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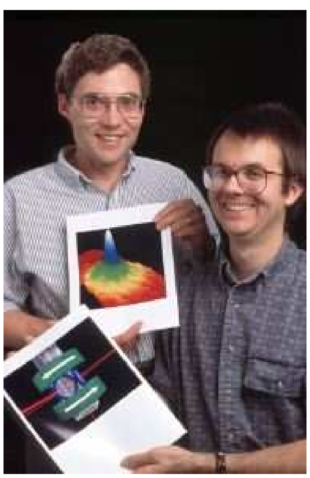
Controlled entanglement of quantum degenerate atoms

Li You

D. L. Zhou
B. Zeng, M. Zhang, Z. Xu (Tsinghua Univ.)
C. P. Sun, (ITP)

$ NASA
$ NSF
$ China NSFB

Bose-Einstein condensation (1995)



C. Wieman and E. Cornell

W. Ketterle

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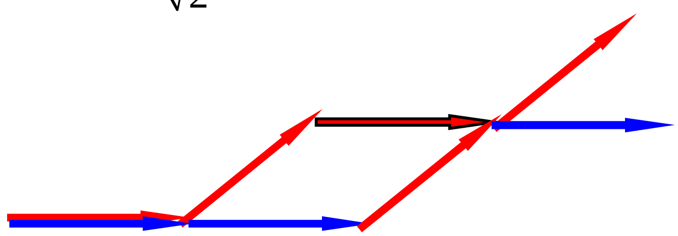
W. M. Itano et al., Phys. Rev. A **41**, 2295 (1990).

$$|\text{N-GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\uparrow,\uparrow,\cdots\rangle + |\downarrow,\downarrow,\downarrow,\cdots\rangle e^{iN\varphi}\right)$$

$$|\text{N}\rangle = \frac{1}{\sqrt{2^N}}\left(|\uparrow\rangle + |\downarrow\rangle e^{i\varphi}\right)^N$$



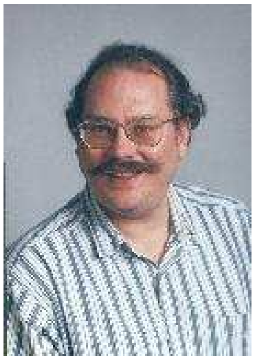
Prognosticatory peril

What is "the application" for quantum degenerate atoms?

1. Understanding and testing for many body theory?
   e.g. quantum phase transitions, BCS-BEC cross-over, High Tc superconductors,
2. Precision measurement or fundamental tests?
   e.g. clocks, edm, parity, atom interferometer
3. We don't care?
   smart people doing wonderful things,
4. **Quantum information science**

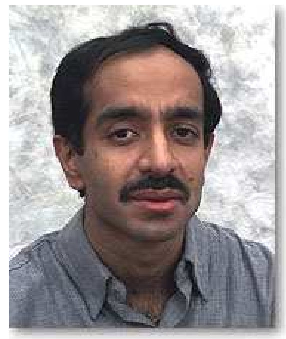
Quantum computing

- factoring on a quantum computer is an "easy" problem (1994)



P. Shor

- Database search (1996)



L.K. Grover

Entangled states

For distinguishable particles

$$\Psi(\vec{r}_A,\vec{r}_B,\cdots)=\left[\sum_{i=1}^{N_A}\alpha_i\phi_i(\vec{r}_A)\right]\otimes\left[\sum_{j=1}^{N_B}\beta_j\psi_j(\vec{r}_B)\right]\otimes\cdots$$

$$=\left[\alpha_1\phi_1(\vec{r}_A)+\alpha_2\phi_2(\vec{r}_A)+\cdots\right]$$

$$\otimes\left[\beta_1\psi_1(\vec{r}_B)+\beta_2\psi_2(\vec{r}_B)+\cdots\right]\otimes\cdots$$

$$\Psi(\vec{r}_A,\vec{r}_B,\cdots)=\sum_{i,j,\cdots=1}^{N_A,N_B,\cdots}\Upsilon_{i,j,\cdots}\,\phi_i(\vec{r}_A)\psi_j(\vec{r}_B)$$

2-particles **Schmidt decomposition** $\Upsilon=U^T D V$

3 or more particles, mixed states, ???

