Many-body physics with exciton-polaritons in semiconductor microcavities

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What is a semiconductor microcavity? What are exciton-polaritons?

The time-dependent Gross-Pitaevskii equation for polaritons.

Stationary states: equation of state and optical bistability effects

The collective excitation spectrum of the polariton gas within the Bogoliubov approximation.

Response to a weak point-like defect: Landau criterion for superfluidity.

Experimental consequences of superfluidity on the emitted light

Conclusions and perspectives
Distributed Bragg reflector (DBR) planar microcavity with a few quantum wells (QW)

- Alternate $\lambda/4$ GaAs/AlAs layers $\rightarrow$ DBR
- Cavity layer $\rightarrow$ confined photonic mode delocalized along cavity plane.
- In-plane photonic dispersion:
  \[
  \omega_C(k) = \omega_C^0 \sqrt{1 + \frac{k^2}{k_1^2}}
  \]

- e and h confined in $\text{In}_x\text{Ga}_{1-x}\text{As}$ QW.
- 2D confinement $\rightarrow$ enhances e-h binding
- Excitons $\rightarrow$ bosons if $n_{exc} a_{Bohr}^2 \ll 1$
- Excitons delocalized along cavity plane.
  Flat exciton dispersion: $\omega_X(k) \approx \omega_X$

Exciton radiatively coupled to cavity-photon only: in-plane $k$ conserved, no coupling to continuum, vacuum Rabi splitting $\Omega_R$. 
The system Hamiltonian

\[ \mathcal{H} = \int dx \sum_{ij=\{X,C\}} \hat{\Psi}^\dagger_i(x) \left[ h_{ij}^0 + V_i(x) \delta_{ij} \right] \hat{\Psi}_j(x) + \frac{g}{2} \int dx \hat{\Psi}^\dagger_X(x) \hat{\Psi}^\dagger_X(x) \hat{\Psi}_X(x) \hat{\Psi}_X(x) + \int dx \left[ k E_p(x,t) \hat{\Psi}^\dagger_C(x) + k^* E_p(x,t) \hat{\Psi}_C(x) \right] + \mathcal{H}_\gamma \]

Single-particle Hamiltonian: \[ h^0 = \begin{pmatrix} \omega_X(-i\nabla) & \Omega_R \\ \Omega_R & \omega_C(-i\nabla) \end{pmatrix} \]

- Eigenmodes of \( h^0 \rightarrow \) superpositions of exciton and photon, called polaritons.
- **Strong coupling** regime (\( \Omega_R \gg \gamma_{X,C} \)): polaritons well resolved
- **Photoluminescence** data: polariton dispersions \( \omega_{LP}(k) \) and \( \omega_{UP}(k) \)
  (figure taken from: Houdré et al., PRL 73, 2043, 1994)
The system Hamiltonian (II)

External potential $V_{x,c}(x)$:
- **wedging** of cavity and/or QW, optical Stark shift of exciton (controlled)
- inhomogeneities in Bragg mirrors, QW interface roughness (disorder)

Exciton-exciton binary interactions:
- experimentally $g > 0$, qualitative agreement with approximate calculations

External driving field $\mathcal{E}_p(x,t)$:
- in **time**: monochromatic at $\omega_p$, pulsed, bichromatic …
- in **space**: plane wave at $k_p$, optical vortex …

Dissipation term $\mathcal{H}_\gamma$:
- radiative decay due to coupling to external photons, background absorption
- decoherence and non-radiative recombination of excitons: scattering on phonons, trapping in localized exciton states
- phenomenological damping rates $\gamma_X \approx \gamma_C$
Experiments on polariton amplification

Pump at "magic angle":
- \((k_p, k_p) \rightarrow (0, 2k_p)\) process is resonant
- probe at \(k = 0\) parametrically amplified
- idler generated at \(k = 2k_p\)

Analog atomic matter wave amplification exp’ts:

