Few-body problem near a narrow Feshbach resonance

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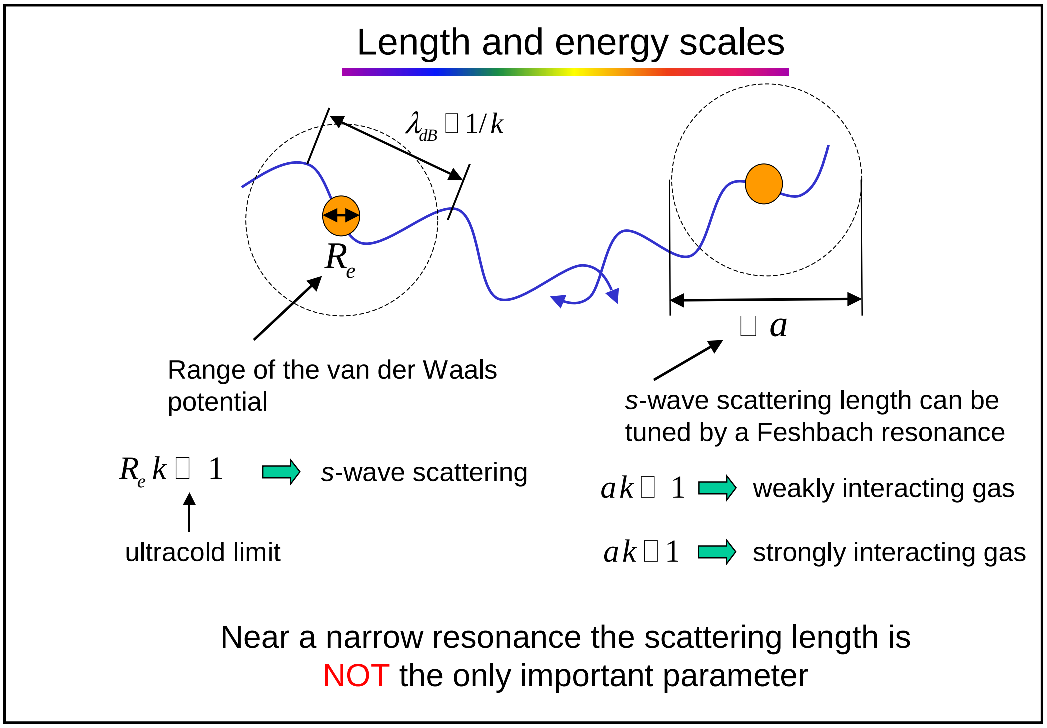
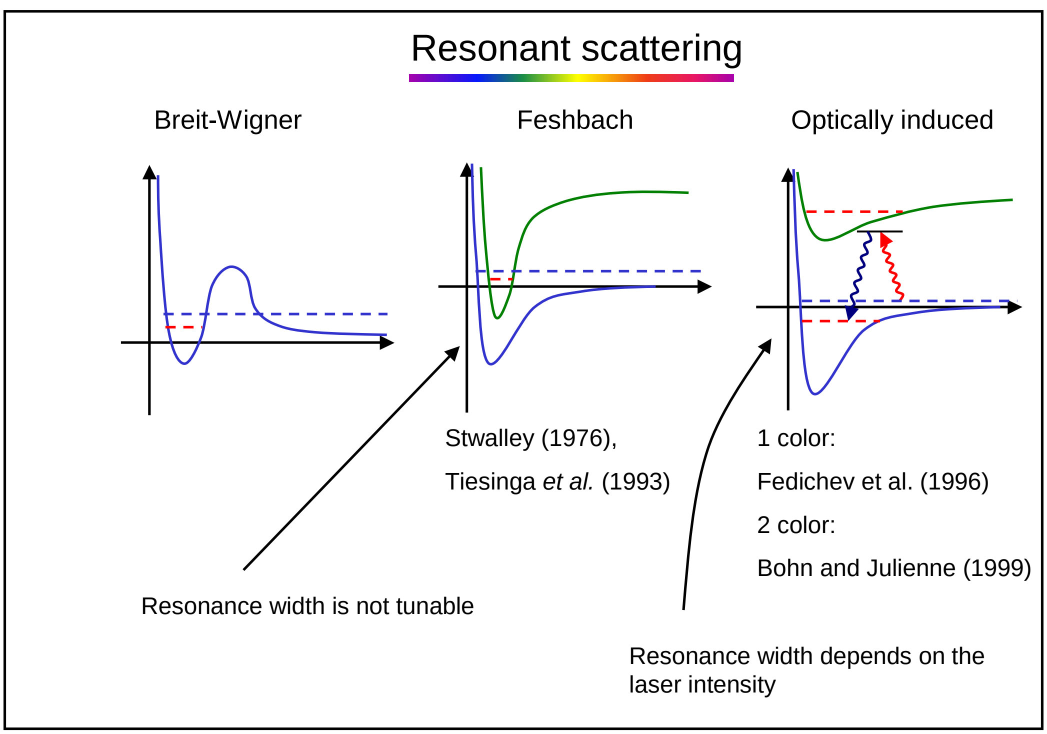
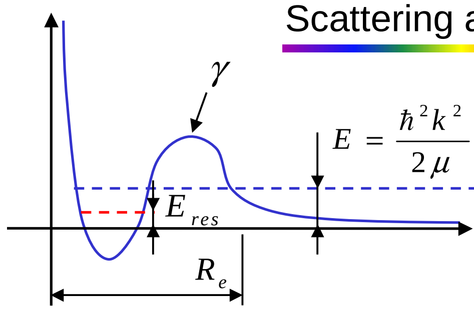
Outline

