ALKALI CASES vs. TRADITIONAL SUPERFLUIDS

Disadvantages:
- almost inevitably inhomogeneous
- v.dilute, so eg mass transport not easily measured.

Advantages:
- geometry can be tailored
- variable parameters
- optical discrimination
- multiple species (eg. tf. of He)
- "near instantaneous" change of parameters

BEC IN A SYSTEM OF BOSONS:

\( \rho_\Psi (\mathbf{r}, \mathbf{r}'; \mathbf{t}) \equiv \sum \rho_\Psi (\mathbf{r}_1, ... \mathbf{r}_N; \mathbf{t}) \Phi_\Psi (\mathbf{r}; \mathbf{r}_1 ... \mathbf{r}_N; \mathbf{t}) \)

\( \phi_\Psi (\mathbf{r}, \mathbf{r}_1 ... \mathbf{r}_N; \mathbf{t}) = \sum \phi_\Psi (\mathbf{r}, \mathbf{r}_1 ... \mathbf{r}_N; \mathbf{t}) \chi_\Psi (\mathbf{r} / \mathbf{r}_1 ... \mathbf{r}_N; \mathbf{t}) \)

\( \Phi_\Psi (\mathbf{r}, \mathbf{r}_1 ... \mathbf{r}_N; \mathbf{t}) \) eigenfunctions

("simple") BEC if one and only one of
\( m_\Psi (\mathbf{t}) \equiv N_\Psi (\mathbf{t}) \sim N, \text{ rest all } 0(3). \text{ Then,}
\)

order parameter \( \Phi_\Psi (\mathbf{r}, \mathbf{t}) \equiv \sqrt{N_\Psi (\mathbf{t})} \chi_\Psi (\mathbf{r}; \mathbf{t}) \)

WHY IS "SIMPLE" BEC THE NORM?

2-state system: \( \hat{\Delta} \hat{\Psi}, \hat{\Delta} \hat{\Psi} \) conjugate variables

\( H \sim \frac{1}{2} \hat{\Delta}^2 + \frac{1}{2} \hat{\Delta}^2 \sim \frac{1}{2} \hat{\Delta}^2 \)

\( \hat{U} / \hbar \to \infty : \text{Four state (eigenfn of } \hat{\Delta} \hat{\Psi}) \)

\( \hat{U} / \hbar \to 0 : \text{current state (eigenfn of } \hat{\Delta} \hat{\Psi}) \)

\( \text{(simple BEC)} \)

Dilute gas:
\( \text{K.E.} = f(\Delta \Psi) \sim \frac{\hbar^2}{2m} n \frac{\Delta \Psi}{n} \)
\( \text{P.E.} = f(\Delta \Psi) \sim g n \sim \frac{g n}{m} \Delta \Psi \)

\( \text{P.E./K.E.} \sim (m \alpha^2 \Delta \Psi)^2 \Rightarrow \text{approx. eigenfn of } \Delta \Psi \)

\( \Delta \Psi \sim \frac{1}{\hbar} \)
Some Examples of Ground States of Bose Systems without "Simple" BEC:

(a) no BEC at all:

- Solid 4He

Mott-insulator phase in optical lattice

- Rapidly moving bosons in QHE regime

(b) "fragmented" BEC:

- Many more one-site + 2-site system in Fock limit

- Attractive interactions

- LPO state of spin-1 bosons

$$KSA^o = \text{spin-1/2}$$

$$\Psi \sim (a_0^+ a_{1,1}^* - a_{0,0}^+ a_{1,1}^+)^{1/2} \mid \text{vac} \rangle$$

Why are spinless, extended Bose cases so rarely fragmented?

(a) Equilibrium: HF approach:

$$E = \sum \xi_n \xi_n^* + \frac{1}{2} U_0 \sum (2 - \delta_{ji}) n_i n_j$$

(b) Nonequilibrium:

- e.g. p → s in annulus:

$$| M, N = M \rangle \sim \int \tilde{F}_{\text{coh}}(\Delta \phi) d(\Delta \phi)$$

- Fock, fragmented simple BEC, chiral

- "best" chiral state always at zero or end of Fock!

The "Mott-Insulator = Superfluid" Transi:

(cf: melting of solid 4He into superfluid phase)

$$| \Psi_M \rangle \sim \bigotimes \varphi(r_n, -r_n) \varphi(r_n, \tau_n) \ldots \varphi(r_n, -r_n)$$

- particle site

$$\Psi_{\text{Sup}} \sim \prod \chi(r_n), \quad \chi(r) \sim \sum \varphi(r - R)$$

- particle sites

In "superfluid" (BEC) state, can take one particle right across sample, hopping on other N-1 times, and Making wave function vanish or become very small.

(c) This is essential to explain of superfluidity

In MI state, this is not true (hence not superfluid)

So: How long does the system take to "find out" that it should be superfluid?

- (and how accurately does it do so?)

- a) time for single atom to move across whole lattice

- b) time to jump to nearest-neighbor site.
Mott Ins'r $\mapsto$ Superfluid, end.

