• Entanglement and precision probing with light and atomic ensembles

Entanglement and precision probing with light and atomic ensembles

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“… . Every atom, impressed with good and with ill, retains at once the motions which philosophers and sages have imparted to it, mixed and combined in ten thousand ways with all that is worthless and base. The air is one vast library, on whose pages are for ever written all that man has ever said or woman whispered.”

Charles Babbage
Ninth Bridgewater Treatise, 1837
Entanglement and precision probing with light and atomic ensembles

Information processing powered by steam.

1832:
Babbage Difference Engine (2000 moving parts)
Mechanical \(\rightarrow\) programmable design (Jacquard-loom)

Quantum Gases
Many-body physics
Quantum Information

Klaus Molmer, U of Aarhus (KITP Quantum Gases Conference 5/13/04)
• Entanglement and precision probing with light and atomic ensembles
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This talk

- Quantum Information with many-atom states
- Entanglement, spin squeezing, precision probing

*Work done in collaboration with:*
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Lars Bojer Madsen, Vivi Petersen, Jacob Sherson,
Klemens Hammerer, Ignacio Cirac, Eugene Polzik

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Hamiltonian evolution vs. measurement dynamics.

Solution of Schrödinger Equation:

\[ \Psi \rightarrow U(t) \Psi \]

Measurement on quantum system:

\[ \Psi \xrightarrow{\text{outcome}} \Psi_m = P_m \Psi / \sqrt{P_m} \]

System must be intact in new state. BEC special case!

Plan for what to do for given, random \( m \):

E.g.: accept only \( m = m_0 \), otherwise try again,

or accept all states (but remember outcome!)

Motivation, what states to make?

Entangled and squeezed states can be used for high precision purposes, quantum communication and computing.

"Schrödinger-strategies" exist, but they may be inefficient, sensitive to noise, or simply impractical – hence we look at measurement strategies.

Teleportation uses measurements,

Knill-Laflamme-Milburn proposal for quantum computing using linear optics + measurements (and feedback).
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A simple example: a pair of atoms

Light does not change atomic state, but measurement of phase shift of light, gives information about the atoms!

Product state, \( (|g> + |f>)(|g> + |f>) \) is changed:

- \( n_t = 0 \) or \( 2 \) \( \rightarrow \) product states \( |gg> \) or \( |ff> \) (try again)
- \( n_t = 1 \) \( \rightarrow \) entangled state \( (|gf> + |fg>) \)

A. Sørensen & KM (2003). (Cabrilo et al, Chris Monroe: experiments)

"Which-atom-decayed entanglement" (Cabrilo et al)

Recent experiments by Chris Monroe (Michigan)
Remote quantum computing:
Repeat until successful (purple atoms)
Local operations \( \rightarrow \) coupling of (red) qubit atoms
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Entanglement of large atomic samples

Interaction $\rightarrow$ shift of optical phase $\sim n_i$

Product state, $(|g> + |f>)^N$ $(|g> + |f>)^N$

Measure phase shift $\rightarrow$ state with better defined $n_{11} + n_{12}$


Demonstrated in Århus (and in Copenhagen) (Polzik)

Gaussian states of atoms and light

• All atoms in $(|\uparrow> + |\downarrow>)~/2 \rightarrow <J_z>=N_x/2$, $<J_x>=<J_y>=0$

  $\text{Var}(J_x)\text{Var}(J_y) = |<J_x>|^2/4 \rightarrow \text{binomial noise (M.U.S.)}$.

  let $p_{at} = J_z/\sqrt{<J_z>}$, $x_{at} = J_y/\sqrt{<J_y>}$, $[x_{at}, p_{at}] = i$

  harmonic oscillator ground state, Gaussian in $x_{at}, p_{at}$

• $x$-polarized light has $<S_{x ph} = N_{ph}/2$, $<S_y> = <S_z> = 0$.

  let $p_{ph} = S_z/\sqrt{<S_z>}$, $x_{ph} = S_y/\sqrt{<S_y>}$, $[x_{ph}, p_{ph}] = i$

  harmonic oscillator ground state, Gaussian in $x_{ph}, p_{ph}$

• Dispersive atom-light interaction:

  $\sigma^+$ ($\sigma^-$) light is phase shifted more by $|\uparrow>$ ($|\downarrow>$) atoms

  $\rightarrow$ Faraday polarization rotation, proportional to $<J_z>$

  $H_{int} = g J_z S_z = \kappa p_{at} p_{ph}$

  $|\uparrow>$ $|\downarrow>$
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Entanglement theory is hard!

"Let us slip into something more comfortable"

Gaussian states !!!

J. Eisert & M. Plenio, quant-ph/0312071 (2003), and others

Gaussian states

State characterized by mean values, \( \mathbf{m} \), and covariance matrix \( \mathbf{y} \). (GP-wave function and Bogoliubov excitations)

Gaussian states (\( \mathbf{m} \) and \( \mathbf{y} \)) transform under \( xx, xp \) and \( pp \) interactions (linear optics, squeezing), decay and losses.

