Many-body physics with exciton-polaritons in semiconductor microcavities

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Outline

- What is a semiconductor microcavity ? What are exciton-polaritons ?

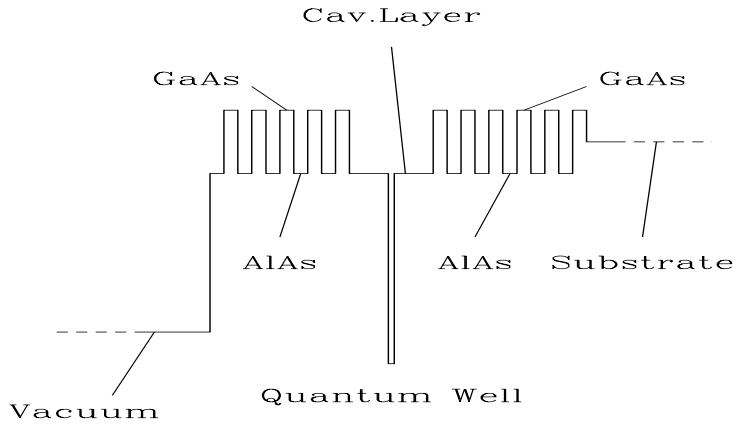
- The time-dependent Gross-Pitaevskii equation for polaritons.
- Stationary states: equation of state and optical bistability effects
- The collective excitation spectrum of the polariton gas within the Bogoliubov approximation.

- Response to a weak point-like defect: Landau criterion for superfluidity.
- Experimental consequences of superfluidity on the emitted light

- Conclusions and perspectives

The physical system

Distributed Bragg reflector (DBR) planar microcavity with a few quantum wells (QW)



- Alternate $\lambda/4$ GaAs/AlAs layers $\longrightarrow$ DBR
- Cavity layer $\longrightarrow$ confined photonic mode delocalized along cavity plane.
- In-plane photonic dispersion: $\omega_C(\mathbf{k}) = \omega_C^0 \sqrt{1 + \mathbf{k}^2/k_z^2}$

- e and h confined in $In_xGa_{1-x}As$ QW.
- 2D confinement $\longrightarrow$ enhances e-h binding
- Excitons $\longrightarrow$ bosons if $n_{exc}\, a_{Bohr}^2 \ll 1$
- Excitons delocalized along cavity plane. Flat exciton dispersion: $\omega_X(\mathbf{k}) \approx \omega_X$

Exciton radiatively coupled to cavity-photon only : in-plane $\mathbf{k}$ conserved, no coupling to continuum, vacuum Rabi splitting Ω_R.

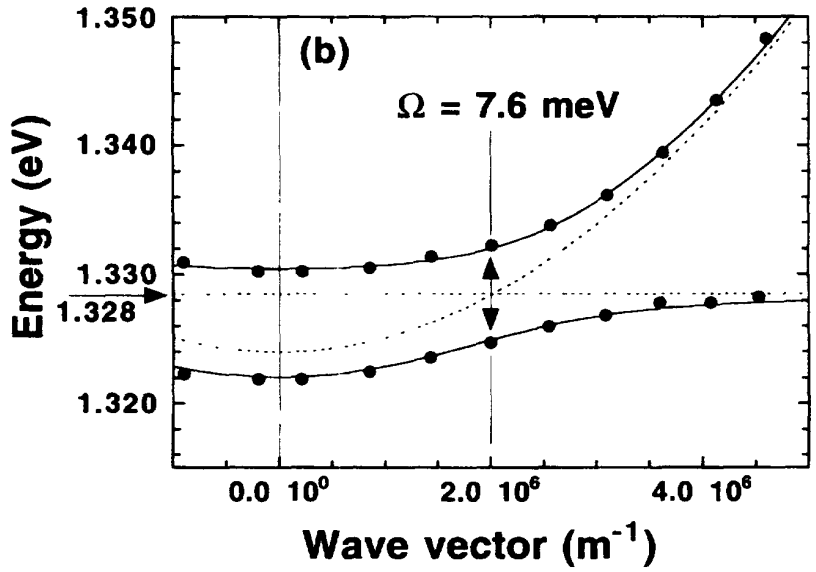
The system Hamiltonian

$$\mathcal{H} = \int d\mathbf{x} \sum_{ij=\{X,C\}} \hat{\Psi}_i^\dagger(\mathbf{x}) \left[\mathbf{h}_{ij}^0 + V_i(\mathbf{x})\,\delta_{ij}\right] \hat{\Psi}_j(\mathbf{x}) + \frac{g}{2} \int d\mathbf{x}\, \hat{\Psi}_X^\dagger(\mathbf{x})\, \hat{\Psi}_X^\dagger(\mathbf{x})\, \hat{\Psi}_X(\mathbf{x})\, \hat{\Psi}_X(\mathbf{x}) +$$

$$+ \int d\mathbf{x} \left[k\, \mathcal{E}_p(\mathbf{x},t)\, \hat{\Psi}_C^\dagger(\mathbf{x}) + k^*\, \mathcal{E}_p^*(\mathbf{x},t)\, \hat{\Psi}_C(\mathbf{x})\right] + \mathcal{H}_\gamma$$

