Dynamics of particles in integrable vortical flow: possible implications for cloud

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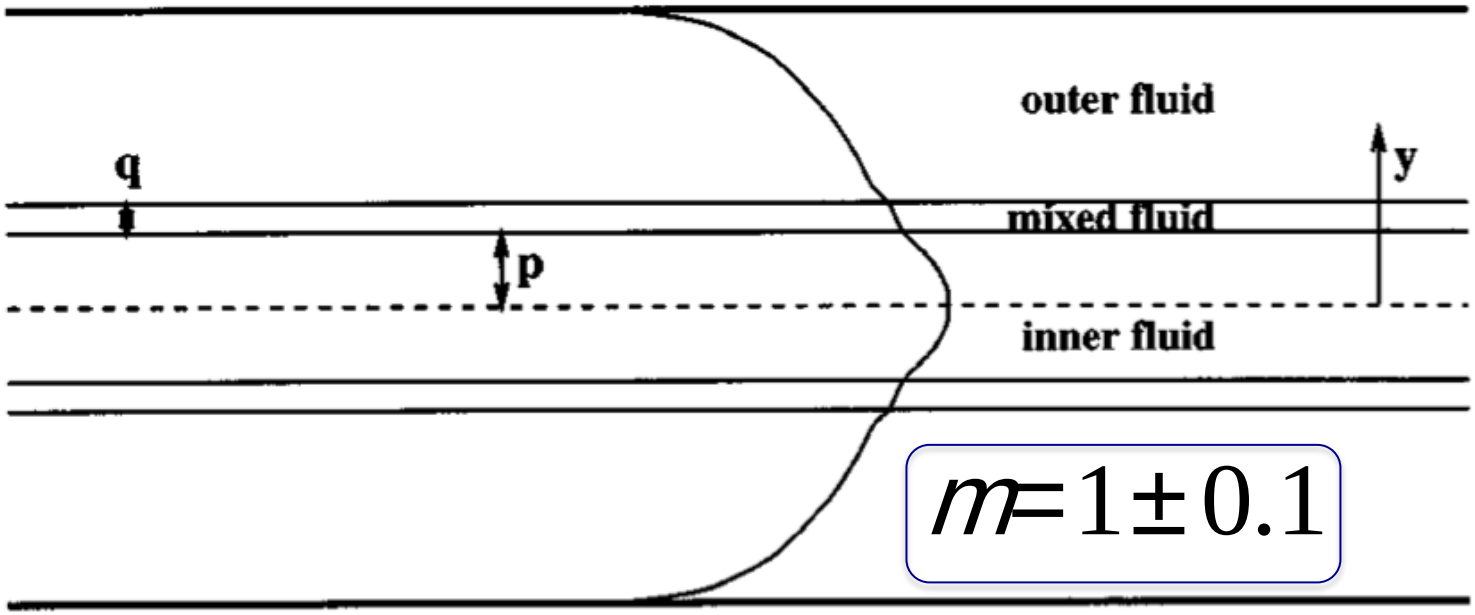
Viscosity-stratified channel flow

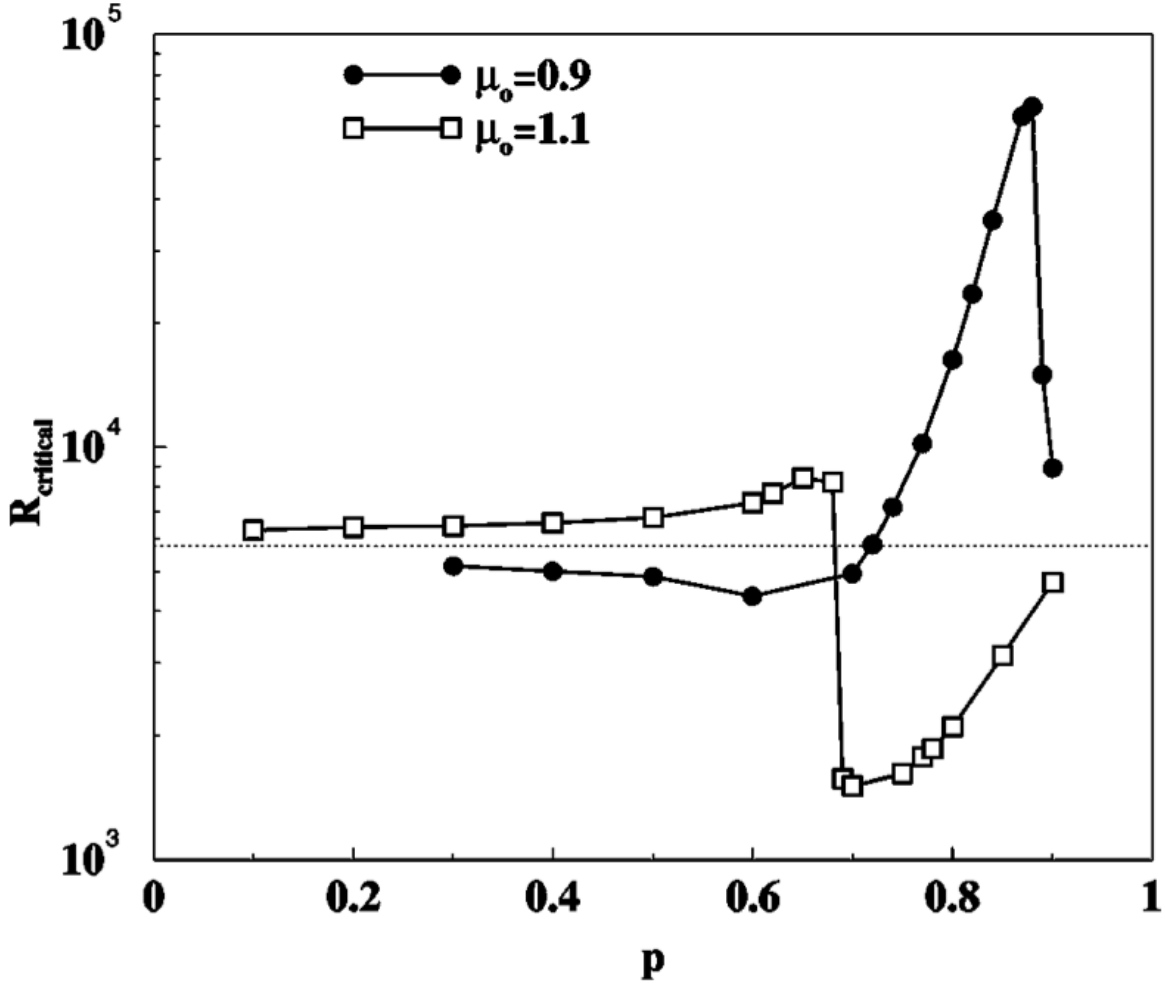


$m=1$

$m=1\pm0.1$

Why should a tiny change in a tiny viscous term matter??

Ranganathan & RG, Phys Fluids (Lett.) 2001



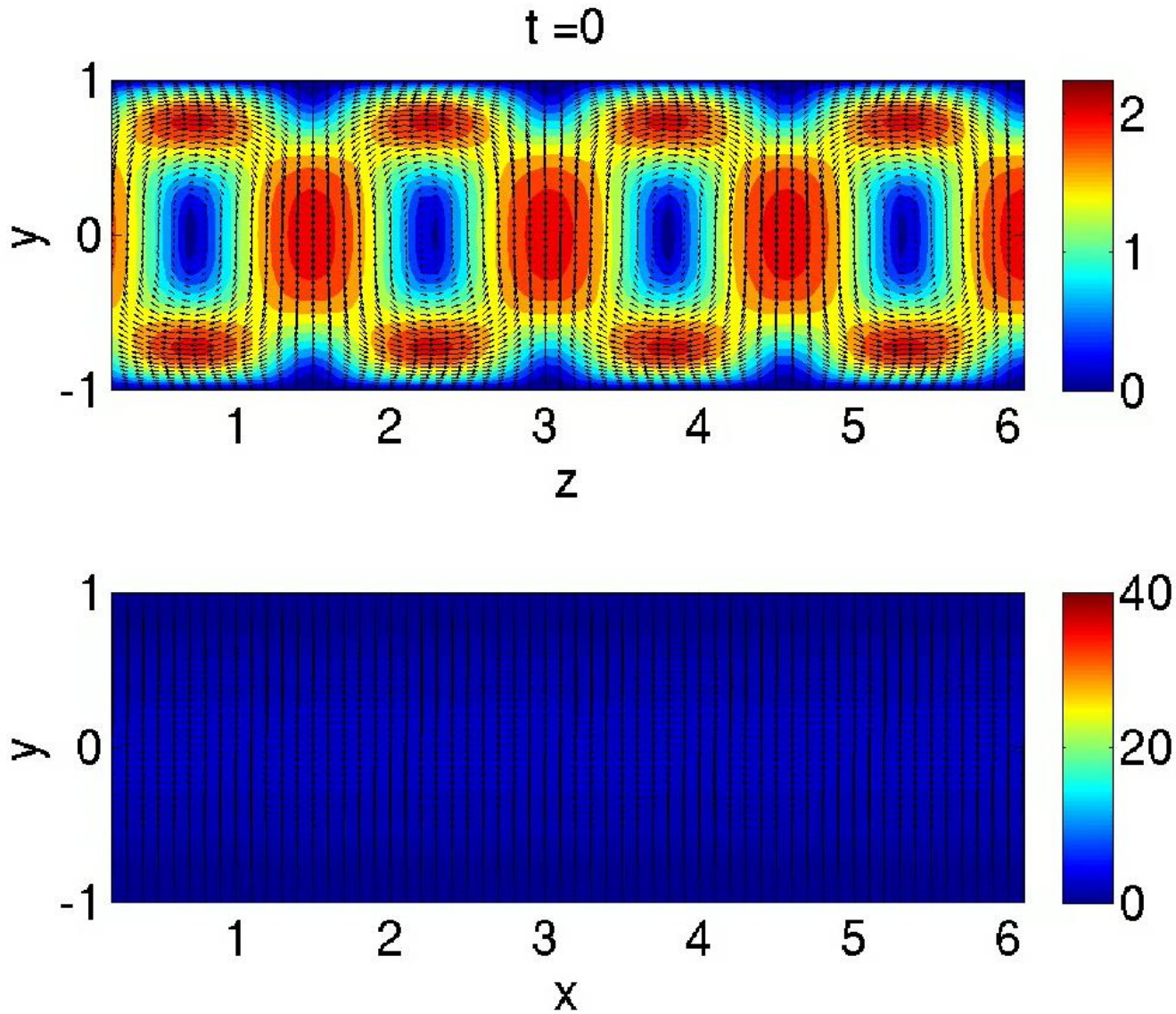
Ranganathan & RG, Phys Fluids 2001

A singular perturbation

Constant viscosity
Channel flow

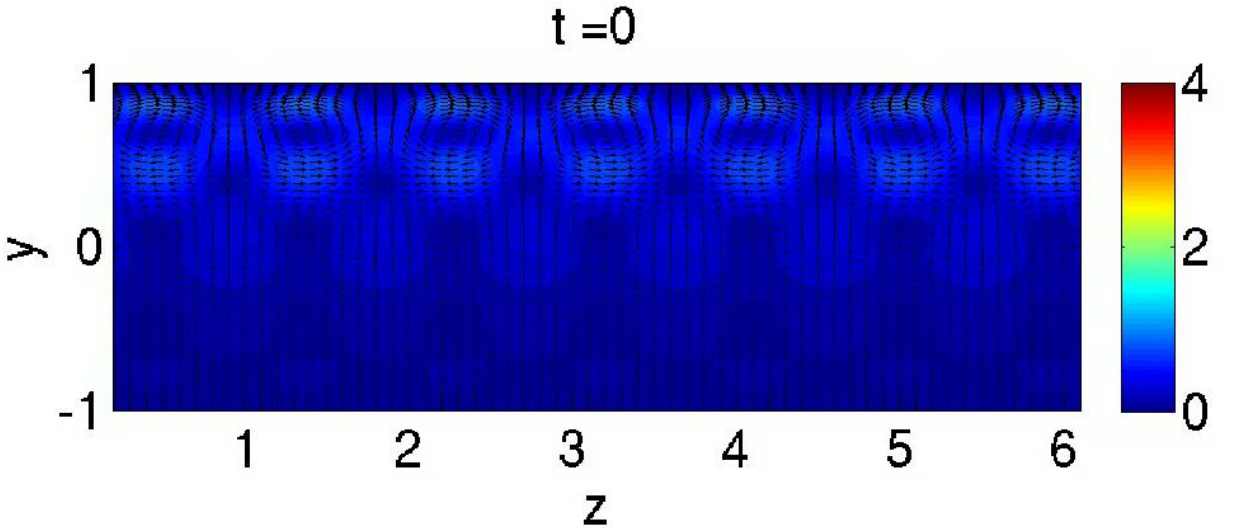
Density is stably
Stratified Ri=0.1
Re=1000

m=1

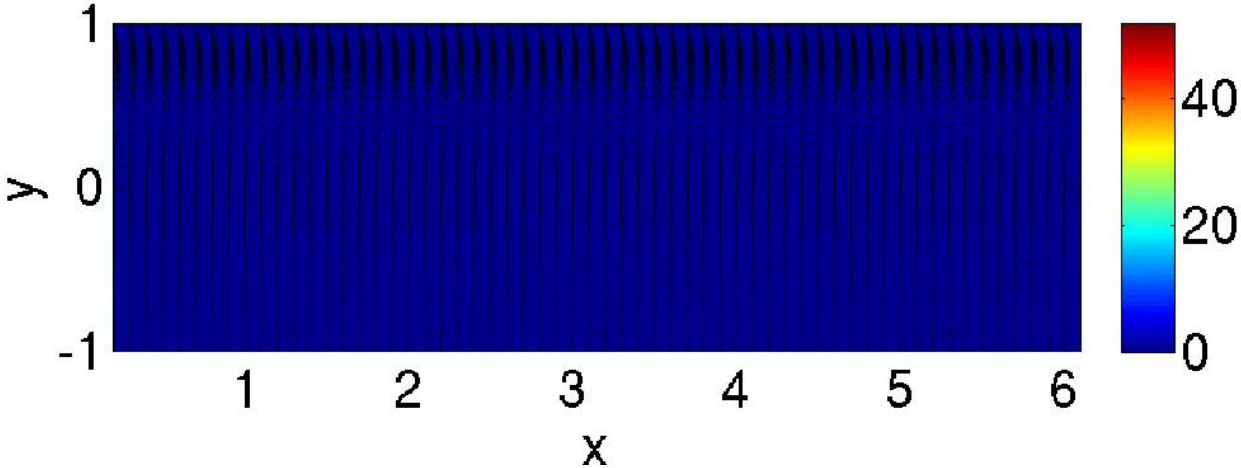


Result of Sharath Jose, with Luca Brandt & RG
See also RG & Sahu, Annual Review of Fluid Mechanics, 2014

Viscosity stratified

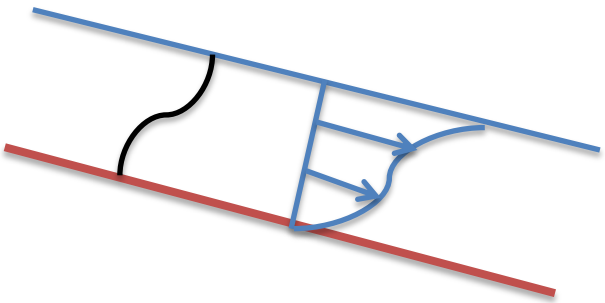


m=1.1



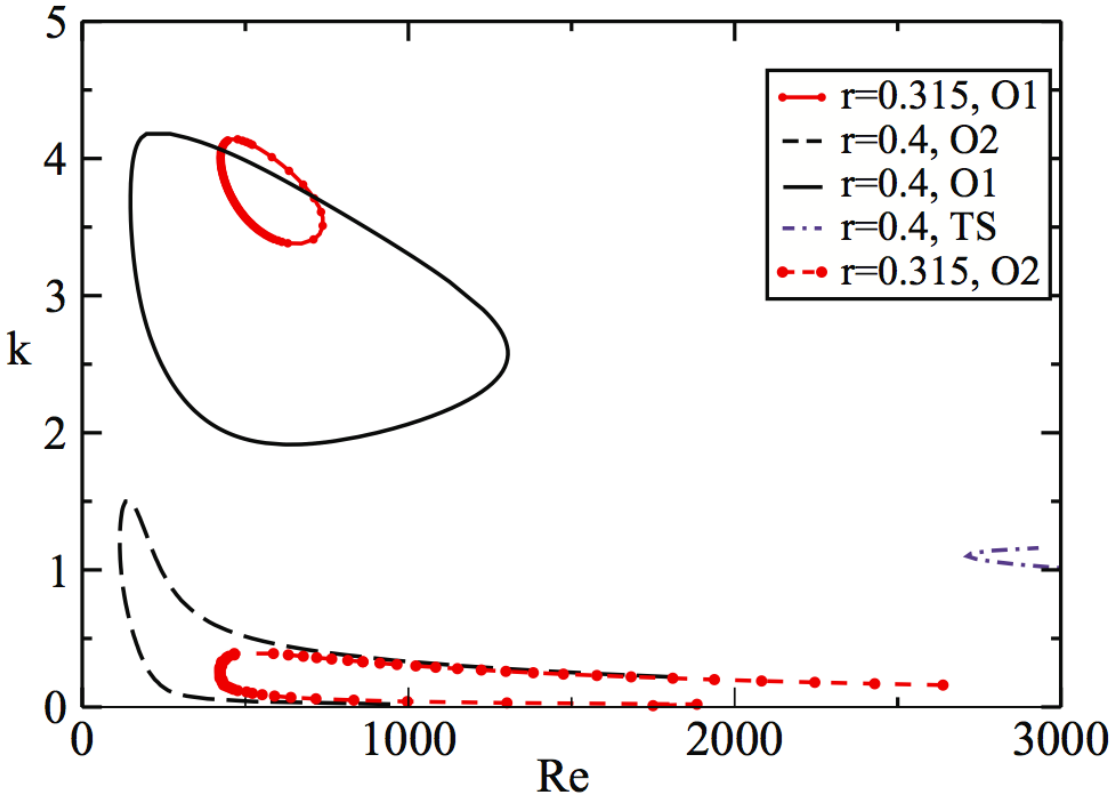
Viscosity stratification:

