sum_s\int_0^{t_0}\frac{dt}{\tau_s}\,[1-f_s(2t-t_0)]$$

The main reason for dephasing to slow down is a large correlation time comparing to the typical traversal time t_0. Then for typical times the correlation function is close to 1, and the phase variance turns out to be small.

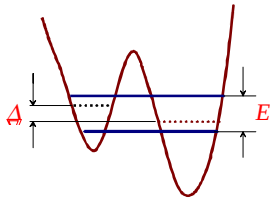
Model for a dynamic defect

two-level tunneling states (TLS)

$\mathcal{H}_{\rm cl}=(\Delta\,\sigma_3-\Lambda\,\sigma_3)/2$

Δ is the diagonal level splitting

Λ is the tunneling amplitude



Two mechanisms of dephasing:

1. Direct transitions between the two TLS's states accompanied by electron-hole pair creation/annihilation

If the energy transfer E is large , the phase relaxation time τ_φ is equal to the typical inelastic relaxation time $\tau_1(E, \Delta)$.

The criterion of "large" E $\quad E\tau_1 >> \hbar.$

For smaller E

effective number of defects – $\bar{N} \sim t/\tau_1$;

correlation function – $f(t) = \cos(Et/\hbar)$;

typical phase shift – $\bar{N}^{1/2} Et/\hbar \rightarrow \tau_\varphi \sim \hbar^{2/3}\tau_1^{1/3}/E^{2/3}$

phase diffusion or wandering $\quad \tau_\varphi^{(1)} = \tau_1 \max\left\{1, (\hbar/E\tau_1)^{2/3}\right\}$

2. Apparently elastic scattering of electrons by a "breathing" scattering potential associated with the dynamic defect.

Estimate the dephasing time:

correlation function $\quad f(t) = e^{-2\gamma|t|}$ $\quad \gamma$ is the the defect transition rate.

$$\tau_\varphi^{(3)} = \max\left\{\tau_3, (\tau_3/\gamma)^{1/2}\right\}$$

The quantitative result for the case of identical fluctuators

$$\delta\sigma = -\frac{e^2}{2\pi^2\hbar}\ln\frac{\tau_\varphi}{\tau}$$

where τ_φ is defined according to the equation

$$\ln\frac{\tau_\varphi}{\tau} \equiv \int_\tau^\infty \frac{dt}{t} e^{-\Gamma_1(t,E,\Lambda)-\Gamma_3(t,E,\Lambda)},$$

$$\Gamma_1(t,E,\Lambda) = \frac{1}{\tau_1}\left[t - \frac{\sin(tE/\hbar)}{E\tau/\hbar}\right],$$

$$\Gamma_3(t,E,\Lambda) = \frac{1}{\tau_3}\left[t - \frac{\hbar}{2\gamma\tau}(1-e^{-2\gamma t/\hbar})\right].$$

Average over different dynamic defects

E and γ can be distributed over a significant range. → distribution function $\mathcal{P}(E,\gamma)$

Two typical model distributions:

1. the so-called "glass-model", GM (Anderson et al., Phillips).

   Δ- distribution is smooth. $\mathcal{P}_\Lambda \propto \Lambda^{-1}$

   exponentially-broad distribution of relaxation rates

2. the "tunneling-states-model" (TM) (Kozub&Rudin)

   more appropriate for crystalline materials.

   Λ are almost the same for all dynamical defects.

   Δ is assumed distributed smoothly within some band.

$$\mathcal{P}_{TM}(E,\Lambda) = \frac{\Theta(E^*-E)}{E^*}\frac{E}{\sqrt{E^2-\Lambda_0^2}}\delta(\Lambda-\Lambda_0).$$

random parameters ⟶ one has to replace $\Gamma_{1,3}$ by the averages

$$\bar{\Gamma}_i(\eta) = \int dE\, d\Lambda\, \mathcal{P}(E,\Lambda)\, \Gamma_i(\eta, E, \Lambda)\,.$$

TM Model: Result

The main conclusion ⟶ there is a temperature region $T \gtrsim T_\alpha$ in which

$$\frac{1}{\tau_\varphi} = \frac{1}{\tau_3}\left(\frac{\Lambda_0}{E^*}\right)\left[\left(\frac{T}{T_\alpha}\right)^{1/3} + \zeta\right], \quad \zeta \approx 1\,.$$

T_α depends on the defect distribution and on electron-defect interaction, τ_3 is the characteristic inelastic relaxation time.

