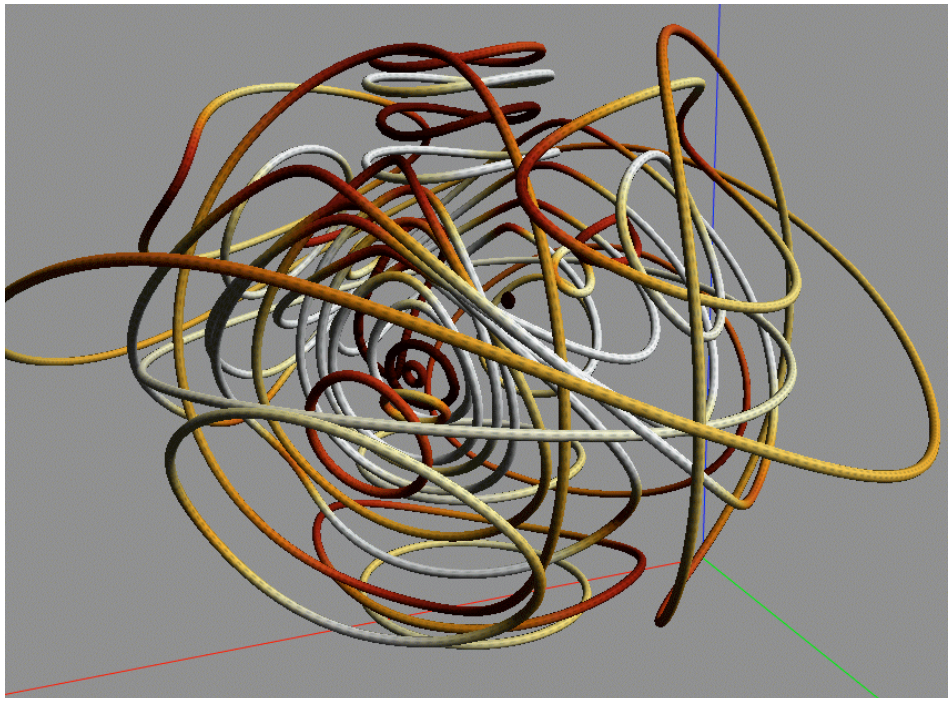
KITP Lunch Talk, Apr 21, 2011

Nature of Overstabilities in Dilute Plasmas



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Special thanks to KITP!

and collaborators Chris Reynolds and Steven Balbus

Role of thermal conduction in dilute plasmas

- ICM plasma is dilute and weakly magnetized-- charged particles are nearly freely streaming along the lines of magnetic field.

$$r_g = \frac{3.1 \times 10^8\,\mathrm{cm}}{Z}\left(\frac{T_g}{10^8\,\mathrm{K}}\right)^{1/2}\left(\frac{m}{m_e}\right)^{1/2}\left(\frac{B}{1\,\mu\mathrm{G}}\right)^{-1}$$

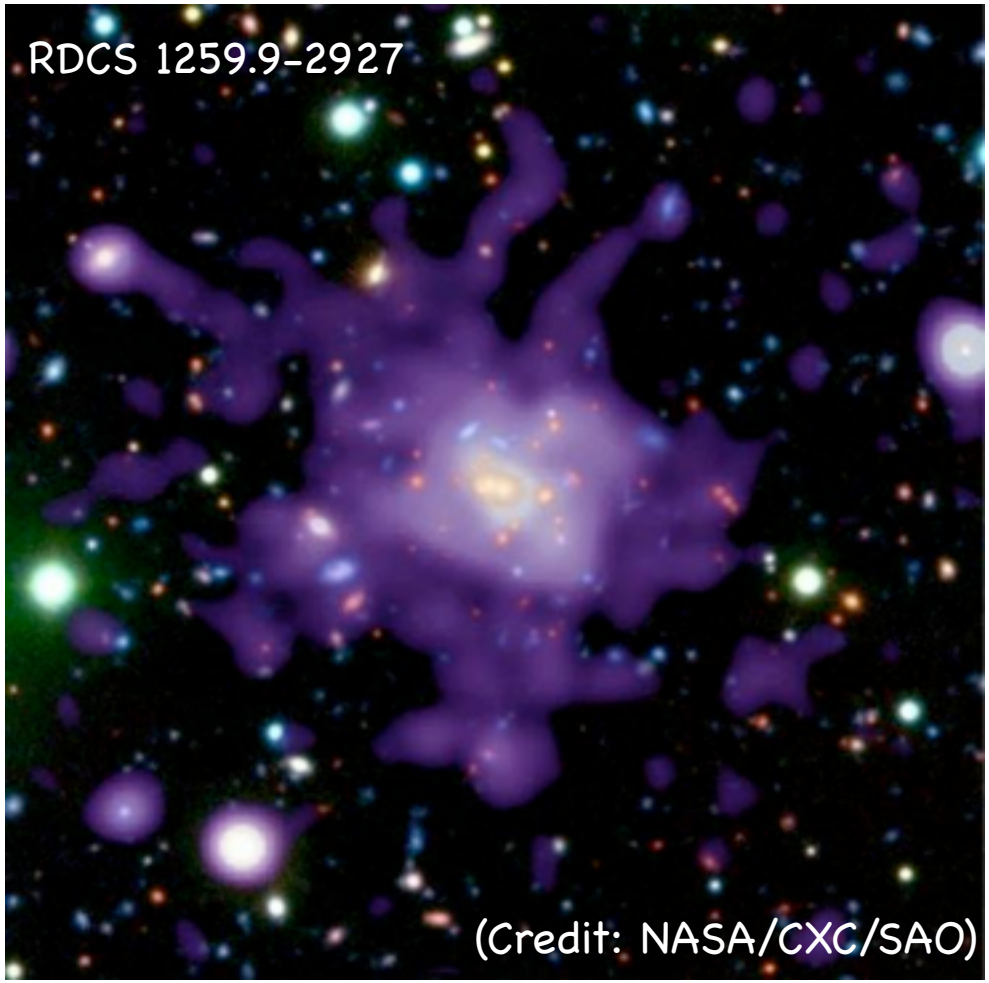
$$\lambda_e = \lambda_i \approx 23\,\mathrm{kpc}\left(\frac{\mathrm{T_g}}{10^8\,\mathrm{K}}\right)^2\left(\frac{\mathrm{n_e}}{10^{-3}\,\mathrm{cm}^{-3}}\right)^{-1}$$

- Anisotropic conduction alters classic condition for convection.

