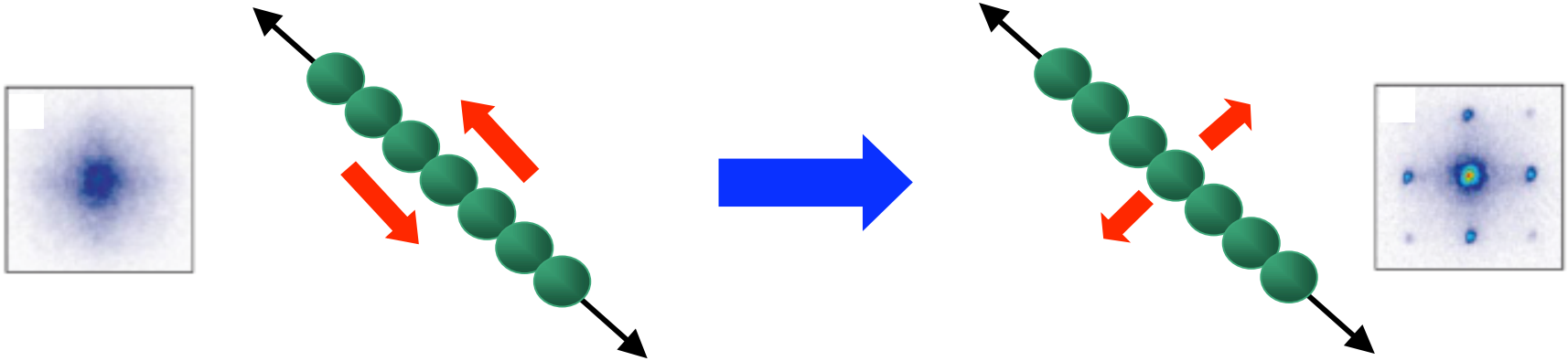
How cold atoms live in (and escape from) 1D



Andrew F. Ho[1], Miguel A. Cazalilla[2] & Thierry Giamarchi[3]

[1] Theory Group, University of Birmingham (UK)

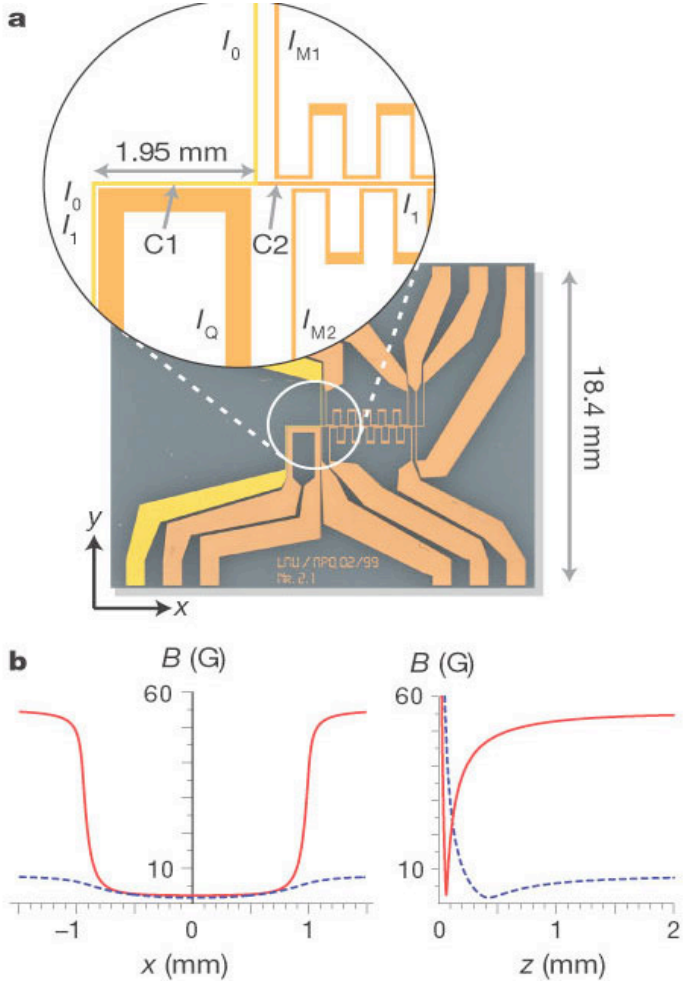
[2] Donostia Int'l Physics Center (DIPC), San Sebastian (Spain)

[3] École de Physique, University of Geneva (Switzerland)

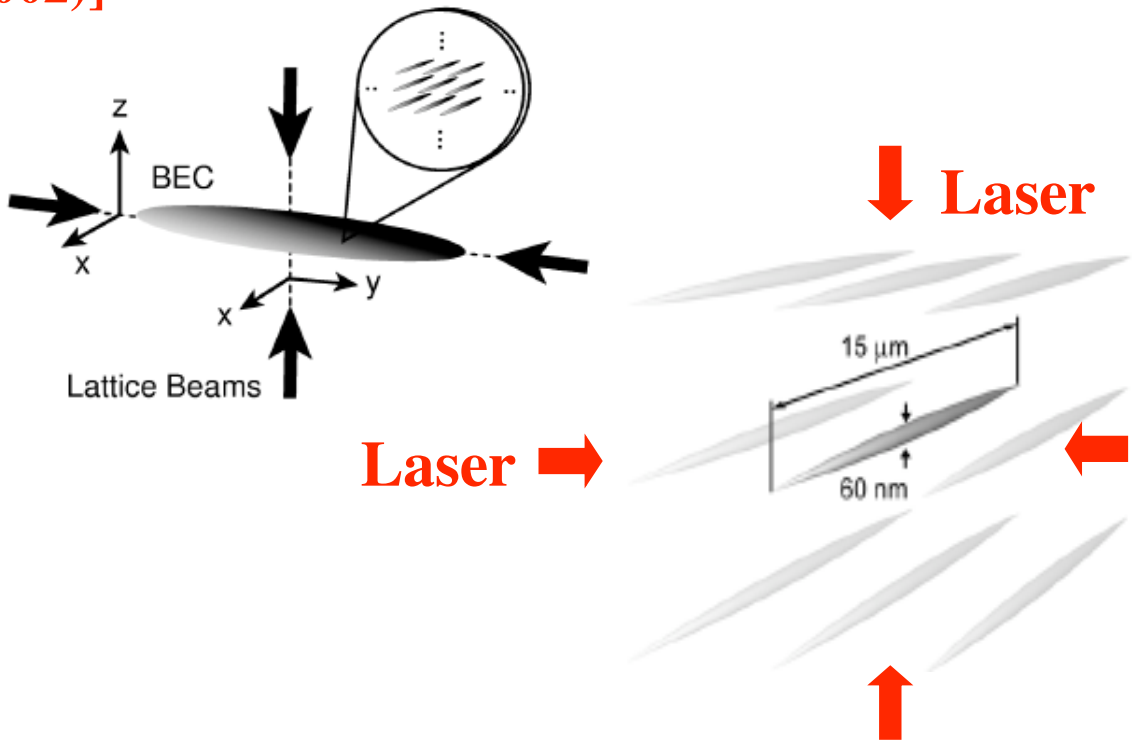
The 1D Cold Atom Zoo

- **Cold atoms on a chip:**

[W. Haensel *et al.* Nature 413 (2002)]