Single-particle Hamiltonian: $\mathbf{h}^0 = \begin{pmatrix} \omega_X(-i\nabla) & \Omega_R \\ \Omega_R & \omega_C(-i\nabla) \end{pmatrix}$

- Eigenmodes of $\mathbf{h}^0 \longrightarrow$ superpositions of exciton and photon, called polaritons.
- Strong coupling regime ($\Omega_R \gg \gamma_{X,C}$): polaritons well resolved
- Photoluminescence data: polariton dispersions $\omega_{LP}(\mathbf{k})$ and $\omega_{UP}(\mathbf{k})$
  (figure taken from: Houdré *et al.*, PRL **73**, 2043, 1994)



The system Hamiltonian (II)

External potential $V_{X,C}(\mathbf{x})$:

- wedging of cavity and/or QW, optical Stark shift of exciton (controlled)
- inhomogeneities in Bragg mirrors, QW interface roughness (disorder)

Exciton-exciton binary interactions:

- experimentally $g > 0$, qualitative agreement with approximate calculations

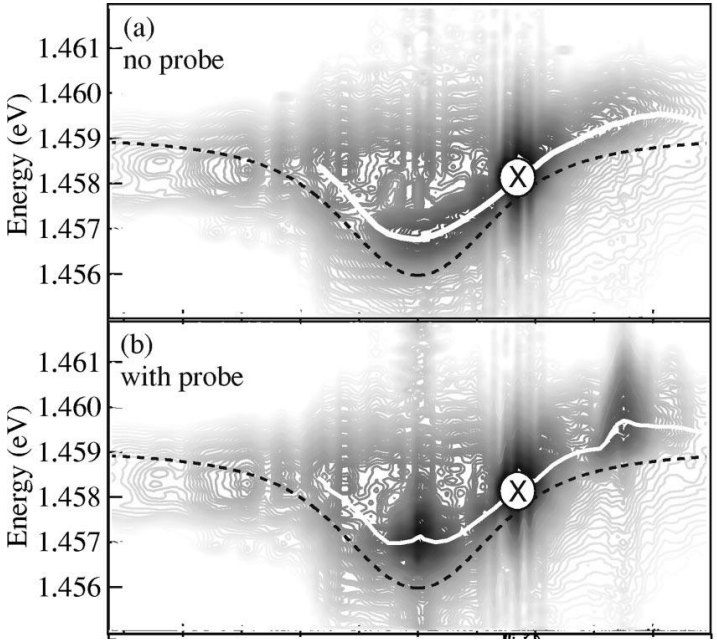
External driving field $\mathcal{E}_p(\mathbf{x}, t)$:

- in time: monochromatic at ω_p, pulsed, bichromatic ...
- in space: plane wave at $\mathbf{k}_p$, optical vortex ...

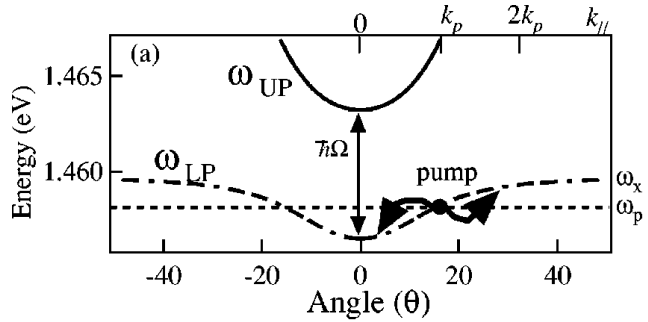
Dissipation term $\mathcal{H}_\gamma$:

- radiative decay due to coupling to external photons, background absorption
- decoherence and non-radiative recombination of excitons: scattering on phonons, trapping in localized exciton states
- phenomenological damping rates $\gamma_X \approx \gamma_C$

Experiments on polariton amplification



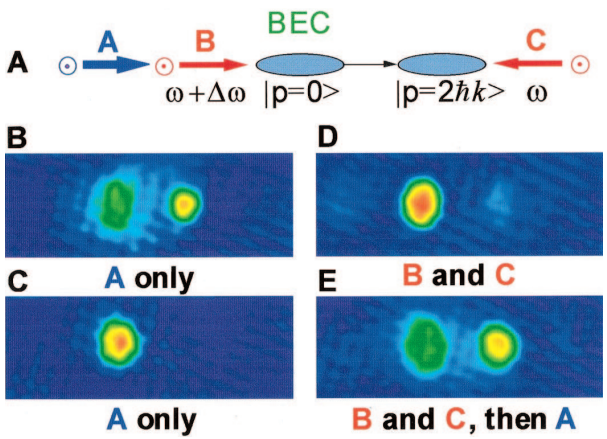
(P. G. Savvidis, *et al.*, PRB **62**, R13278, 2000)



Pump at "magic angle":

- $(\mathbf{k}_p,\ \mathbf{k}_p) \longrightarrow (0,\ 2\mathbf{k}_p)$ process is resonant
- probe at $\mathbf{k} = 0$ parametrically amplified
- idler generated at $\mathbf{k} = 2\mathbf{k}_p$

Analog atomic matter wave amplification exp'ts:



(M. Kozuma, Science **286**, 2309, 1999)