Quantum state of a condensate

$$\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\cdots)=\phi(\vec{r}_1)\phi(\vec{r}_2)\phi(\vec{r}_3)\cdots$$

GP equation (coarse-grained mean-field)

$$\left[-\frac{\hbar^2\nabla^2}{2M}+V_{\text{ext}}(\vec{r})+a_{sc}N\left|\phi(\vec{r})\right|^2\right]\phi(\vec{r})=\mu\phi(\vec{r})$$

Two-atom scattering state $\Psi(\vec{r}_1,\vec{r}_2)\xrightarrow{|\vec{r}_1-\vec{r}_2|\to\infty}1-\frac{a_{sc}}{|\vec{r}_1-\vec{r}_2|}$

Number fluctuations, Phase diffusions, Goldstone modes,

Phys. Rev. Lett. **77**, 3489 (1996); **90**, 140404 (2003).

Correlated states

Condensed matter physics of strongly correlated systems in atomic quantum gases

1. Feshbach resonances, unitary limit,
2. Fast rotations, Hall states
3. Optical lattices, Mott transitions

equilibrium states: large degeneracies near ground state

Manybody entanglement

correlated states from controlled dynamics

Atomic quantum gases

1. Many atoms (internal and external degrees of freedom),
2. Controlled interactions
3. Highly efficient quantum detections

Identical particle entanglement

Single particle (mode) entanglement ?

$$\left(a^{\dagger}+b^{\dagger}\right)|0,0\rangle_{+,-}\rightarrow\frac{1}{\sqrt{2}}\left(|1,0\rangle_{+,-}+|0,1\rangle_{+,-}\right)$$

Separable for the modes,
but particle entangled?

$$a^{\dagger N}b^{\dagger N}\,|0,0\rangle\rightarrow|N,N\rangle$$

$$\rightarrow[\varphi_a(\vec{r}_A)\varphi_b(\vec{r}_B)+\varphi_b(\vec{r}_A)\varphi_a(\vec{r}_B)]^{\otimes N}$$

L.-M. Duan, J. I. Cirac, and P. Zoller, Phys. Rev. A **65**, 033619 (2002)

Two fermions

Schmidt expansion in terms of orthogonal Slater determinant

$$\left|\Psi_F\right\rangle = \sum_{i,j}^{N} \omega_{ij} f_i^\dagger f_j^\dagger \,|0\rangle$$

$$= 2 \sum_{k}^{\leq N/2} Z_k \, f_{2k-1}^{'\dagger} f_{2k}^{'\dagger} \,|0\rangle$$

John Schliemann, Daniel Loss, A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001).

John Schliemann, J. I. Cirac, M. Ku\'s, M. Lewenstein, and Daniel Loss, Phys. Rev. A **64**, 022303 (2001).

Is this useful entanglement or correlation ?

Two bosons

Schmidt expansion in terms of orthogonal 2-boson modes

$$\left|\Psi_B\right\rangle = \sum_{i,j}^{N} \beta_{ij} b_i^\dagger b_j^\dagger \,|0\rangle$$

$$= \sqrt{2} \sum_{k}^{N} B_k b_k^{'\dagger} b_k^{'\dagger} \,|0\rangle$$

$$= B_1 \,|2,0,0,\cdots\rangle + B_2 \,|0,2,0,\cdots\rangle + B_3 \,|0,0,2,\cdots\rangle + \cdots$$

R. Pauskauskas and L. You, Phys. Rev. A **64**, 042310 (2001).

Y.S. Li, B. Zeng, X.S. Liu, and G. L. Long, Phys. Rev. A **64**, 054302 (2001).