- **2D optical lattices:**



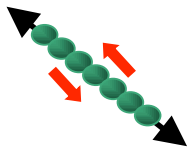
[M. Greiner *et al.* PRL 87 (2001),

H. Moritz, PRL 91 (2003),

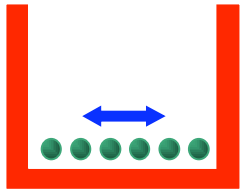
T. Stoferle *et al.* PRL 92 (2004)]

Outline:

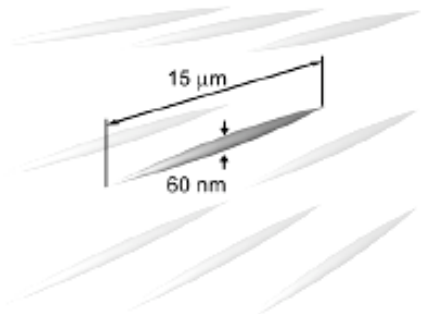
- Life in one dimension: Luttinger liquids



- Life in a trap: bosonic atoms in a 1D box

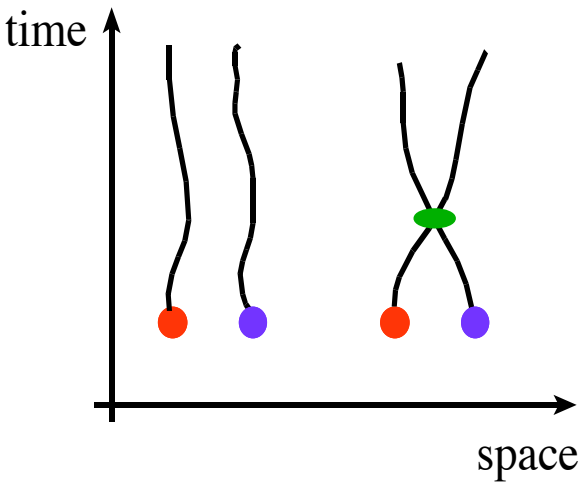


- Experiments in 3D and 2D optical lattices



- Escaping from 1D: Phases of a 2D optical lattice

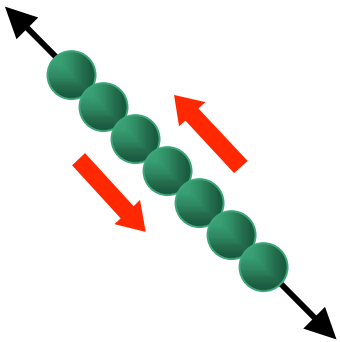
A crash course in Luttinger liquids (1)



- **What is it made of ? Bosons or Fermions?**

"In 1D [...] the symmetry of the wave function cannot be tested by a continuous change of coordinates that exchanges particles without close approach (collision). Thus interaction and statistics effects cannot be separated."

[FDM Haldane, PRL 47 (1981)]



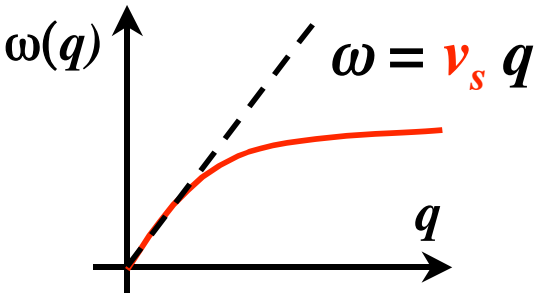
- **Collective modes exhaust the low-energy spectrum:**

$$H = \frac{\hbar}{2\pi} \int dx \left[v_J \left(\partial_x \phi\right)^2 + v_N \left(\partial_x \theta\right)^2 \right]$$

phase stiffness (v_J) — **density stiffness** (v_N)

A crash course in Luttinger liquids (2)

- **Collective modes have linear dispersion:**



$$\omega(q \to 0) = v_s q \qquad v_s = \sqrt{v_J\, v_N}$$

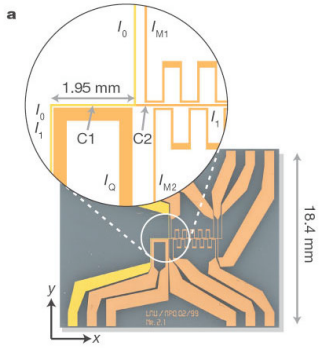
$$K = \sqrt{\frac{v_J}{v_N}}$$

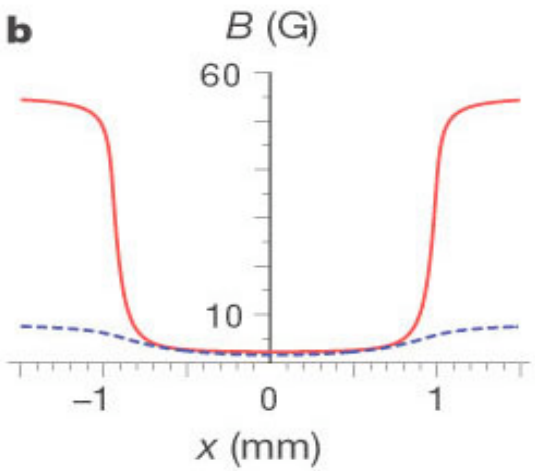
- **The road map: range of *K* for the Bose-Hubbard model**

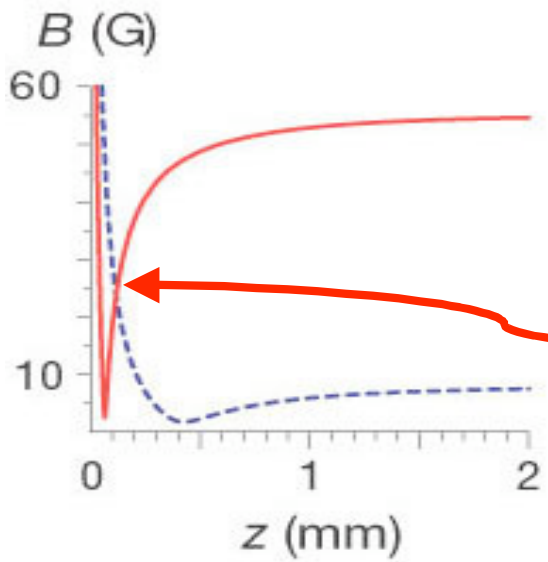
[T. Giamarchi, "Quantum Physics in One dimension", Oxford University Press (2003)]