Related problems:

A. Higgs-Kibble mechanism in cosmology
   analog simulations of atom (?) in sup. $^3$He (some)
   $\Psi$: (a) interpret? of raw data open to question
   (b) normal-superfluid contrast apparently essential (and absent in early Universe)

B. "Supersolid" behavior of solid (?) $^4$He
   (Chen & Kim, Nature (2004))

\[ \begin{array}{cccccccc}
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\end{array} \]

destroyed by very small (10-100 ppm) of $^3$He!

Can one introduce "chemical impurities" in optical-lattice problem?

"Pseudo-BEC" (Cooper Pairing) in a

\[
\begin{align*}
\rho^v(r, r_i, r_i'; t) & \equiv \sum_{\nu} \sum_{\nu'} \left| \psi^v_\nu(r, r_i; r_i', t) \right|^2 \\
\psi^v_\nu(r, r_i; r_i', t) & \equiv \langle \psi^\dagger(r, r_i) \psi^\dagger(r_i') \psi^\dagger(r_i') \psi(r_i') \rangle(t) \\
& \equiv \sum_i \chi_i^v(r, r_i; t) \chi_i^v(r_i', r_i'; t) \\
\end{align*}
\]

For unrestricted no. of single-particle states,

\[
\sum \chi_i^v(r, r_i; t)^2 \leq N
\]

max. value of any $\chi_i^v$ is $N/2$.

(Fermi sea: all $n_i^v = 1$ or 0)

"Simple" Pseudo-BEC if one and only one

of eigenvalues is $O(N)$, rest $O(1)$

BCS theory:

\[ \chi^v_0(r, r_i; t) \propto F(r, r_i) \]

\[ F(p) \equiv \frac{\sin k_F p \rho_{\text{ext}} - \Phi(\rho_{\text{ext}})}{\hbar k_F} \]

\[ \frac{N_0(T)}{N} \approx \Delta \approx \frac{T_c}{T_F} \]

$\leq 1$ in BCS limit
Why "Simple" Pseudo-BEC (i.e. no fragments)?

Not Trivial! (cf. Gorkov & Grotch 196)

Generally, if 2 states possible and nonzero scattering between them, single fermion state found in thermodynamic limit.

Ex. ^3K^-A:

poss. pair states have same orbital dependence, but can be |111> or |111>.

Which is correct:

(a) \( \Phi_N \sim (|111> + e^{i\phi}|111>)^{1/2} \) column

(b) \( \Phi_N \sim |111>^{1/2} \) Fock

Relevant terms in Hamiltonian:

- polarization energy \( S^z/2k \) from \( \langle S^z \rangle = 0 \)
- dipole interaction \( g_3(1-\langle Aq^3 \rangle) \) in Fock state

in thermodynamic limit:

\( \chi \sim N, g_3 \sim N \)

\( \Rightarrow \) column state always favored

\( \uparrow \): for small enough samples, Fock wins!

(\( \Rightarrow \) theory of NMR must be re-done)

The "BEC-BCS Crossover" Problem

1. What is optimal measure of "degree of condensation" \( f \)?
   (All reasonable measures give \( f \sim 1 \) in BEC limit)

   \( \beta_N \) state

   \( S_{\text{max}}, T=0 \)

   max. eigenvalue of \( \hat{\beta}_N \)

   0 \( \sim T_c/T_F \) \( \ll 1 \)

   "superfluid" response

   0 \( \sim 1 \)

   "degree of pairing" \( \sum \hat{n}_k \hat{n}_{-k} \sim 1-0(T_c/T_F) \)

   Which of the above (if any) is measured by various setups in the crossover region?

2. Equilibrium state:

   - is single-channel model adequate (for real-life FR) in (a) strong-repulsion limit
     (b) weak-repulsion limit?

   - is the "naive" Eagles estimate
     (a) qualitatively correct over wide crossover region? (no: CMB, PSS)
     (b) qualitatively ("topologically") correct?

   \( \star \) Can we bound \( T_c/N_0(T) \)?
3. Nonequilibrium (magnetic-field-driven) effects:

- \( t/\Omega(\hbar\Omega) \) — inverse gap freq.
- \( t_{\text{pair}} \) — time to change \( N_0 \) at cond. \( \hbar\Omega \)
- \( t_{\text{rew}} \) — time for internal equilibrium of quasiparticle gas.
- \( t_{\text{rew}} \) — time for thermal equilibrium with "bath" (of dressed photons in red cavity)

If sweep is fast compared to \( t_{\text{pair}} \) and \( t_{\text{rew}} \), but
slow compared to \( t/\Omega \) (at relevant \( \hbar\Omega \)), would provide
photon imprint of "degree of pairing".
If slow compared to all of \( t_{\text{rew}}, t_{\text{pair}}, t/\Omega \), should
consume energy (?)

4. NMR behavior:

To extend that \( H(s) = f(\tau) - \frac{1}{2} \cdot k_{\text{eff}} \).

Resonance must be \( \delta \)-function at Larmor frequency
\( k_{\text{eff}} \) (of suppressed \( k_{\text{LA}} \)).

5. MISC: quench in BCS limit, p-wave.