Inseparable correlations beyond what is required from exchange symmetry among identical particles

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Entanglement and spin squeezing of three bosons in two modes

L. You

D. L. Zhou
B. Zeng, Z. Xu (Tsinghua Univ.)

Three distinguishable qubits

Generalized Schmidt decomposition:

$$l_0 |000\rangle + l_1 e^{ij} |100\rangle + l_2 |101\rangle + l_3 |110\rangle + l_4 |111\rangle$$

A. Acin, A. Andrianov, L. Costa, E. Jane, J.I. Latorre, and R. Tarrach, Phys. Rev. Lett. **85**, 1560(2000).

3-qubir entanglement, SLOCC,

Two types of entanglement:

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$$

W. Dur, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314(2000).

Three bosons in two modes

Standard form:

$$r\,|000\rangle + se^{ij}\left(|100\rangle + |010\rangle + |001\rangle\right) + t\,|111\rangle$$

Single particle basis state (unitary transformation),

Two types of entanglement:

$$|\mathrm{GHZ}\rangle = z_a\,|aaa\rangle + z_b\,|bbb\rangle$$

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3\left(1+2|\langle a\,|\,b\rangle|^2\right)}}\left(|abb\rangle + |bab\rangle + |bba\rangle\right)$$

Creating maximally entangled atomic states in a condensate

Li You

L. You, Phys. Rev. Lett. **90**, 030402 (2003)

$$|\text{N-GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\uparrow,\uparrow,\cdots\rangle + |\downarrow,\downarrow,\downarrow,\cdots\rangle\right)$$

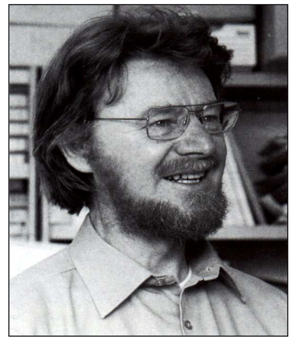
B. Zeng, D. L. Zhou, P. Zhang, Z. Xu, and L. You, Phys. Rev. A **68**, 042316 (2003).

$$\langle \text{N-GHZ}|\,\rho_{N-particle}\,|\text{N-GHZ}\rangle > \frac{1}{2}$$

EPR and Bell

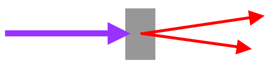
EPR, Phys. Rev. 47, 777 (1935);
J. S. Bell, Physics (Long Island City, NY) 1, 195 (1964).
A. Aspect, J. Dalibard, Roger, PRL **49**, 1804 (1982)

$$|\text{BELL}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$



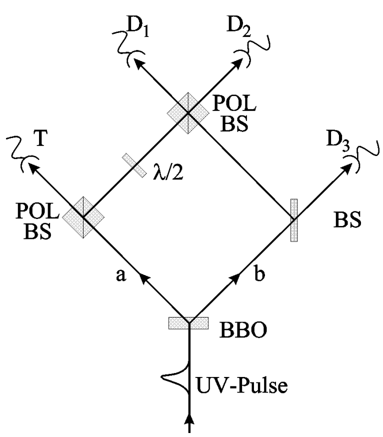
Bell

Bi-photons (SPDC)



Z. Y. Ou & L. Mandel, PRL. 61, 50 (1988)
Y. H. Shih & C. O. Alley, PRL. 61, 2921 (1988)
Kwiat & Zeilinger et. al, PRL. 75, 4337 (1995)

Greenberger-Horne-Zeilinger



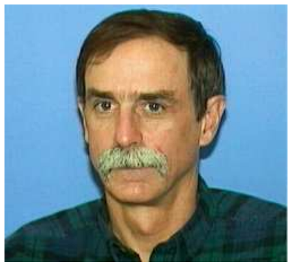
A. Zeilinger et. al, PRL **82**, 1345 (1999).