When your cage is too small: mesoscopic LL's

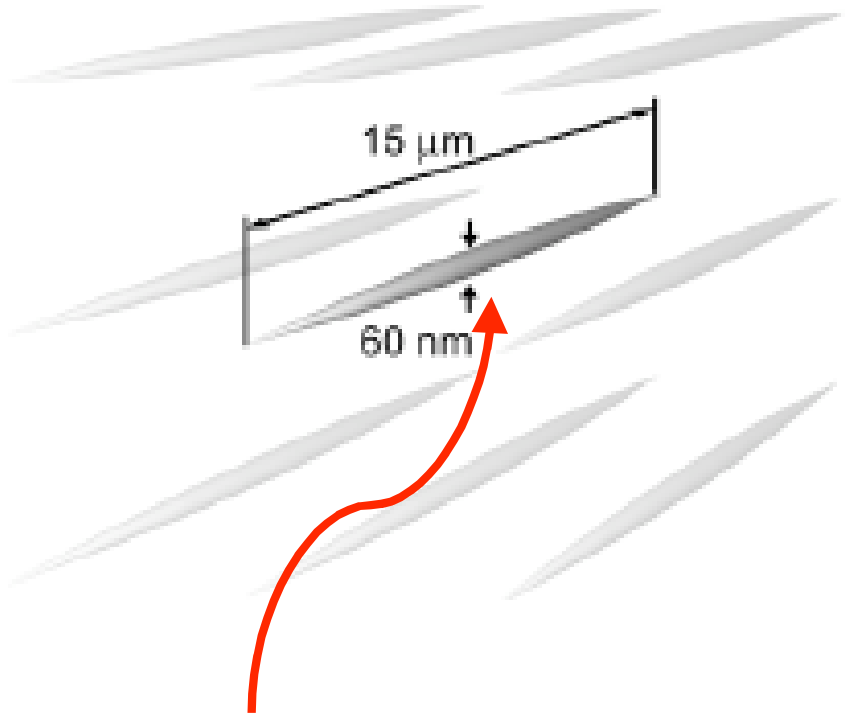
- **Cold atoms on a chip:**







- **2D Optical lattices:**



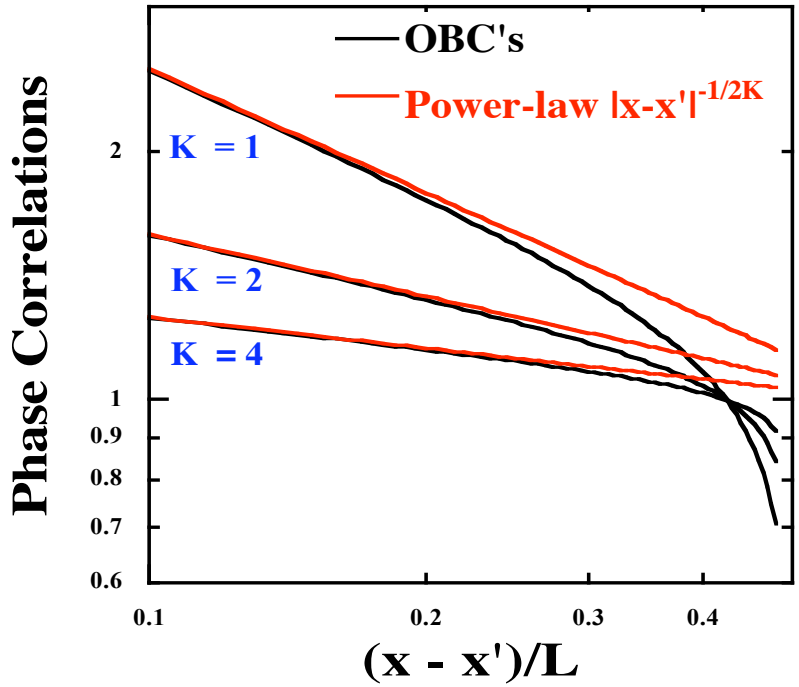
$N_0 \sim 10$ to 10^3 atoms

A toy model: bosonic atoms in a 1D box

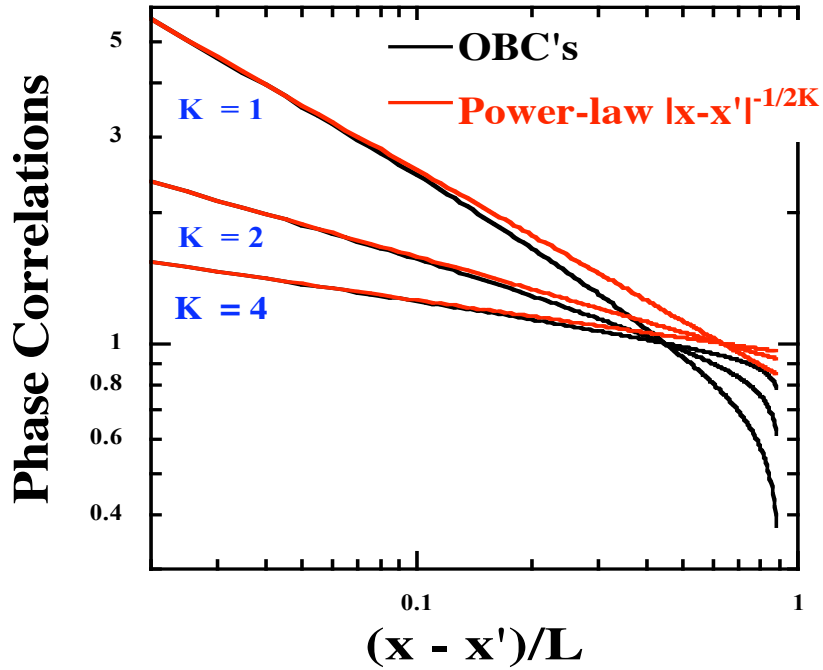
- **Phase correlations:** $\langle \Psi^\dagger(x)\Psi(x')\rangle \simeq \rho_0 \, \langle e^{-i\phi(x)} \, e^{i\phi(x')}\rangle$

[Thermodynamic limit (T = 0) $g_1(x) = \langle \Psi^\dagger(x)\Psi(0)\rangle = \dfrac{A}{x^{1/2K}}$]

x '/L = 0.5



x '/L = 0.1

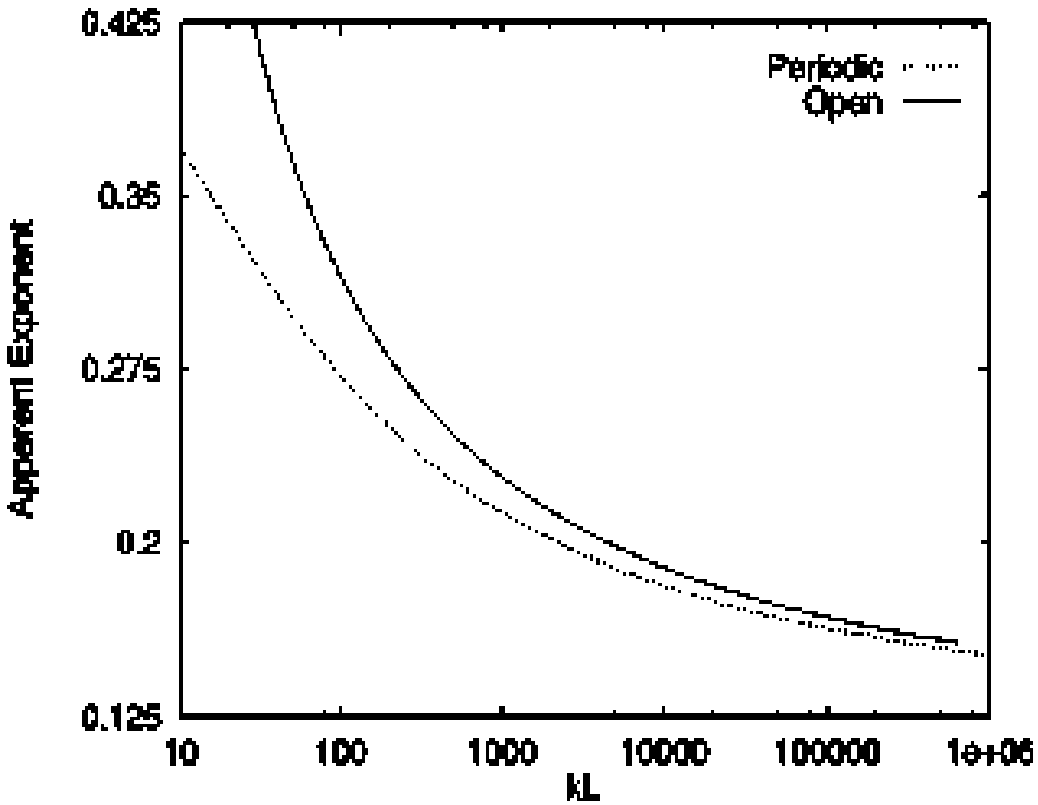


▼ = x

● = x'