At $T \ll T_\alpha$ the decoherence rate τ_φ^{-1} rapidly decreases with temperature.

Estimates

We use numbers, obtained from experiments on zero-bias anomalies in point contacts. Based on these estimates and taking $P_d \approx 10^{34}$ erg^{-1}cm^{-3}, $\sigma_{\rm in} \approx 10^{-15}$ cm^2, $v_F \approx 10^8$ cm/s, and $\Lambda_0 \approx 10$ mK we obtain $\tau_\Lambda \approx 10^{-9}$ s.

According to the estimates, at temperatures larger than $T_\Lambda \approx \Lambda_0 \approx 10$ mK one expects temperature-independent contribution of resonant processes.

For the relaxation channel, one obtains $T_\alpha \approx T_\beta \approx 10$ mK. Consequently, at $T \gtrsim T_\alpha \approx T_\Lambda \approx 10$ mK one expects that dephasing rate obeys the above equation with $\tau_\Lambda \approx 10^{-9}$ s.

Correlation with the diffusion coefficient

Surprising feature observed in 3D disordered metals

– the bigger the conductance – the shorter the decoherence time,

$\tau_\phi : 1/D$

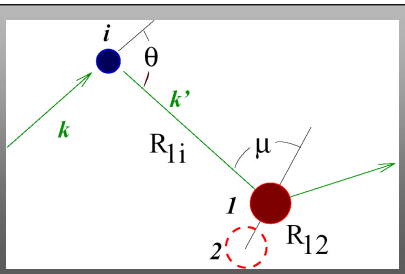
Our explanation:

Friedel oscillations of the electron density near dynamical defects due to electrons scattered by adjacent point defects.

Adiabatic renormalization of the site energy ε_{1i} of one of TS component due to conduction electrons scattered by some defect i

$$\varepsilon_{1i} = V_1 \Re \left[\sum_{\mathbf{k}} \frac{f_i(\theta)}{R_{1i}} \frac{e^{ikR_{1i}(1-\cos\theta)}}{1+e^{(\varepsilon_{\mathbf{k}}-\varepsilon_F)/k_BT}} \right]$$

Here $\theta = \angle\{\mathbf{k}, \mathbf{R}_{1i}\}$, f_i is the scattering amplitude by the ith defect, $\mathbf{R}_{1j}$ is the vector connecting the sites 1 and i, while V_1 is the potential of the defect 1.

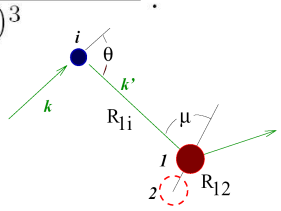


Order-of-magnitude estimate: $$\varepsilon_{1i} \approx -\frac{|V|^2}{\varepsilon_F}\,\frac{\cos(2k_F R_{1i})}{(k_F R_{1i})^3}\,.$$

Now let us consider a TS formed by the site 1 and some state 2, such as $R_{12} \ll R_{1i}, R_{2i}$. Then the effective two-level system acquires the diagonal splitting Δ_i given by the expression

$$\Delta(\mathbf{R}_i,\mu) \approx \frac{2|V|^2}{\varepsilon_F}\,\frac{\sin(k_F R_{12}\mu)\cdot\sin 2k_F R_i}{(k_F R_i)^3}\,.$$

Here $\mathbf{R}_i = \mathbf{R}_{1i} \approx \mathbf{R}_{2i}$, $\mu = \cos\angle\{\mathbf{R}_{12}, \mathbf{R}_i\}$



The probability to find a TS with the splitting Δ is then

$$W(\Delta) = 2\pi n_d \int R^2\,dR \int_{-1}^{1} d\mu\,\delta\left[\Delta - \Delta(\mathbf{R},\mu)\right]$$

Here n_d is the density of defects

A straightforward analysis shows that there is a characteristic energy

$$E^* = \frac{|V|^2 n_d}{\varepsilon_F k_F^3} \approx \frac{1}{2\pi}\,\frac{\hbar}{\tau_{el}} \sim \frac{\hbar D}{v_F^2}$$

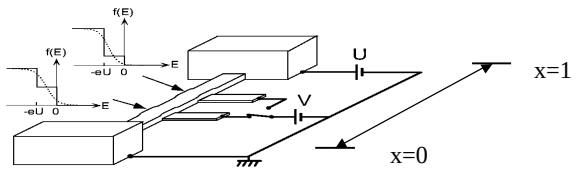
In this way way explain the experimental results.