4-photons

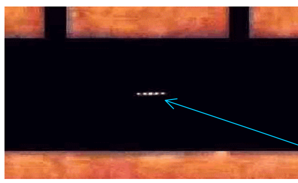
J.-W. Pan, M. Daniell, S. Gasparoni, G. Weihs, and A. Zeilinger
Phys. Rev. Lett. **86**, 4435 (2001)

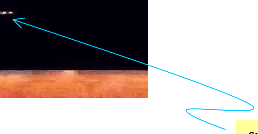
4-particle entanglement

Wineland



Monroe



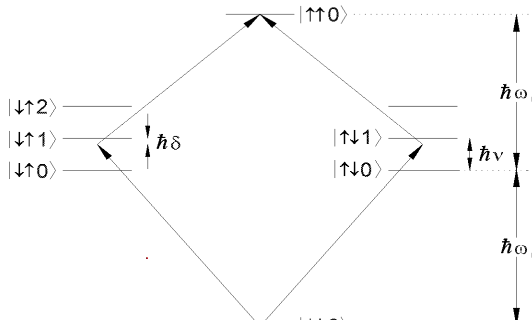


D. J. Wineland, and C. I. Monroe *et. al*,

NATURE **404**, 256 (2000)

Molmer proposal

Molmer and Sorensen,
PRL **82**, 1835 (1999).

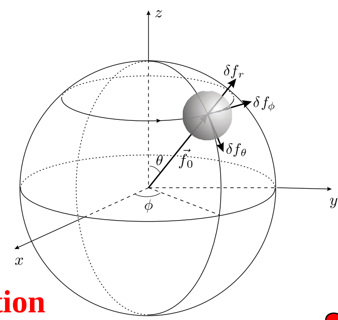


$$\sum_{i<j}^{N} \frac{\hbar}{2} \Omega_R \sigma_x^{(i)} \otimes \sigma_x^{(j)}$$

$$= \frac{\hbar}{2} \Omega_R \left(|\downarrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow| \right)_i \left(|\downarrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow| \right)_j \propto J_x^2$$

N-GHZ $|\uparrow,\uparrow,\uparrow,\uparrow,\cdots\rangle + |\downarrow,\downarrow,\downarrow,\downarrow,\cdots\rangle$

Bloch sphere & pseudo spin



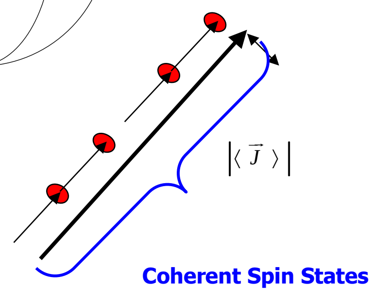
$$J = \frac{N}{2}$$

2 mode condensate
Schwinger representation

$$J_x = \frac{1}{2}\left(b^\dagger a + a^\dagger b\right)$$

$$J_y = -\frac{i}{2}\left(b^\dagger a - a^\dagger b\right)$$

$$J_z = \frac{1}{2}\left(b^\dagger b - a^\dagger a\right)$$

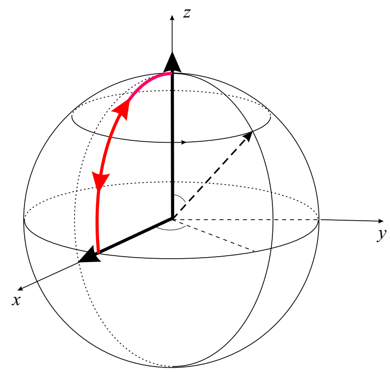


Atom collisions

$$H = u J_z^2,$$

$$u = \left(a_{aa} + a_{bb} - 2a_{ab}\right) \int d\vec{r} \left|\varphi_{a/b}(\vec{r})\right|^4$$

SU(2) rotations



$J_x,\quad J_y,\quad J_z,\quad J_z^{\,2}$

Can be used to generate any symmetric state

Two mode condensate

Single atom Raman coupling

$H = \Omega J_y$

is readily implemented with external laser pulses

Step 3 **Step 1**

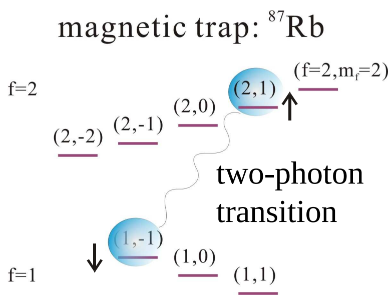
$$U(t) \approx e^{i\frac{\pi}{2}J_y} e^{-iuJ_z^2 t/\hbar} e^{-i\frac{\pi}{2}J_y} = e^{-iuJ_x^2 t/\hbar}$$