The polariton Gross-Pitaevskii equation

$$i\,\frac{d}{dt}\begin{pmatrix}\psi_X(\mathbf{x})\\ \psi_C(\mathbf{x})\end{pmatrix}=\begin{pmatrix}0\\ k\,\mathcal{E}_p(\mathbf{x},t)\end{pmatrix}+\left[\mathbf{h}^0+\begin{pmatrix}V_X(\mathbf{x})+g|\psi_X(\mathbf{x})|^2-i\,\gamma_X & 0\\ 0 & V_C(\mathbf{x})-i\,\gamma_C\end{pmatrix}\right]\begin{pmatrix}\psi_X(\mathbf{x})\\ \psi_C(\mathbf{x})\end{pmatrix}$$

Differences with ordinary GPE:

- driving and dissipative terms: system is out of equilibrium
- two coupled Bose fields with different mass: cavity-photonic ψ_C and excitonic ψ_X

Homogeneous case $V_{C,X}=0$, monochromatic plane-wave excitation $\mathcal{E}_p(\mathbf{x},t)=\mathcal{E}_p\,e^{i\mathbf{k}_p\mathbf{x}}\,e^{-i\omega_p t}$.

Look for plane-wave stationary solutions of the form: $\psi_{X,C}(\mathbf{x},t)=\psi^{ss}_{X,C}\,e^{i\mathbf{k}_p\mathbf{x}}\,e^{-i\omega_p t}$,

$$\begin{cases}\left(\omega_X(\mathbf{k}_p)-\omega_p-i\gamma+g\,|\psi^{ss}_X|^2\right)\psi^{ss}_X+\Omega_R\,\psi^{ss}_C=0\\[2ex] \left(\omega_C(\mathbf{k}_p)-\omega_p-i\gamma\right)\psi^{ss}_C+\Omega_R\,\psi^{ss}_X=-k\,\mathcal{E}_p\end{cases}$$

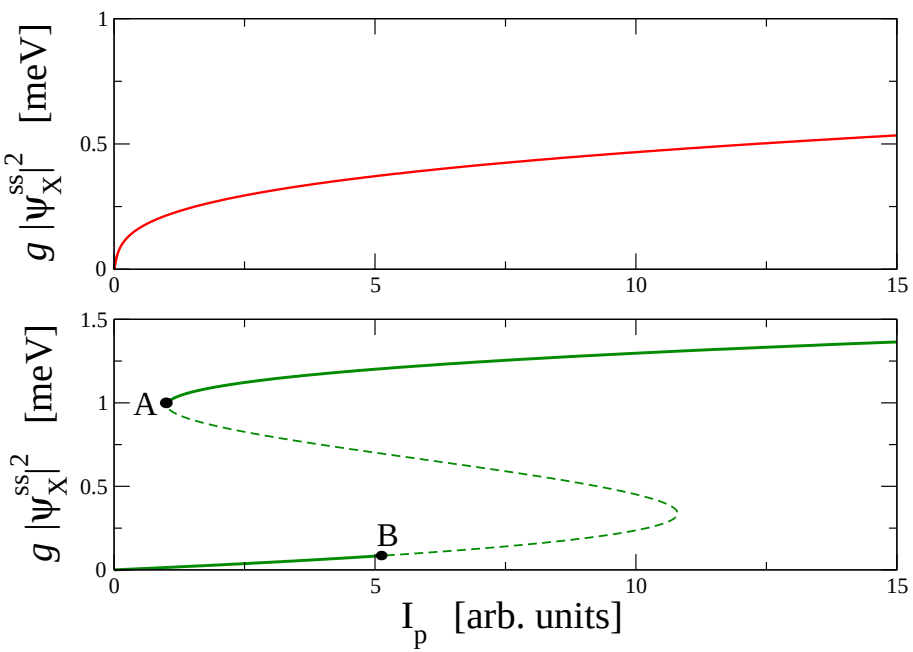
is the equation of state: oscillation frequency not fixed by $\mu=\mu(n)$ but tunable with ω_p.

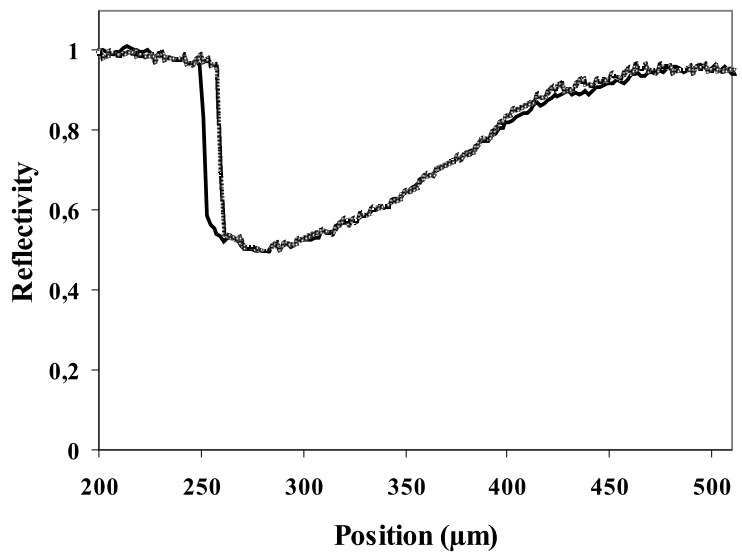
The equation of state

Equation of state relates internal intensity $|\psi^{ss}_{X,C}|^2$ to incident one $I_p \propto |\mathcal{E}_p|^2$.