Now destabilizes either way



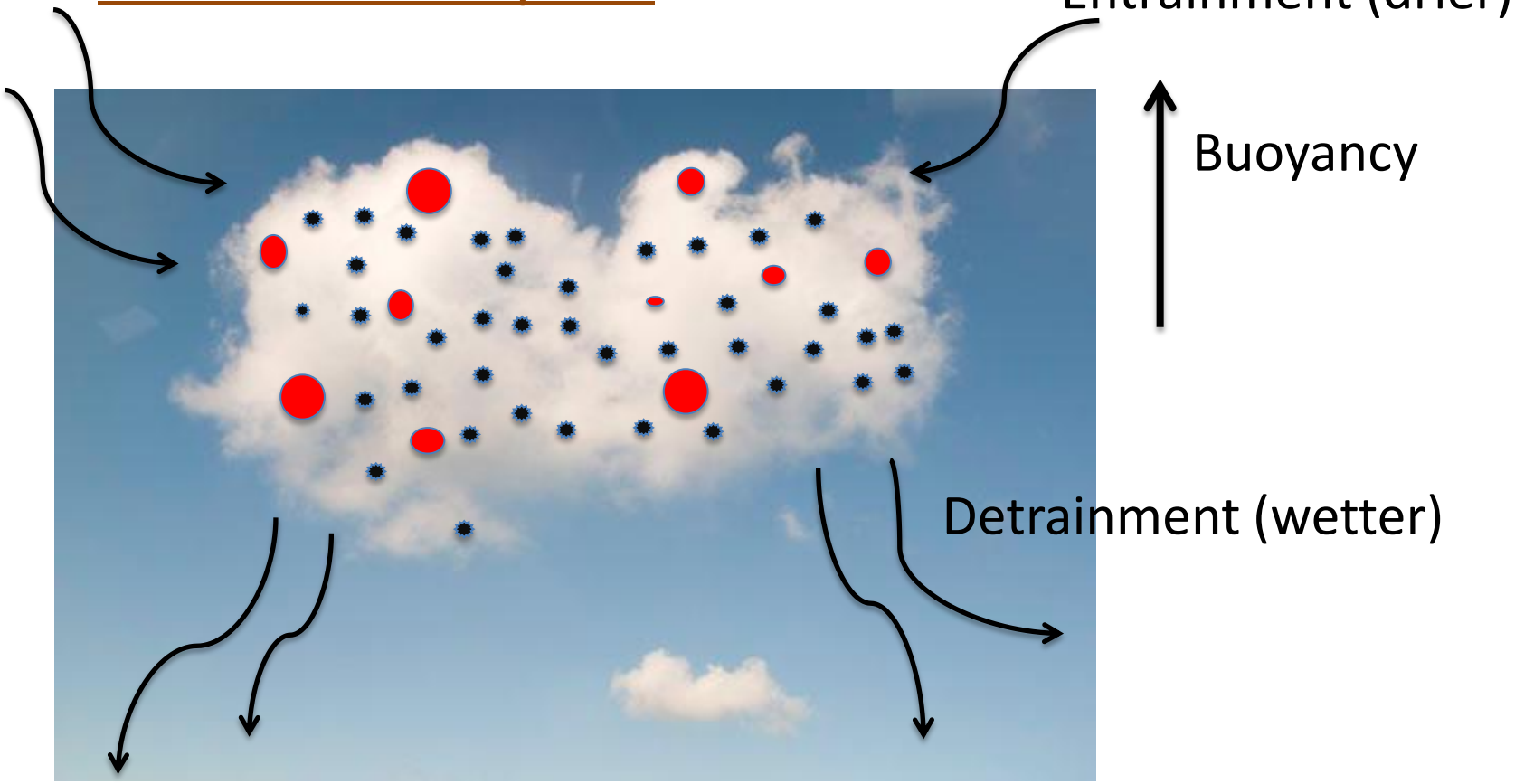
More instability = more mixing?

m=1.5



Usha, Tammisola & RG, Phys Fluids 2013

A rain-cloud hopeful



frangardino.com

Particles:

Water vapor: particles leave, condense

Aerosol: some concentration above Kohler critical radius

Water droplets: nucleate, grow, leave, cluster, maybe glaciate ……

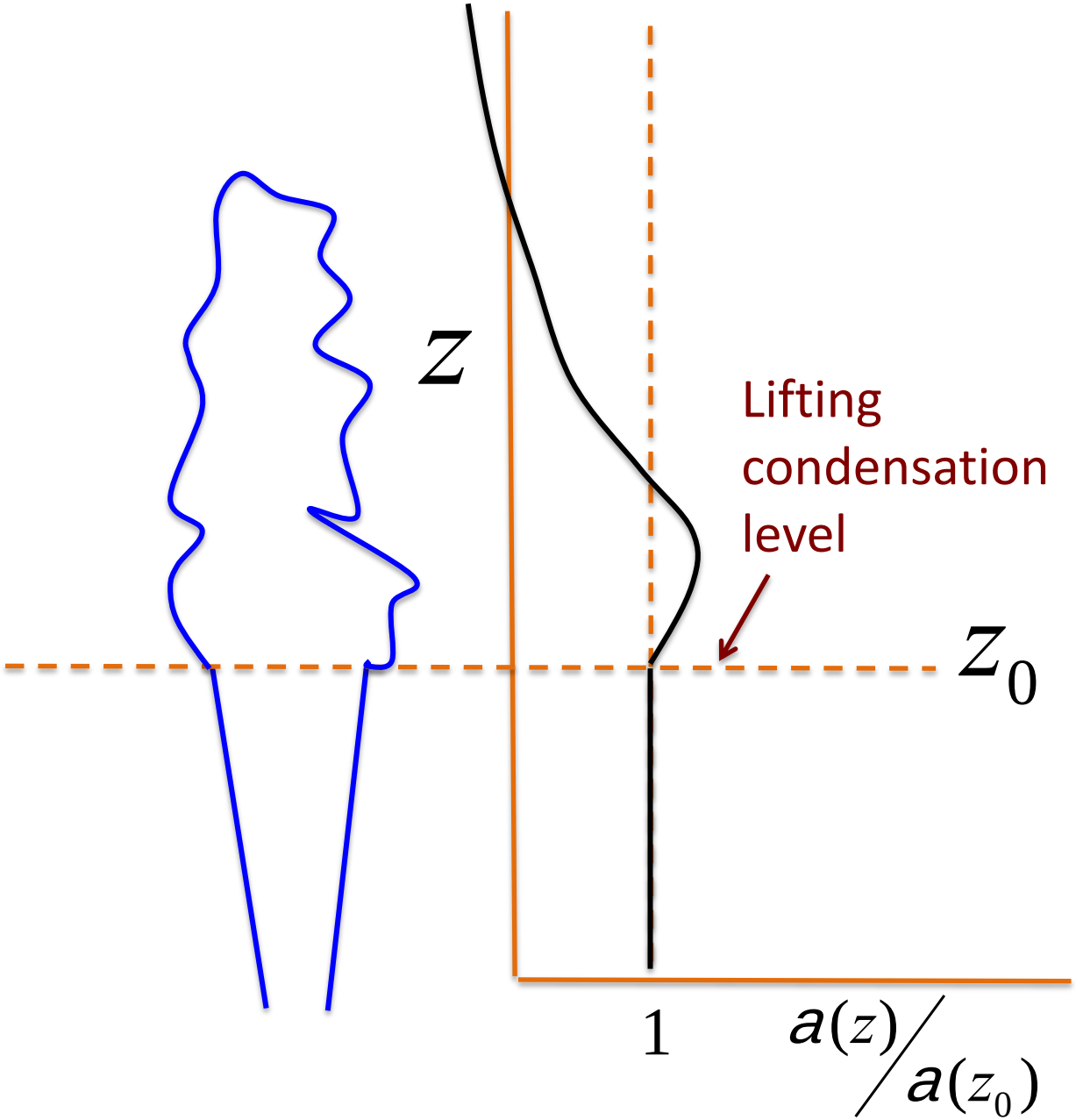
Yet another hot turbulent shear flow?

http://www.colourbox.com/preview/3264534-405165-the-smoking-chimneys-of-a-factory-against-a-blue-sky-white-smoke-rises-from-chimneys-on.jpg

But entrainment kills

Entrainment

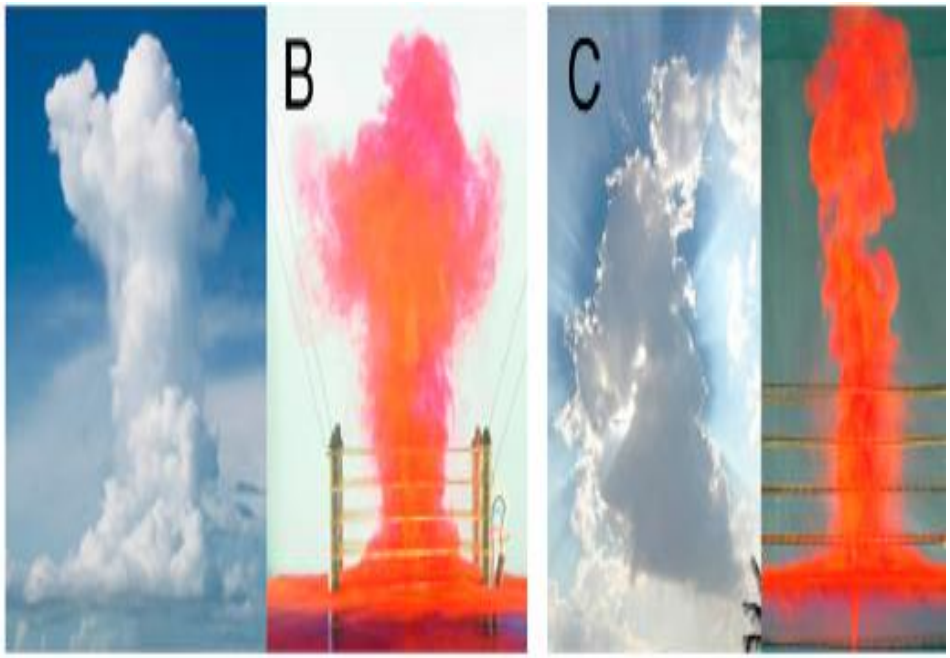
Mangalyaan



Ref: R Narasimha

Real clouds, lab 'clouds'

Experiments of Narasimha group, see e.g. PNAS 2011



Microphysics:

Phase change -
Deliquescence,
Nucleation, Condensation,
Collisions and Coalescence.

Aerosols, CCN, Droplets

Latent Heat release:

Volumetric heat source,
Saturation pressure p(T),
Stable and unstable ambient stratification

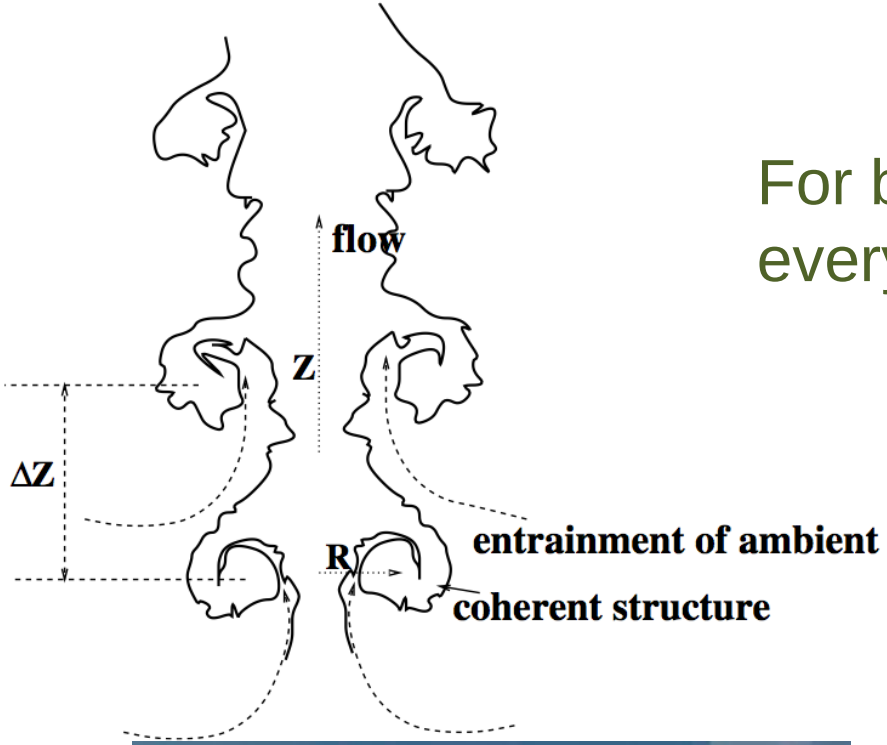
Entrainment cut-off:

Role of buoyancy and body forces,
'Similarity Drift' of shapes and widths,
(from Kaminski, Tait, Carazzo)

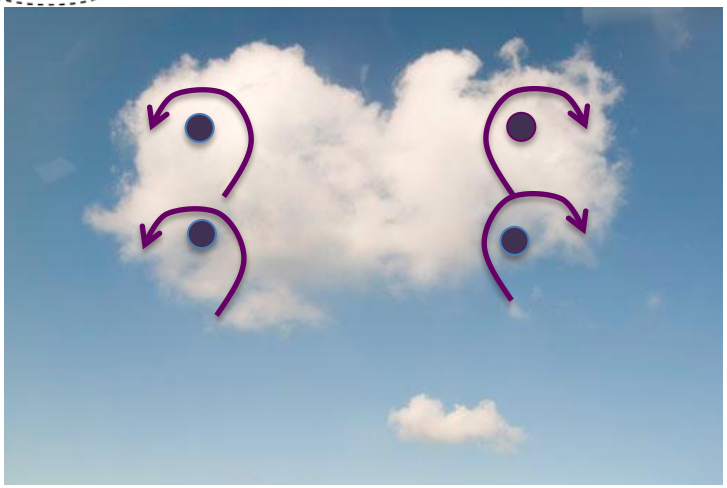
Large scale fields

Of velocity and temperature,
Vorticity in the shear layer

A toy flow: point vortices and particles



For big scales:
everybody is a tracer



<u>Point vortices:</u>

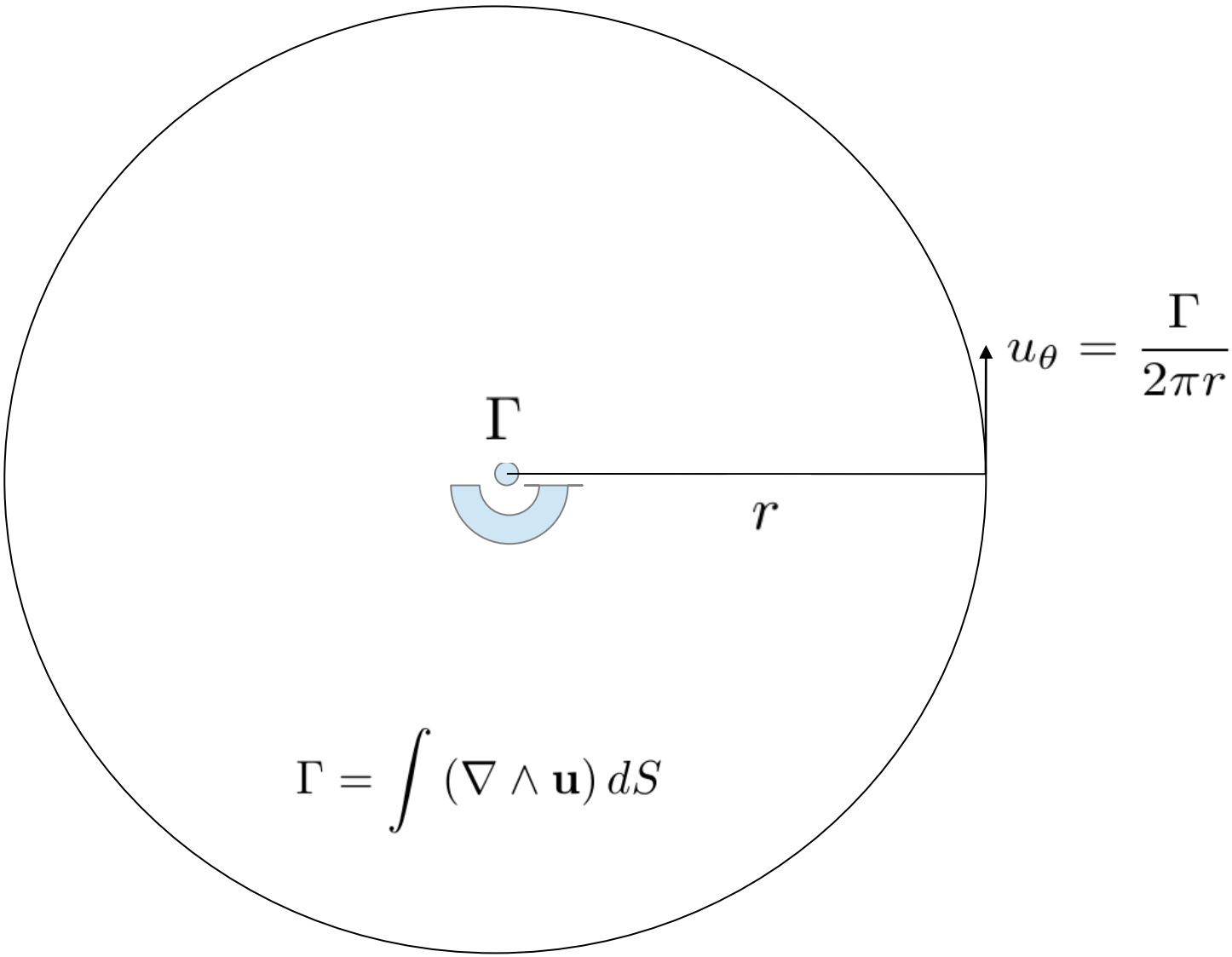
Two vortices

Four vortices

Two rows, periodic

Many vortices, random:
clubs of two and three

Point vortices



Inertial particles

$$\rho_p \dot{\mathbf{v}} = \rho_f \frac{D\mathbf{u}}{Dt} + (\rho_p - \rho_f)\,\mathbf{g} - \frac{9\nu\rho_f}{2a^2}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\Delta\mathbf{u}\right) - \frac{\rho_f}{2}\left[\dot{\mathbf{v}} - \frac{D}{Dt}\left(\mathbf{u} + \frac{a^2}{10}\Delta\mathbf{u}\right)\right]$$
$$-\frac{9\rho_f}{2a}\sqrt{\frac{\nu}{\pi}}\int_0^t \frac{ds}{\sqrt{t-s}}\left[\dot{\mathbf{v}}(s) - \frac{d}{ds}\left(\mathbf{u} + \frac{a^2}{6}\Delta u\right)_{\mathbf{x}=\mathbf{x}(s)}\right]$$