[MAC, Europhys. Lett. 59 (2002) & J. Phys. B 37 (2004)]

Momentum distribution in a 1D finite LL



[S. Eggert *et al.* PRL 26 (1996)]

- **Fermi Hubbard model U = +∞**

(away from half-filling)

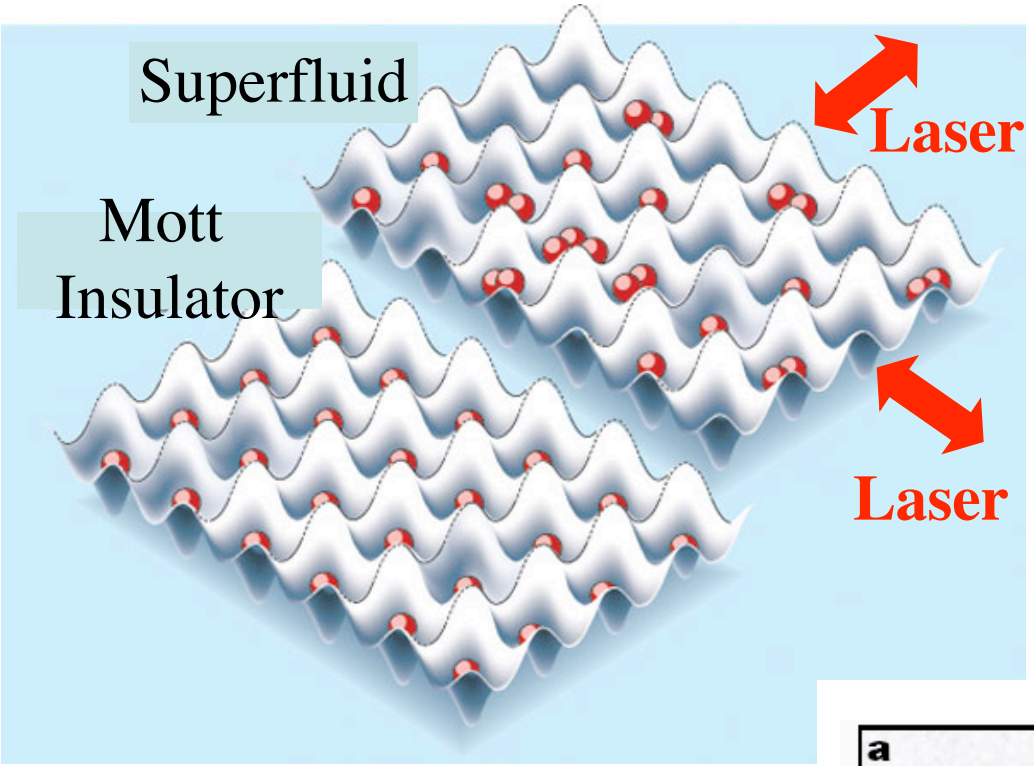
$$\alpha_{\rm app}(kL) = \frac{dn(k,L)}{d\ln kL}$$

$$n(k,L) \sim |k|^{\alpha_{\rm app}}$$

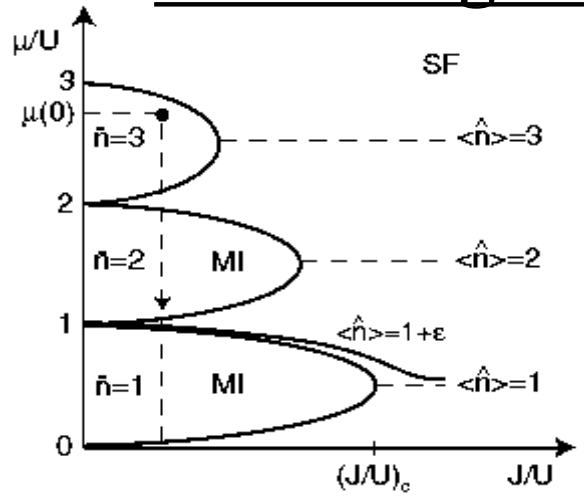
$$\alpha(L\to\infty) = 0.125$$

- **Momentum distribution:** $n(p,L) = (\rho_o L)^{1-\frac{1}{2K}} I(pL)$

Experiments: 3D optical lattices

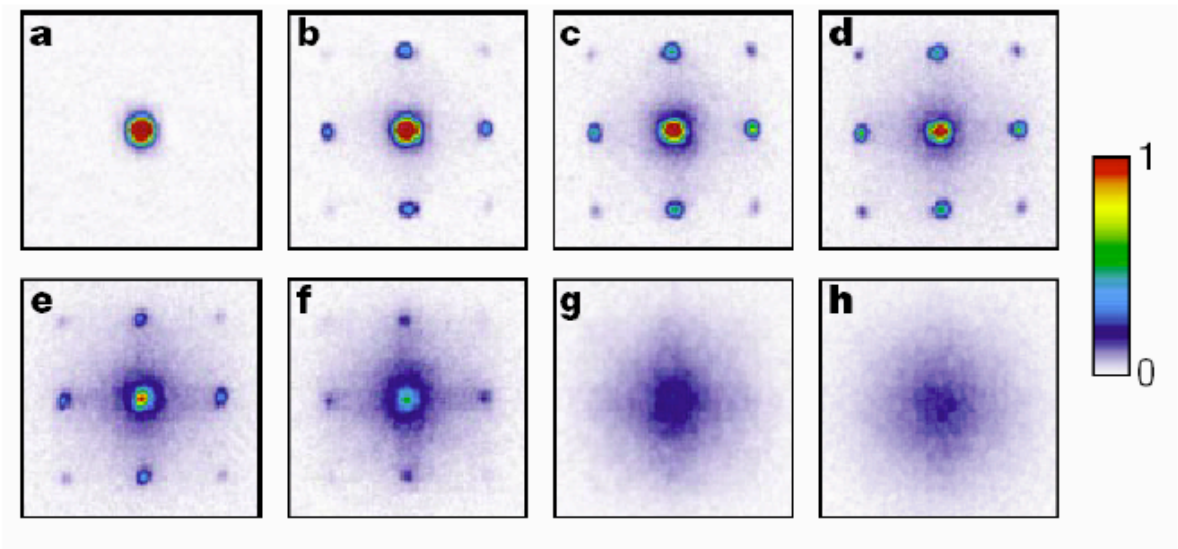


- **Phase diagram:**



[MPA Fisher *et al.* PRB 40 (1989)]

Superfluid to Mott insulator transition in a 3D optical lattice



[D. Jaksch *et al.* PRL 81 (1998)]
[M Greiner *et al.* Nature, 415 (2002)]

Excitation spectrum: Bragg spectroscopy (3D)