(Balbus 00)

$$\text{Schwarzschild criterion } (\partial S/\partial z > 0)$$

- What are the implications for the ICM?



MHD instabilities and overstabilities

(Balbus & Reynolds 10)

b_z \ $\frac{dT}{dz}$	<0	>0
≈0	MTI	g-mode driven overstable by radiative cooling
≈1	g-mode driven overstable by conductive heat flux	HBI

MHD equations of magnetized plasma

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \boldsymbol{v}) = 0,$$

mass

$$\rho \frac{D\boldsymbol{v}}{Dt} = \frac{(\nabla\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi} - \nabla P + \rho \boldsymbol{g},$$

momentum

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}),$$

induction

$$\frac{D \ln P\rho^{-\gamma}}{Dt} = -\frac{\gamma-1}{P}\left[\nabla\cdot\boldsymbol{Q} + \rho\mathcal{L}\right],$$

entropy

$$\boldsymbol{Q} = -\chi \boldsymbol{b}(\boldsymbol{b}\cdot\nabla)T$$

$$\rho\mathcal{L} \simeq 2\times 10^{-27} n_e^2 T^{1/2}\ \mathrm{erg\ cm^{-3}\ s^{-1}},$$

$$\chi \simeq 6\times 10^{-7} T^{5/2}\ \mathrm{erg\ cm^{-1}\ s^{-1}\ K^{-1}}.$$

Spitzer conduction

Local WKB perturbations (Wentzel-Kramers-Brillouin)

plane wave disturbances $\propto \quad \exp(\sigma t + i\boldsymbol{k}\cdot\boldsymbol{r})$

dispersion relation $\quad \sigma^3 + a_1\sigma^2 + a_2\sigma + a_3 = 0$

stable solutions require $\quad a_1 > 0, \quad a_3 > 0, \quad a_1 a_2 > a_3.$

$a_1 > 0$ thermal stability (Field criterion)

$a_3 > 0$ stable to thermal/heat flux driven instabilities

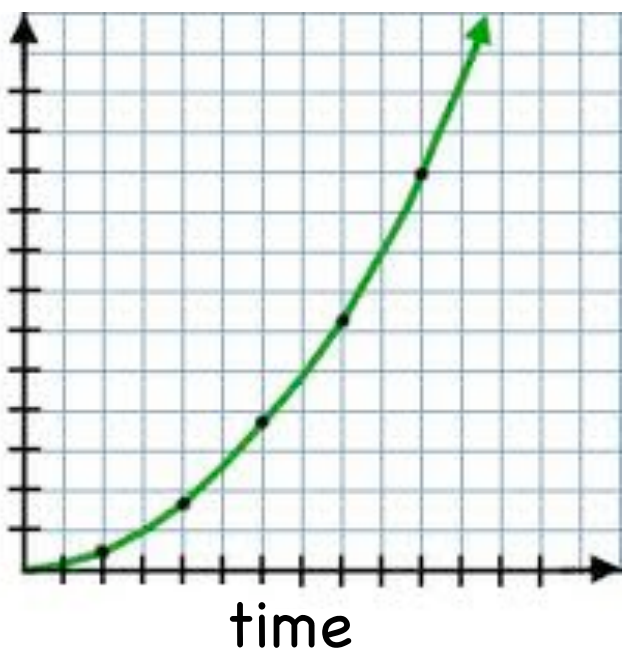
$a_1 a_2 > a_3$ stable to cooling/heat flux driven overstabilities

“Instability” vs. “overstability”

small perturbation $\propto \exp(\sigma t + i\boldsymbol{k}\cdot\boldsymbol{r})$

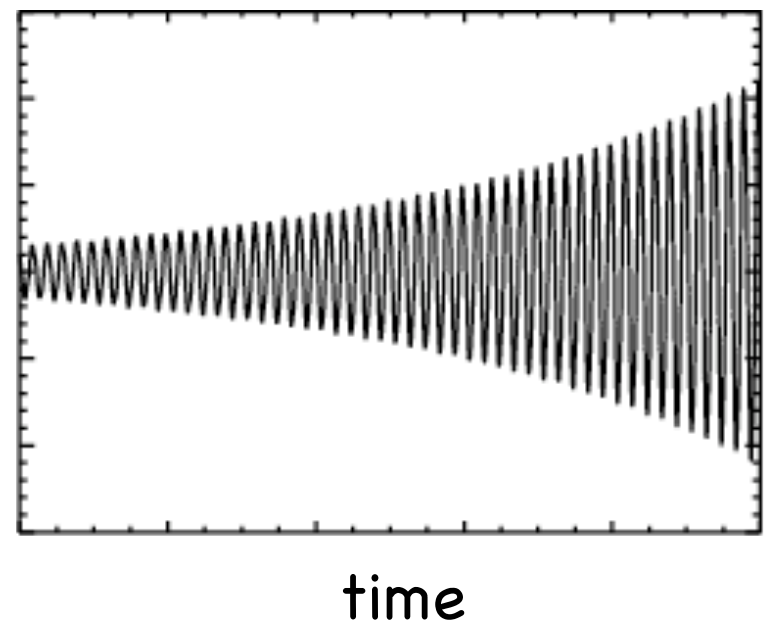
if σ is ...