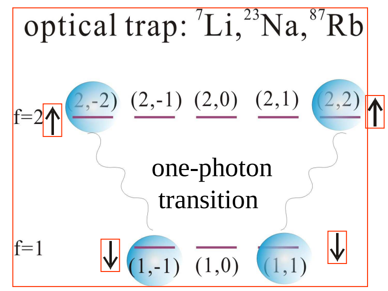
Step 2

$|\Omega| \gg |u| N$

Two mode model

D. S. Hall et al., Phys. Rev. Lett. **81**, 4532 (1998)





Nikuni and Williams, **cond-mat/0304095**

Mott state

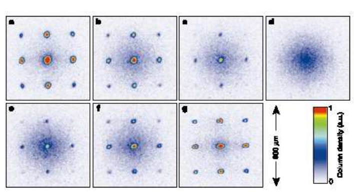
Condensate in a periodic potential

J, single atom tunneling rate of the nearest neighbor wells

u, on-site collisional interaction

Quantum Phase Transition

$$\frac{|u|}{J} \approx z \times 2.6,$$



M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, and I. Bloch, Nature **413**, 44 (2002).

D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).

A slow process

$$u=\left(a_{aa}+a_{bb}-2a_{ab}\right)\int d\vec{r}\left|\varphi_{a/b}(\vec{r})\right|^4$$

small for Rb87 $a_{aa} \approx a_{bb} \approx a_{ab}$

For an optical lattice with $\omega_{trap} \approx (2\pi)30(\mathrm{kHz})$

$$u \approx (2\pi)20(\mathrm{Hz}) \Rightarrow \text{N-GHZ in 10 (milli-second)}$$

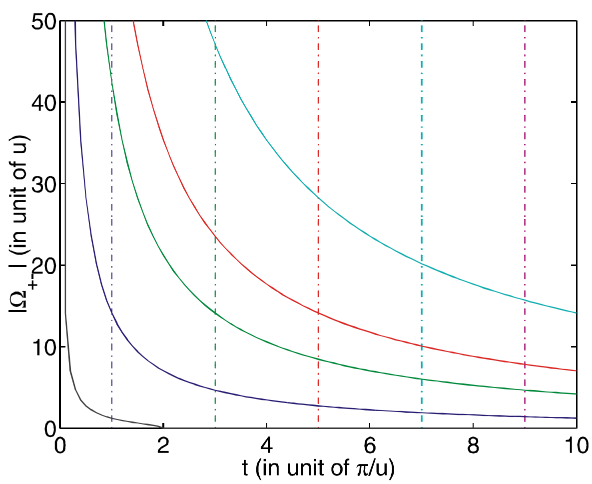
Inelastic collisional decay of population and

Elastic collisional decay of M by populating other Zeeman states

Life-time of 2-body dipolar loss > 6 second
Life-time of 3-body inelastic collision loss > 200 mini-second (<5 atoms)