- **Bose-Hubbard model:**

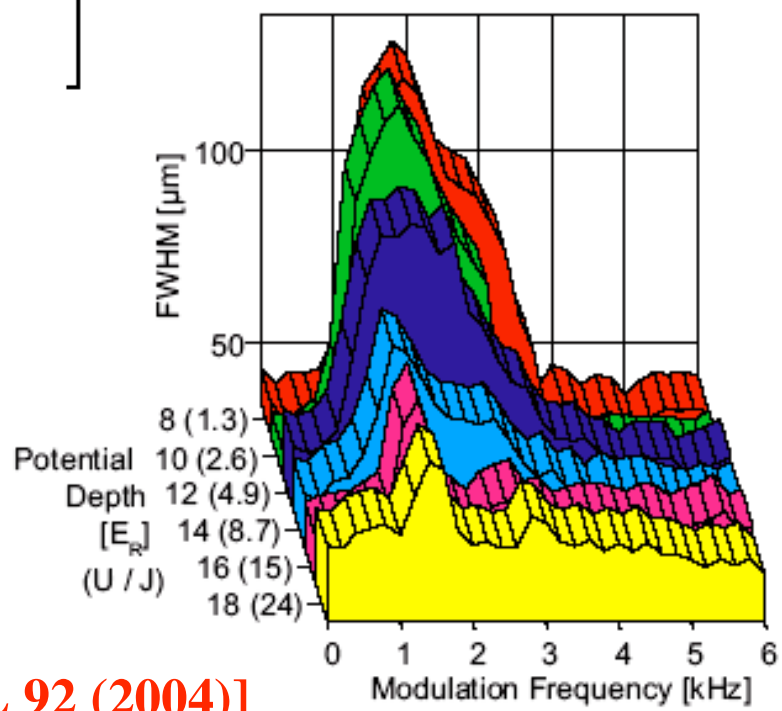
$$H_{\mathrm{BH}} = \sum_{\mathbf{R},m}\left[-\frac{J_x}{2}\left(b^\dagger_{m+1}(\mathbf{R})b_m(\mathbf{R}) + b^\dagger_m(\mathbf{R})b_{m+1}(\mathbf{R})\right) + \epsilon_m(\mathbf{R})b^\dagger_m(\mathbf{R})b_m(\mathbf{R})\right]$$

$$- J\sum_{\langle\mathbf{R},\mathbf{R}'\rangle,m} b^\dagger_m(\mathbf{R})b_m(\mathbf{R}') + U\sum_{\mathbf{R},m} b^\dagger_m(\mathbf{R})b^\dagger_m(\mathbf{R})b_m(\mathbf{R})b_m(\mathbf{R})$$

$$J_\alpha\left(\frac{V_{0\alpha}}{E_R} \gg 1\right) = \frac{4E_R}{\sqrt{\pi}}\left(\frac{V_{0\alpha}}{E_R}\right)^{1/4}\exp\left[-2\left(\frac{V_{0\alpha}}{E_R}\right)^{1/2}\right]$$

$$(\alpha = x, y, z)$$

Bragg spectrum for $J_x = J$



- **2-photon Bragg spectroscopy:**

$$V_{0x} \to V_{0x}(t) = [V_{0x} + A_{\mathrm{mod}}\sin(2\pi\nu_{\mathrm{mod}}t)]$$

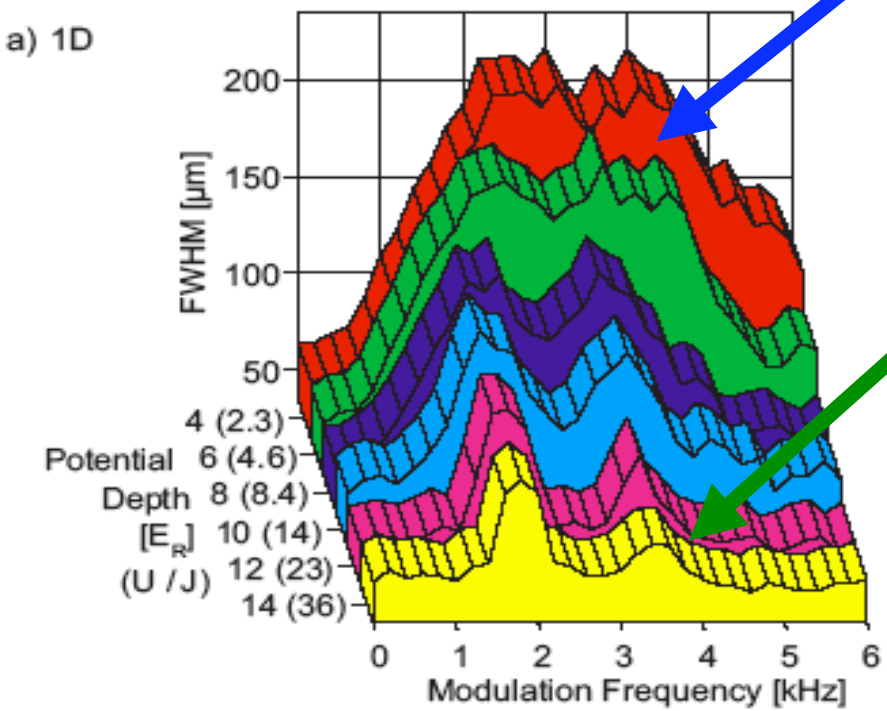
(i.e. modulate the axial optical potential)

[T. Stoferle *et al.* PRL 92 (2004)]

Exc. spectrum: Bragg spectroscopy (J_x >> J)

• $V_{0x,y} = 30\ E_R$:

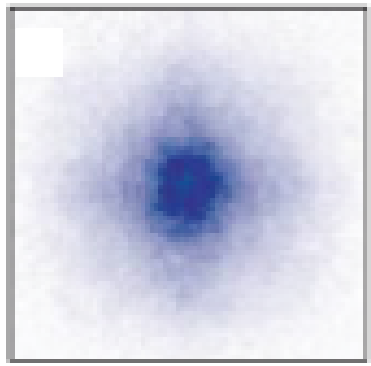
Bragg spectrum for J_x >> J



$$\left(\frac{J_x}{J}\right)_{\max} \simeq 25$$

Broad Spectrum: 1D SF (LL)

Discrete features: 1D MI

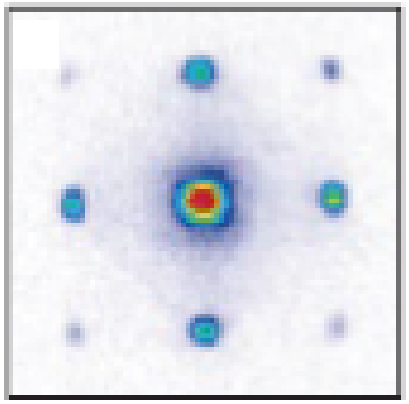


[T. Stoferle *et al.* PRL 92 (2004)]

Excitation spectrum: deconfinement!

