How cold atoms live in (and escape from) 1D

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The 1D Cold Atom Zoo

• **Cold atoms on a chip:**
  

• **2D optical lattices:**
  
  [M. Greiner *et al.* *PRL* **87** (2001),
  H. Moritz, *PRL* **91** (2003),
  T. Stoferle *et al.* *PRL* **92** (2004)]
Outline:

• Life in one dimension: Luttinger liquids

• Life in a trap: bosonic atoms in a 1D box

• Experiments in 3D and 2D optical lattices

• Escaping from 1D: Phases of a 2D optical lattice
A crash course in Luttinger liquids (1)

• What is it made of? Bosons or Fermions?

“In 1D [...] the symmetry of the wave function cannot be tested by a continuous change of coordinates that exchanges particles without close approach (collision). Thus interaction and statistics effects cannot be separated.”

[FDM Haldane, PRL 47 (1981)]

• Collective modes exhaust the low-energy spectrum:

\[
H = \frac{\hbar}{2\pi} \int dx \left[ v_J (\partial_x \phi)^2 + v_N (\partial_x \theta)^2 \right]
\]

phase stiffness
density stiffness
A crash course in Luttinger liquids (2)

• Collective modes have linear dispersion:

\[ \omega(q) = v_s q \]

\[ \omega(q \to 0) = v_s q \]

\[ v_s = \sqrt{\frac{v_J}{v_N}} \]

\[ K = \sqrt{\frac{v_J}{v_N}} \]

• The road map: range of \( K \) for the Bose-Hubbard model

\[ K = 1 \text{ (Tonks gas)} \quad U/J = +\infty \]

\[ K \gg 1 \text{ (Quasi-condensate)} \quad U/J \to 0 \]

When your cage is too small: mesoscopic LL’s

- Cold atoms on a chip:

- 2D Optical lattices:

\[ N_0 \sim 10 \text{ to } 10^3 \text{ atoms} \]
A toy model: bosonic atoms in a 1D box

- **Phase correlations**: \( \langle \Psi^\dagger(x) \Psi(x') \rangle \approx \rho_0 \langle e^{-i\phi(x)} e^{i\phi(x')} \rangle \)

[Thermodynamic limit (T = 0)]

\[
g_1(x) = \langle \Psi^\dagger(x) \Psi(0) \rangle = \frac{A}{x^{1/2K}}
\]

\( x'/L = 0.5 \)

\( x'/L = 0.1 \)

\( \nabla = x \)

\( \bullet = x' \)

Momentum distribution in a 1D finite LL

- **Fermi Hubbard model** $U = +\infty$
  
  (away from half-filling)

$$\alpha_{\text{app}}(kL) = \frac{dn(k, L)}{d \ln kL}$$

$$n(k, L) \sim |k|^\alpha_{\text{app}}$$

$$\alpha(L \to \infty) = 0.125$$

[S. Eggert et al. PRL 26 (1996)]

- **Momentum distribution**: $n(p, L) = (\rho_0 L)^{1 - \frac{1}{2k}} I(pL)$
Experiments: 3D optical lattices

Superfluid to Mott insulator transition in a 3D optical lattice

Superfluid

Mott Insulator

Phase diagram:

Experiments:
3D optical lattices

Laser

[Laser]

[D. Jaksch et al. PRL 81 (1998)]


[MPA Fisher et al. PRB 40 (1989)]
Excitation spectrum: Bragg spectroscopy (3D)

• Bose-Hubbard model:

\[ H_{BH} = \sum_{R,m} \left[ -\frac{J_x}{2} \left( b_{m+1}^\dagger(R)b_m(R) + b_m^\dagger(R)b_{m+1}(R) \right) + \epsilon_m(R)b_m^\dagger(R)b_m(R) \right] - J \sum_{\langle R,R' \rangle,m} b_m^\dagger(R)b_m(R') + U \sum_{R,m} b_m^\dagger(R)b_m^\dagger(R)b_m(R)b_m(R) \]

\[ J_\alpha \left( \frac{V_{0\alpha}}{E_R} \gg 1 \right) = \frac{4E_R}{\sqrt{\pi}} \left( \frac{V_{0\alpha}}{E_R} \right)^{1/4} \exp \left[ -2 \left( \frac{V_{0\alpha}}{E_R} \right)^{1/2} \right] \]

\( J_\alpha \quad (\alpha = x, y, z) \)

Bragg spectrum for \( J_x = J \)

• 2-photon Bragg spectroscopy:

\[ V_{0x} \rightarrow V_{0x}(t) = [V_{0x} + A_{\text{mod}} \sin (2\pi \nu_{\text{mod}} t)] \]

(i.e. modulate the axial optical potential)

[ T. Stoferle et al. PRL 92 (2004)]
Exc. spectrum: Bragg spectroscopy ($J_x \gg J$)

- $V_{0x,y} = 30 E_R$:
  Bragg spectrum for $J_x \gg J$

Broad Spectrum: 1D SF (LL)

Discrete features: 1D MI

\[
\left( \frac{J_x}{J} \right)_{\text{max}} \approx 25
\]

[T. Stoferle et al. PRL 92 (2004)]
Excitation spectrum: deconfinement!