... real and >0, instability



velocity perturbation

... complex and |σ|>0, overstability



$\kappa \equiv \chi T / P$

diffusivity (cm^2/s) in terms of Spitzer conductivity

Stability criteria

thermal stability (Field criterion):

$$\boxed{T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa > 0,}$$

thermal/heat flux driven instabilities:

$$\left(\frac{\gamma-1}{\gamma}\right)\kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\left((1-2b_z^2)k_\perp^2 + 2b_x b_z k_x k_z\right)\frac{g}{k^2}\frac{d\ln T}{dz} + (\boldsymbol{k}\cdot\boldsymbol{v_A})^2 a_1 > 0.$$

radiative cooling/heat flux driven overstabilities:

$$\boxed{T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} > 0,}$$

$\mathcal{R}$ = f (dlnT, dlnp) < 1 "reduction factor"

Interplay of thermal stability and overstabilities

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa > 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} > 0$$

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa < 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} > 0$$

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa > 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} < 0$$

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa < 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} < 0$$

Interplay of thermal stability and overstabilities

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa > 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} > 0.$$

✔

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa < 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} > 0.$$

✗

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa > 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} < 0.$$

✔

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa < 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} < 0.$$

✔

Interplay of thermal stability and overstabilities

$$T\Theta_{T|P} + (\boldsymbol{k}$$

$$T\Theta_{T|P}$$

nothing happens here

✔

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b}$$

$$T\Theta_{T|P}$$

unphysical solution

✗

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa > 0,$$

$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} < 0$$

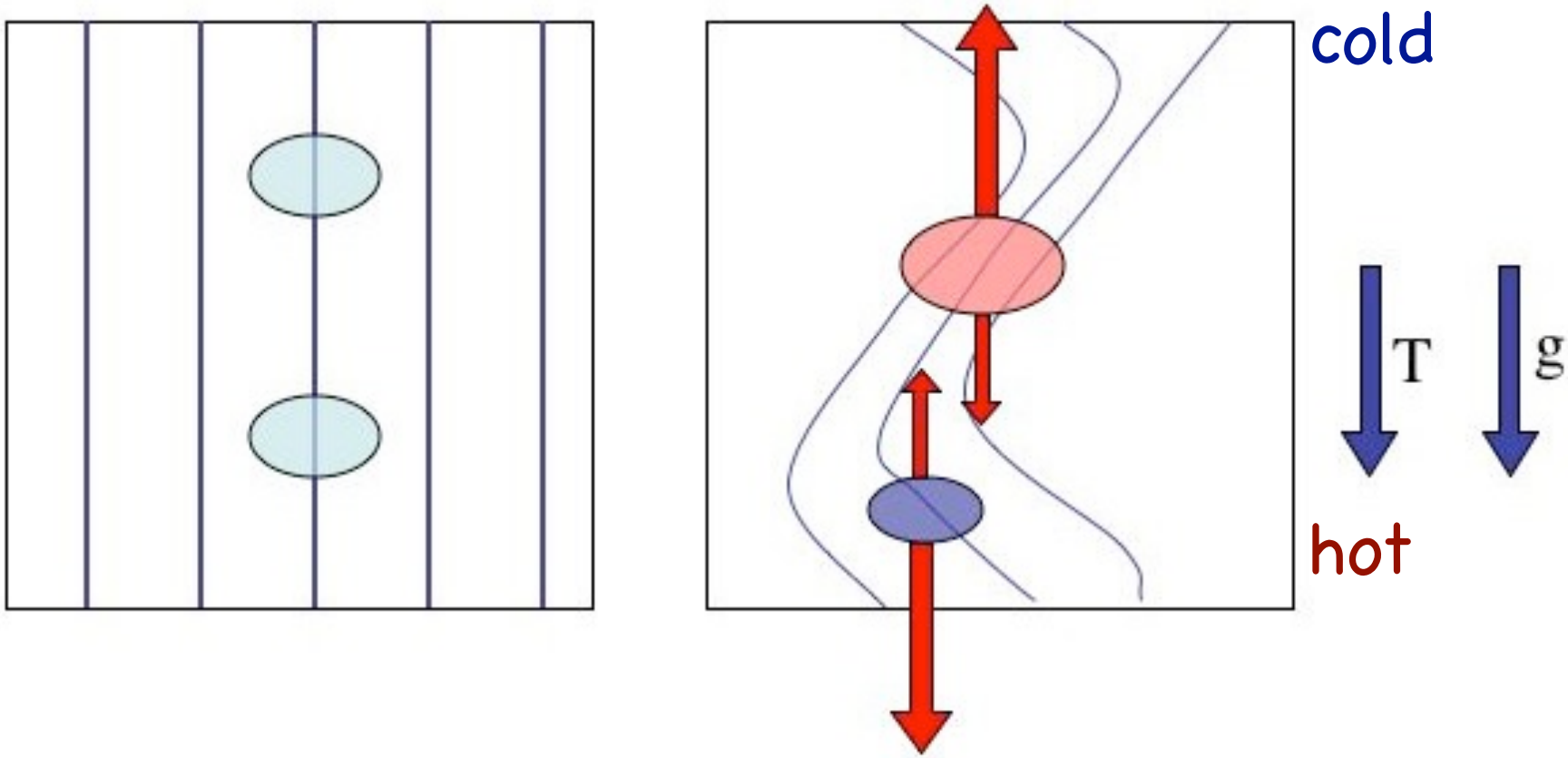
✔

$$T\Theta_{T|P} + (\boldsymbol{k}\cdot\boldsymbol{b})^2\kappa < 0,$$

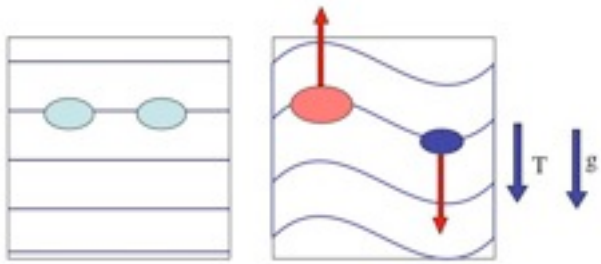
$$T\Theta_{T|P} + \kappa(\boldsymbol{k}\cdot\boldsymbol{b})^2\mathcal{R} < 0$$

✔

Heat-flux Driven Overstability (HFO)