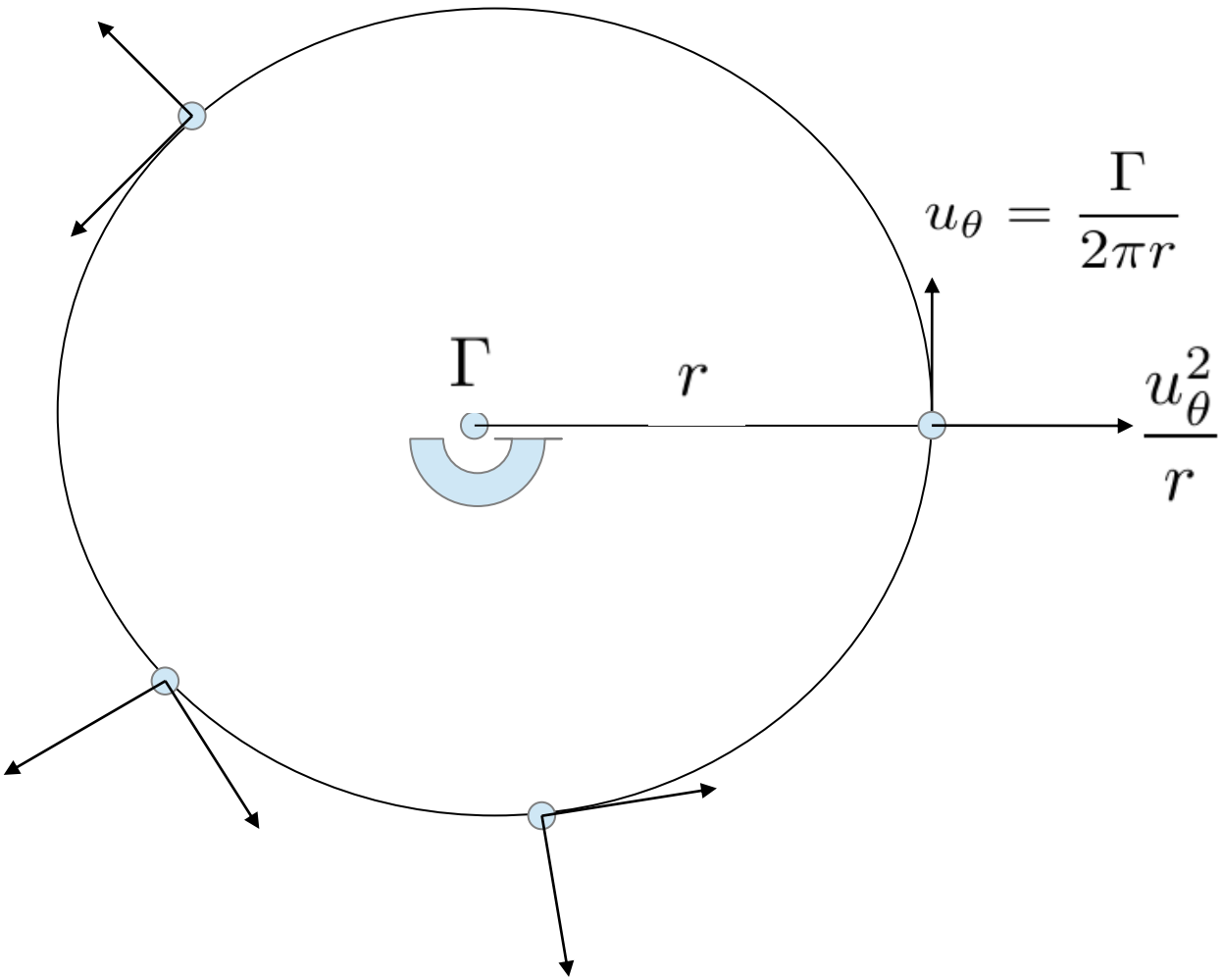
Maxey & Riley, 1983, Phys. Fluids

$$\boxed{\tau = \frac{2}{9}\frac{a^2}{\nu}\frac{\rho_p}{\rho_f}}$$

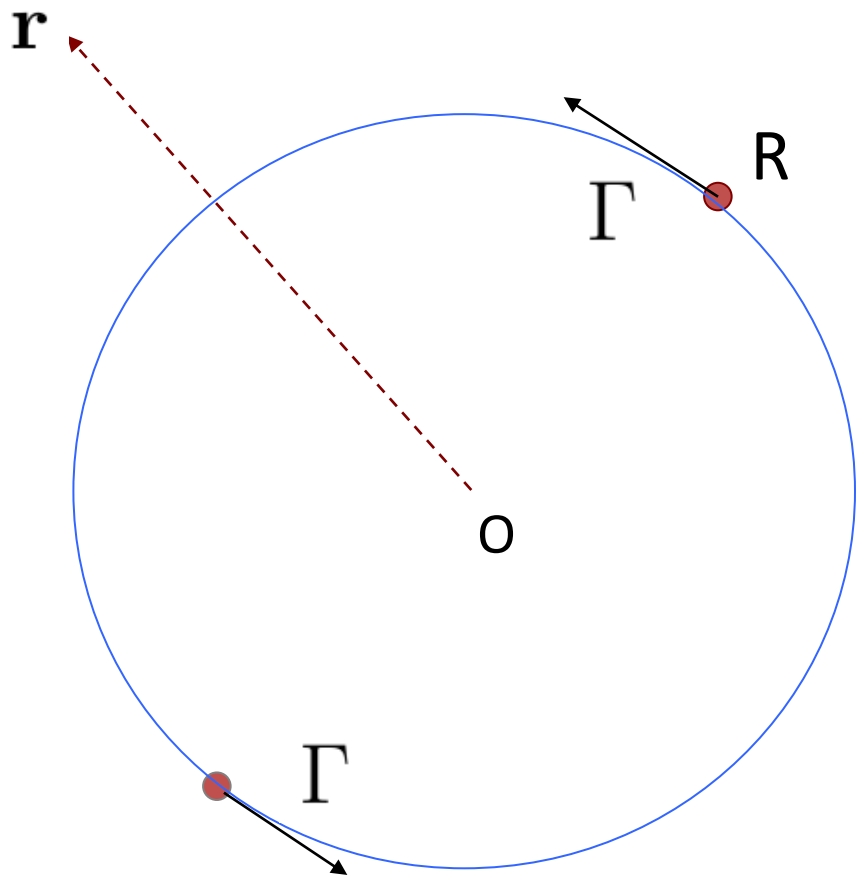
$$St = \frac{t}{T}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$
$$\frac{d\mathbf{v}}{dt} = -\frac{1}{St}(\mathbf{v} - \mathbf{u}) + \beta\frac{D\mathbf{u}}{Dt}$$

Heavy particles



Two vortices

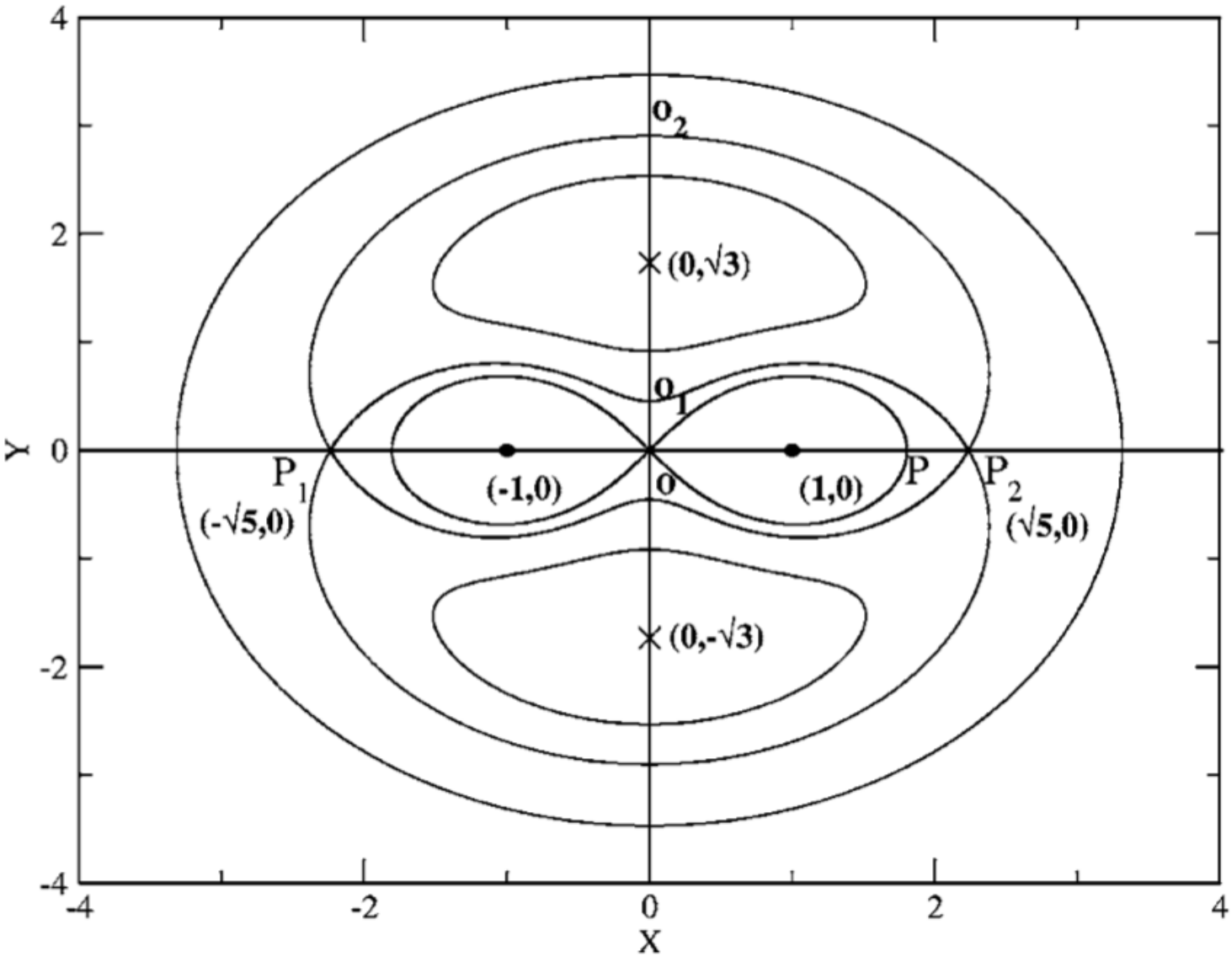


$$T = \frac{4\pi^2 R^2}{\Gamma}$$

$$St = \frac{\tau}{T}$$

$$\mathbf{u}_{\mathrm{lab}} = \frac{\Gamma}{2\pi}\mathbf{e}_z \times \left(\frac{\mathbf{r}-\mathbf{R}}{|\mathbf{r}-\mathbf{R}|^2} + \frac{\mathbf{r}+\mathbf{R}}{|\mathbf{r}+\mathbf{R}|^2} \right)$$

Two vortices: fixed points in a rotating frame



Inertial particle clustering

$$\mathbf{v} = \mathbf{u} - \tau \frac{D\mathbf{u}}{Dt} + \mathcal{O}\left(\tau^2\right)$$

$$\nabla \cdot \mathbf{v} = -\tau \nabla \cdot (\mathbf{u} \cdot \nabla \mathbf{u}) = -\tau \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} = \tau Q$$

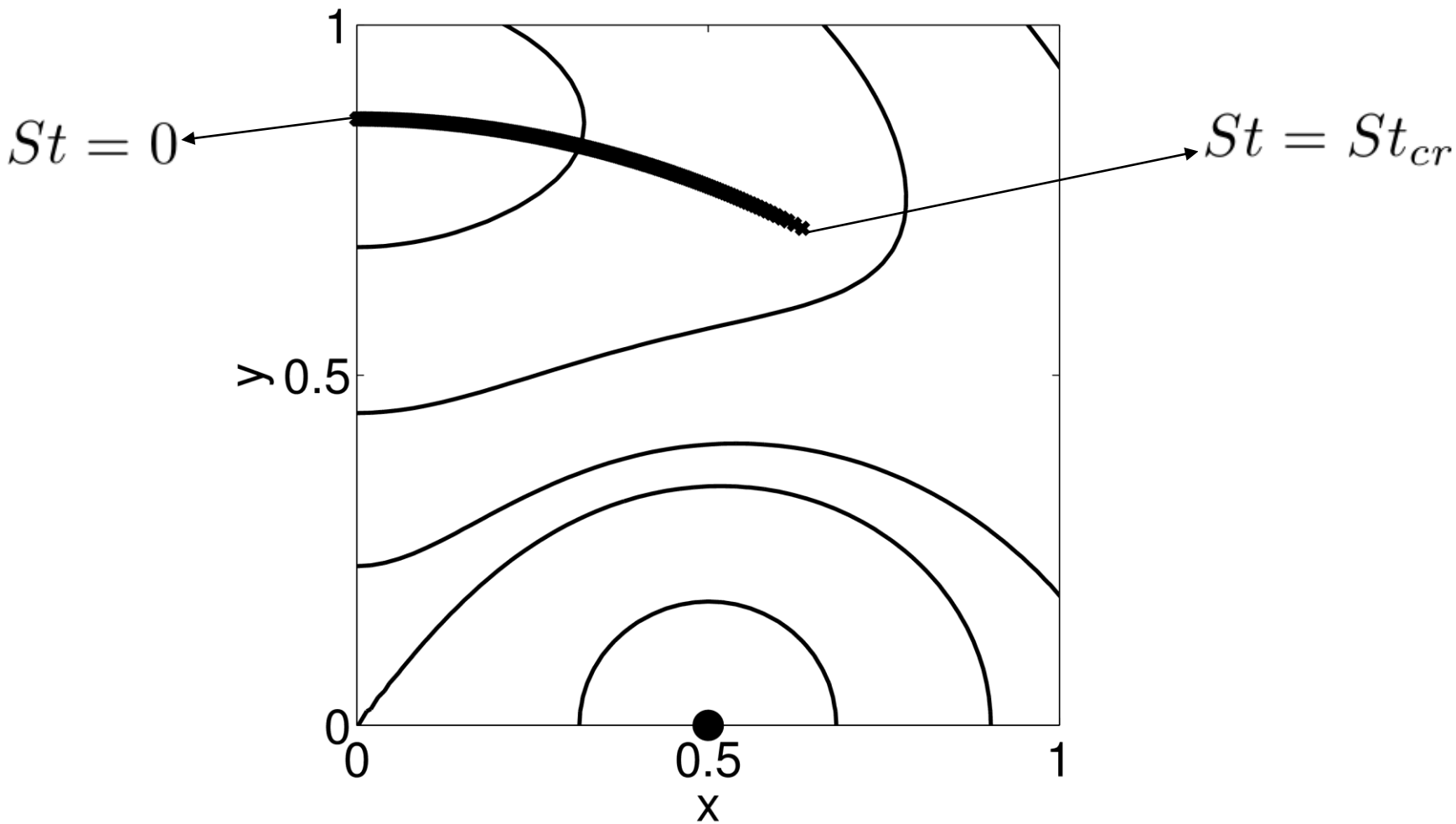
Okubo-Weiss $Q = \Omega^2 - S^2$

$Q > 0$ For an elliptic fixed point

Sapsis & Haller, 2010, Chaos

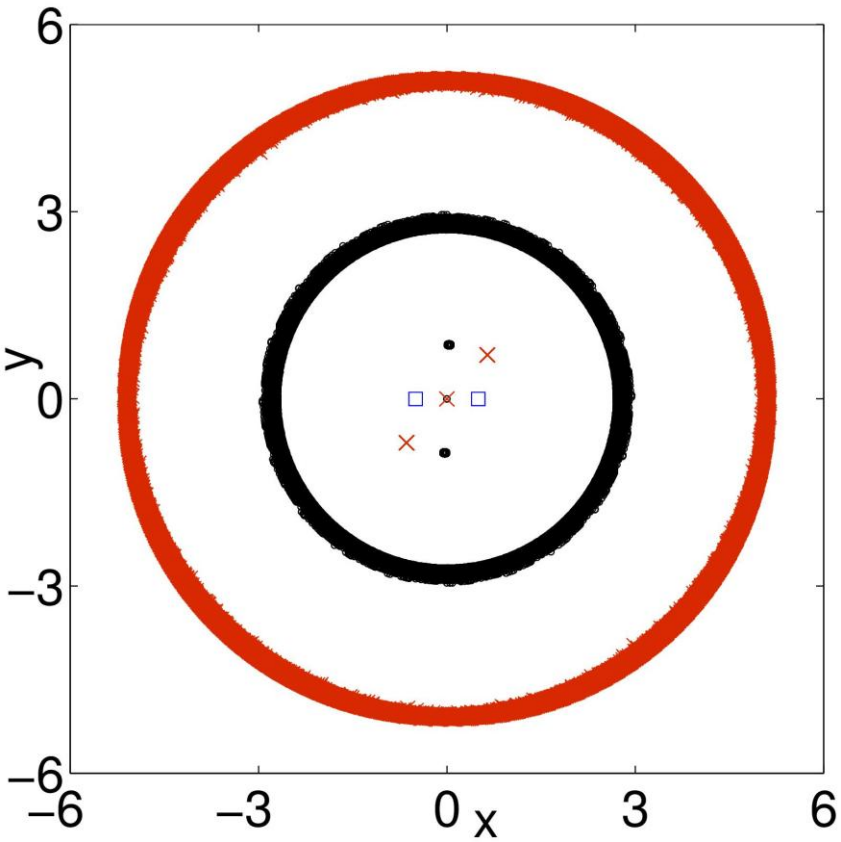
Fixed points

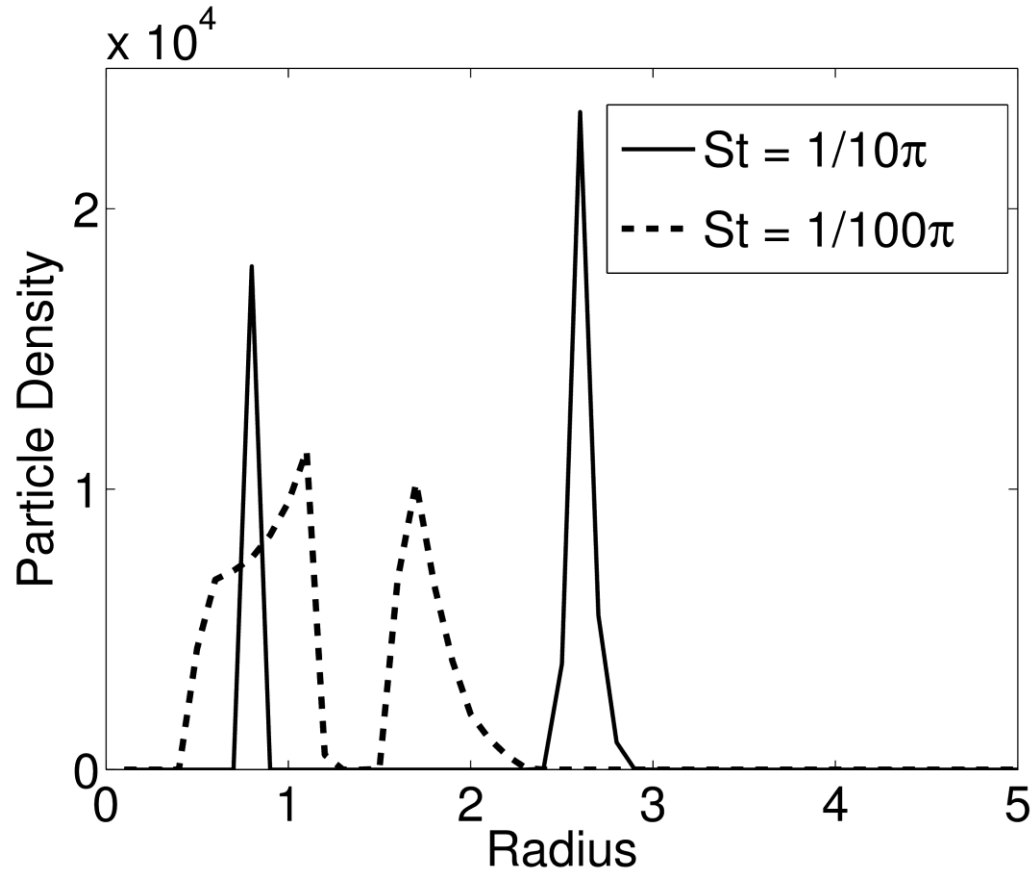
$$\mathbf{v}_{\mathrm{lab}} = \Omega \times \mathbf{r} \qquad \mathbf{a}_{\mathrm{lab}} = \frac{\mathbf{u}_{\mathrm{lab}} - \mathbf{v}_{\mathrm{lab}}}{St} = -\Omega^2 \mathbf{r}$$