- **3D Superfluid**

- **1D Mott Insulator**

\[
\frac{J_x}{J} \approx 10
\]

- \( V_0 = 20 \, E_R \): 3D SF to 1D MI

By reducing the axial hopping intertube coherence is destroyed!!

[ T. Stoferle et al. PRL 92 (2004)]
Where does the phase transition takes place?

Large quantum depletion!!

\[
\left( \frac{U}{zJ} \right)_{1D} \approx 1.9
\]

\[
\left( \frac{U}{zJ} \right)_{3D} \approx 5.8
\]

[ C. Kollath et al. PRA 69 (2004)]

[ T. Stoferle et al. PRL 92 (2004)]
2D optical lattices: effective low-energy theory

Through “bosonization”:

\[ H_{\text{eff}} = \frac{\hbar v_s}{2\pi} \sum_R \int_0^L dx \left[ \frac{1}{K} \left( \partial_x \theta_R(x) \right)^2 + K \left( \partial_x \phi_R(x) \right)^2 \right] \]

\[ + \frac{\hbar v_g u}{2\pi a^2} \sum_R \int_0^L dx \cos(2\theta_R(x) + \delta \pi x) \]

\[ - \frac{\hbar v_g J}{2\pi a^2} \sum_{\langle R, R' \rangle} \int_0^L dx \cos(\phi_R(x) - \phi_{R'}(x)) \]

“Mott” potential: localizes atoms

Josephson coupling: delocalizes atoms

[AFH, MAC & T Giamarchi, PRL 92 (2004)]
2D optical lattices: phase diagram at $T = 0$

- Renormalization-group flow:

$$\frac{dg_F}{d\ell} = \frac{g_J^2}{K},$$
$$\frac{dg_J}{d\ell} = \left(2 - \frac{1}{2K}\right)g_J + \frac{g_J g_F}{2K},$$
$$\frac{dg_u}{d\ell} = (2 - K)g_u,$$
$$\frac{dK}{d\ell} = 4g_J^2 - g_u^2 K^2,$$
$$\ell \approx \ln \mu / T$$

[AFH, MAC & T Giamarchi, PRL 92 (2004)]
2D optical lattice of finite tubes: phase diagram

Array of atomic ‘quantum dots’

- Quantum phase Hamiltonian ($g_u = 0$):

$$H_{QP} = -E_J \sum_{\langle R, R' \rangle} \cos(\phi_{0R} - \phi_{0R'})$$

$$+ \frac{E_C}{2} \sum_R (N_R - N_0)^2 - \mu \sum_R N_R$$

$$E_J \approx JN_0^{1-\frac{1}{2\pi}}$$

$$E_C = \frac{\hbar \pi v_s}{KL}$$

- Incommensurate fillings:

[AFH, MAC & T Giamarchi, PRL 92 (2004)]
2D optical lattices: 3D Superfluid (BEC) phase

- **Mean-field theory**: condensate fraction $\psi_0^2(T = 0) \sim \rho_0 \left( \frac{J}{\mu} \right)^{1/(4K - 1)}$

- **Variational approach**: momentum distribution at $T = 0$

\[
\frac{n(Q, q)}{|w(Q)|^2} \simeq \psi_0^2 \delta(Q) \delta(q) + \frac{\pi b^2 \psi_0^2 / 2K}{\left[ q^2 + (v_{\perp} Q / v_s)^2 \right]^{1/2}},
\]

transverse velocity:

\[v_{\perp} \sim \mu b (J/\mu)^{2K/(4K - 1)}/\hbar\]

- **RPA**: condensation temperature and excitation spectrum

\[
\left( \frac{2\pi T_c}{\hbar v_s \rho_0} \right)^{2 - 1/2K} = f(K) \frac{4J}{\hbar v_s \rho_0}
\]

\[\omega(Q)\]

\[\omega_+\]

\[\omega_-\] (Goldstone)

[AFH, MAC & T Giamarchi, PRL 92 (2004)]
• 2D optical lattice: phase diagram

![Phase diagram for 2D optical lattice showing Anisotropic 3D SF (BEC) phase with phase boundaries and labels for different γ values: (γ = +∞), (γ = 8), (γ = 3.5), (γ = 2).]