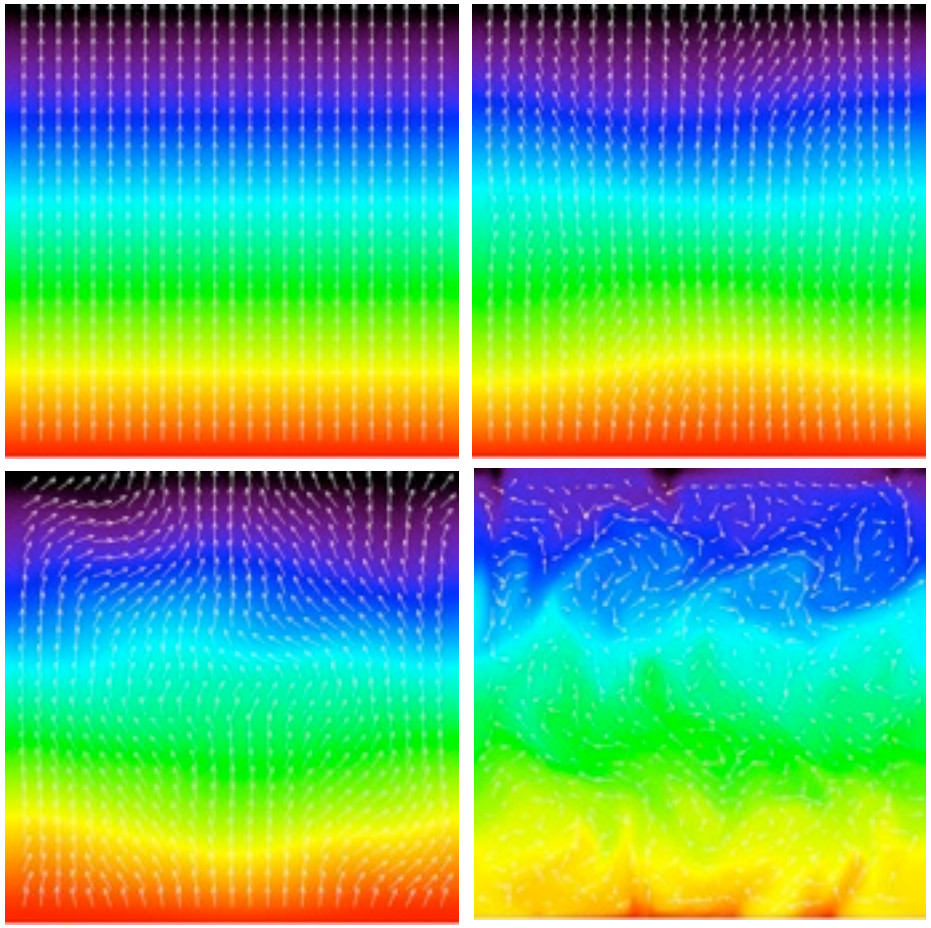


MTI



Thermally stable and overstable to HFO

$\omega_{dyn} > \omega_{cond}$



~2500 t_{dyn}

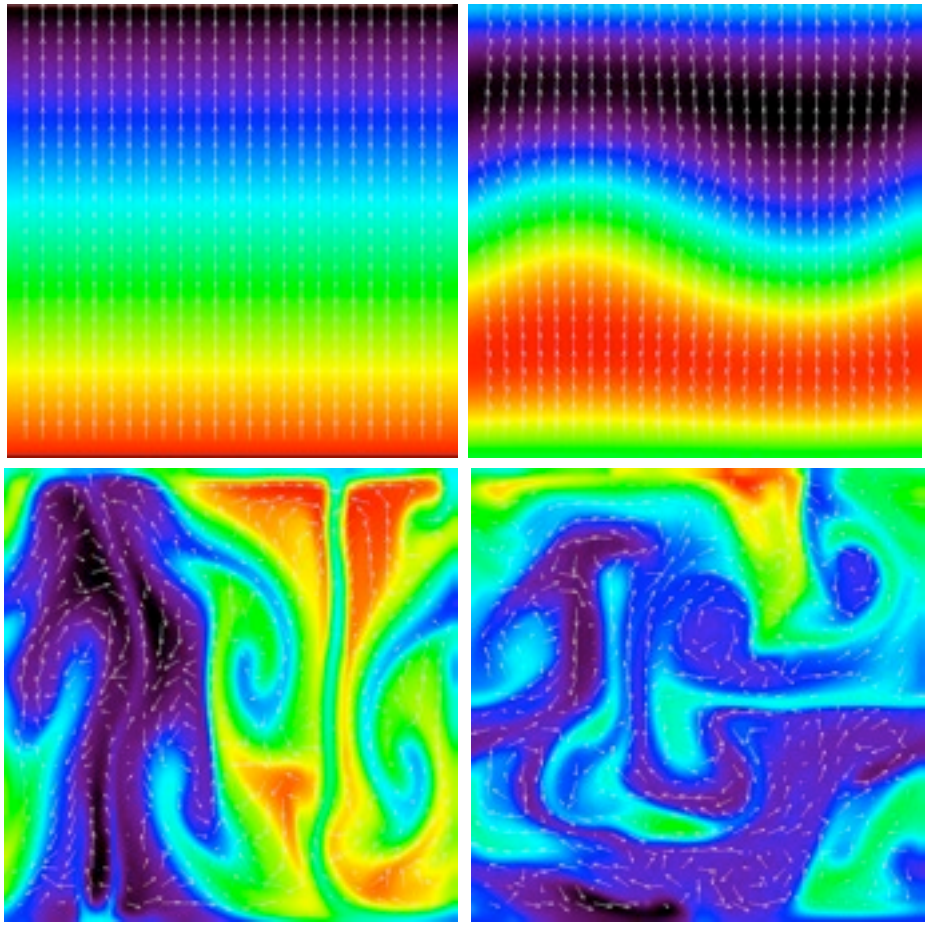
cold