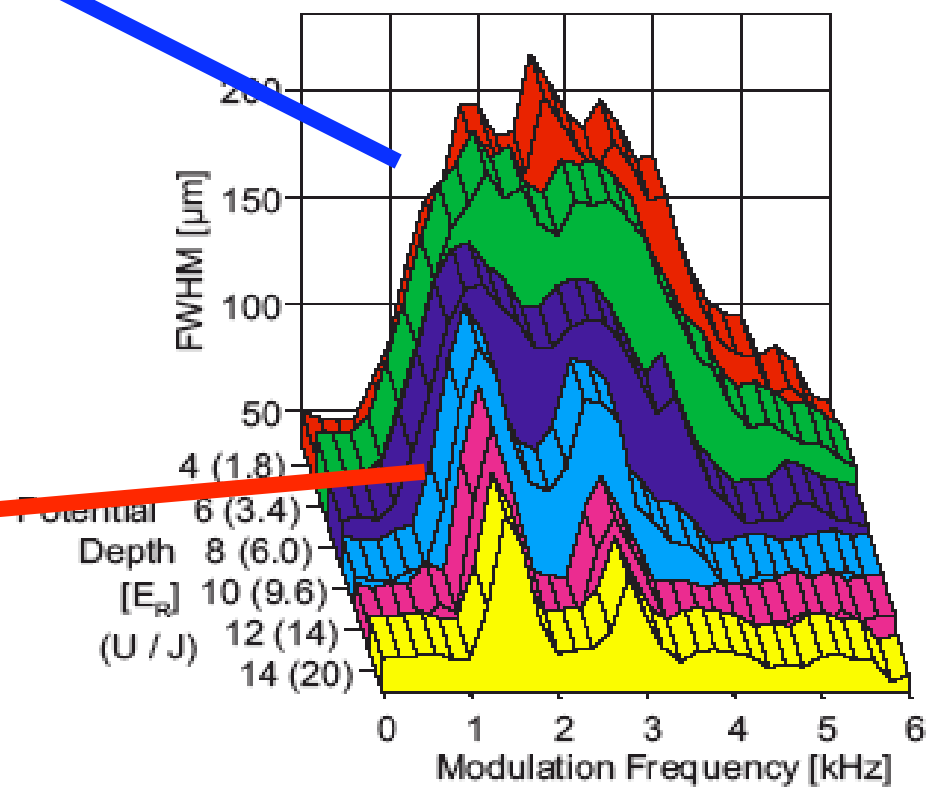
[T. Stoferle *et al.* PRL 92 (2004)]

- **3D Superfluid**

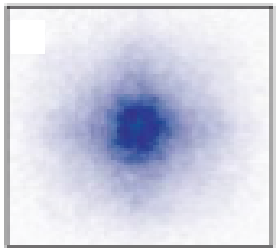


- **$V_0 = 20\ E_R$:**

3D SF to 1D MI



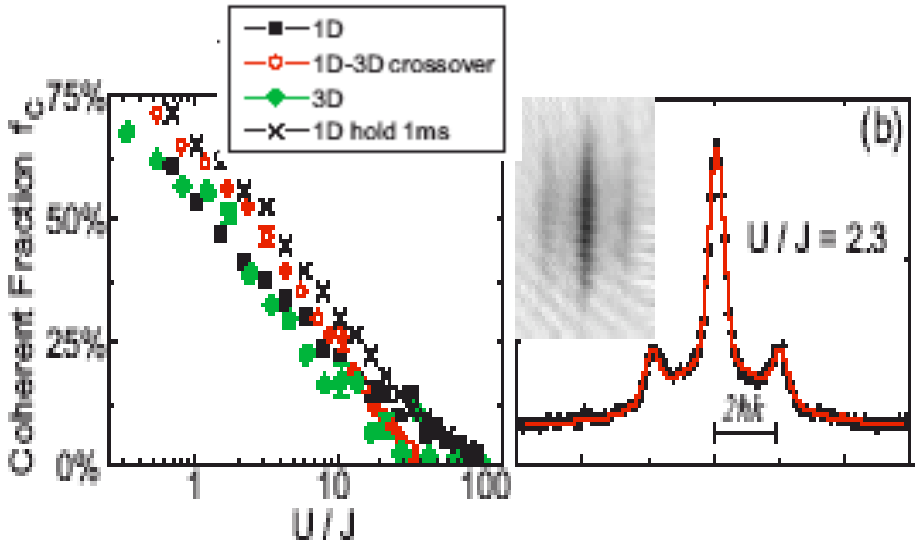
- **1D Mott Insulator**

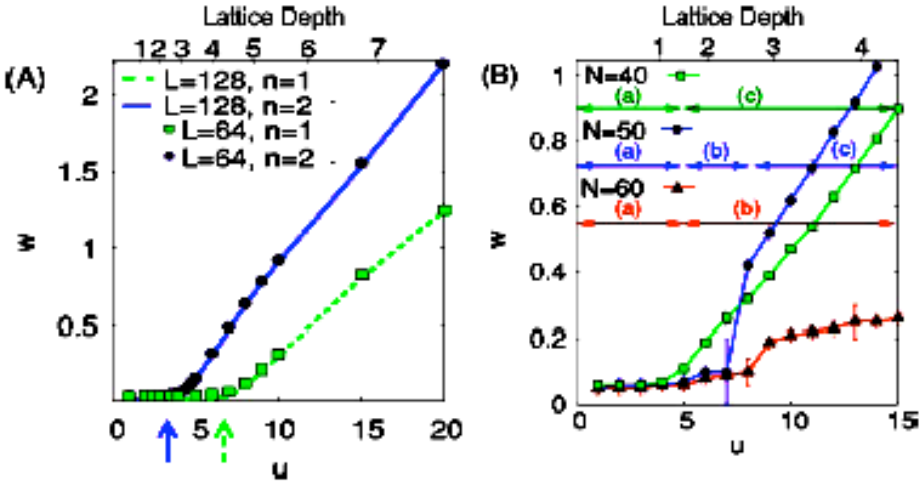


$$\frac{J_x}{J} \simeq 10$$

By reducing the axial hopping intertube coherence is destroyed!!

Where does the phase transition takes place?





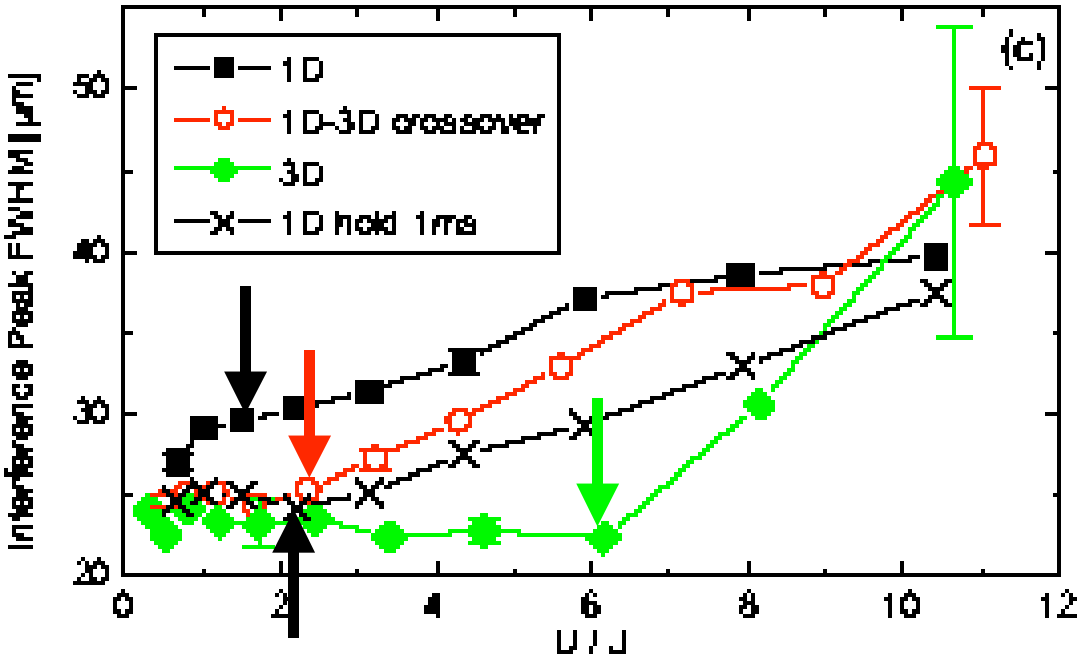
[C. Kollath *et al.* PRA 69 (2004)]