E. A. Burt *et. al.*, Phys. Rev. Lett. **79**, 337 (1997).

Constant drive of 2 modes

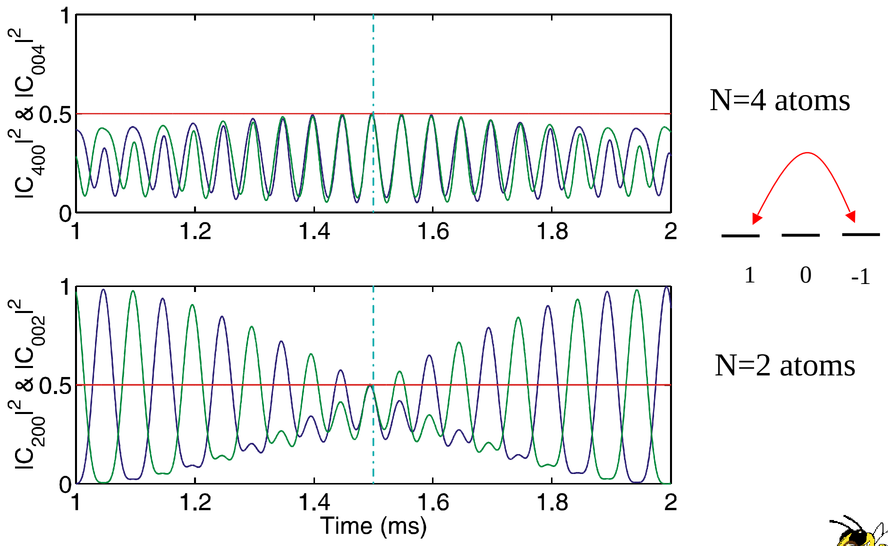


$$H = uS_z^2 + \Omega S_y$$

$$(ut) = (2k+1)\pi$$

$$(ut)\sqrt{1+2\left(\frac{\Omega}{u}\right)^2} = 2m\pi$$

4- & 2-GHZ



Summary

- Maximally entangled pairs, triplets, quartiles, …
- Massive parallelism and common addressing
- Efficient $\left|\frac{\Omega}{u}\right| \gg 1, \quad \geq 100$ for up to 4-atoms
- Collision stable, (superposition of eigen-states)

$$\frac{1}{\sqrt{2}}\left(|\uparrow,\uparrow,\uparrow,\dots\rangle + |\downarrow,\downarrow,\downarrow,\dots\rangle\right)$$

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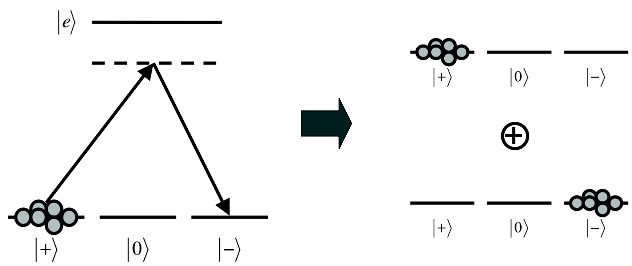
Quantum Zeno subspaces and maximally entangled states in a spin-1 condensate

Li You

M. Zhang (Tsinghua Univ.)

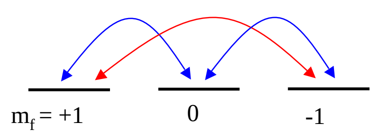
PRL, **91**, 230404 (2003)



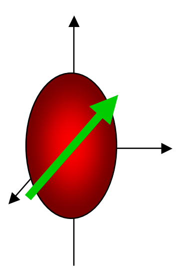
Spin-1 condensate

A vector field

$$\vec{\hat{\Psi}} = \begin{pmatrix} \hat{\Psi}_+ \\ \hat{\Psi}_0 \\ \hat{\Psi}_- \end{pmatrix}$$



$$c_0 = \frac{4\pi\hbar^2}{3M}(a_0 + 2a_2), \quad c_2 = \frac{4\pi\hbar^2}{3M}(a_2 - a_0)$$



Single mode approximation

$$\Psi_\alpha \equiv a_\alpha(t)\phi(\vec{r})$$

Phys. Rev. A **66**, 011601 (2002).

Schwinger representation

SU(3) of three bosonic modes

$$a_+,\ a_0,\ a_-$$

$$J_+ = \sqrt{2}\left(a_+^\dagger a_0 + a_0^\dagger a_-\right)$$

$$J_- = J_+^\dagger$$

$$J_z = a_+^\dagger a_+ - a_-^\dagger a_-$$

Casimir relation ?

Law et al., PRL **81**, 5257 (1998);
Ho and Yip, PRL **84**, 4031 (2000).

$$N = a_+^\dagger a_+ + a_0^\dagger a_0 + a_-^\dagger a_-$$

$$A = \left(a_0^2 - 2a_+a_-\right)/\sqrt{3}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2 = N(N+1) - 3A^\dagger A$$

SU(2) symmetric term, e.g. J^2

Do squeeze SU(2) subspaces !