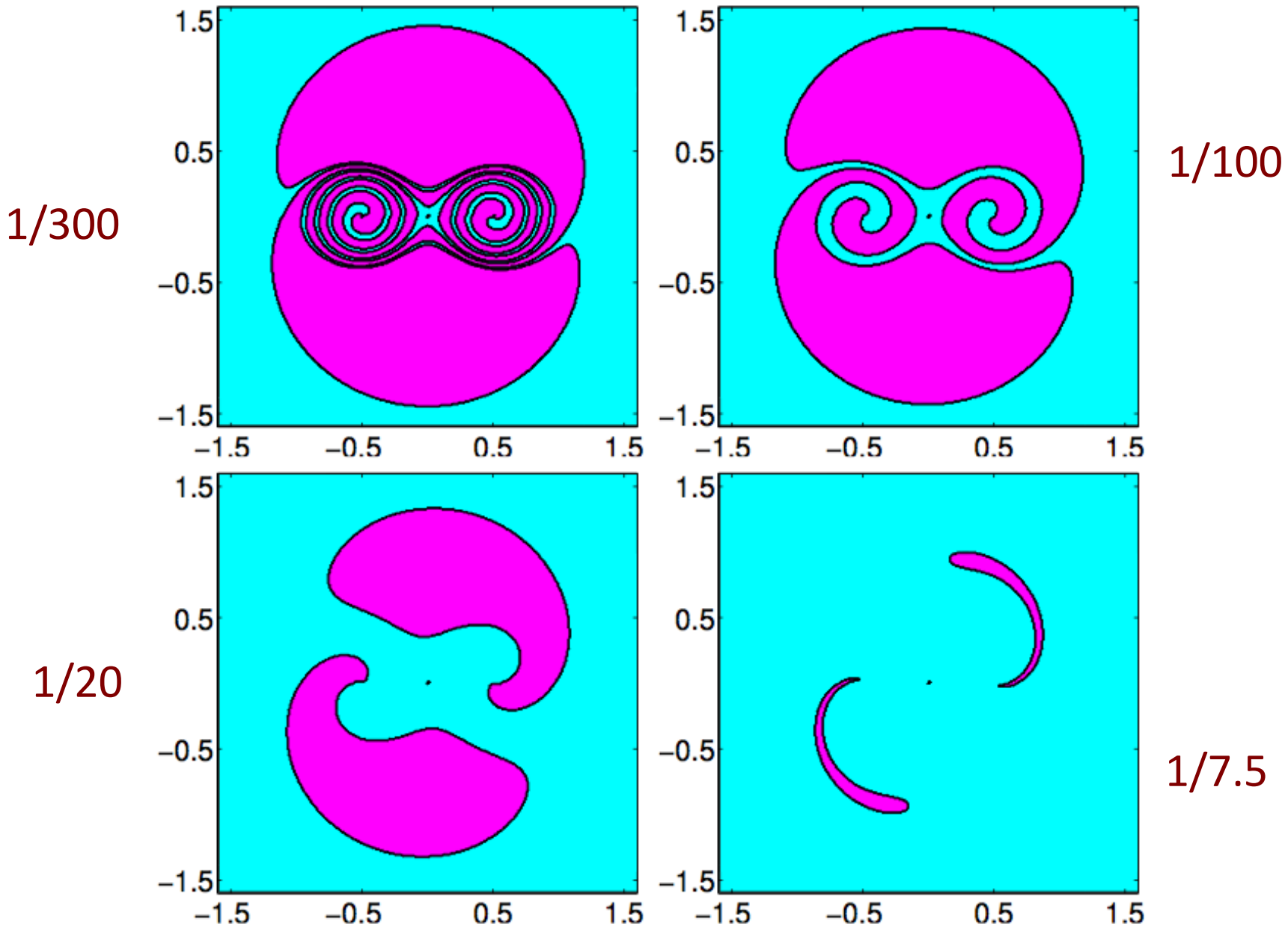
Ravichandran, Perlekar, RG, revision submitted