T g

hot

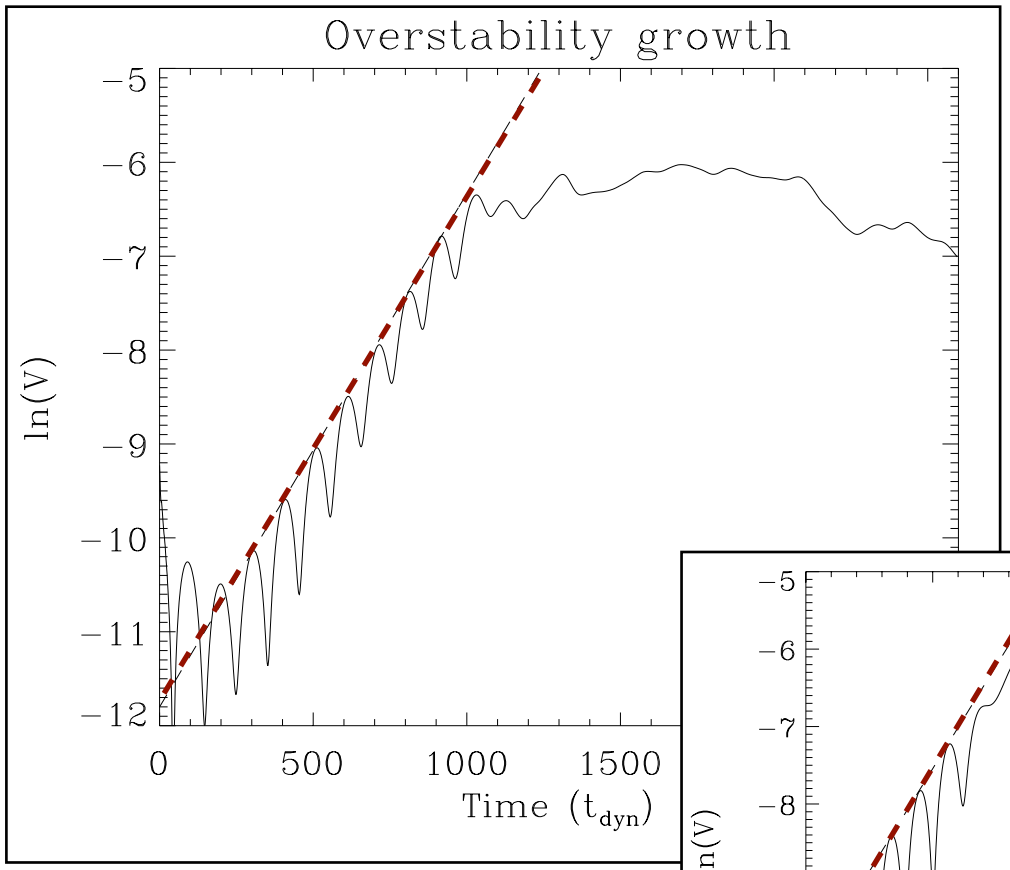
Thermally unstable and overstable to HFO

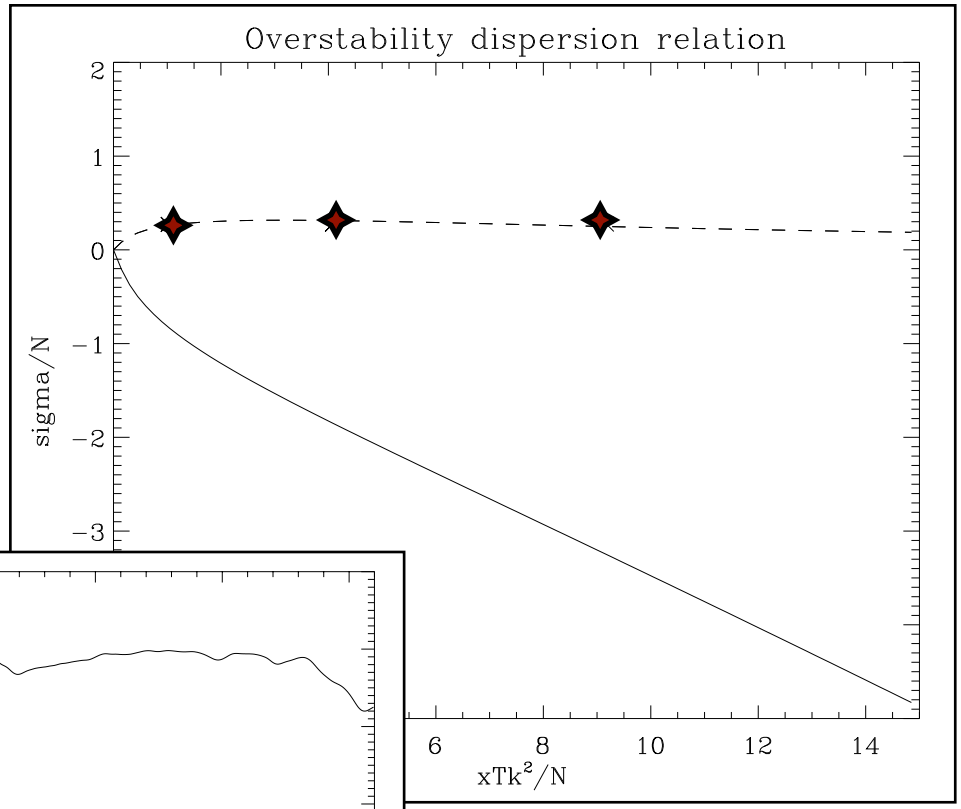
$\omega_{dyn} \gtrsim \omega_{cool} > \omega_{cond}$