SU(3) decomposition

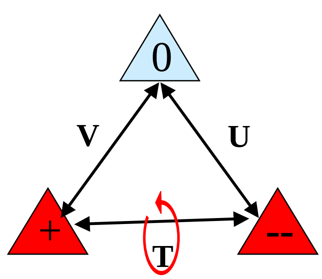
$$S^2 = 4T_3^2 + \frac{1}{2}(N-\varepsilon_+)(N-\varepsilon_-) - 2(Y-Y_0)^2 + G_Y$$

$$G_Y = 2\left(V_+U_+ + h.c.\right)$$

Phys. Rev. A **66**, 033611 (2002).

Gell-Mann decomposition

$$T_+ = a_+^\dagger a_-;\quad T_3 = (N_+ - N_-)/2$$
$$V_+ = a_+^\dagger a_0;\quad V_3 = (N_+ - N_0)/2$$
$$U_+ = a_-^\dagger a_0;\quad U_3 = (N_- - N_0)/2$$
$$Y = (N_+ + N_- - 2N_0)/3$$



Quantum Zeno subspace

P. Facchi and S. Pascazio, Phys. Rev. Lett. **89**, 080401 (2002).

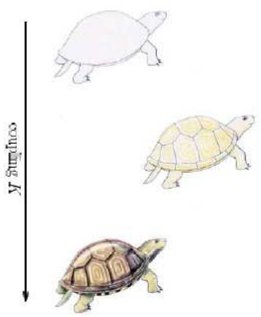
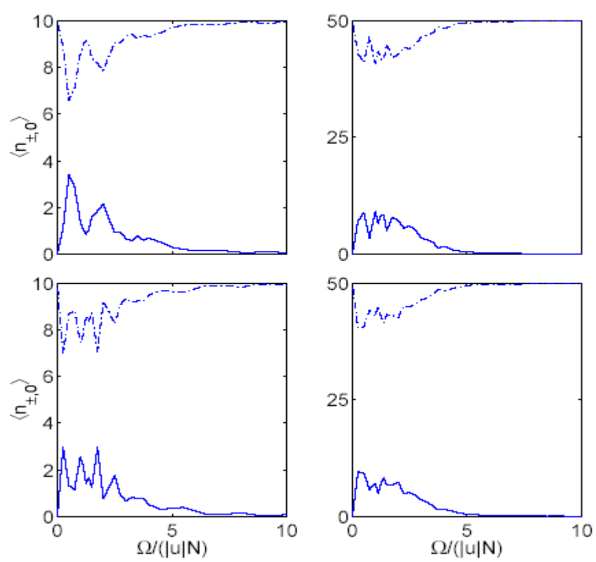


FIG. 1 (color online). The Hilbert space of the system: an effective superselection rule appears as the coupling K to the apparatus is increased.