Large quantum depletion!!

$$\left(\frac{U}{zJ}\right)_{1D} \simeq 1.9$$

$$\left(\frac{U}{zJ}\right)_{3D} \simeq 5.8$$

[T. Stoferle *et al.* PRL 92 (2004)]



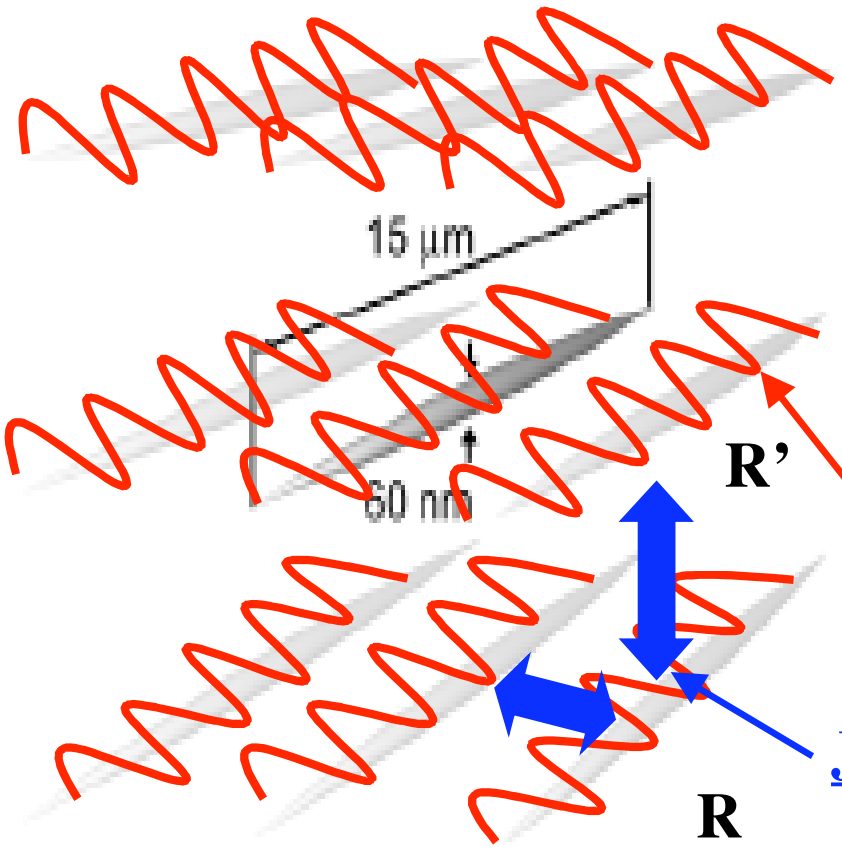
2D optical lattices: effective low-energy theory

- **Through "bosonization" :**

$$H_{\text{eff}} = \frac{\hbar v_s}{2\pi} \sum_{\mathbf{R}} \int_0^L dx \left[\frac{1}{K} \left(\partial_x \theta_{\mathbf{R}}(x)\right)^2 + K \left(\partial_x \phi_{\mathbf{R}}(x)\right)^2 \right]$$

$$+ \frac{\hbar v_s g_u}{2\pi a^2} \sum_{\mathbf{R}} \int_0^L dx \, \cos\left(2\theta_{\mathbf{R}}(x) + \delta\pi x\right)$$

$$- \frac{\hbar v_s g_J}{2\pi a^2} \sum_{\langle \mathbf{R},\mathbf{R}' \rangle} \int_0^L dx \, \cos\left(\phi_{\mathbf{R}}(x) - \phi_{\mathbf{R}'}(x)\right)$$



"Mott" potential: localizes atoms

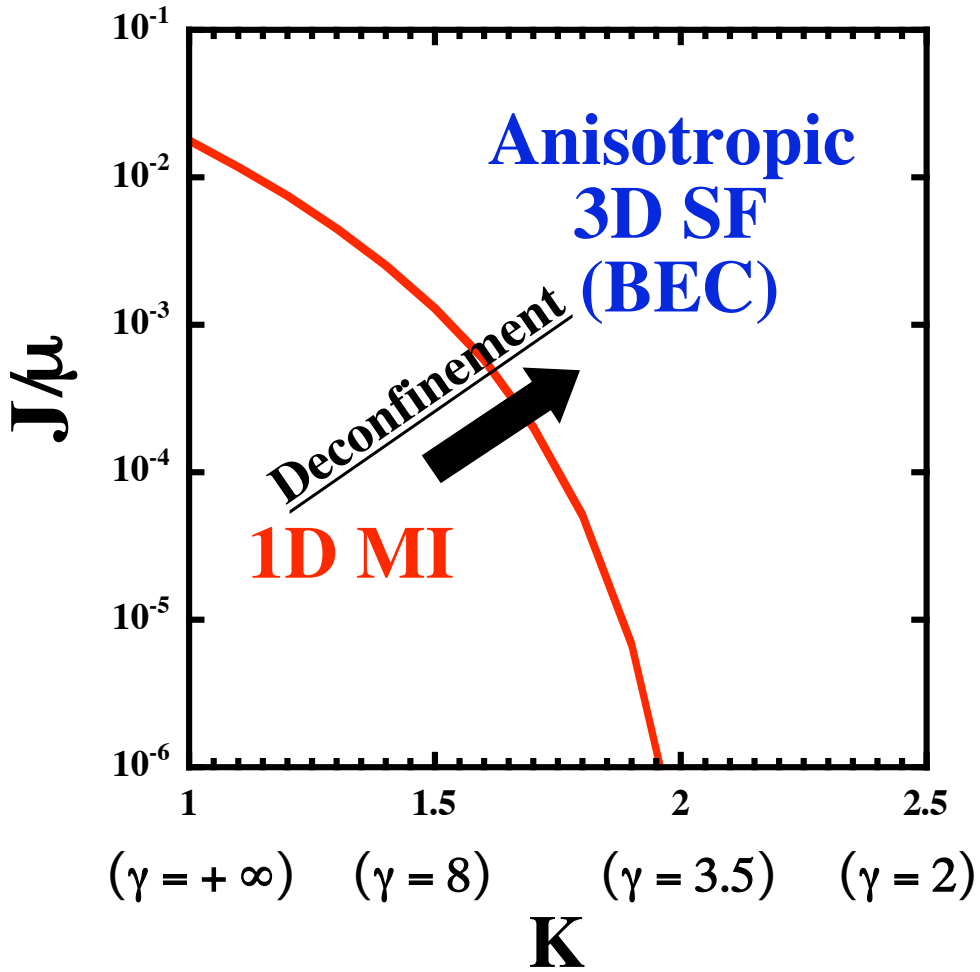
Josephson coupling: delocalizes atoms

[AFH, MAC & T Giamarchi, PRL 92 (2004)]