Conclusions

* Dynamical defects (TLS) in disordered metals can be responsible for apparent "saturation" of the temperature dependence of τ_ϕ
* At very low temperatures the dephasing rate vanishes.
* Two mechanism of TLS-induced decoherence are important:
  - direct inelastic scattering of electrons by dynamical defects;
  - breaking of the time-reversal symmetry by non-stationary scattering potential
* Model of tunneling states with diagonal disorder allows one to obtain reasonable numbers and to explain correlation between τ_ϕ and conductance.

What is not understood yet?

Relation between dephasing and energy relaxation rates

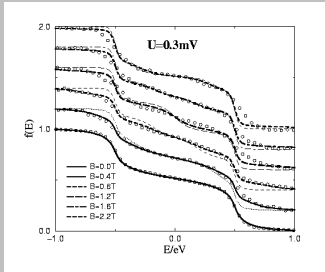


Pothier et al., 1997

Pierre et al., 2001

Magnetic field:

Anthore et al., 2001



Though the I-V curves were qualitatively explained

Kaminskii & Glazman, 2001
Göppert, YG, Altshuler & Grabert, 2002

there are quantitative descrepancies:
Explanation of energy relaxation would require more magnetic impurities, than it is necessary to explain dephasing rate.

Appendix: First-principle calculation

Spinless electrons which scatter against tunneling defects.

The Hamiltonian

$$\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_{\rm d} + \sum_{\vec{p}} \epsilon_p c^+_{\vec{p}} c_{\vec{p}} + \tilde{\mathcal{H}}_{\rm int} \tag{3}$$

where

$$\tilde{\mathcal{H}}_{\rm d} = (\Delta\,\sigma_3 - \Lambda\,\sigma_1)/2 \tag{4}$$

$$\tilde{\mathcal{H}}_{\rm int} = \frac{1}{2}\sum_{\vec{p}\vec{p}_1,n} \left(\hat{\imath}\,\tilde{V}^+_{\vec{p}\vec{p}_1} + \sigma_3 \tilde{V}^-_{\vec{p}\vec{p}_1}\right) c^+_{\vec{p}} c_{\vec{p}_1}\, e^{i(\vec{p}-\vec{p}_1)\cdot\vec{r}_n/\hbar}, \tag{5}$$

$V^{\pm} = V_l \pm V_r$ represent components of a short-range defect potential in the "left" and "right" positions. Estimates for $\tilde{V}^{\pm}$ were given by several authors, e. g., by J. Black and by Y. Imry.

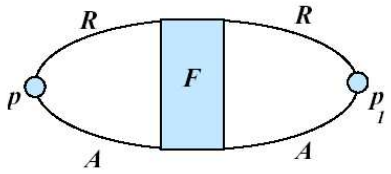
After the transform which makes $\tilde{\mathcal{H}}_{\rm d}$ diagonal we arrive at the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_n E_n\,\sigma_3 + \sum_{\vec{p}} \epsilon_p c^+_{\vec{p}} c_{\vec{p}} + \frac{1}{2}\sum_{\vec{p}\vec{p}_1,n} \Big\{ \hat{\imath}\, V^+_{\vec{p}\vec{p}_1} + \left(\frac{\Lambda_n}{E_n}\sigma_1 + \frac{\Delta_n}{E_n}\sigma_3\right) V^-_{\vec{p}\vec{p}_1} \Big\} c^+_{\vec{p}} c_{\vec{p}_1}\, e^{i(\vec{p}-\vec{p}')\cdot\vec{r}_n/\hbar}. \tag{6}$$

$\Rightarrow$ two processes of electron-defect interaction described by the items proportional to σ_1 and σ_3, respectively.

They correspond to the two mechanisms discussed above.