(M. Kozuma, Science 286, 2309, 1999)
The polariton Gross-Pitaevskii equation

\[ i \frac{d}{dt} \begin{pmatrix} \psi_X(x) \\ \psi_C(x) \end{pmatrix} = \begin{pmatrix} 0 \\ k \mathcal{E}_p(x) \end{pmatrix} + \begin{pmatrix} \hbar^0 + \begin{pmatrix} V_X(x) + g|\psi_X(x)|^2 - i\gamma_X & 0 \\ 0 & V_C(x) - i\gamma_C \end{pmatrix} \end{pmatrix} \begin{pmatrix} \psi_X(x) \\ \psi_C(x) \end{pmatrix} \]

Differences with ordinary GPE:

- driving and dissipative terms: system is out of equilibrium
- two coupled Bose fields with different mass: cavity-photonic \( \psi_C \) and excitonic \( \psi_X \)

Homogeneous case \( V_{C,X} = 0 \), monochromatic plane-wave excitation

\[ \mathcal{E}_p(x) = \mathcal{E}_p \ e^{i k_p x} e^{-i \omega_p t}. \]

Look for plane-wave stationary solutions of the form:

\[ \psi_{X,C}(x,t) = \psi_{X,C}^{gs} \ e^{i k_p x} e^{-i \omega_p t}, \]

\[ \begin{cases} 
(\omega_X(k_p) - \omega_p - i\gamma + g|\psi_{X}^{gs}|^2)\psi_{X}^{gs} + \Omega_R \psi_C^{gs} = 0 \\
(\omega_C(k_p) - \omega_p - i\gamma)\psi_{C}^{gs} + \Omega_R \psi_X^{gs} = -k \mathcal{E}_p 
\end{cases} \]

is the equation of state: oscillation frequency not fixed by \( \mu = \mu(n) \) but tunable with \( \omega_p \).
The equation of state

Equation of state relates internal intensity $|\psi_{X,C}^{ss}|^2$ to incident one $I_p \propto |E_p|^2$. Transmitted intensity $I_t \propto \gamma_C |\psi_C^{ss}|^2$:

- optical limiting for $\omega_p < \omega_{LP}(k_p)$
- optical bistability for $\omega_p > \omega_{LP}(k_p)$. Dashed branches $\rightarrow$ unstable.

FIG. 6: All curves are drawn as a function of the position $(x, y)$ in the transverse plane. a) and b): nonlinear energy shift proportional to the Gaussian intensity distribution of the excitation spot and linear shift due to the cavity wedge, for two positions of the spot on the sample: $X=245$ μm and $X=180$ μm ($X=0$ corresponds to zero exciton-cavity detuning). c) and d): reflectivity for the parameters of a) and b) respectively. The reflectivity resonance is obtained when the nonlinear shift compensates exactly for the linear shift, i.e., at the intersection between the two curves of Fig. a) and b). The low-intensity resonance (a straight line) can be seen on the edge of plot d). The unshifted resonance is at $X=270$ μm.

FIG. 7: Upper figure: reflected intensities (in arbitrary units) as a function of the spot position on the sample (the origin of the axis is arbitrary), for several values of the input power $I_{in}: 1, 2, 3, 4, 5$ and 6 mW. The laser wavelength is 831.32 nm, resonant with the lower polariton at $\Delta \omega = 1.5$ meV. Bistability appears at $I_{in} = 2.8$ mW. Lower figure: hysteresis cycle for the curve $I_{in} = 6$ mW. The two curves correspond to the two directions for the scan of the spot position on the sample. (Exp’t: A. Baas et al., PRA 69, 023809, 2004)
Field variation from stationary state:  \( \delta \vec{\phi} = (\delta \phi_X(x, t), \delta \phi_C(x, t), \delta \phi_X^*(x, t), \delta \phi_C^*(x, t))^T \)