Fixed points

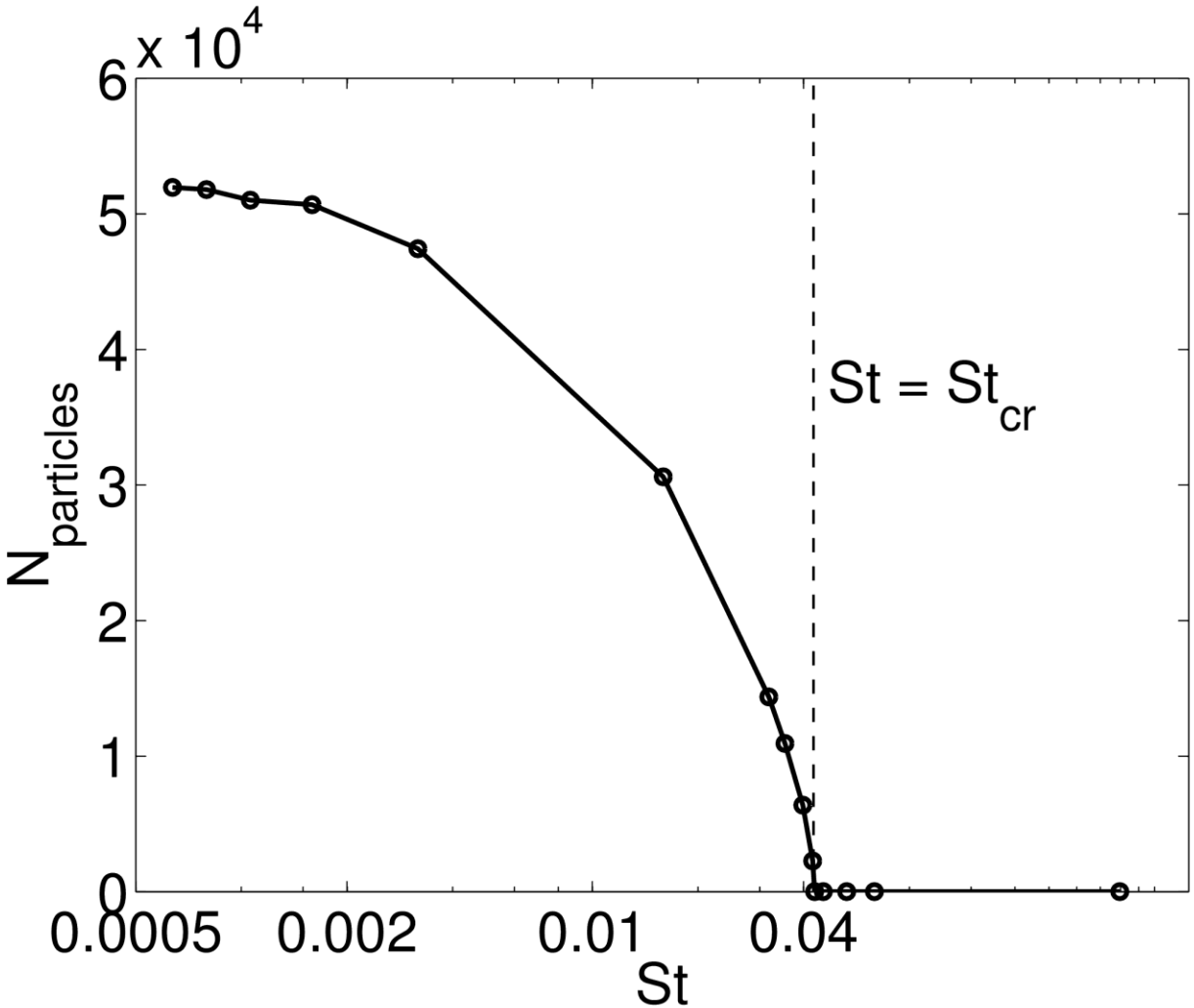




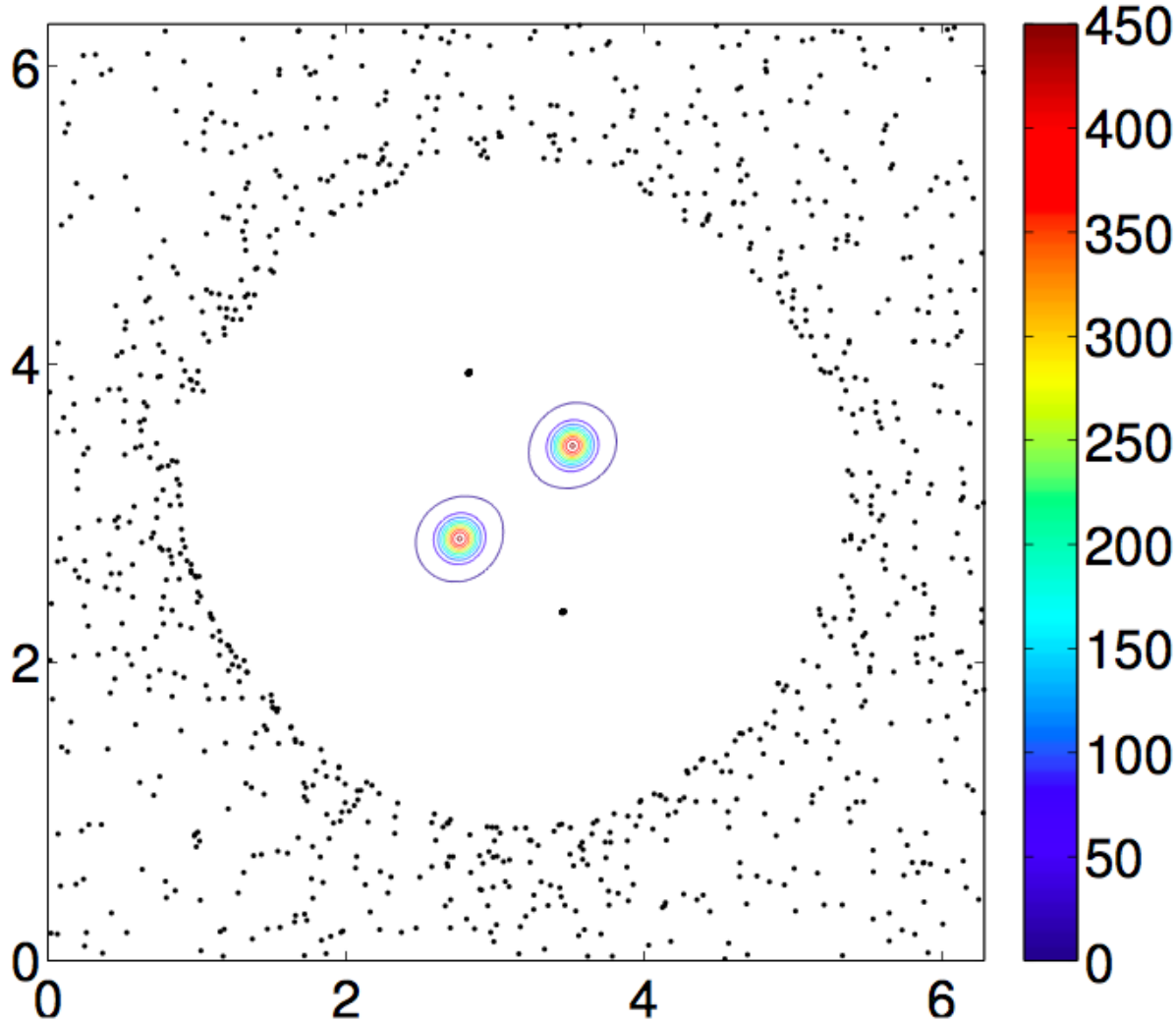
Basin boundaries



Basin area



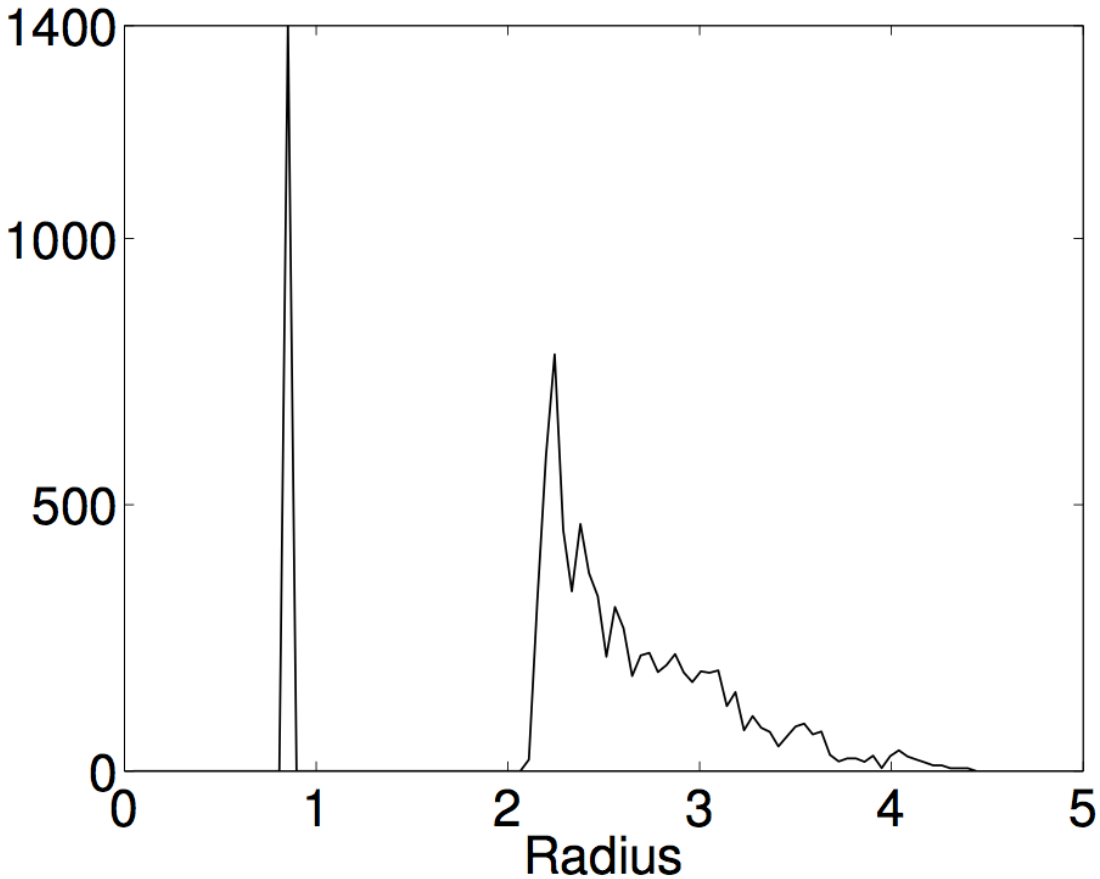
Viscous simulations



$St = 0.027$

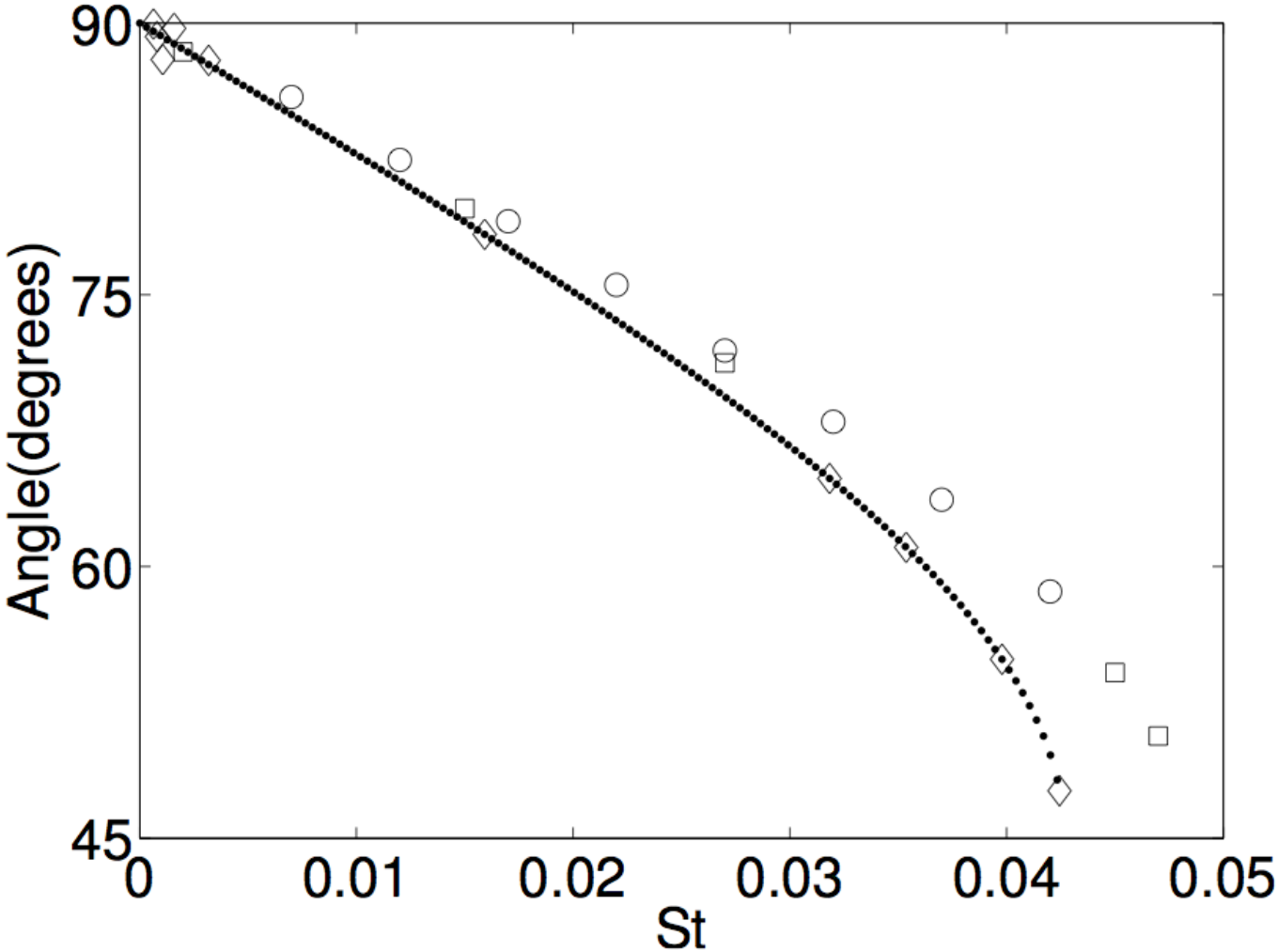
See also Angilella Physica D 2010

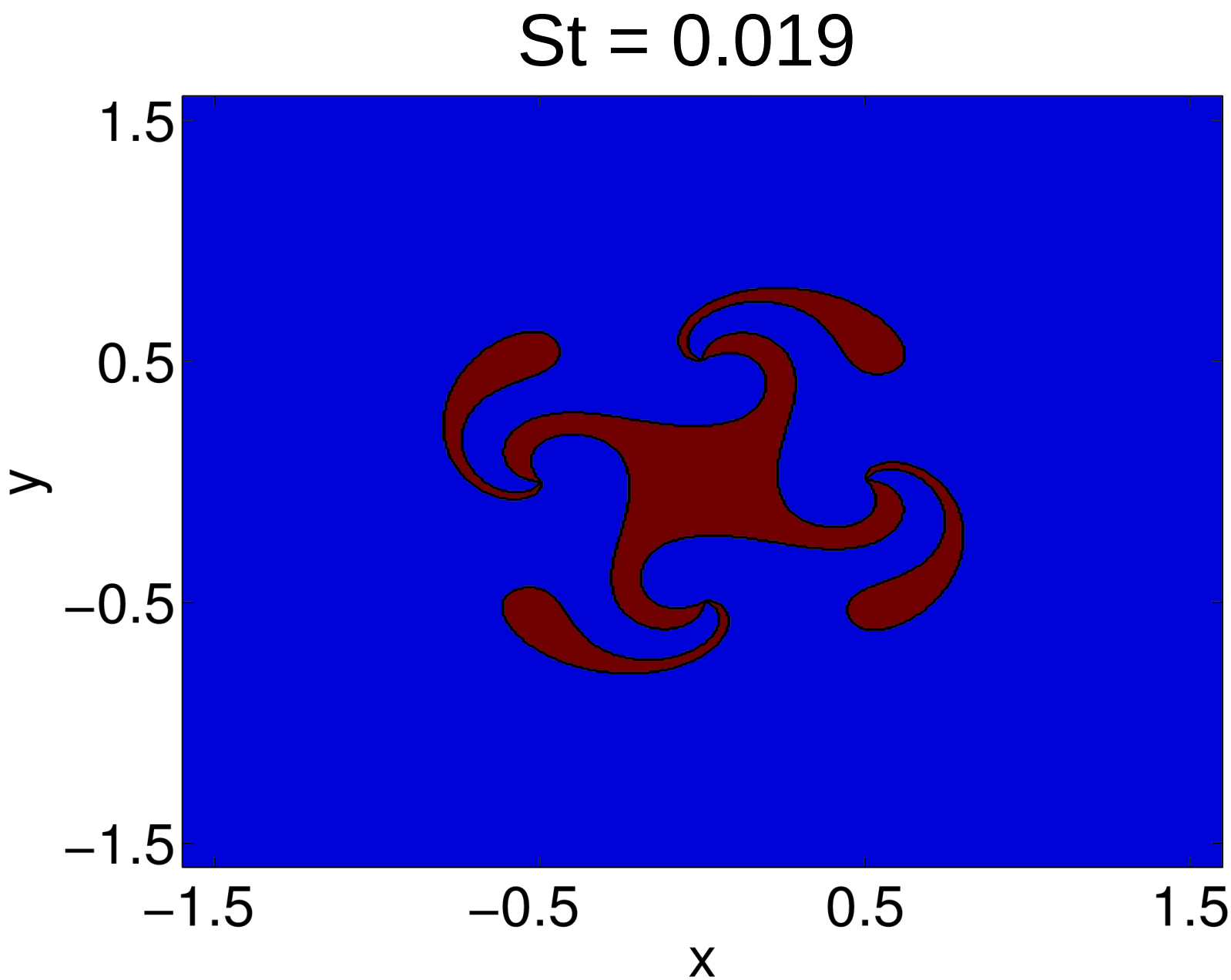
Particle density at a later time

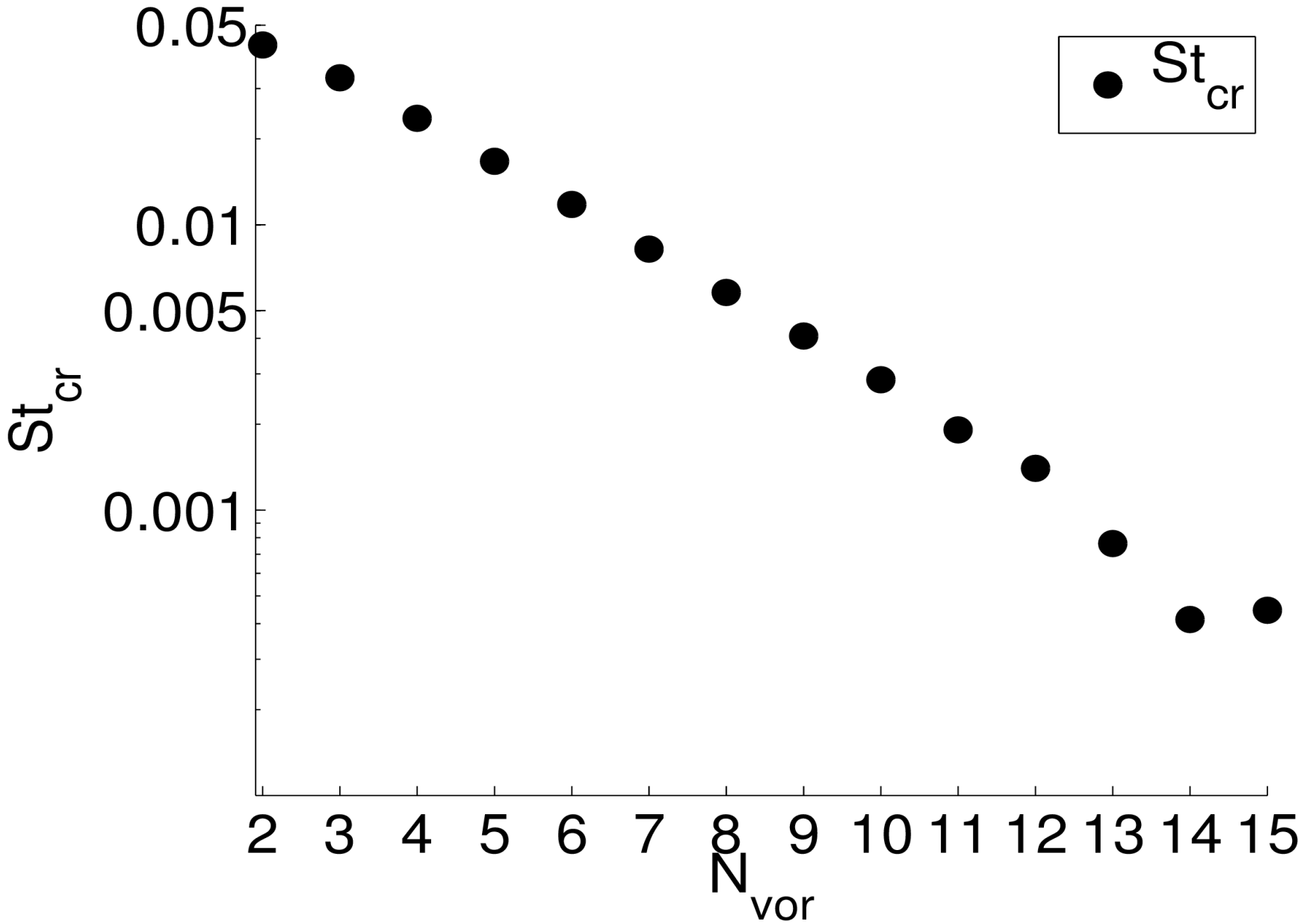


$St = 0.027$

Comparison: analytical, point vortex, DNS




St = 0.019
1.5
0.5
−0.5
−1.5
y
−1.5
−0.5
0.5
1.5
x


0.05
0.01
0.005
0.001
St_cr
2 3 4 5 6 7 8 9 10 11 12 13 14 15
N_vor
St_cr

Fixed points: linear stability

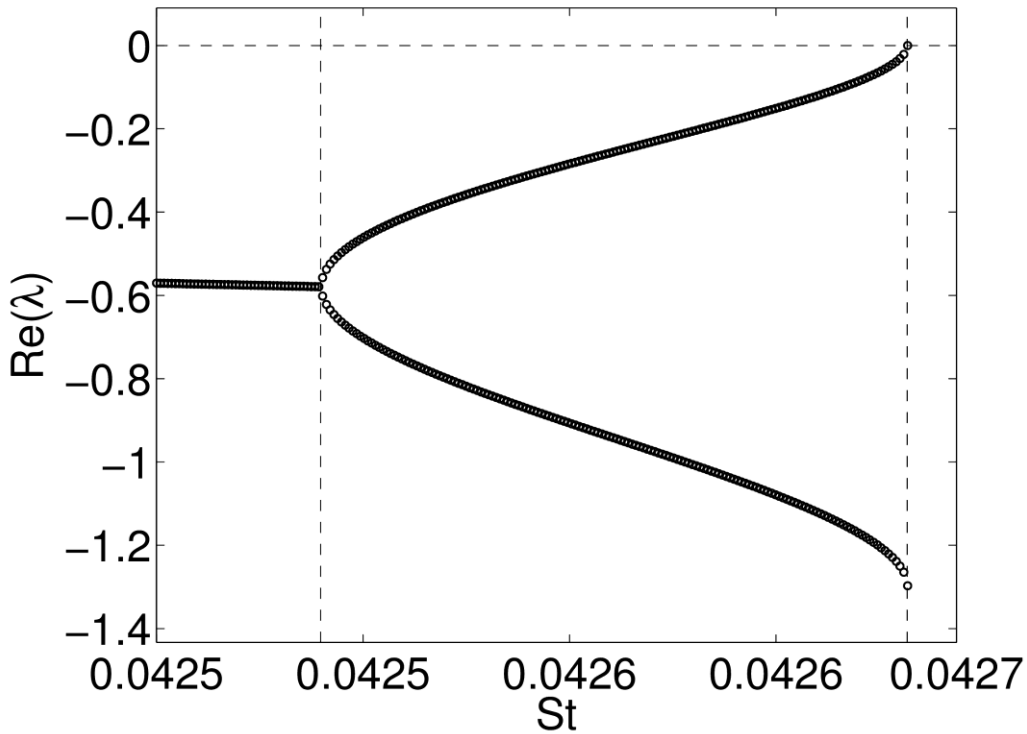
in the rotating frame

$$\hat{\mathbf{v}}_{fp} = 0 \text{ and } \hat{\mathbf{u}}_{fp} = -St(\Omega^2 \hat{\mathbf{r}})$$

$$\frac{d\left(\delta\hat{\mathbf{x}}\right)}{d\hat{t}} = \delta\hat{\mathbf{v}}$$

$$\frac{d\left(\delta\hat{\mathbf{v}}\right)}{d\hat{t}} = \frac{\delta\hat{\mathbf{u}} - \delta\hat{\mathbf{v}}}{St} - 2\mathbf{\Omega} \times \left(\delta\hat{\mathbf{v}}\right) + \Omega^2 \delta\hat{\mathbf{r}}$$

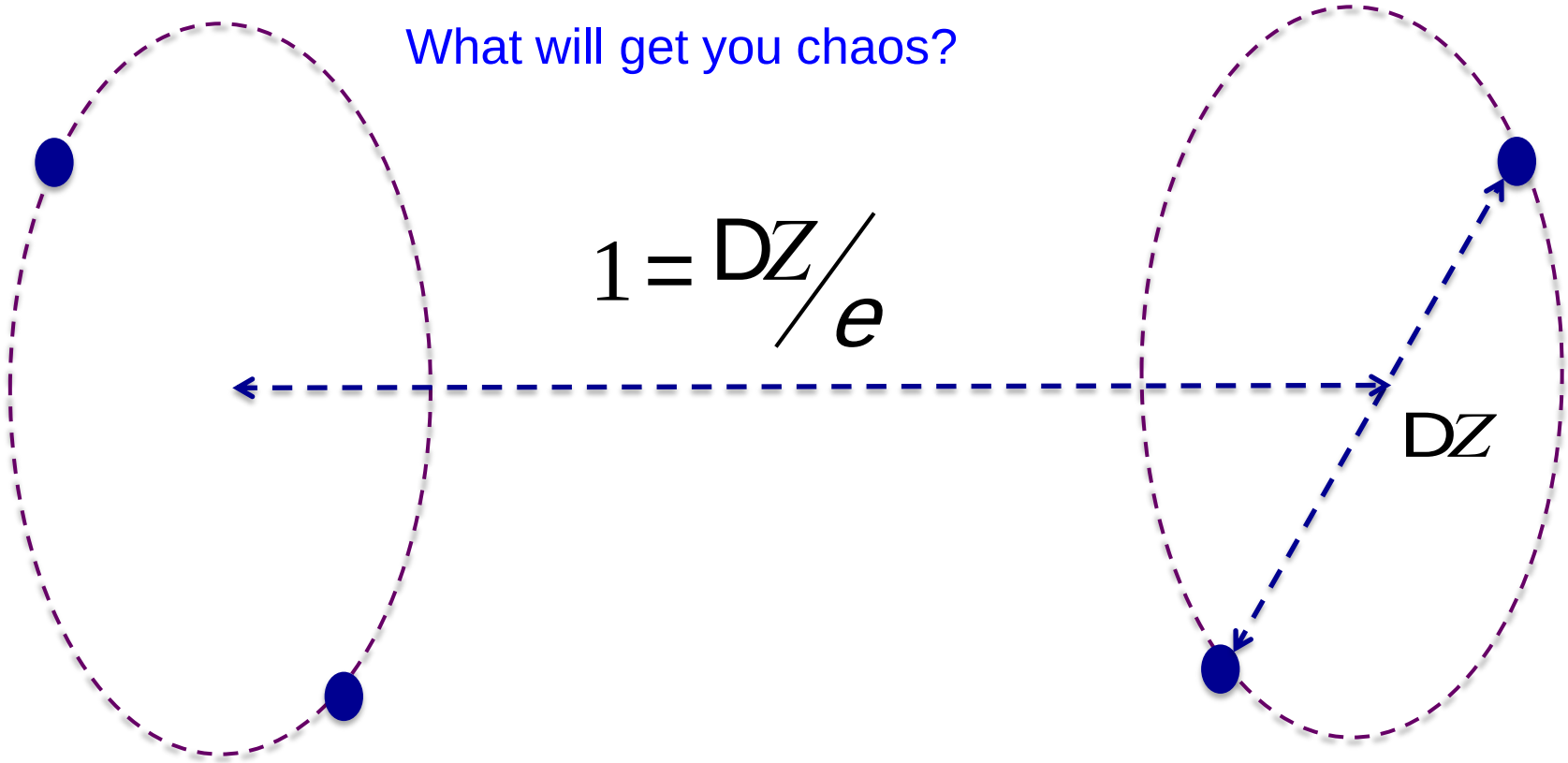
Fixed points: linear stability



Unforeseen clustering into a moving “fixed” point

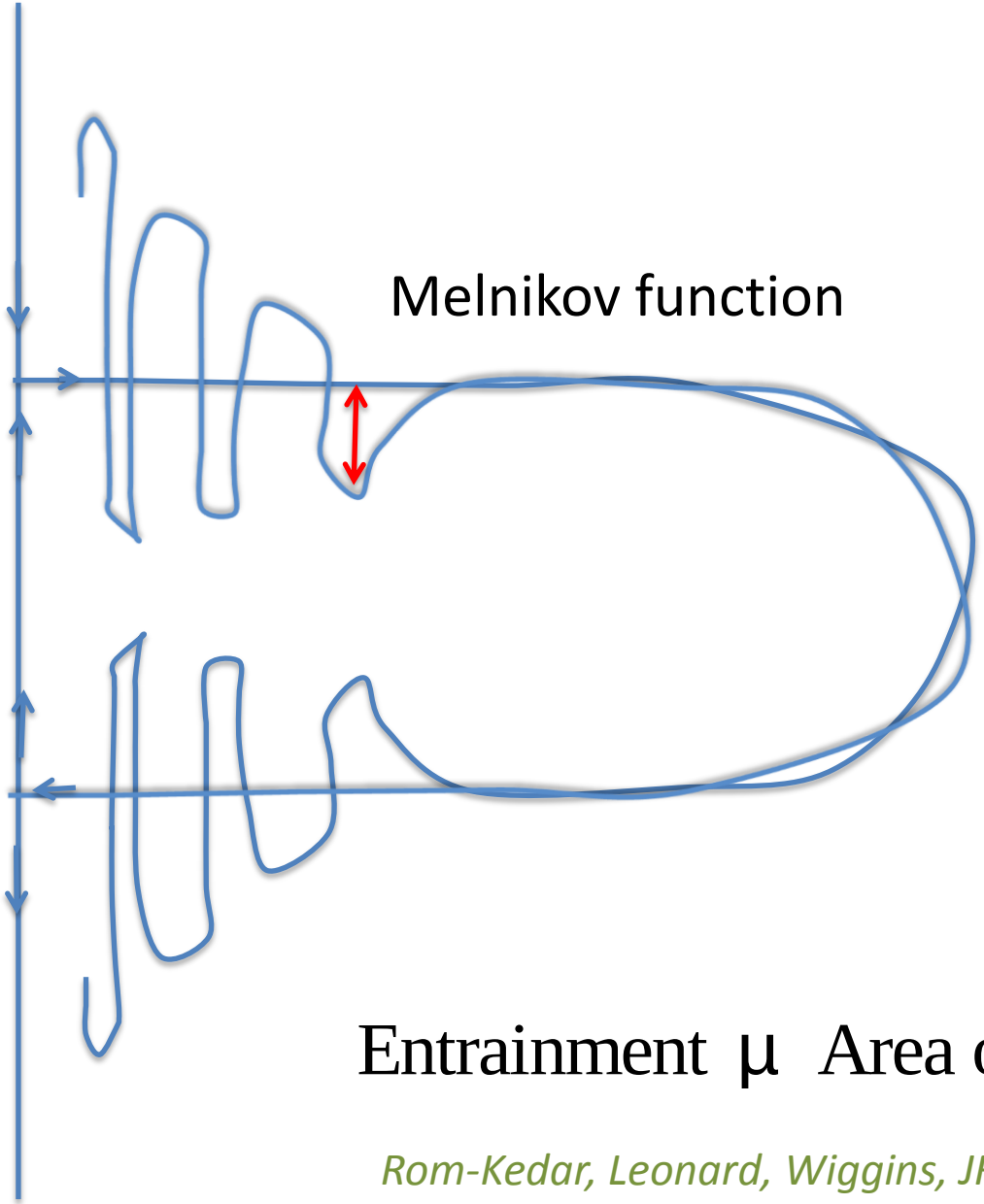
Inviscid, non-diffusive, 2D

What will get you chaos?