Quantum contribution to conductance



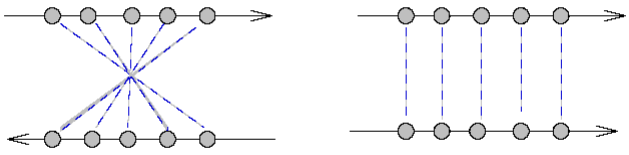
$$\delta\sigma = \frac{e^2}{2m^2}\int (dp)\,(dq)p^2 \int \left(-\frac{dn}{d\varepsilon}\right) d\varepsilon \int \frac{d\omega}{2\pi i} N(\omega)$$
$$\times G_R(\varepsilon,\vec{p})G_A(\varepsilon,\vec{p})\,F(\varepsilon,\omega,\vec{p},\vec{q}-\vec{p})$$
$$\times G_R(\varepsilon+\omega,\vec{q}-\vec{p})G_A(\varepsilon+\omega,\vec{q}-\vec{p})\,. \qquad (7)$$

Here $(dp) \equiv d^2p/(2\pi\hbar)^2$,

$n(\varepsilon)$ is the Fermi function,

$N(\omega)$ is the Planck function,

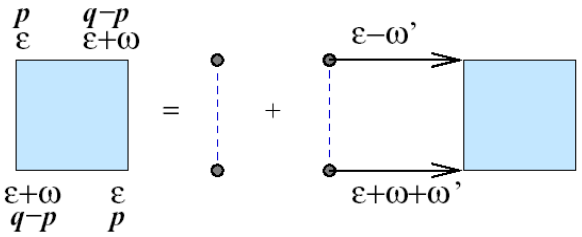
$F(\varepsilon,\omega,\vec{p},\vec{p}_1)$ is a sum of the maximally-crossed diagrams – Cooperon.



Dashed lines - "propagators" of dynamic defects.

The Cooperon is a sum of a ladder in the particle-particle channel.

It satisfies the following Dyson equation



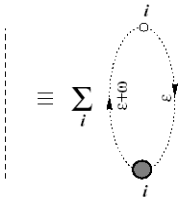
filled square – the Cooperon,

thick lines – the Green's functions averaged over the defect positions,

as well as over the states of the thermal bath,

dashed lines – propagators for electron scattering against dynamic defects.

The propagator can be expressed as a loop graph where dotted lines represent Green's functions for a dynamic defect.



Since the interaction Hamiltonian (6) contain the items of three types ($\propto \hat{1}, \sigma_1, \sigma_3$), each propagator consists of a sum of three items.

Propagators are derived by the Abrikosov technique developed for the Kondo effect – a dynamic defect is interpreted as a pseudo-Fermion with the Green's function

$$g_{\pm}(\epsilon) = (\epsilon \mp E/2 - \lambda + i\delta)^{-1}, \qquad (8)$$

where λ is an auxiliary "chemical potential".

To remove extra unphysical states, at the initial stage $\lambda \to \infty$.

As a result, the retarded propagator in the σ_1-channel is

$$\mathcal{D}_1^R(\omega) = -\tanh\frac{E}{2T}\left(\frac{1}{\omega - E + i\delta} - \frac{1}{\omega + E + i\delta}\right), \quad \delta \to +0\,. \quad (9)$$

The propagator for the σ_3-channel is (cf. with Maleev's expression for glasses)

$$\mathcal{D}_3^R(\omega) = \frac{1}{T\cosh^2(E/2T)}\frac{2i\gamma}{\omega + 2i\gamma}\,. \quad (10)$$

Here

$$\gamma(\Lambda, E) = \left(\frac{\Lambda}{E}\right)^2 \gamma_0(E)\,, \quad \gamma_0(E) = \frac{\chi E}{\tanh E/2T}\,, \quad (11)$$

where $\chi = 0.01 - 0.3$ is dimensionless constant dependent on the matrix element $V^{(1)}$ where $\gamma_0(E)$ has the meaning of *maximum* hopping rate for the systems with a given interlevel spacing (J. Black).

For the *1*-channel we define the propagator as

$$\mathcal{D}_0^R(\omega) = \frac{\nu}{2T}\left(\frac{1}{\omega + \nu + i\delta} - \frac{1}{\omega - \nu + i\delta}\right), \quad \delta, \nu \to +0\,. \quad (12)$$

The propagators do not include the electron-defect coupling constant, hence each propagator should be multiplied by $|W^{(i)}|^2$ where

$$W^{(0)} = V^+\,,\ W^{(1)} = (\Lambda/E)V^-\,,\ W^{(3)} = (\Delta/E)V^-\,.$$

Then, summation over different dynamic defects should be performed.