- Introduction
  - ✓ Length and energy scales in ultracold gases
  - ✓ Strongly interacting or weakly interacting?
- Feshbach resonances
  - ✓ When is the width important?
  - ✓ Can a short range potential look like a long range one?
  - ✓ Two-body bound state
- Three-body physics near a narrow resonance
  - ✓ Atom-dimer scattering
  - ✓ Three-body recombination


Length and energy scales
$\lambda_{dB} \sim 1/k$
R_e
$\sim a$
Range of the van der Waals potential
s-wave scattering length can be tuned by a Feshbach resonance
$R_e k \ll 1$
s-wave scattering
ultracold limit
$ak \ll 1$
weakly interacting gas
$ak \gg 1$
strongly interacting gas
Near a narrow resonance the scattering length is
NOT the only important parameter


Resonant scattering
Breit-Wigner
Feshbach
Optically induced
Stwalley (1976),
Tiesinga et al. (1993)
1 color:
Fedichev et al. (1996)
2 color:
Bohn and Julienne (1999)
Resonance width is not tunable
Resonance width depends on the laser intensity

Scattering amplitude

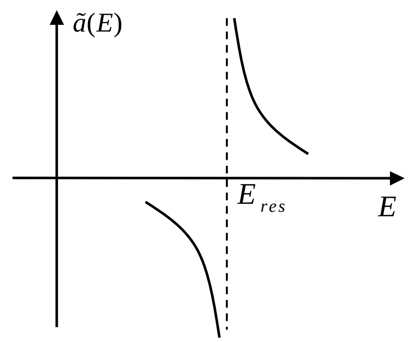


$$f(E) \approx -\frac{\hbar\gamma}{\sqrt{2\mu}(E - E_{res} + i\gamma\sqrt{E})}$$

valid for $kR_e \ll 1$ and for $|f| \gg |a_{bg}| \sim R_e$

$$f(k) \approx -\frac{1}{\underbrace{1/a + R^* k^2}_{1/\tilde{a}(E)} + ik}$$

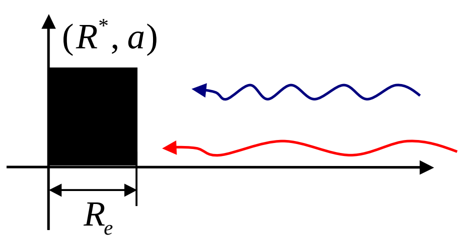
$$\tilde{a}(E) = \frac{\hbar\gamma}{\sqrt{2\mu}(E - E_{res})} = \frac{1}{1/a + R^* k^2}$$



$$f(k) \approx -\frac{1}{1/a + R^* k^2 + ik}, \quad kR_e \ll 1$$

$$\tilde{a}(E) = \frac{1}{1/a + R^* k^2}$$

$a = \infty$ ⇨ $\tilde{a}(k)k = (R^* k)^{-1}$



$k \gg 1/R^*$ ⇨ off-resonant

$k \ll 1/R^*$ ⇨ resonant

Strong momentum dependence of the scattering amplitude – similar to the Ramsauer effect

“New” effective range is $\sim R^*$ (strictly speaking, $r_0 = -2R^*$)

We are interested in the case of NARROW resonances ⇔ $R^* \gg R_e$

What is R^* in reality?