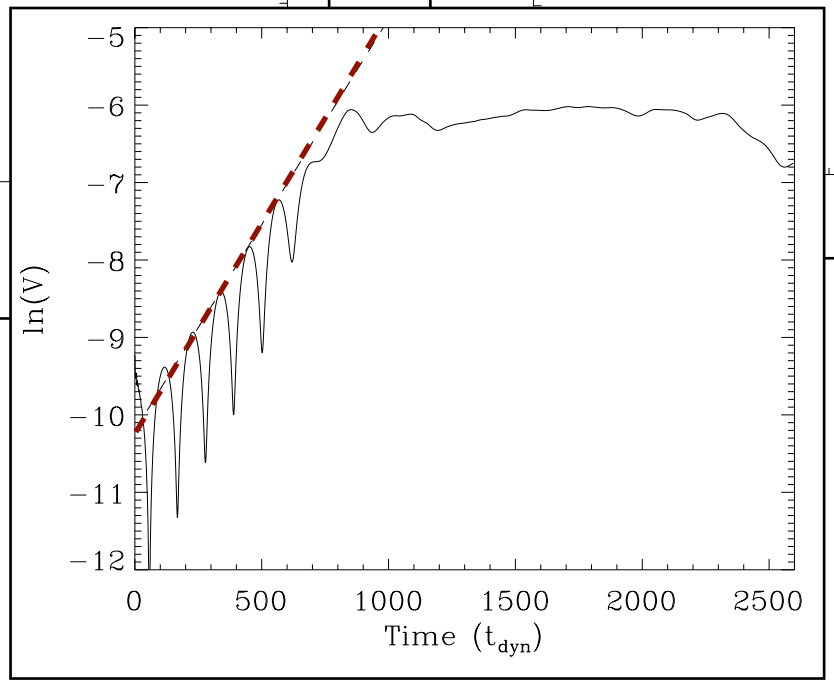


~200 t_{dyn}

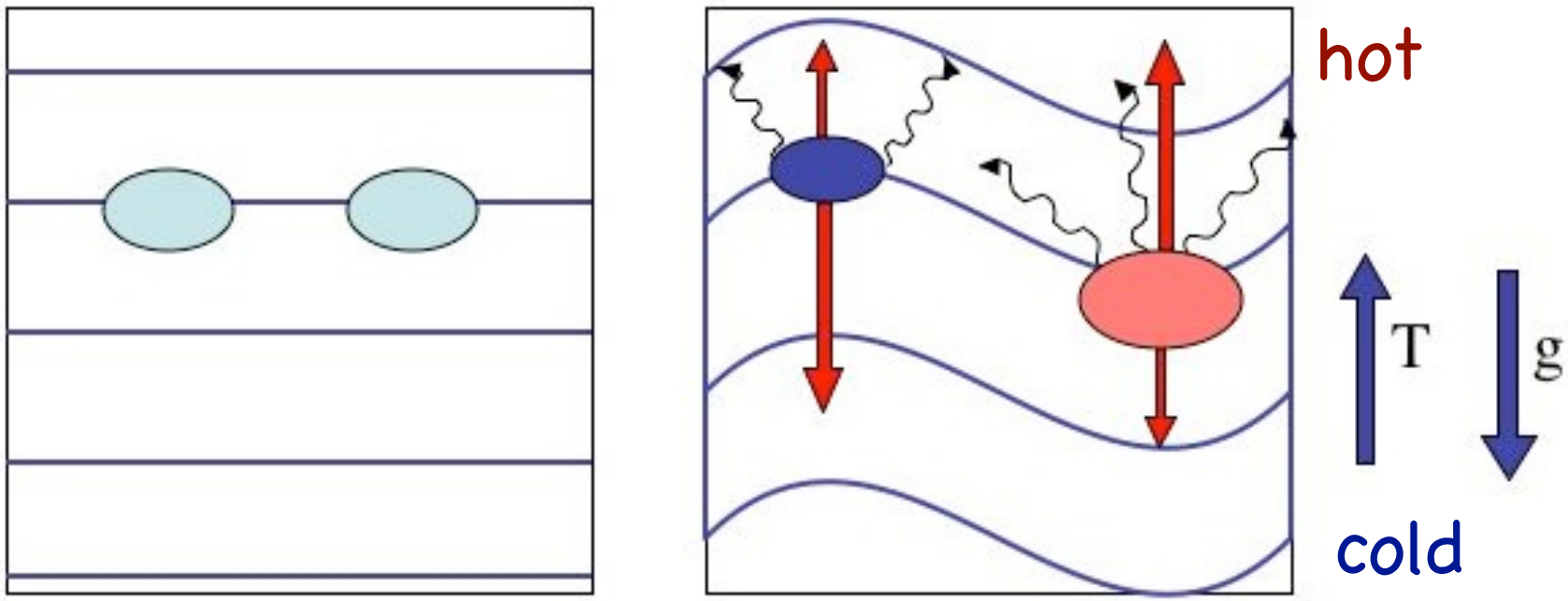
HFO growth and dispersion relation



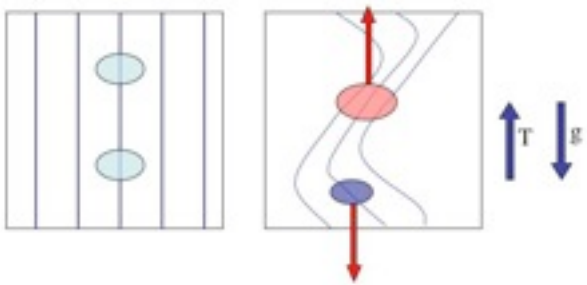




Overstability driven by radiative cooling (RCO)