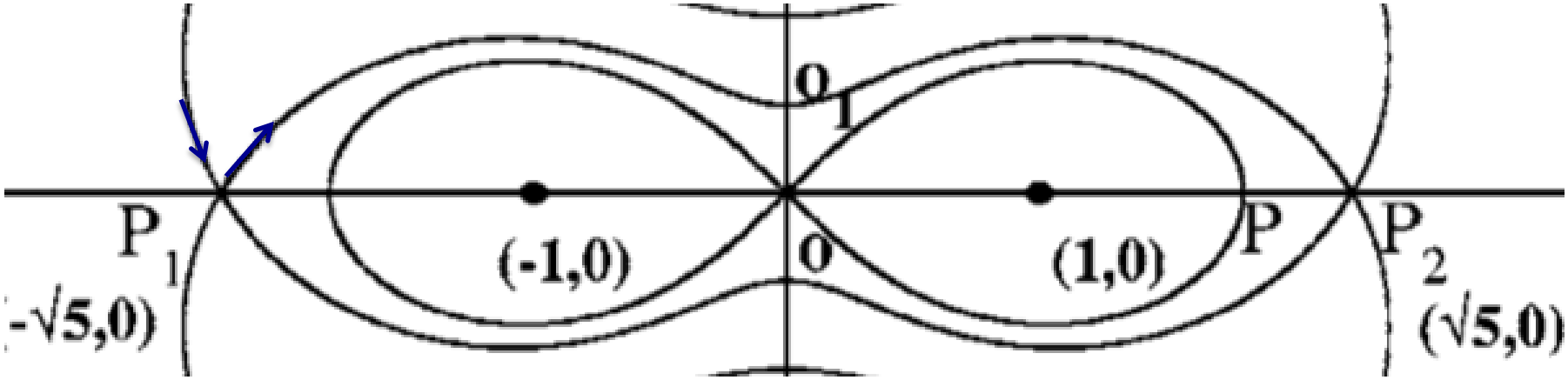
Leapfrogging

$$u_f = \sum_i \frac{\Gamma_i}{r_i} e_{\wedge i}$$

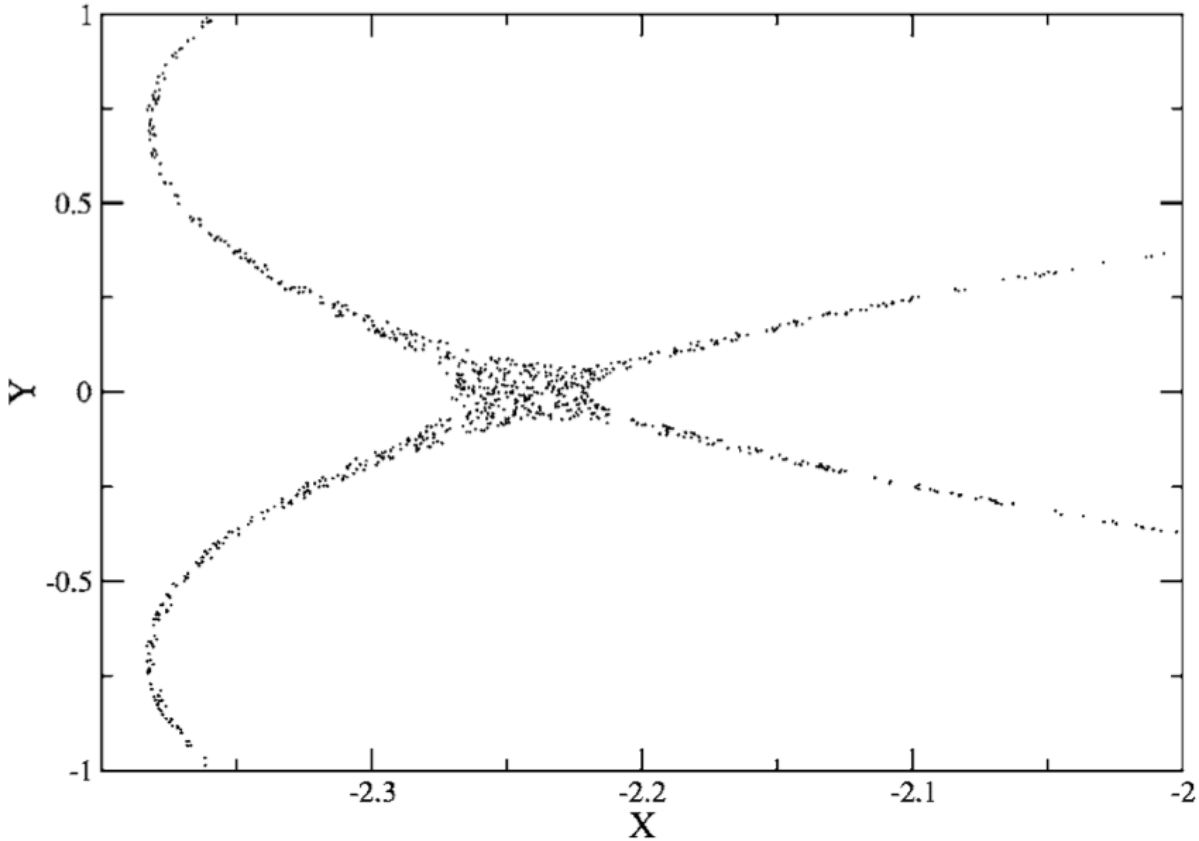


Entrainment μ Area of lobe

Rom-Kedar, Leonard, Wiggins, JFM, 1993


O
I
P
1
(-1,0)
O
(1,0)
P
P
2
(-√5,0)
(√5,0)

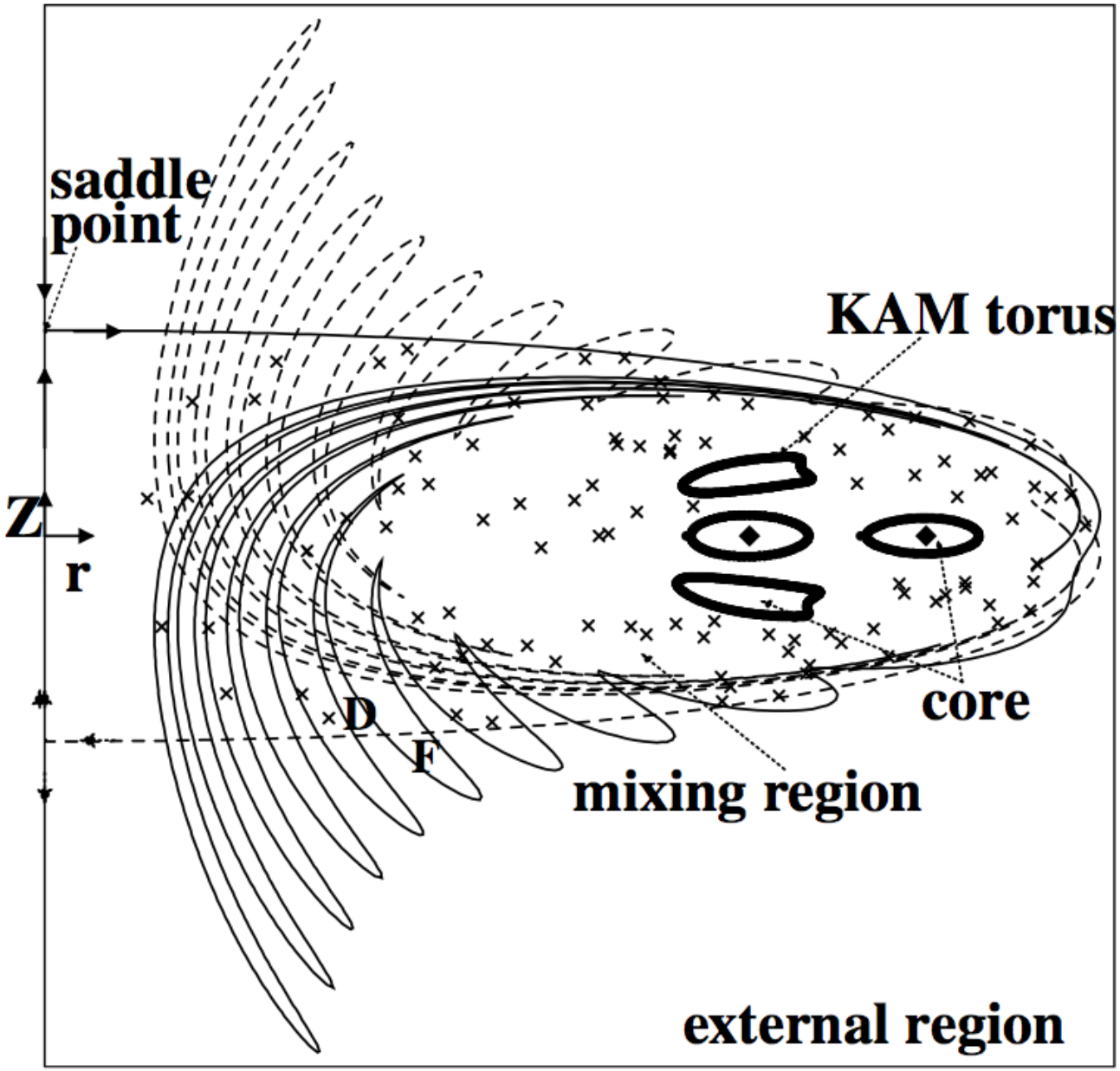
Particles near two vortices, perturbed by two far-away vortices



$$E \mu e^2$$

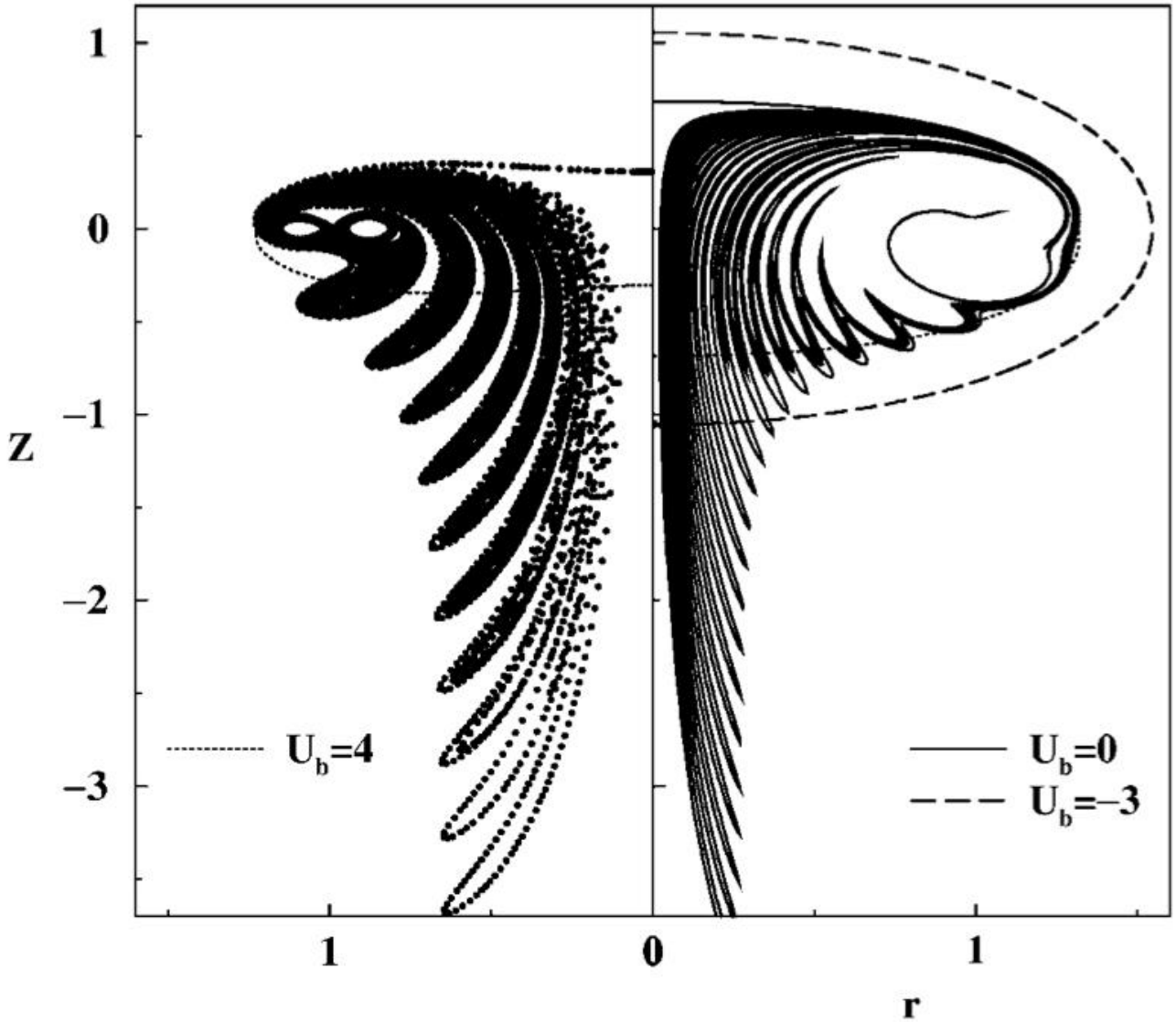
A measure of chaotic region

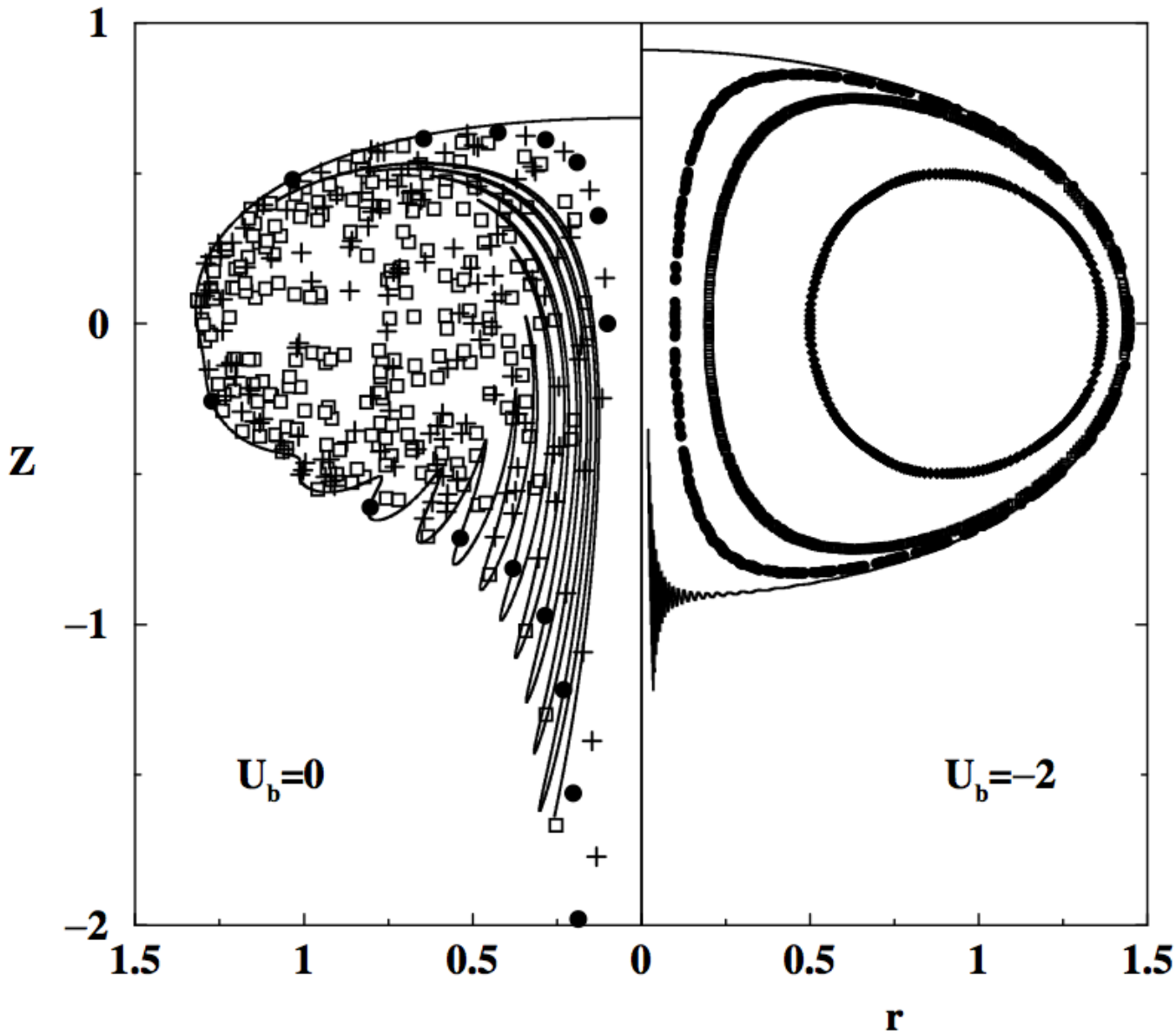
Muralidharan, Srinivas & RG, Phys Rev E 2005