Transmitted intensity $I_t \propto \gamma_C |\psi^{ss}_C|^2$:

- optical limiting for $\omega_p < \omega_{LP}(\mathbf{k}_p)$
- optical bistability for $\omega_p > \omega_{LP}(\mathbf{k}_p)$. Dashed branches $\longrightarrow$ unstable.





(Exp't: A. Baas *et al.*, PRA **69**, 023809, 2004)

Bogoliubov theory

Field variation from stationary state: $\delta\vec{\phi} = \left(\delta\phi_X(\mathbf{x},t), \delta\phi_C(\mathbf{x},t), \delta\phi_X^*(\mathbf{x},t), \delta\phi_C^*(\mathbf{x},t)\right)^T$

Linear response to weak perturbation: $i\frac{d}{dt}\delta\vec{\phi} = \mathcal{L}\cdot\delta\vec{\phi} + \vec{f}_d$.

Bogoliubov matrix:

$$\mathcal{L} = \begin{pmatrix} \omega_X + 2g\,|\psi_X^{ss}|^2 - \omega_p - i\gamma & \Omega_R & g\,\psi_X^{ss\,2}\,e^{2i\mathbf{k}_p\mathbf{x}} & 0 \\ \Omega_R & \omega_C(-i\nabla) - \omega_p - i\gamma & 0 & 0 \\ -g\,\psi_X^{ss\,*\,2}\,e^{-2i\mathbf{k}_p\mathbf{x}} & 0 & -\left(\omega_X + 2g\,|\psi_X^{ss}|^2\right) + \omega_p - i\gamma & -\Omega_R \\ 0 & 0 & -\Omega_R & -\omega_C(-i\nabla) + \omega_p - i\gamma \end{pmatrix}$$

Eigenvalues of $\mathcal{L}$ give Bogoliubov mode frequencies $\omega_{LP}^{\pm}(\mathbf{k})$ and $\omega_{UP}^{\pm}(\mathbf{k})$.

Particle- and hole-like $\pm$ branches symmetrical under $\mathbf{k} \to 2\,\mathbf{k}_p - \mathbf{k}$ and $\omega \to 2\,\omega_p - \omega$.

Mode dispersion depends on interaction energy $\delta\omega_{MF} = g\,|\psi_X^{ss}|^2$ and on pump frequency ω_p.

Approximate analytical results

Interaction energy $\delta\omega_{MF} = g\,|\psi_X^{ss}|^2 \ll \omega_{UP} - \omega_{LP} \implies$ only LP effectively excited

Parabolic approximation for low $\mathbf{k}$: $\omega_{LP}(\mathbf{k}) \approx \mathbf{k}^2/2m_{LP}$

$$\omega_{LP}^{\pm}(\mathbf{k}) \simeq \omega_p + (\mathbf{k}-\mathbf{k}_p)\cdot\mathbf{v}_p \pm \sqrt{\left(2\,\delta\omega_{MF} + \frac{(\mathbf{k}-\mathbf{k}_p)^2}{2m_{LP}} - \Delta_p\right)\left(\frac{(\mathbf{k}-\mathbf{k}_p)^2}{2m_{LP}} - \Delta_p\right)} - i\gamma$$

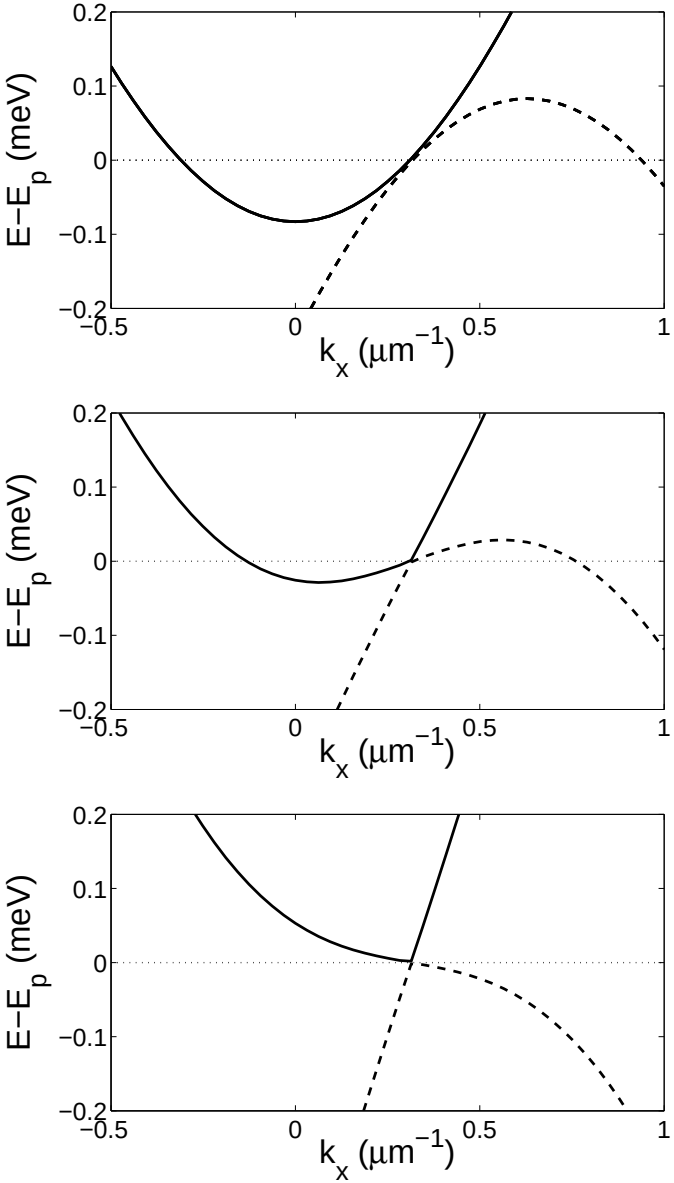
where effective detuning $\Delta_p = \omega_p - \omega_{LP}(\mathbf{k}_p) - \delta\omega_{MF}$.