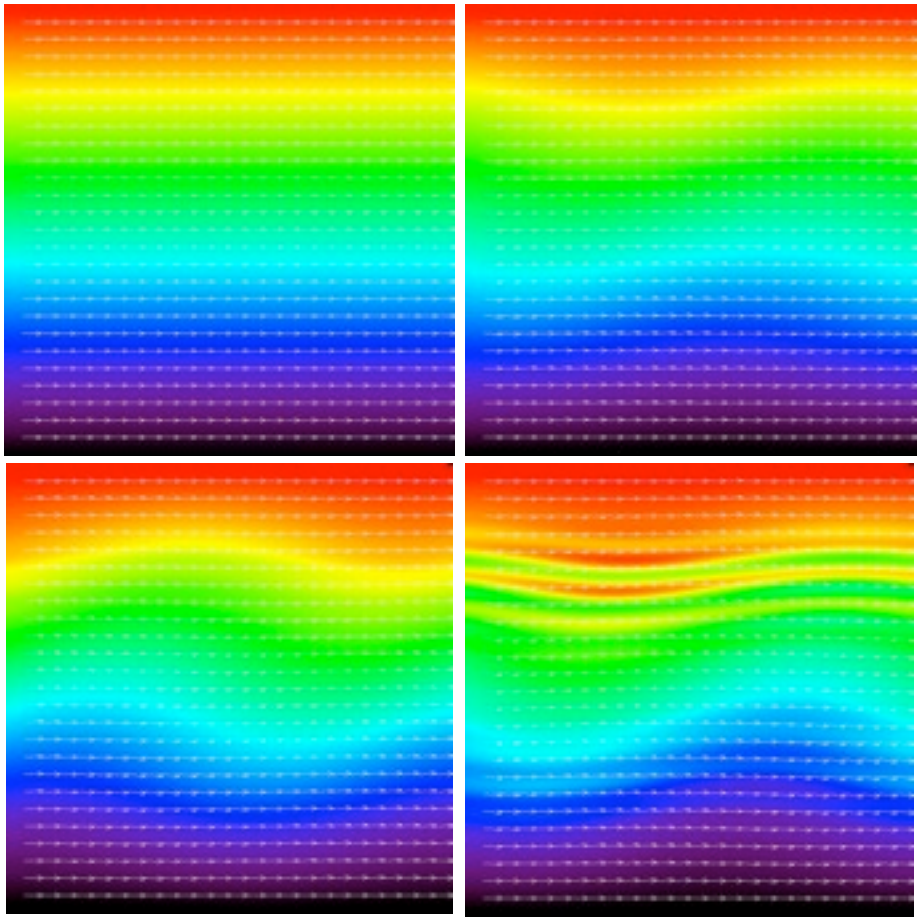


HBI



Thermally stable and overstable to RCO

$\omega_{cool} > \omega_{dyn} \approx \omega_{cond}$



~400 t_{dyn}

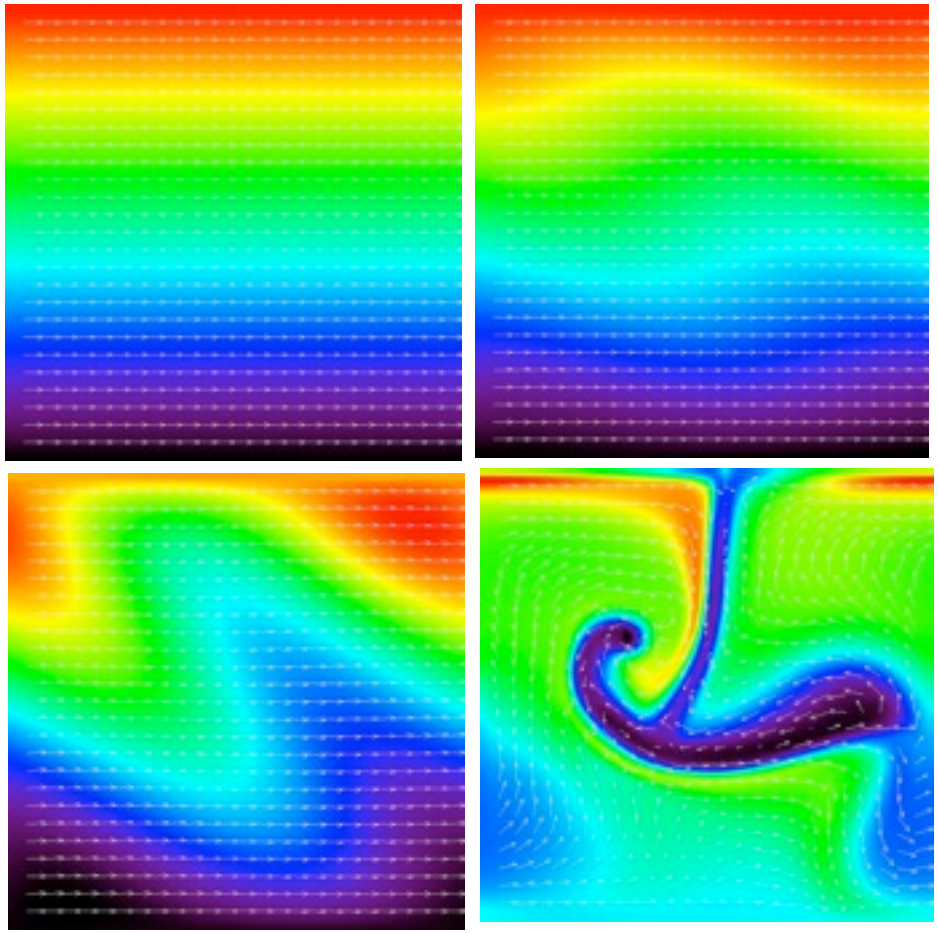
cold

T ⇧ g ⇩

hot

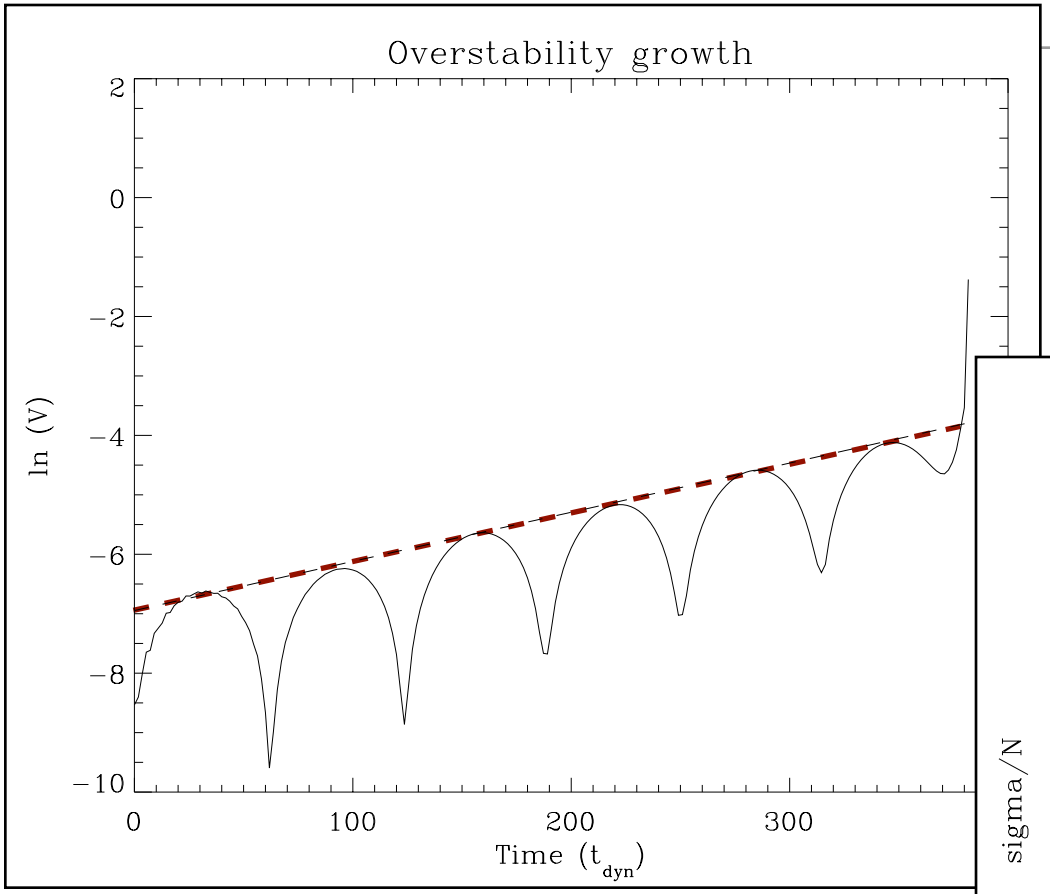
Thermally unstable and overstable to RCO

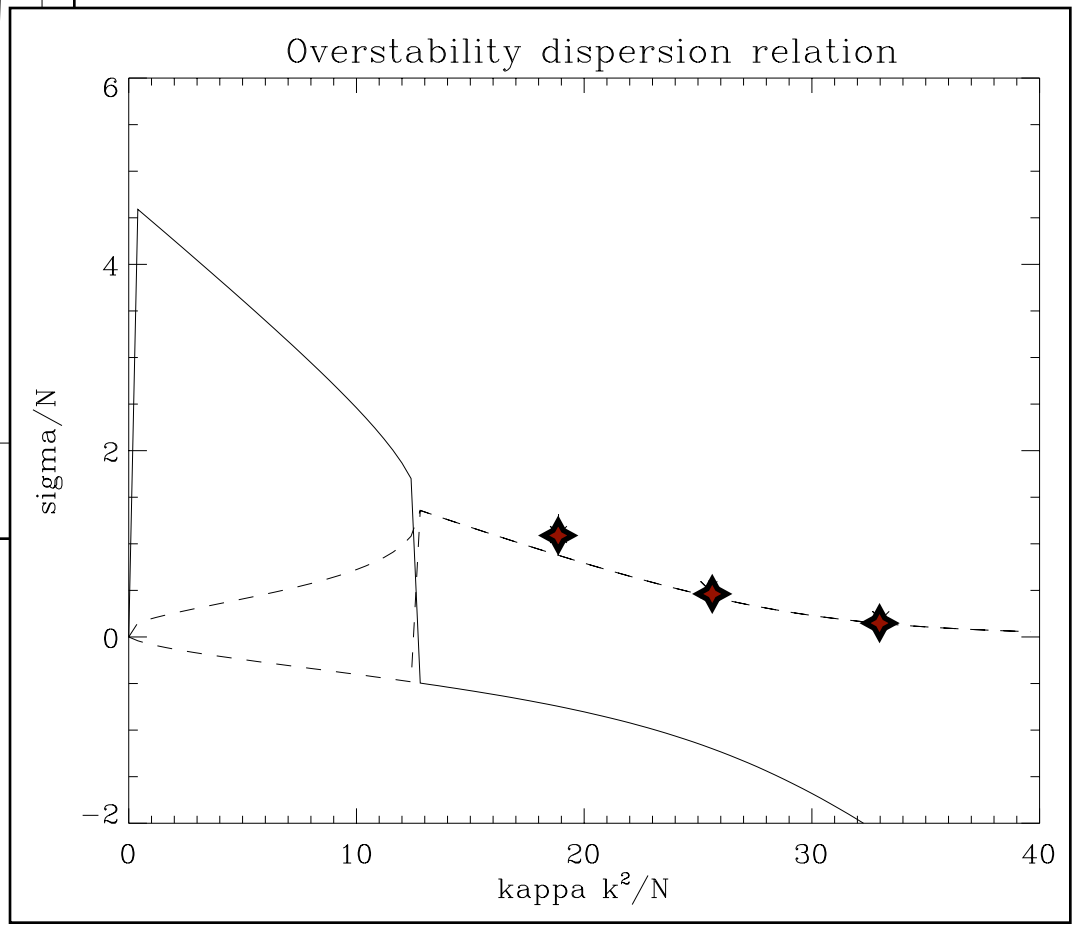
$\omega_{cool} > \omega_{dyn} > \omega_{cond}$



~100 t_{dyn}

RCO growth and dispersion relation





Conclusions & prospects

- MHD overstabilities related to the well known MHD instabilities.
- Analytic theory: Need for thorough understanding of plasma processes (e.g., anisotropic viscosity).
- Simulations: Understanding the relative importance of individual plasma instabilities/overstabilities and their connection.
- Observations: Spectro-polarimetric measurements, measurements of temperature profiles and metallicity in cluster outskirts, measurements of turbulence, other yet to be realized methods...