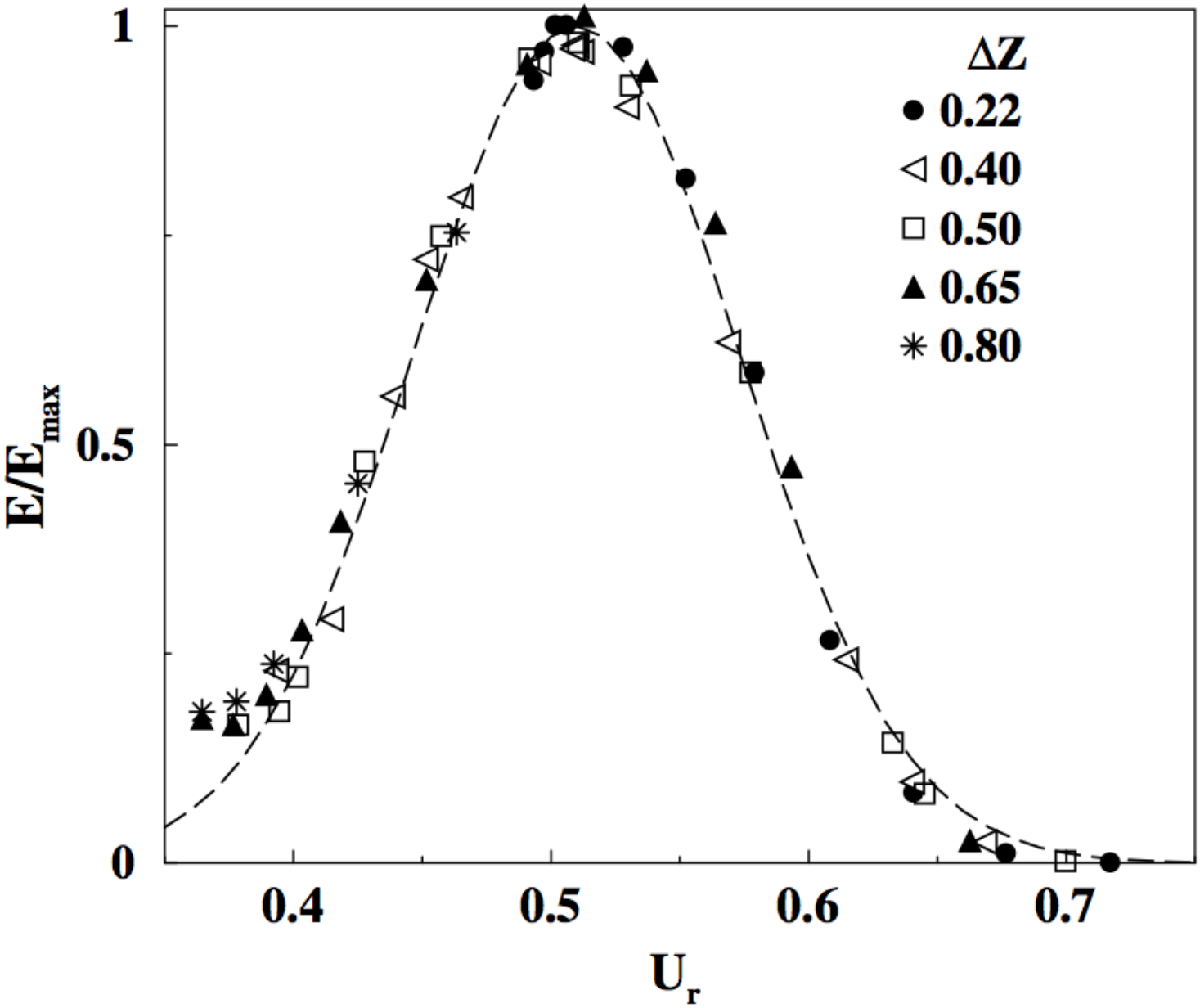


See e.g. RG, Phys. Rev. Lett. 2002

U_b: Buoyancy velocity




1
0
−1
−2
z
1.5
1
0.5
0
0.5
1
1.5
r
U_b=0
U_b=−2


1
0.5
0
E/E_max
0.4
0.5
0.6
0.7
U_r
ΔZ
0.22
0.40
0.50
0.65
0.80

Water droplets

$$u_f = \sum_i \frac{\Gamma_i}{r_i} e_{\wedge i}$$

$$\frac{du_p}{dt} = \frac{1}{t}\left(u_f - u_p\right)$$

$$t = \frac{2 r_p a^2}{9 \nu r_f}$$

$$St = \frac{t}{T_f}$$

For a~ microns, St ~ a range (10^{-3} to 10^{-1})

Local turbulence time scale ~ a range

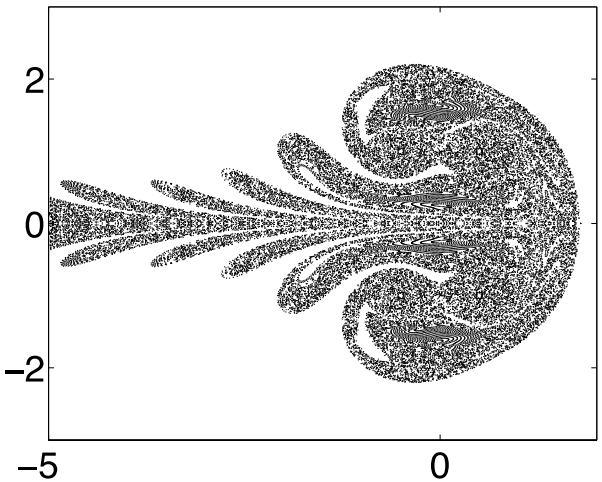
Cloud life time ~ 1000 sec

Particle clustering ~ 100 sec

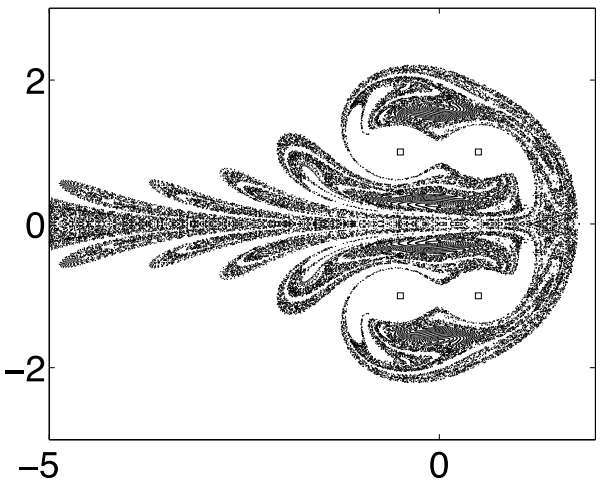
Particle growth ~ 10-100 sec

Four vortex: particle distribution after some time periods

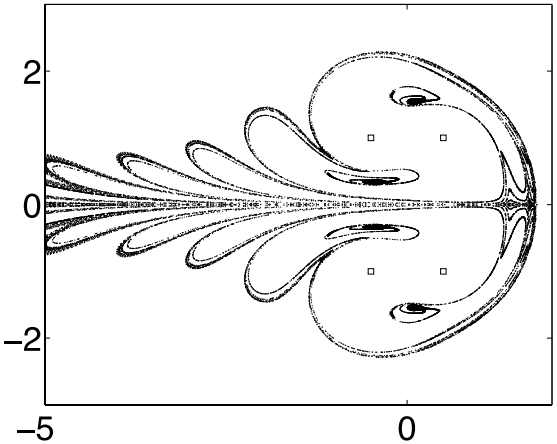
St=0, T=5



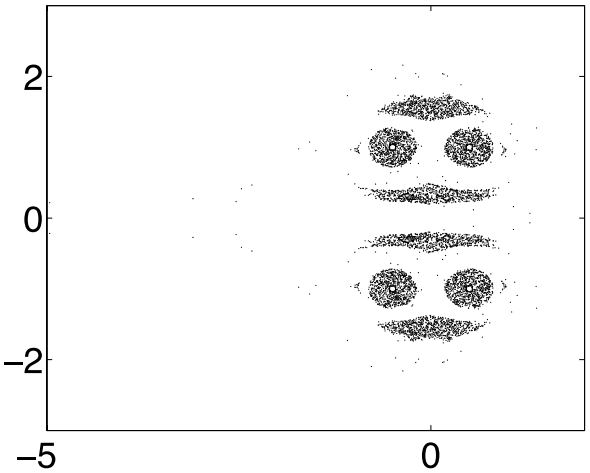
St=1/300



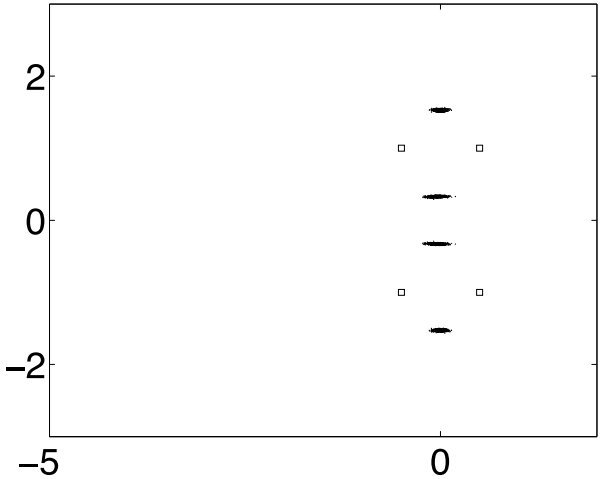
St=1/20



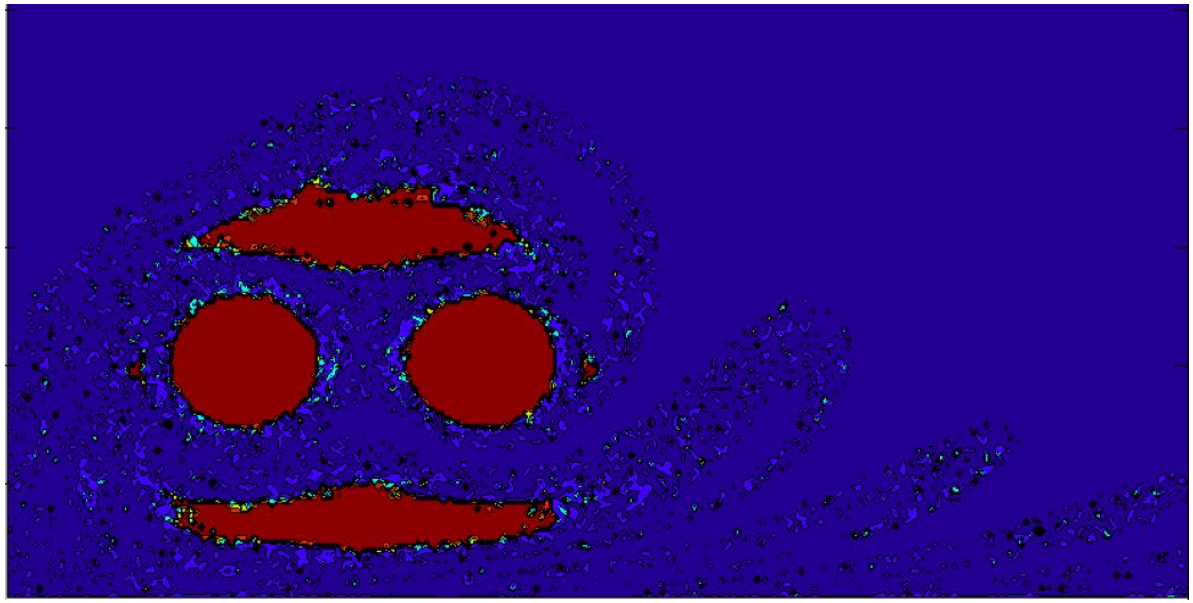
St=0, T=100



St=1/300, T=100

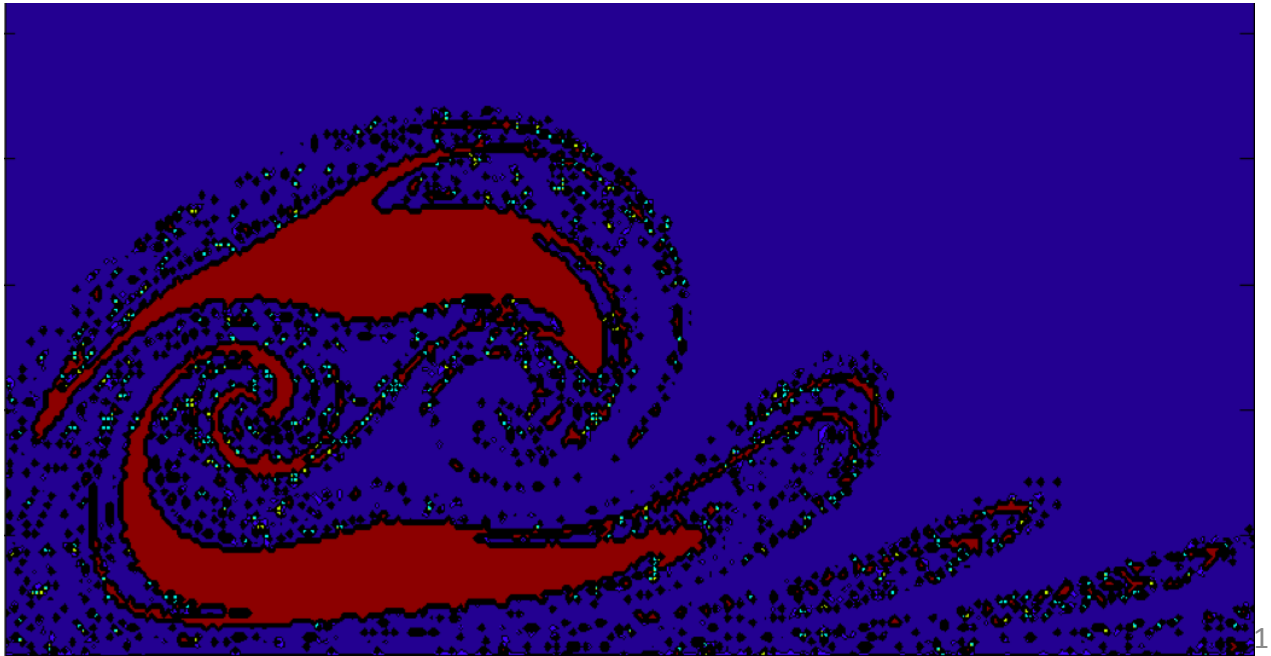


Particles remaining with the flow, ``stable manifold''

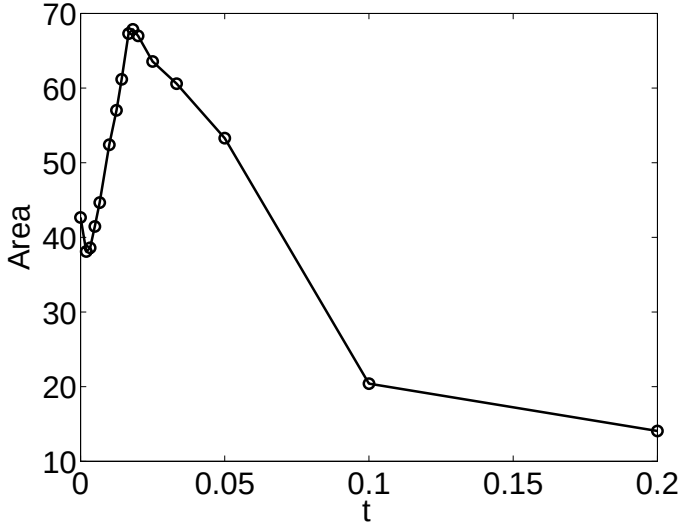


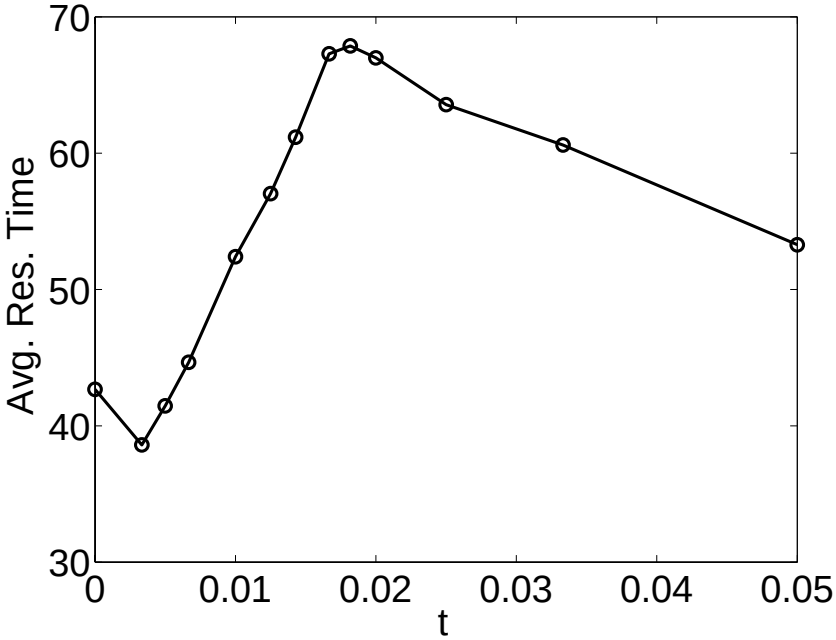
St=0

St=0.01



Competition between two attractors: increasing and decreasing basin boundaries





$$s = \frac{r}{r_s} - 1, \quad y \equiv \int_0^t s\,dt, \quad \dot{a} = a\frac{s}{a}$$