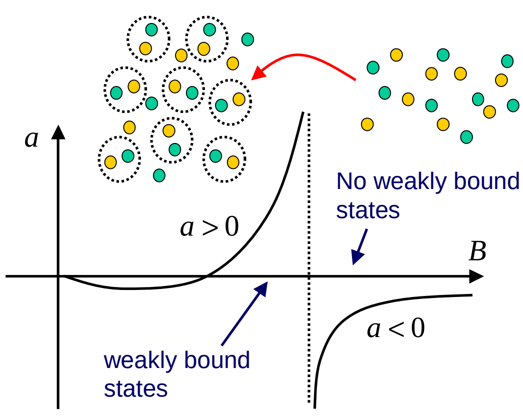
$$R^* = \frac{\hbar^2}{2\mu a_{bg} \frac{\partial E_{res}}{\partial B} \Delta_B}$$

	R_e [Å]	B_0 [G]	Δ_B [G]	$\partial E_{res}/\partial B$	a_{bg} [Å]	R^* [Å]
^{6}Li	30	543.25	0.1	$2\mu_B$	32	19000
^{23}Na	45	907	1	$3.7\mu_B$	33	260
^{87}Rb	85	1007.4	0.17	$2.5\mu_B$	60	320
^{133}Cs	100	19.8	0.005	$0.55\mu_B$	160	13000

In all examples $R^* \gg R_e, a_{bg}$

It would be interesting to study these "long range forces" in the context of many body physics

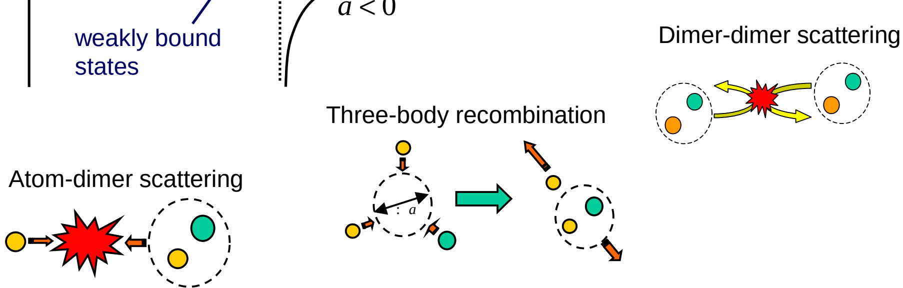
Atom molecule conversion



Observation of molecules in Fermi gases - JILA, ENS, Innsbruck, Rice, MIT (2003)

and in Bose gases – MIT, Garching, Innsbruck (2003)

Bose-Fermi molecules?



Weakly bound molecular state

Weakly bound ⟺ $|E_b| = \hbar^2\kappa^2/2\mu \ll \hbar^2/2\mu R_e^2$ ⟺ $1/\kappa \gg R_e$

$$f(k) \approx -\frac{1}{1/a + R^* k^2 + ik}, \quad kR_e \ll 1$$

position of the pole ⇩

$$\kappa = (\sqrt{1+4R^*/a} - 1)/2R^*$$

⇩ residue at the pole

$$\varphi_b(r) = \frac{1}{\sqrt{1+2\kappa R^*}}\sqrt{\frac{\kappa}{2\pi}}\frac{e^{-\kappa r}}{r}$$

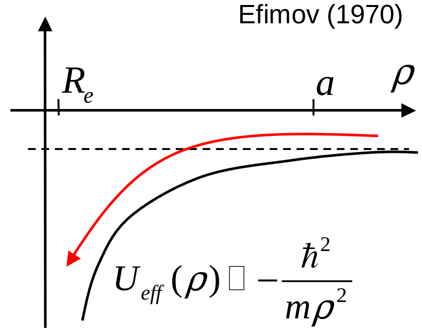
Small detuning ⟺ $a \gg R^*$ ⇨ $1/\kappa \approx a$, open channel

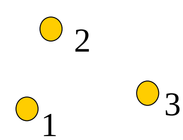
Intermediate detuning ⟺ $R_e \ll a \ll R^*$ ⇨ $1/\kappa \approx \sqrt{R^* a} \gg R_e$, closed channel

Three-body problem (symmetric case)

Large a, independent of energy

Efimov (1970)





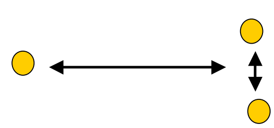
$\rho = \sqrt{(r_{12}^2 + r_{13}^2 + r_{23}^2)/3}$ - hyperradius