(Kasevich, Chapman, Sengstock: \( p_{\alpha}p_{\alpha} \))

Gaussian states (\( \mathbf{m} \) and \( \mathbf{y} \)) transform under measurements of \( x \)'s and \( p \)'s (Stern-Gerlach and homodyne detection).

K. Hammerer et al, quant-ph/0312156
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Gaussian distributions (classical)

\[ y = \text{vector of } (x\text{'s and } p\text{'s) with mean values } m. \]
\[ y = \text{matrix of covariances, } y_{ij} = 2<(y_i-m_i)(y_j-m_j)> . \]
\[ P(y) = \mathcal{N} \exp(-(y-m)^T y^{-1}(y-m)) \]

(cf. \( P(y) = \mathcal{N} \exp(-(y-m)^2/2\sigma^2) \) for single variable).

Transformation of Gaussian

QUANTUM CASE:
When we measure an \( x \), its conjugate becomes completely undetermined, Assign infinite variance to \( p \) and remove its correlations with all other elements, and update as before

Linear transformation \( y \to Sy \):
\( m \to Sm \), and \( y \to S y S^T \), e.g., a rotation.

Measurement of one component \( y_2 \) of \( y = (y_1, y_2) \):
\[ P(y_1) \sim \exp(- (y_1-m_1)^T (y_1-m_1) - \text{linear terms in } y_1) \]
New covariance block: \( y_{11} \to y_{11} - y_{12}(y_{22})^{-1}y_{21} \),
New mean value: \( m_1 \to m_1 + y_{12}(y_{22})^{-1}(y_2^2-m_2) \)
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Transformation of Gaussian states

Linear transformation
\[ \mathbf{y} \rightarrow S \mathbf{y} : \]
\[ \mathbf{m} \rightarrow S \mathbf{m}, \]
and \( \mathbf{y} \rightarrow S \mathbf{y} S^T \), e.g.,
a rotation.

Measurement of one component \( \mathbf{y}_2 = \mathbf{y}_2^* \) of \( \mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2) \):

New covariance block: \( \mathbf{y}_{11} \rightarrow \mathbf{y}_{11} - \mathbf{y}_{12} (\mathbf{y}_{22})^{-1} \mathbf{y}_{21} \),
New mean value: \( \mathbf{m}_1 \rightarrow \mathbf{m}_1 + \mathbf{y}_{12} (\mathbf{y}_{22})^{-1} (\mathbf{y}_2^* - \mathbf{m}_2) \)

Loss of light \( \gamma_{\text{ph}} \rightarrow (1-\epsilon)\gamma_{\text{ph}} + \epsilon \)
Atomic decay: \( \gamma_{\text{at}} \rightarrow (1-\eta\Delta t)\gamma_{\text{at}} + 2(\eta\Delta t) \)

Gaussian Quantum states II (loss and decay)

Loss of light:
\[ \gamma_{\text{ph}} \rightarrow (1-\epsilon)\gamma_{\text{ph}} + \epsilon \]

Atomic decay:
\[ \gamma_{\text{at}} \rightarrow (1-\eta\Delta t)\gamma_{\text{at}} + 2(\eta\Delta t) \]
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Update of gaussian atomic state

\[ \gamma = \begin{pmatrix} \gamma_A & 0 \\ 0 & I \end{pmatrix} \rightarrow \begin{pmatrix} \gamma_A & \gamma_{AF} \\ \gamma_{FA} & \gamma_F \end{pmatrix} \rightarrow \begin{pmatrix} \gamma_A & 0 \\ 0 & I \end{pmatrix} \]

Result: Spin squeezing
Entanglement of samples

Continuous limit

Frequent probing
(weak pulses/short segments of cw beam):
Update becomes continuous

Differential equation for covariance matrix

This, so-called, Ricatti equation is non-linear

… but it can sometimes be solved!
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Continuous limit

Frequent probing
(weak pulses/short segments of cw beam):
Update becomes continuous

Differential equation for covariance matrix

This, so-called, Ricatti equation is non-linear

… but it can sometimes be solved!

Atomic spin squeezing due to optical probing.

For the simple atom-light example (binomial distribution):

$$\frac{d}{dt} \text{Var}(p_{at}) = -2\kappa^2 \text{Var}(p_{at})^2 \Rightarrow \text{Var}(p_{at}) = 1/(2 + 2\kappa^2 t)$$

In presence of atomic decay $\eta$ ($\varepsilon<<1$):

$$\frac{d}{dt} \text{Var}(p_{at}) = -2\kappa^2 e^{-\eta t} \text{Var}(p_{at})^2 - \eta \text{Var}(p_{at}) + \eta e^{\eta t}$$

Optimum of $\text{Var}(p_{at}) \sim \sqrt{\eta/(\kappa^2)} \sim 1/\sqrt{N_{at}}$
Occurring at $t=1/(\sqrt{\eta} \kappa) << 1/\eta$
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Update of γ is deterministic but displacement (mean value) varies from shot to shot.