2D optical lattices: phase diagram at T = 0

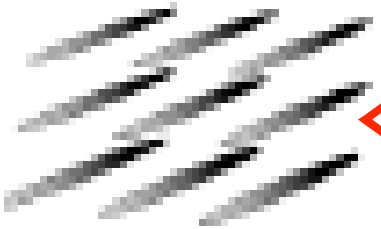
- **Renormalization-group flow:**

$$
\begin{aligned}
\frac{dg_F}{d\ell} &= \frac{g_J^2}{K}, \\
\frac{dg_J}{d\ell} &= \left(2 - \frac{1}{2K}\right) g_J + \frac{g_J g_F}{2K}, \\
\frac{dg_u}{d\ell} &= (2 - K)\, g_u, \\
\frac{dK}{d\ell} &= 4g_J^2 - g_u^2 K^2, \\
\ell &\approx \ln \mu/T
\end{aligned}
$$



[AFH, MAC & T Giamarchi, PRL 92 (2004)]

2D optical lattice of finite tubes: phase diagram



Array of atomic 'quantum dots'

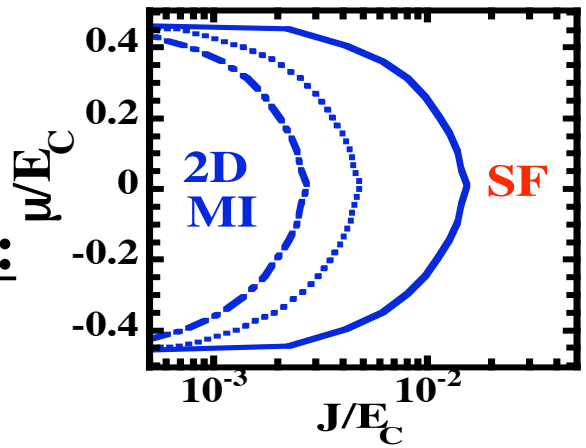
- **2D optical lattice:** phase diagram

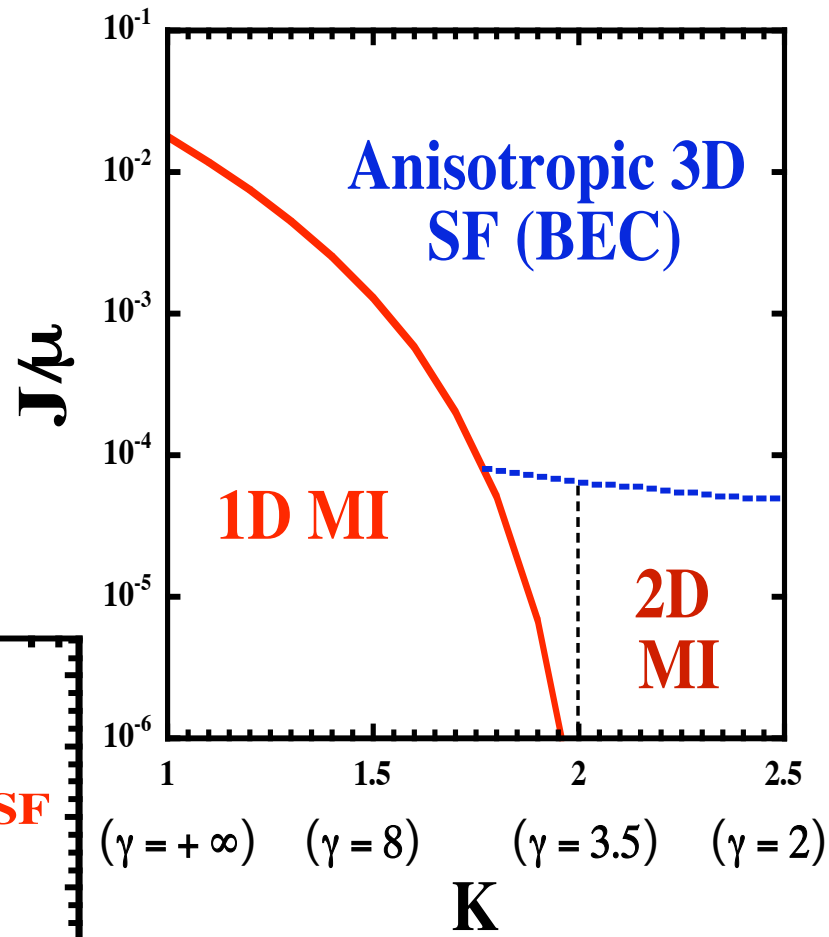
- **Quantum phase Hamiltonian** ($g_u = 0$) :

$$H_{QP} = -E_J \sum_{\langle \mathbf{R},\mathbf{R}' \rangle} \cos\left(\phi_{0\mathbf{R}} - \phi_{0\mathbf{R}'}\right) + \frac{E_C}{2} \sum_{\mathbf{R}} (N_{\mathbf{R}} - N_0)^2 - \mu \sum_{\mathbf{R}} N_{\mathbf{R}}$$

$$E_J \simeq J N_0^{1-\frac{1}{2K}}$$

$$E_C = \frac{\hbar \pi v_s}{KL}$$





- **Incommensurate fillings:**

[AFH, MAC & T Giamarchi, PRL 92 (2004)]

2D optical lattices: 3D Superfluid (BEC) phase

- **Mean-field theory:** condensate fraction $\psi_0^2(T=0) \sim \rho_0 \left(\frac{J}{\mu}\right)^{1/(4K-1)}$

- **Variational approach:** momentum distribution at T = 0

$$\frac{n(\mathbf{Q},q)}{|w(\mathbf{Q})|^2} \simeq \psi_0^2 \delta(\mathbf{Q})\delta(q) + \frac{\pi b^2 \psi_0^2/2K}{\left[q^2 + (v_\perp \mathbf{Q}/v_s)^2\right]^{1/2}},$$

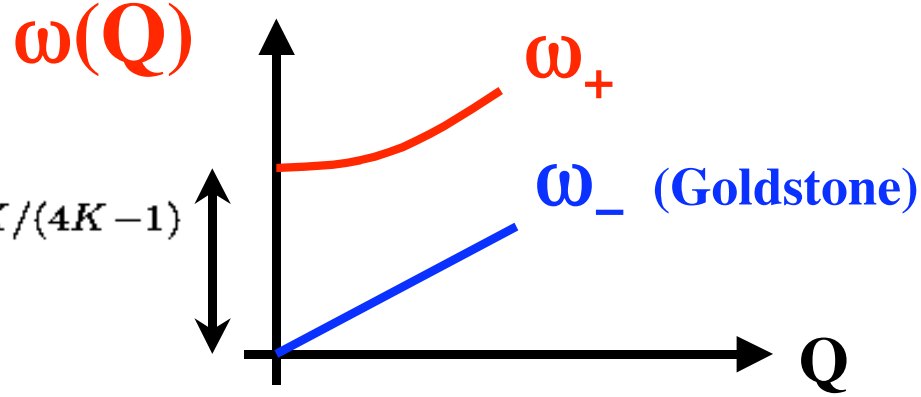
transverse velocity:

$$v_\perp \sim \mu b (J/\mu)^{2K/(4K-1)}/\hbar$$

- **RPA :** condensation temperature and excitation spectrum

$$\left(\frac{2\pi T_c}{\hbar v_s \rho_0}\right)^{2-1/2K} = f(K)\frac{4J}{\hbar v_s \rho_0}$$

$$\Delta_+ \sim \mu \left(\frac{J}{\mu}\right)^{2K/(4K-1)}$$



[AFH, MAC & T Giamarchi, PRL 92 (2004)]

• **2D optical lattice:** **phase diagram**