Different regimes depending on sign of Δ_p and on value of $\delta\omega_{MF}$.

With respect to standard Bogoliubov:

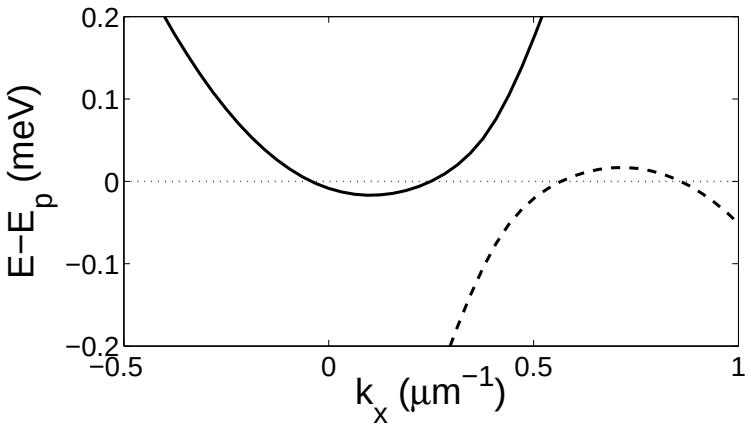
- tilting term $(\mathbf{k}-\mathbf{k}_p)\cdot\mathbf{v}_p$ due to flow velocity
- detuning term due to effective detuning Δ_p

Bogoliubov dispersion (I) : resonant case $\Delta_p = 0$

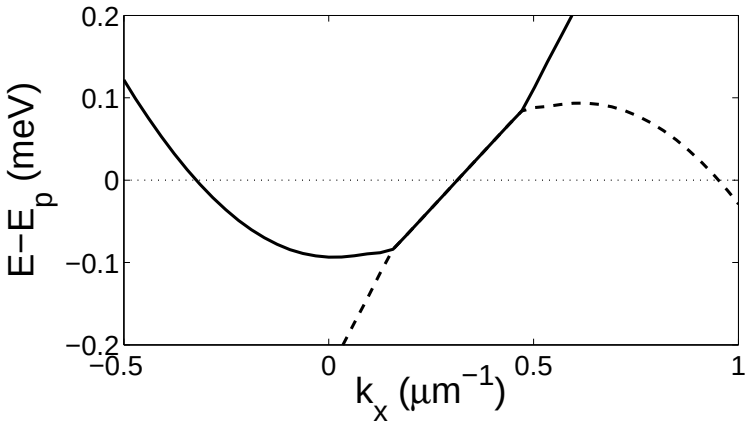


- $\Delta_p = 0$ at inversion point A on hysteresis curve
- Particle- and hole-like $\pm$ branches touch at $\mathbf{k}_p$
- Standard Bogoliubov dispersion tilted by $\mathbf{v}_p$
- Linear regime $\delta\omega_{MF} \approx 0$: almost parabolic LP polariton dispersion
- Interaction energy $\delta\omega_{MF}$ and sound velocity $c_s = \sqrt{\delta\omega_{MF}/m_{LP}}$ increasing from top to bottom
- Group velocity $v_g^{l,r} = v_p \pm c_s$. In particular, v_g^l changes sign when $c_s = v_p$ (Landau criterion).

Bogoliubov dispersion (II) : non-resonant case $\Delta_p \neq 0$



- Right of A on upper branch of hysteresis curve: $\Delta_p < 0$
- Branches no longer touch at $\mathbf{k}_p$, possibility of full gap.



- On lower branch of hysteresis curve: $\Delta_p > 0$
- Argument of square root is negative for :

$$\Delta_p > \frac{(\mathbf{k}-\mathbf{k}_p)^2}{2m_{LP}} > \Delta_p - 2\,\delta\omega_{MF}$$

- Branches stick together
- Imaginary parts split, dynam. instability if $\delta\omega_{MF} > \gamma$.