The resulting equation for $F(\varepsilon,\omega,\vec{p},\vec{q}-\vec{p}_1)$ obtained by a proper analytical continuation of the Matsubara Green's functions, has the form

$$F(\varepsilon,\omega,\vec{p},\vec{q}-\vec{p}) = \mathcal{D}(\omega) - \int \frac{(dp_1)\,d\omega'}{2\pi i} F(\varepsilon,\omega',\vec{p}_1,\vec{q}-\vec{p})\mathcal{D}(\omega-\omega')$$
$$\times G^R(\varepsilon+\omega-\omega',\vec{p}_1')\,G^A(\varepsilon+\omega',\vec{q}-\vec{p}_1)\,[N_0(\omega')-N_0(\omega'-\omega)] \quad (13)$$

where $\mathcal{D}(\omega) \equiv \sum_{is} |W_s^{(i)}|^2 \left[\mathcal{D}_i^R(\omega) - \mathcal{D}_i^A(\omega)\right]$.

$q\ell \ll 1$ $\quad \ell = v_F\tau$ is the electron mean free path, while τ is the electron life time

$\tau^{-1} = \tau_e^{-1} + \tau_1^{-1} + \tau_3^{-1}$

τ_e^{-1} is a sum ow the contributions of static and dynamic defects

$\tau_1^{-1} = 2\pi\rho n_d(\Lambda/E)^2|V_d^-|^2/\hbar\,, \quad \tau_3^{-1} = 2\pi\rho n_d(\Delta/E)^2|V_d^-|^2/\hbar\,.$

We transform (13) to the form of the diffusion equation. Using the inequalities

$$p_F\ell/\hbar \gg 1,\ \hbar\omega \ll T$$

and expressing results in terms of a new function

$$\mathcal{F}(\varepsilon,\vec{q},\omega) \equiv \frac{F(\varepsilon,\omega,\vec{p},\vec{q}-\vec{p})}{\omega(1-i\tau\vec{q}\cdot\vec{v})} \quad (14)$$

we obtain the equation

$$(1+Dq^2\tau)\mathcal{F}(\varepsilon,q,\omega) = \frac{\Phi(\omega)}{4\pi\rho}$$
$$-T\int \frac{d\omega'}{(2\pi i)(\omega-2\omega'+i/2\tau)} \mathcal{F}(\varepsilon,q,\omega')\Phi(\omega-\omega')\,. \quad (15)$$

Here $D = v_F\ell/2$ is the diffusion constant, while $\Phi(\omega) \equiv \mathcal{D}(\omega)/\omega$

In the time representation

$$(1+Dq^2\tau)\mathcal{F}(\varepsilon,q,t) = \frac{\Phi(\varepsilon,t)}{2\tau T\rho} + \int_{-\infty}^{t} \frac{dt'}{\tau} e^{(t'-t)/\tau} \mathcal{F}(\varepsilon,q,t')\Phi(\varepsilon,2t-t')\,,$$

$$\Phi(\varepsilon,t) \equiv \frac{\tau}{\tau_e} + \frac{\tau}{\tau_1}\cos\frac{Et}{\hbar} + \frac{\tau}{\tau_3} e^{-2\gamma t/\hbar}\,.$$

Exact solution:

$$\mathcal{F}(0,q) = \int_{-\infty}^{0} \frac{dt'}{\tau}\,\Phi(t')\,e^{\vartheta(t')}\,,$$

$$\vartheta(t) = Dq^2 t + \left[\frac{t}{\tau_1} - \frac{\sin(Et/\hbar)}{E\tau_1/\hbar}\right] + \left[\frac{t}{\tau_3} - \frac{\hbar}{2\gamma\tau_3}(e^{2\gamma t/\hbar}-1)\right],\ t<0\,.$$

The final result can be formulated as

$$\delta\sigma = -\frac{e^2}{2\pi^2\hbar}\ln\frac{\tau_\varphi}{\tau}$$

where τ_φ is defined according to the equation

$$\ln\frac{\tau_\varphi}{\tau} \equiv \int_1^\infty \frac{d\eta}{\eta} e^{-\Gamma_1(\eta,E,\Lambda)-\Gamma_3(\eta,E,\Lambda)}\,, \qquad (16)$$