Linear response to weak perturbation:  \( i \frac{d}{dt} \delta \vec{\phi} = \mathcal{L} \cdot \delta \vec{\phi} + \vec{f}_d. \)

Bogoliubov matrix:

\[
\mathcal{L} = \begin{pmatrix}
\omega_X + 2g |\psi_X|^2 - \omega_p - i\gamma & \Omega_R & g \psi_X^s e^{2ik_p x} & 0 \\
\Omega_R & \omega_C(-i\nabla) - \omega_p - i\gamma & 0 & 0 \\
-g \psi_X^s e^{2ik_p x} & 0 & -(\omega_X + 2g |\psi_X|^2) + \omega_p - i\gamma & -\Omega_R \\
0 & 0 & -\Omega_R & -\omega_C(-i\nabla) + \omega_p - i\gamma \\
\end{pmatrix}
\]

Eigenvalues of \( \mathcal{L} \) give Bogoliubov mode frequencies \( \omega^{\pm}_LP(k) \) and \( \omega^{\pm}_UP(k) \).

Particle- and hole-like \( \pm \) branches symmetrical under \( k \to 2k_p - k \) and \( \omega \to 2\omega_p - \omega \).

Mode dispersion depends on interaction energy \( \delta \omega_{MF} = g |\psi_X^s|^2 \) and on pump frequency \( \omega_p \).
Approximate analytical results

Interaction energy  $\delta \omega_{MF} = g |\psi_{X}^{2s}|^2 \ll \omega_{UP} - \omega_{LP}$  \implies  only LP effectively excited

Parabolic approximation for low $k$ : $\omega_{LP}(k) \approx k^2/2m_{LP}$

$$\omega_{LP}^{\pm}(k) \simeq \omega_p + (k - k_p) \cdot v_p \pm \sqrt{\left(2 \delta \omega_{MF} + \frac{(k - k_p)^2}{2m_{LP}} - \Delta_p\right)\left(\frac{(k - k_p)^2}{2m_{LP}} - \Delta_p\right) - i\gamma}$$

where effective detuning  $\Delta_p = \omega_p - \omega_{LP}(k_p) - \delta \omega_{MF}$.

Different regimes depending on sign of $\Delta_p$ and on value of $\delta \omega_{MF}$.

With respect to standard Bogoliubov:

- tilting term  $(k - k_p) \cdot v_p$  due to flow velocity
- detuning term due to effective detuning  $\Delta_p$
**Bogoliubov dispersion (I): resonant case** \( \Delta_p = 0 \)

- \( \Delta_p = 0 \) at inversion point \( A \) on hysteresis curve
- Particle- and hole-like \( \pm \) branches touch at \( k_p \)
- Standard Bogoliubov dispersion tilted by \( v_p \)
- Linear regime \( \delta \omega_{MF} \approx 0 \): almost parabolic LP polariton dispersion
- Interaction energy \( \delta \omega_{MF} \) and sound velocity \( c_s = \sqrt{\delta \omega_{MF}/m_{LP}} \) increasing from top to bottom
- Group velocity \( v_{g}^{l,r} = v_p \pm c_s \). In particular, \( v_{g}^{l} \) changes sign when \( c_s = v_p \) (Landau criterion).
Bogoliubov dispersion (II): non-resonant case $\Delta_p \neq 0$

- Right of $A$ on upper branch of hysteresis curve: $\Delta_p < 0$
- Branches no longer touch at $k_p$, possibility of full gap.

- On lower branch of hysteresis curve: $\Delta_p > 0$
- Argument of square root is negative for:
  $$\Delta_p > \frac{(k-k_p)^2}{2m_{LP}} > \Delta_p - 2\delta\omega_{MF}$$
- Branches stick together
- Imaginary parts split, dynam. instability if $\delta\omega_{MF} > \gamma$.