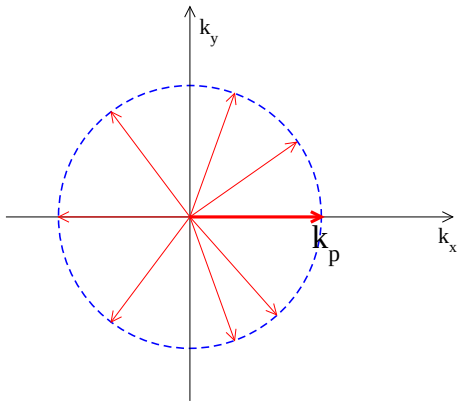
Optics $\longrightarrow$ parametric instability. Quantum fluids $\longrightarrow$ modulational instability

Response to weak, point-like, static defect

$$\vec{f}_d = \left(V_X(\mathbf{x})\, \phi_X^{ss}(\mathbf{x}),\ V_C(\mathbf{x})\, \phi_C^{ss}(\mathbf{x}),\ -V_X(\mathbf{x})\, \phi_X^{ss\,*}(\mathbf{x}),\ -V_C(\mathbf{x})\, \phi_C^{ss\,*}(\mathbf{x}) \right)^T$$

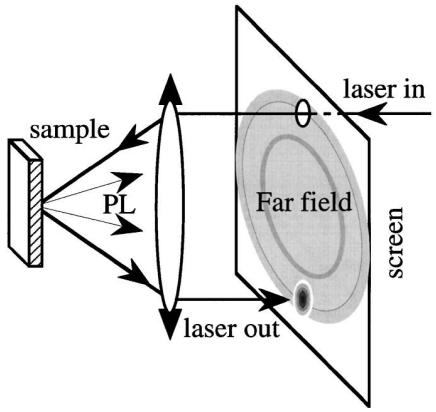
- Induced perturbation $\delta\vec{\phi}_d = -\mathcal{L}^{-1} \cdot \vec{f}_d$
- Bogoliubov modes at ω_p resonantly excited: Čerenkov emission of propagating sound waves
- Damping of sound waves over a length scale $\ell_{prop} \approx v_g/\gamma$.
- Excitation of non-resonant modes: localized, not propagating perturbation
- Disordered ensemble of defects: similar **k**-space pattern with speckle-like intensity modulation due to polaritonic coherence.

The resonant Rayleigh scattering (RRS)

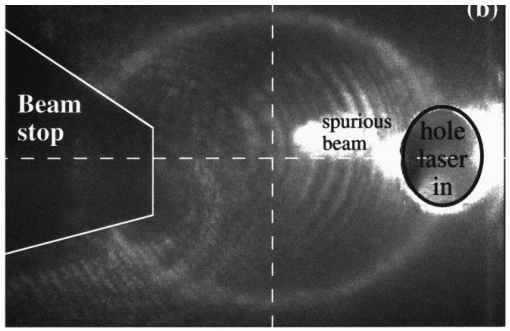


⟵ Scheme of RRS process

Experimental set-up ⟶

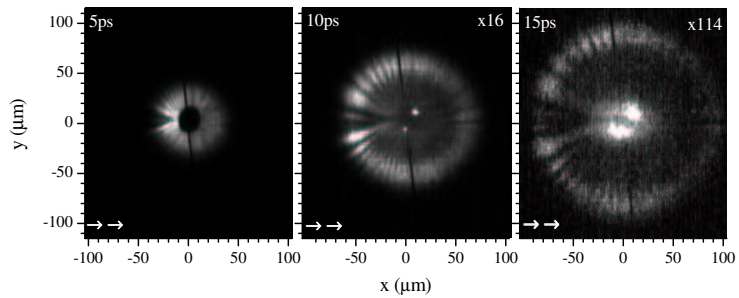


k-space $I(\mathbf{k}) \propto |\tilde{\psi}_C(\mathbf{k})|^2$: far–field image



(taken from: R. Houdré *et al.*, PRB **61**, 13333R, 2000)