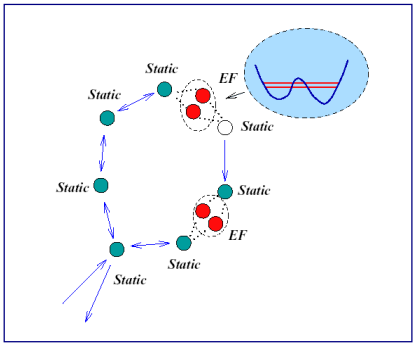
$$\Gamma_1(\eta,E,\Lambda) = \frac{\tau}{\tau_1}\left[\eta - \frac{\sin(\eta\,E\tau/\hbar)}{E\tau/\hbar}\right],$$

$$\Gamma_3(\eta,E,\Lambda) = \frac{\tau}{\tau_3}\left[\eta - \frac{\hbar}{2\gamma\tau}(1-e^{-2\eta\gamma\tau/\hbar})\right],$$

where $\eta = t/\tau$. This equation is obtained by the integration over q.

Appendix: Model of Random Telegraph Noise

Consider N dynamic defects, or *elementary fluctuators*, EFs, on the trajectory with the total traversal time t_0 having M scatterers.



length of the trajectory

$$\mathcal{L}_0 = \tau / v_F = \sum_{s=1}^{M} \left| \boldsymbol{R}_{s+1}^{(0)} - \boldsymbol{R}_s^{(0)} \right| .$$

$$\boldsymbol{R}_j(t) = \boldsymbol{R}_j^{(0)} + \boldsymbol{u}_j(t) .$$

Random telegraph process

$$u_j(t) \equiv a_j \, \xi_j(t) , \quad \xi_j(t) = \pm 1 , \quad \langle \xi_j(t) \xi_{j'}(t') \rangle = \delta_{jj'} \, e^{-2\Gamma_j |t-t'|} .$$

Then the time-dependent contribution to the length is

$$(\delta \mathcal{L})_j(t) = l_j \, \xi_j(t) , \quad l_j \equiv (\boldsymbol{v}_j \cdot \boldsymbol{a}_j)/v .$$

For a given defect j, the phase difference is just

$$(\delta \Phi)_j(t_0) = (p_F l_j / \hbar) \left[\xi(t_j) - \xi(t_0 - t_j) \right] .$$

Average over the telegraph process

$$k(t_0) = \left\langle e^{iJ\,[\xi(t)-\xi(t_0-t)]} \right\rangle_{\rm RTP}, \quad J \equiv p_F l/\hbar\,.$$

$$k(t,t_0) = 2\cos^2 J + 2\sin^2 J\, e^{-2\Gamma|t_0-2t|}\,.$$

Average over different EFs

Holtzmark procedure, $\left\langle e^{i(\Delta\varphi)} \right\rangle_{\rm EF} = e^{-W(t,t_0)}$

$$W(t,t_0) = \langle 1 - k(t,t_0)\rangle_{\rm EF} = 2\left\langle \sin^2 J \left[1 - e^{-2\Gamma|t_0-2t|}\right]\right\rangle_{\rm EF}\,.$$

number of active EFs → $\mathcal{P}_0\, E_{\rm eff}$ times the "contact volume" $\sigma v_F t_0$

$E_{\rm eff} = \min E^*, T$ σ is the scattering cross section.

average over the positions of the defects along the trajectories.

$$\mathcal{F} \equiv \frac{2}{t_0}\int_0^{t_0/2} dt\, \left(1 - e^{-2\Gamma(t_0-2t)}\right) = 1 - \frac{1-\exp(-2\Gamma t_0)}{2\Gamma t_0}\,. \qquad (19)$$

average over $\mathcal{P}(\Gamma)$

we get $W = (t_0/\tau_\varphi)^{3/2}$ with

$$\frac{1}{\tau_\varphi} \approx 10\,(P_0 \Lambda \sigma v_F)^{2/3} \left(\frac{p_F a}{\hbar}\right)^{4/3} \left(\frac{\chi T}{\hbar}\right)^{1/3} \propto T^{1/3}\,.$$