Optics $\rightarrow$ parametric instability. Quantum fluids $\rightarrow$ modulational instability.
Response to weak, point-like, static defect

\[
\tilde{f}_d = \begin{pmatrix} V_X(x) \phi_X^s(x), & V_C(x) \phi_C^s(x), & -V_X(x) \phi_X^s(x), & -V_C(x) \phi_C^s(x) \end{pmatrix}^T
\]

- Induced perturbation \( \delta \tilde{\phi}_d = -\mathbf{L}^{-1} \cdot \tilde{f}_d \)

- Bogoliubov modes at \( \omega_p \) resonantly excited: Čerenkov emission of propagating sound waves

- Damping of sound waves over a length scale \( \ell_{prop} \approx v_g/\gamma \).

- Excitation of non-resonant modes: localized, not propagating perturbation

- Disordered ensemble of defects: similar k-space pattern with speckle-like intensity modulation due to polaritonic coherence.
The resonant Rayleigh scattering (RRS)

**Scheme of RRS process**

**Experimental set-up**

**k-space** \( I(k) \propto |\tilde{\psi}_C(k)|^2 : \text{far-field image} \)

**Real space** \( I(x) \propto |\psi_C(x)|^2 : \text{near-field image} \)

(taken from: R. Houdré et al., PRB 61, 13333R, 2000)

(taken from: W. Langbein, proceedings of ICPS 26, 2002)

Discard incoherent photoluminescence by spectral selection of coherently emitted intensity at \( \omega_p \)
Far-field emission pattern (I): resonant case $\Delta_p = 0$

**Linear regime:**
- peak at $k_p$ due to unscattered light
- resonant Rayleigh scattering (RRS) ring

**Nonlinear regime, $v_p > c_s$:**
- RRS ring transformed into $\infty$-like shape
- lobes touch at $k_p$, low-k lobe (+ branch) more intense than high-k lobe (− branch)

**Nonlinear regime, $v_p < c_s$:**
- Landau criterion predicts superfluidity
- lobes disappear, much weaker emission
Far-field emission pattern: non-resonant case $\Delta_p \neq 0$

Case $\Delta_p < 0$:
- gap appears in between the two lobes

Case $\Delta_p > 0$:
- lobes are pushed closer
- enhancement of RRS intensity in merging region around $k_p$.
- precursor of parametric instability.
Near-field emission pattern: resonant case $\Delta p = 0$

**Linear regime:**
- diffused light has parabolic wavefronts

**Nonlinear regime, $v_p > c_s$:**
- sound waves form Čerenkov cone.
  - Its aperture: $\sin \theta = c_s/v_p$
- parabolic precursors ahead

**Nonlinear regime, $v_p < c_s$:**
- localized perturbation
- system superfluid: fluid flows around impurity without dissipation

The polariton fluid is moving to the right
Conclusions

A novel quantum Bose gas: exciton-polaritons in a semiconductor microcavity

- **Disadvantages:**
  - polaritons have finite lifetime, mostly due to extrinsic effects:
    * radiative decay of cavity-photon through cavity mirrors
    * absorption and dephasing due to sample imperfections
  - little is known about exciton-exciton scattering physics

- **Good points:**
  - New physics: driven-dissipative Bose system intrinsically out of equilibrium
  - Long-range coherence imprinted by driving laser. No need for complicate cooling procedure.
  - Exciton-exciton interactions are important $\delta \omega_{MF} \gg \gamma$
  - External trapping potential applicable
  - Exciton and photon spin degrees of freedom
  - Easy in-situ diagnostic through emitted light:
    * real- and k-space intensity distributions, higher-order correlation functions
    * time evolution with streak cameras (ps resolution).
Future developments

- Calculations have been performed for realistic experimental parameters for III-V semiconductor microcavities. How to take advantage of larger vacuum Rabi splitting of II-VI microcavities?

- What is the best way of applying an external trapping potential to polaritons?

- Better characterization of exciton-exciton interactions, in particular for different spin states: first experimental results available, but not yet fully understood.

- Physics of inhomogeneous systems: oscillation modes in external potential.