Entanglement of two gases
GEoF

Rotations → EPR state

Loss → optimum:

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Probing of classical parameter

Optical transmission \( \rightarrow \)
Existence and location of atom.
But experimental signal is noisy!
How well can we localize the atom?
\( \leftarrow \) exp. MPQ

Probing of a classical magnetic field

• B-field causes spin precession \( \Psi(t) \)
• Faraday rotation of polarization \( \sim < \Psi(t) | J_z | \Psi(t) > \sim B_y \)
• Detection of light
  what is \( B_y \), what is the error-bar?

• Light detection is random
• Stochastic evolution of atomic quantum state
• Bayesian update for classical probability \( P(B) \):
  \[ P(\text{random signal} \mid B) \rightarrow P(B \mid \text{signal}) \] (H. Mabuchi)
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Probing of a classical magnetic field
(K.M & L. Madsen, quant-ph/0402158)

Our approach:
Treat atoms AND light AND B field as a quantum system
Covariance matrix for \((B, x_{at}, P_{at}, x_{ph}, P_{ph})\).

\[
\Delta B(t)^2 = \frac{(1 + \kappa^2 t) \Delta B_0^2}{1 + \kappa^2 t + 2 \kappa^2 \mu^2 (\Delta B_0)^2 t^3 + \frac{1}{6} \kappa^4 \mu^2 (\Delta B_0)^2 t^4}
\]

Long times: \(\Delta B \sim 1/(N_{at} t^{3/2})\)
• Independent of \(\Delta B_0\)
• not as \(1/\sqrt{N_{at}}, 1/\sqrt{t}\)

fT-magnetometry with atomic probe
(Recent experiments in Nature 422, 596 (2003))

\(\Delta B\) as function of time.
(2 \(10^{12}\) Cs atoms, \(\mu W\) laser power
2 mm\(^2\) cross section, \(\kappa^2 = 1.8 \times 10^6\) s\(^{-1}\))

Lower solid curve: analytical result
Upper solid curve: include spon. em.
(\(\eta = 1.8\) s\(^{-1}\) -GHz detuning)
Dashed curves: polarization squeezed
optical probe

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More field components: divide and conquer

Mean spin along $x$:
$J_z, J_y$ sensitive to $B_y, B_z$,
but we cannot measure both!
Divide gas in two, orient along $+/- x$,
and measure sums and differences
of $z, y$ components (EPR-pair).
Local use of entanglement !!!

Same scaling for long times:
$\Delta B_{x,y} \sim 1/(N_{at} t^{3/2})$
(t $\rightarrow$ t/2)

Measure three B-field components
divide gas in 6, and probe 4 commuting obs.

Same scaling for long times:
$\Delta B_{x,y,z} \sim 1/(N_{at} t^{3/2})$
(t $\rightarrow$ t/3, $N_{at} \rightarrow$ 2/3 $N_{at}$)
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**Outlook I, Gaussian states.**

- Many atoms and many photons are "easy experiments" (classical fields, homodyne detection)
- Many atoms and many photons are "easy theory" (readily generalized at low cost to more samples/fields)
- Gaussian states: squeezing, entanglement, … and also: finite bandwidth sources, finite bandwidth detection
- Gaussian states unify quantum and classical variables: classical B-field + atoms + light probe
  → other observables: interferometry, … .

**Outlook II, Gaussian states.**

- Condensates are good atomic systems for precision probing: well localized, good optical depth, … .
- Condensate and fermion physics with Gaussian states
- Gross-Pitaevskii equation + Bogoliubov excitations
- Probing and dynamics/interaction
- Current project: quantum state tomography from spatial densities recorded at different times
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**Outlook III, non-gaussian states**

- Gaussian states $\rightarrow$ qubits
  conditioned: cf. example with ions, squeezed light on atoms
  unconditioned: (B. Kraus, I. Cirac, quant-ph/0307158)
- Distillation of Gaussian states requires non-gaussian intermediate state
  non-homodyne detection: photon counting
  J. Wenger et al, quant-ph/0402192:

![Diagram of squeezed state generation](image)

**Is parameter estimation by ‘quantization’ correct?**

1. Any classical variable is a quantum variable, that happens to be tractable by classical theory (QM is still correct).
   Here QND measurement: conjugate variable $\pi_B$ not relevant.
2. The "quantum state" is a representation of our knowledge about a system.
   it does not underestimate variances (it is correct)
   more knowledge would be a "hidden variable"

The conditioned mean and the quantum variance is the correct estimator of the parameter!
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<table>
<thead>
<tr>
<th>Conditioned estimator (mean)</th>
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<tbody>
<tr>
<td>Random photocurrent $I(t)$</td>
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<tr>
<td>- varies from shot to shot</td>
</tr>
<tr>
<td>- different $B_{\text{est}} = \langle B \rangle$</td>
</tr>
<tr>
<td>- converges ($\text{Var}(B) \to 0$)</td>
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</tbody>
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- Upper curve:
  - Simple estimator
  - Integrated current
  - $Q \sim B \mu^2 t^{3/6}$

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