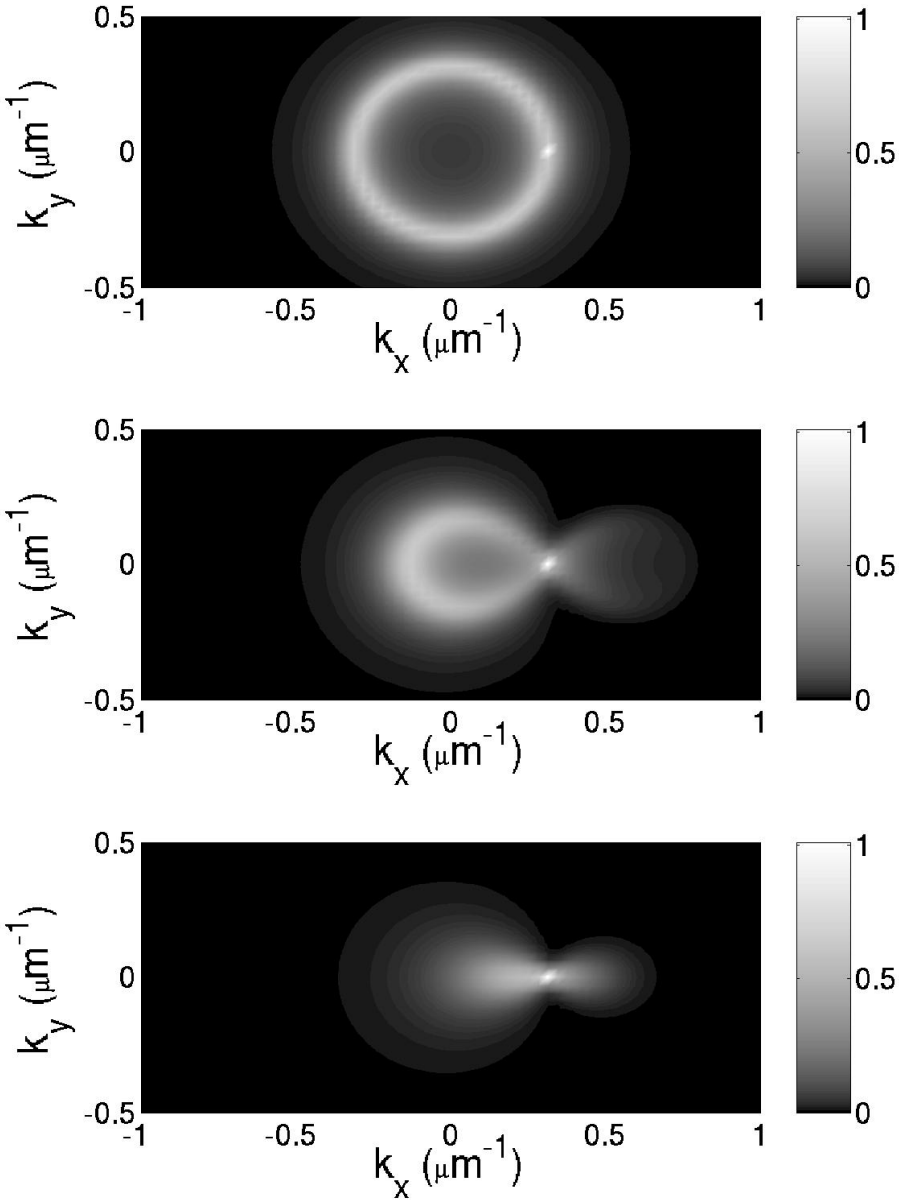
Real space $I(\mathbf{x}) \propto |\psi_C(\mathbf{x})|^2$: near–field image



(taken from: W. Langbein, proceedings of ICPS 26, 2002)

Discard incoherent photoluminescence by spectral selection of coherently emitted intensity at ω_p

Far-field emission pattern (I): resonant case $\Delta_p = 0$



Linear regime:

- peak at $\mathbf{k}_p$ due to unscattered light
- resonant Rayleigh scattering (RRS) ring

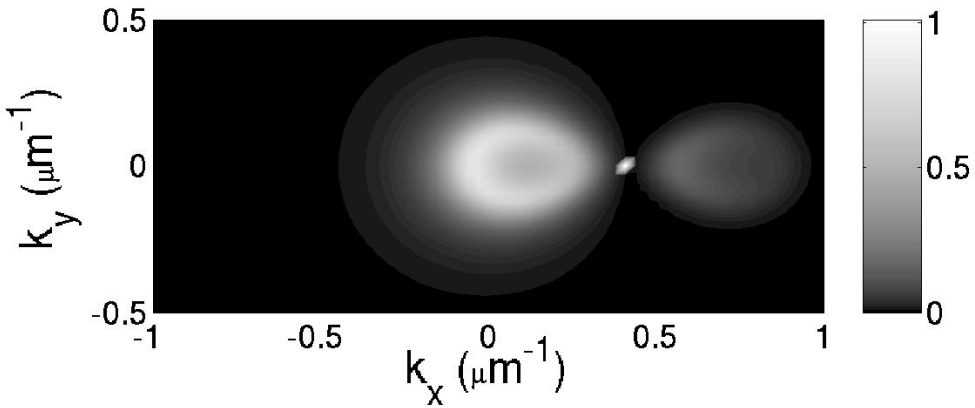
Nonlinear regime, $v_p > c_s$:

- RRS ring transformed into ∞-like shape
- lobes touch at $\mathbf{k}_p$, low-$\mathbf{k}$ lobe ($+$ branch) more intense than high-$\mathbf{k}$ lobe ($-$ branch)

Nonlinear regime, $v_p < c_s$:

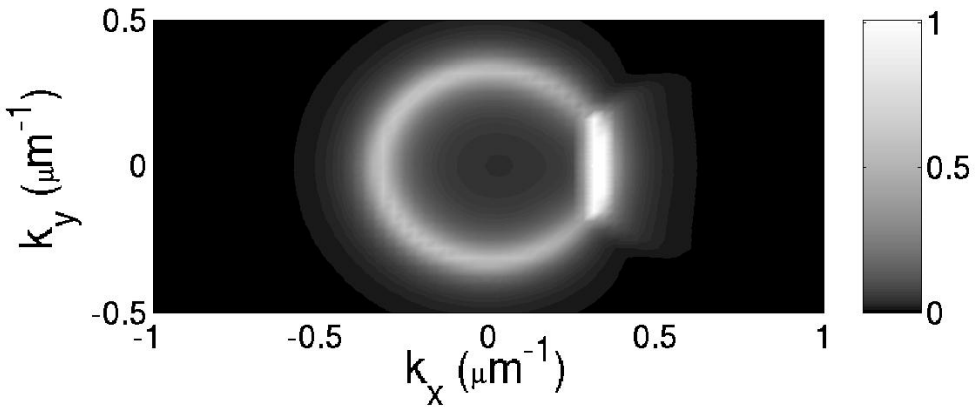
- Landau criterion predicts superfluidity
- lobes disappear, much weaker emission