ρ^{-2}- attraction

⇩

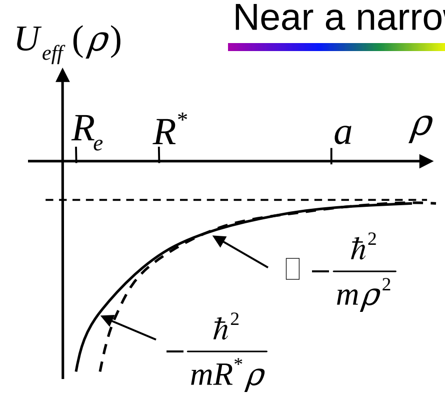
"Fall of a particle to the center"

Energy stored in the pairwise interaction transfers to the kinetic energy of motion along the hyperradius



The transferred energy is huge $\sim U_{eff}(R_e) \sim \frac{\hbar^2}{mR_e^2} \sim 1\text{mK}$

Near a narrow resonance



At $\rho < R^*$ collision energies in pairs of atoms become "off-resonant"

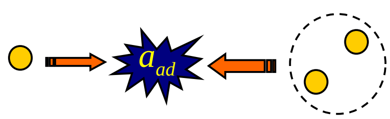
At $a \gg R^*$ the role of the three-body parameter is played by R^*, which is a two-body observable.

Regime of intermediate detuning $(a \ll R^*)$ is fundamentally different

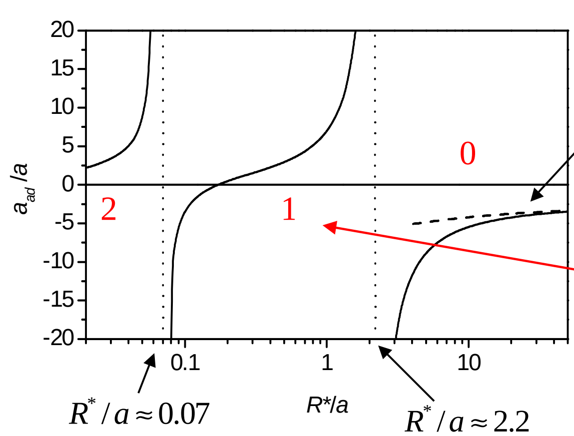
Here Efimov's physical picture is significantly modified!

Three-body observables are determined uniquely by R^* and a

Cold atom-dimer scattering



We generalize the method of Skorniakov and Ter-Martirosian for narrow resonances

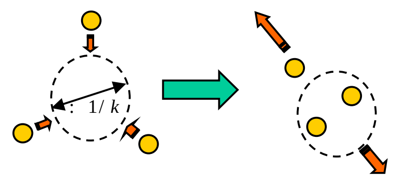


$$\frac{a_{ad}}{a} \approx -\frac{8}{3} - \frac{44}{9}\sqrt{\frac{a}{R^*}}$$

Small parameter $\sqrt{a/R^*}$

Number of Efimov (weakly bound trimer) states

Three-body recombination to a weakly bound state



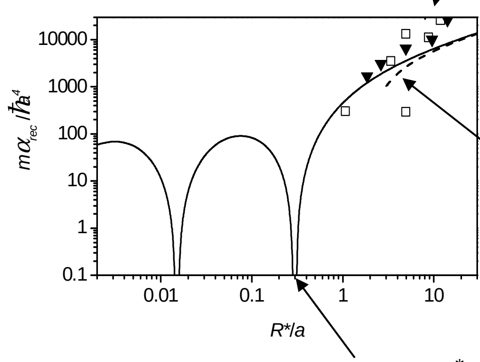
Three-body recombination rate

$$\nu_{rec} = \alpha_{rec} n^2$$

$$\alpha_{rec} = C\hbar a^4 / m$$

$C = 3.9$ Fedichev *et al.*, (1996)

$C < 68.4$ Nielsen and Macek, Esry *et al.*, (1999), Bedaque *et al.*, (2000)



MIT experiment
Stenger *et al.* (1999)

$$\alpha_{rec} \approx 192\sqrt{3}\pi^2 \frac{\hbar}{m}\sqrt{a^7 R^*}$$

$a = 3.3R^*$ - no recombination!

Summary

- Narrow Feshbach resonances have features of long-range potentials
- Two- and three-body physics is fundamentally different in the regimes of small ($a>>R^*$) and intermediate ($R_e<<a<<R^*$) detuning
- In any case three-body observables are determined by *a* and *R** - both are low-energy two-body observables
- In the case of a narrow resonance the effective 3-body attraction is weaker than in the case of a wide one – reduced relaxation to deeper bound states?

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