Constant atmosphere

$$V\dot{r} = -f r V - r_w n_w 4\pi a^2 \dot{a}$$

$$r_s \ddot{y} = -f r_s(\dot{y}+1) - \frac{4\pi r_w n_w}{V} a^2 \dot{a}$$

$$b \equiv \frac{4\pi r_w n_w}{r_s V}$$

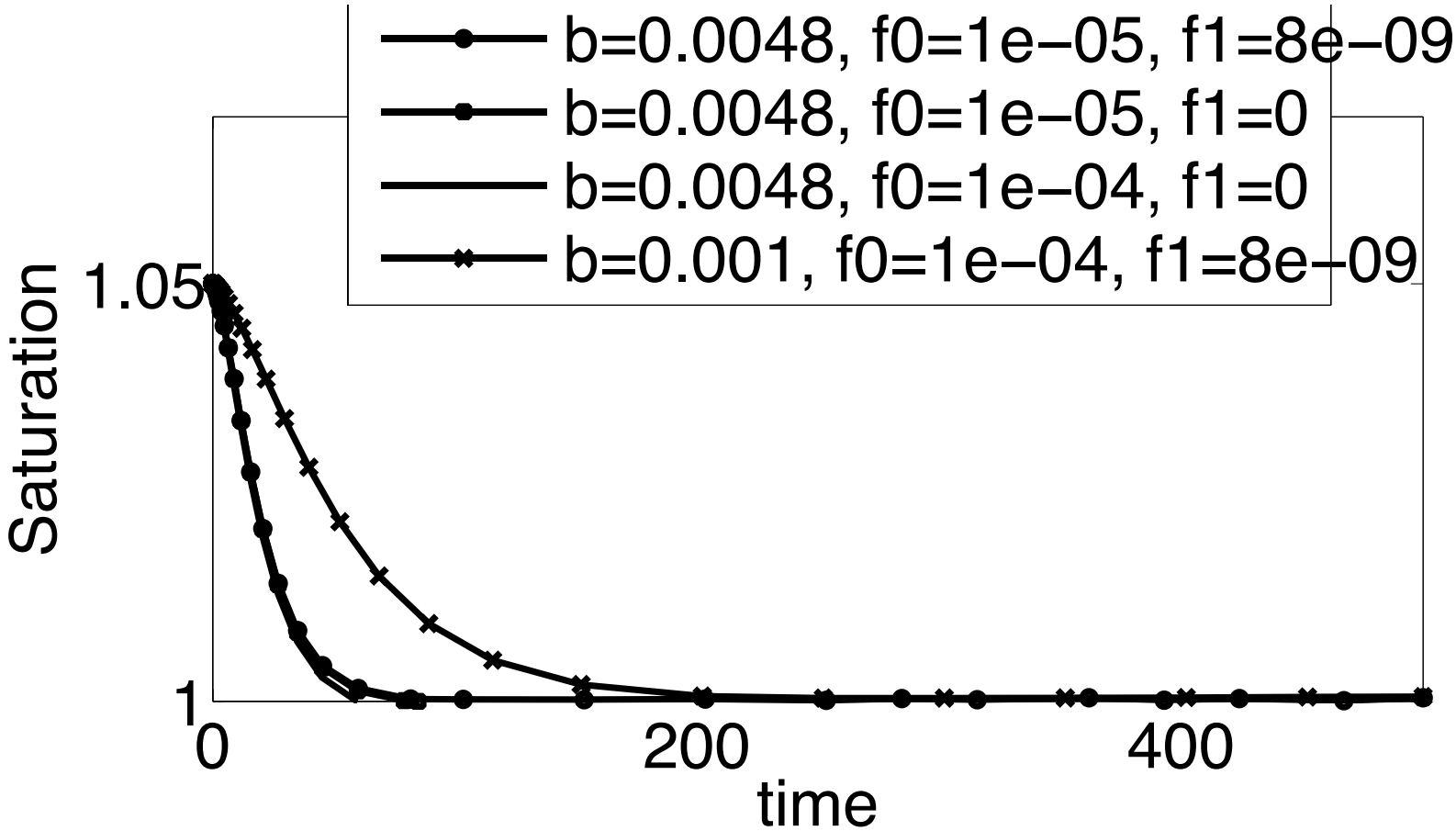
Effect of n_w is smaller for inertial case

$$\ddot{y} = -f(\dot{y}+1) - b\dot{y}(a_0^2 + 2ay)^{1/2}$$

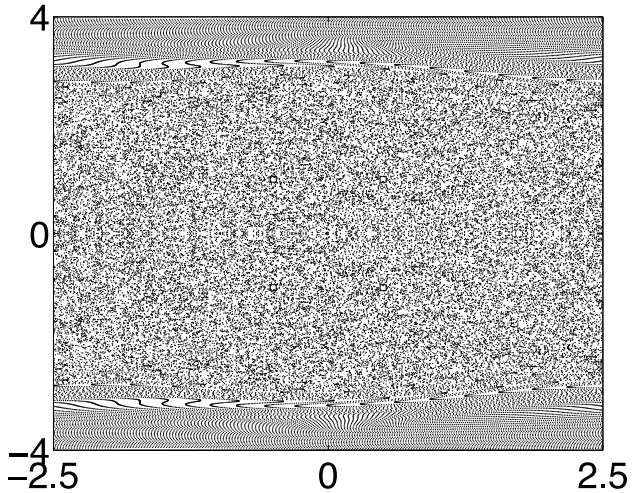
$$\frac{dz}{dx} + \frac{fa}{z} = cx^{1/2} - f$$

x,z linearly related to y,s

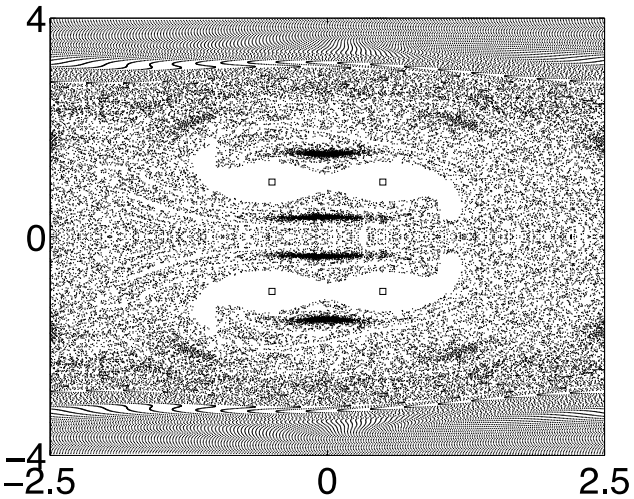
Toy clouds can last 10^1 to 10^4 seconds
reach metres to kilometres
Inertia helps



St=0, water vapor, aerosol

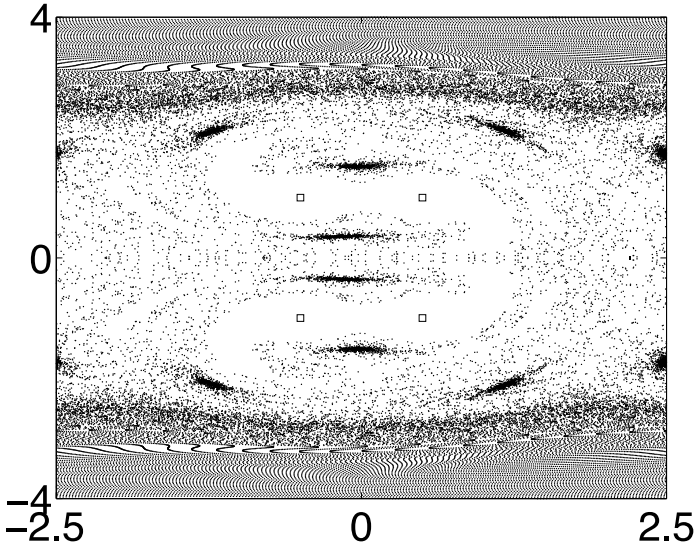


St=1/300

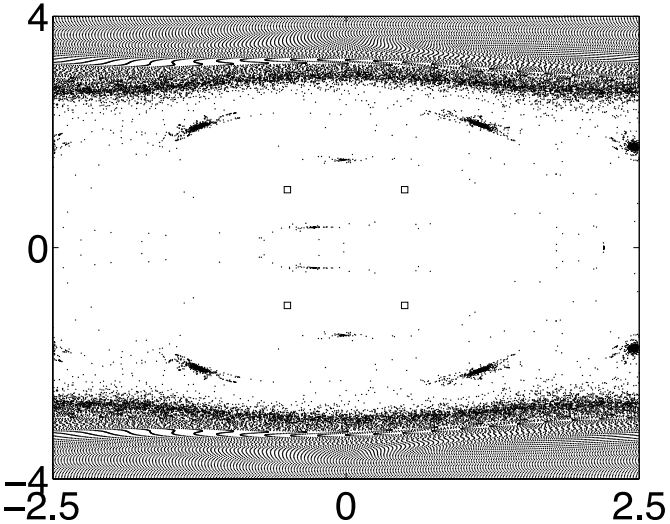


Periodic flow, at long times

St=1/100

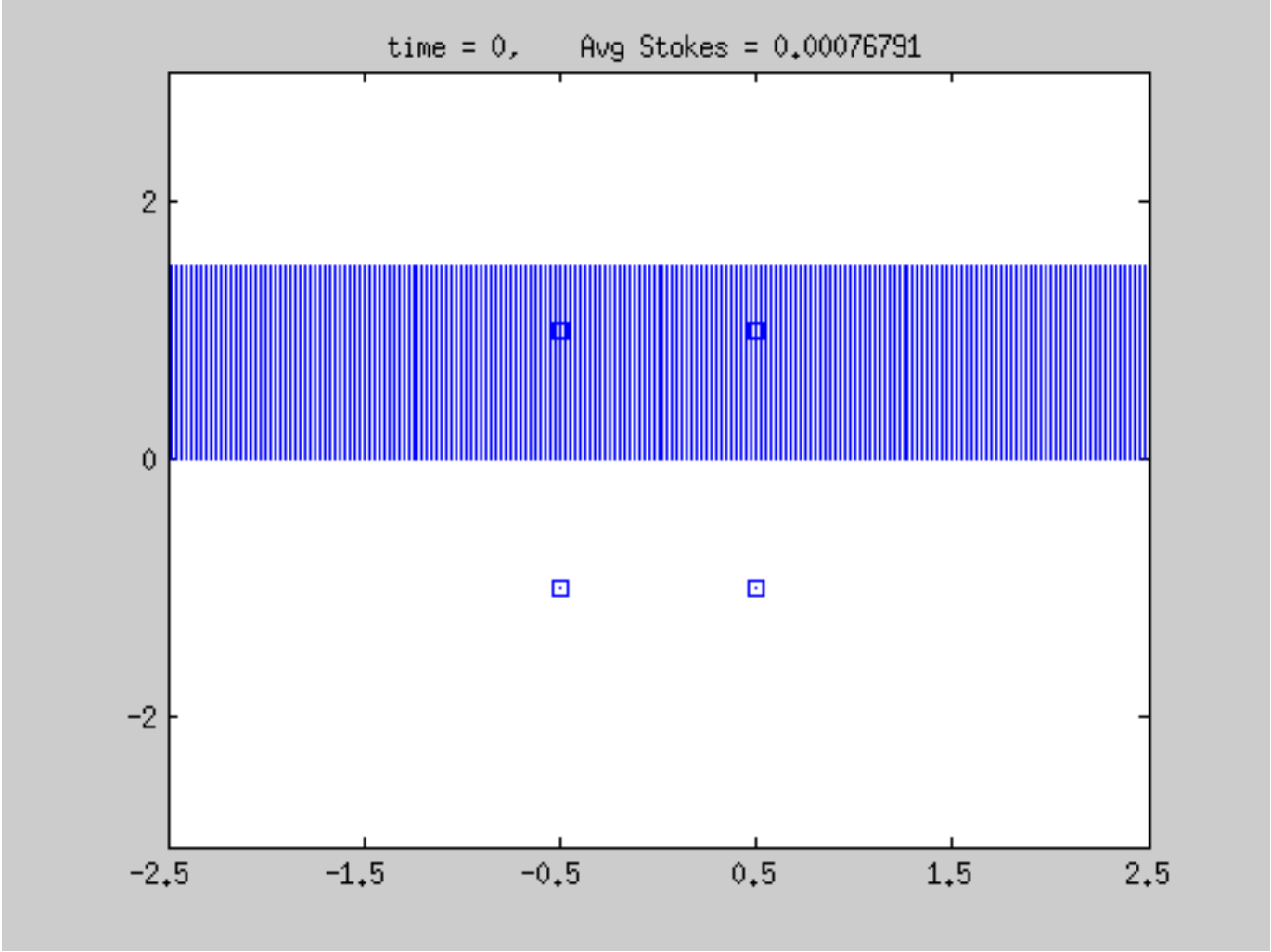


St=1/50

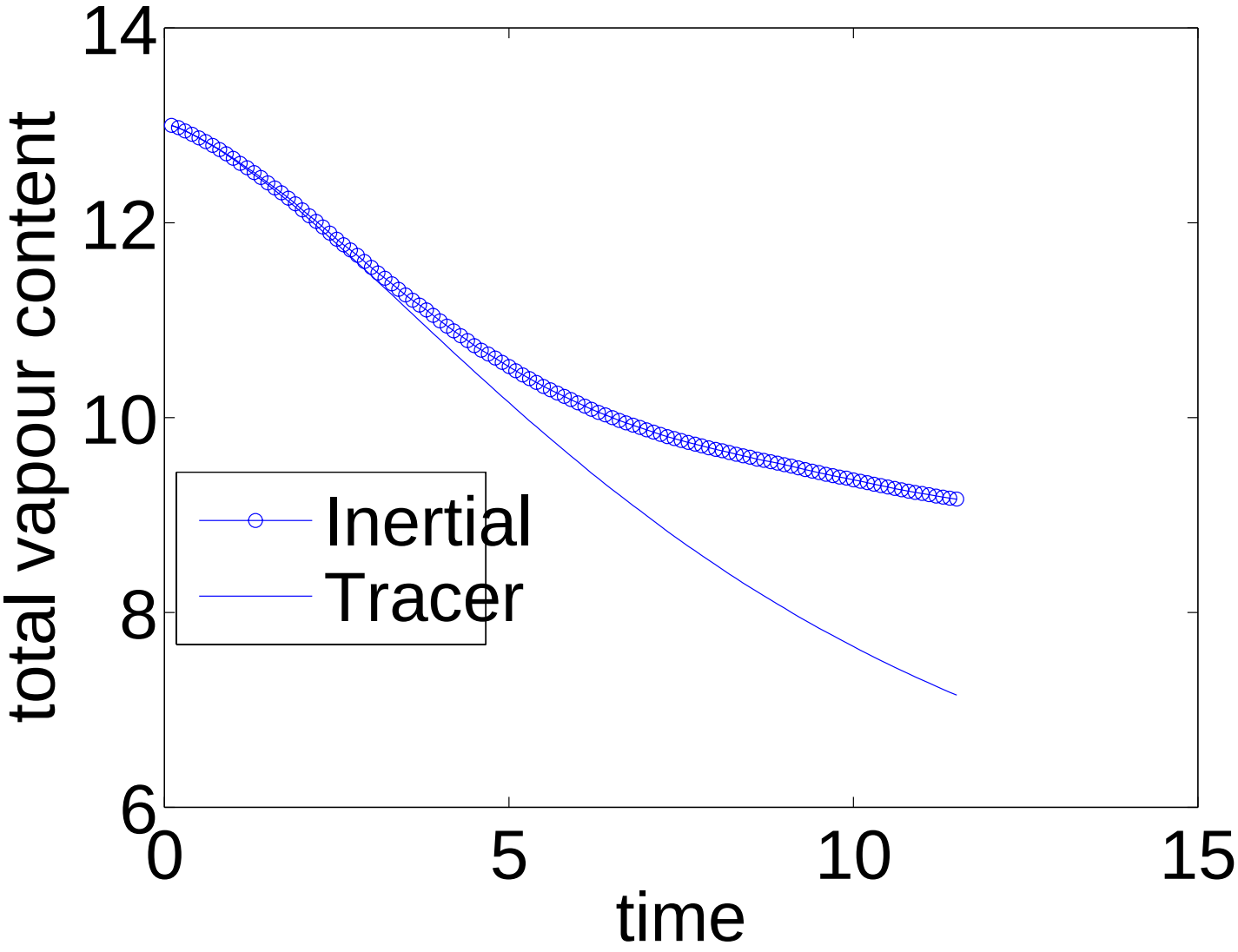


Inertial particles, growing

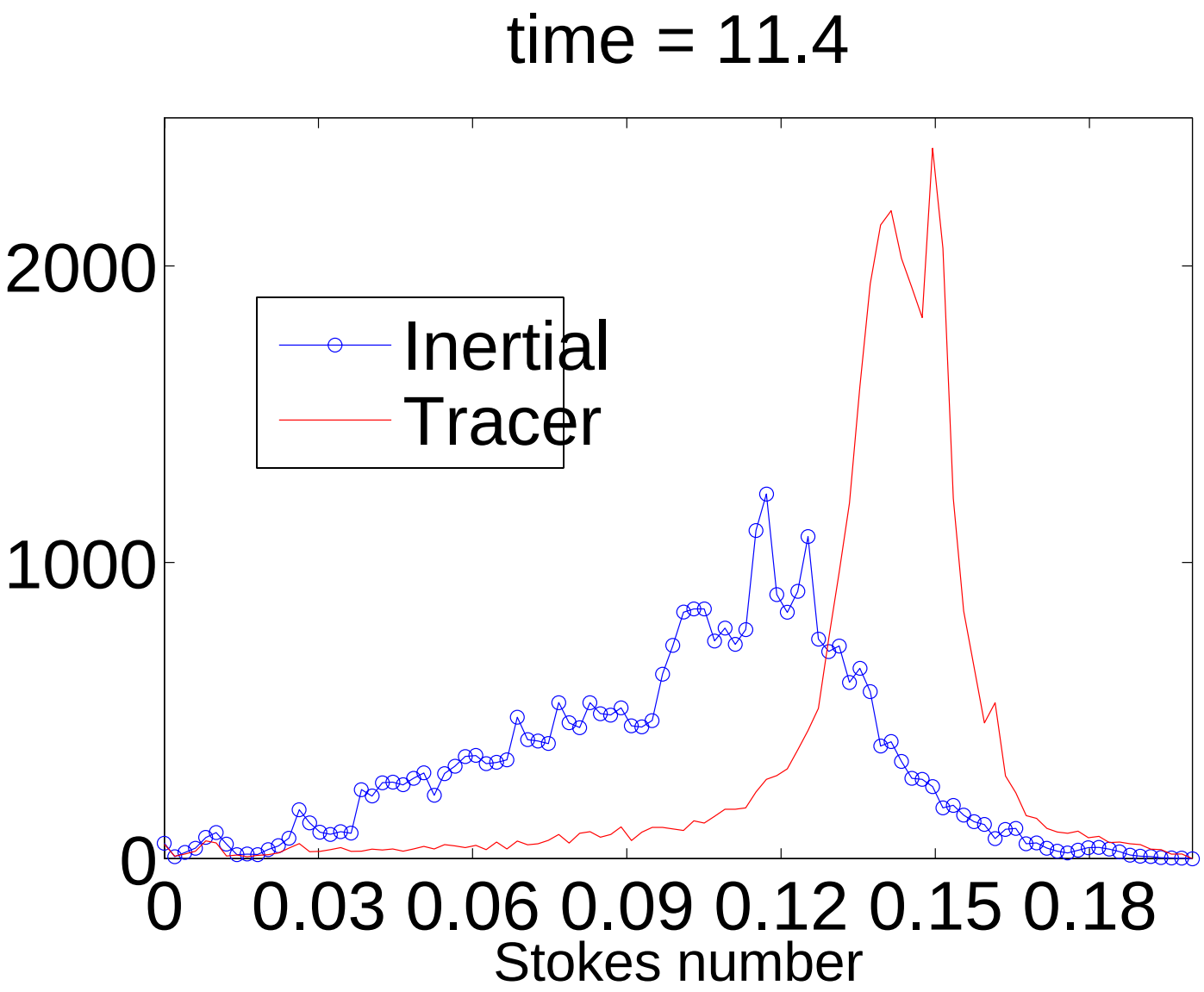
$$\frac{dt}{dt} \mu\ a \frac{da}{dt} \mu\ \frac{s}{a}$$



s depleting suitably



Inertia helps life-span?


time = 11.4
2000
1000
0
0
0.03
0.06
0.09
0.12
0.15
0.18
Stokes number
Inertial
Tracer

Conclusions:

Proposing a “toy-est” model to describe

Cloud life-span as affected by

Vortex interactions

Buoyancy

Particle inertia

Unexpected stable “fixed” points