Far-field emission pattern: non-resonant case $\Delta_p \neq 0$



Case $\Delta_p < 0$:

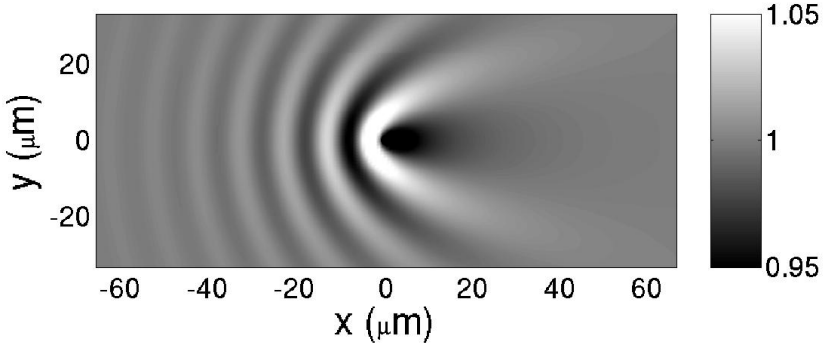
- gap appears in between the two lobes



Case $\Delta_p > 0$:

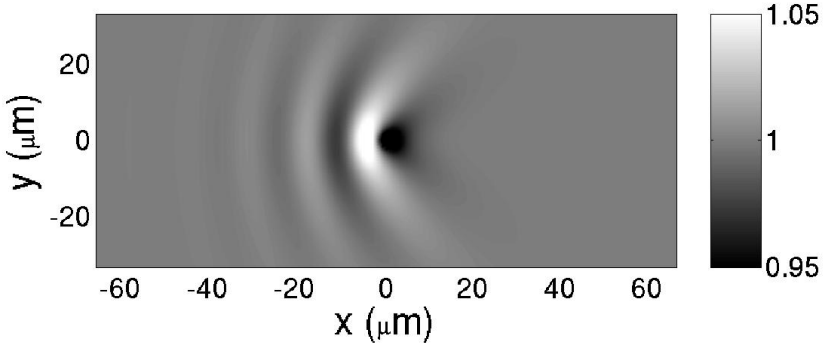
- lobes are pushed closer
- enhancement of RRS intensity in merging region around $\mathbf{k}_p$.
- precursor of parametric instability.

Near-field emission pattern: resonant case $\Delta_p = 0$



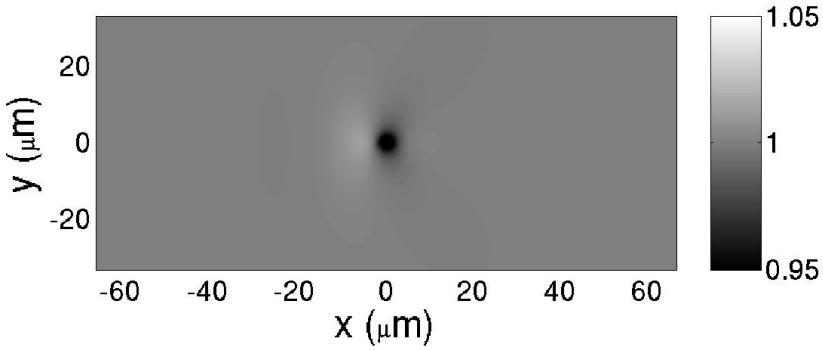
Linear regime:

- diffused light has parabolic wavefronts



Nonlinear regime, $v_p > c_s$:

- sound waves form Čerenkov cone.
  Its aperture: $\sin\theta = c_s/v_p$
- parabolic precursors ahead



Nonlinear regime, $v_p < c_s$:

- localized perturbation
- system superfluid: fluid flows around impurity without dissipation

The polariton fluid is moving to the right

Conclusions

A novel quantum Bose gas : exciton-polaritons in a semiconductor microcavity

- **Disadvantages:**
  - polaritons have finite lifetime, mostly due to extrinsic effects:
    - radiative decay of cavity-photon through cavity mirrors
    - absorption and dephasing due to sample imperfections
  - little is known about exciton-exciton scattering physics

- **Good points**:
  - New physics: driven-dissipative Bose system intrinsically out of equilibrium
  - Long-range coherence imprinted by driving laser. No need for complicate cooling procedure.
  - Exciton-exciton interactions are important $\delta\omega_{MF} \gg \gamma$
  - External trapping potential applicable
  - Exciton and photon spin degrees of freedom
  - Easy in-situ diagnostic through emitted light:
    - real- and **k**-space intensity distributions, higher-order correlation functions
    - time evolution with streak cameras (ps resolution).

Future developments

- Calculations have been performed for realistic experimental parameters for III-V semiconductor microcavities. How to take advantage of larger vacuum Rabi splitting of II-VI microcavities ?

- What is the best way of applying an external trapping potential to polaritons ?

- Better characterization of exciton-exciton interactions, in particular for different spin states: first experimental results available, but not yet fully understood

- Physics of inhomogeneous systems: oscillation modes in external potential