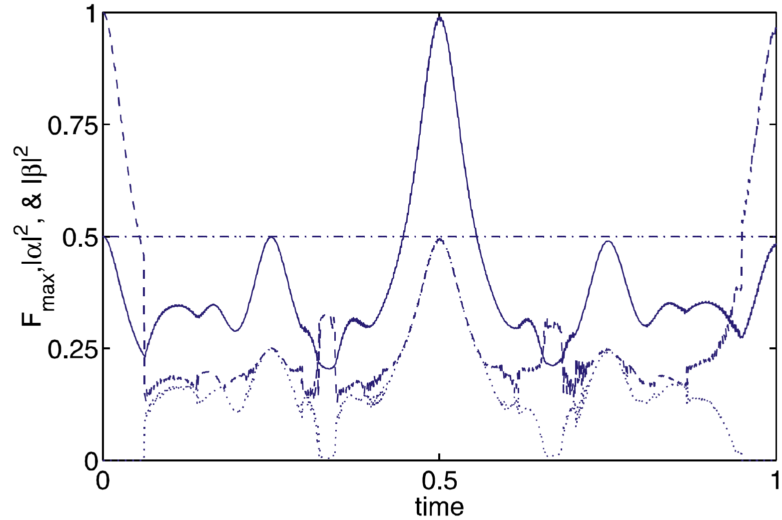
$$H_K = H + K\,H_{\text{measurement}}$$

$$\Omega\, T_y$$

Formation of a Zeno subspace



Perfect N-GHZ



N=20 atoms

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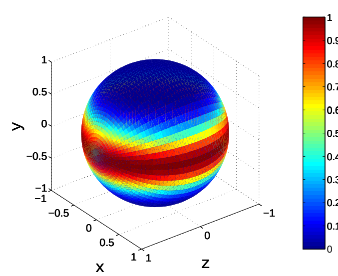
N-qubit entanglement from the J_y^2-type collective interaction

L. You

D. L. Zhou
B. Zeng, Z. Xu (Tsinghua Univ.)
C. P. Sun (ITP)



Motivations

1. strong restrictions on the initial states: all qubits in one single orbit orthogonal to y-direction

2. an even-odd parity: even and odd qubits need different single-body interactions

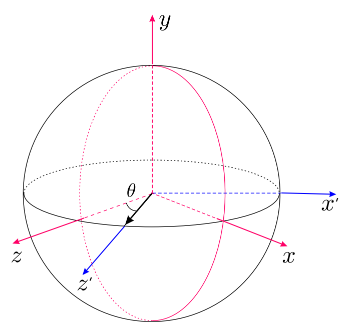
Geometrical representation

Initial separate state:

$$|\psi\rangle = \prod_{k=1}^{N} \left(\cos\frac{\theta}{2}|0\rangle^{k} + \sin\frac{\theta}{2}|1\rangle^{k} \right)$$

Interaction type:

$$H_I \propto \sum_{j,k=1,j<k}^{N} \sigma_y^{(j)} \sigma_y^{(k)} \sim J_y^2$$



Proper state basis:

$$|0\rangle_{x'}^{i} = \cos\frac{\theta_i + \frac{\pi}{2}}{2}|0\rangle^{(i)} + \sin\frac{\theta_i + \frac{\pi}{2}}{2}|1\rangle^{(i)}$$

$$|1\rangle_{x'}^{i} = -i\sin\frac{\theta_i + \frac{\pi}{2}}{2}|0\rangle^{(i)} + i\cos\frac{\theta_i + \frac{\pi}{2}}{2}|1\rangle^{(i)}$$

N-GHZ'

Unitary evolution:

$$S = \exp\left(-iN(N-1)\frac{\pi}{4}\right)\exp\left(i(N-1)\frac{\pi}{4}\sum_{j}^{N}\sigma_y^{(j)}\right)\exp\left(-i\frac{\pi}{4}\sum_{j,k=1,j<k}^{N}\sigma_y^{(j)}\sigma_y^{(k)}\right)$$

$$= \sum_{i,j=1,i<j}^{N}\frac{1}{2}\left(I+\sigma_y^{(i)}+\sigma_y^{(j)}-\sigma_y^{(i)}\sigma_y^{(j)}\right)$$

Final N-GHZ state at selected times:

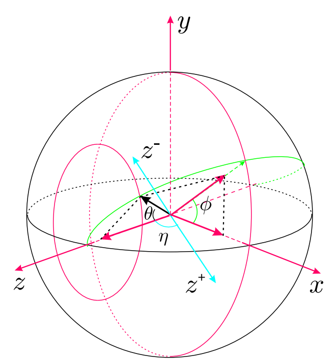
$$|\psi_M\rangle = \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{N}|0\rangle_{x'}^{i}+\prod_{i=1}^{N}|1\rangle_{x'}^{i}\right)$$

H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86,** 910 (2001).

Optimal Projection

General separate N-qubit state:

$$|\psi\rangle = \prod_{k=1}^{N}\left(\cos\frac{\theta}{2}|0\rangle^{k}+\sin\frac{\theta}{2}e^{i\phi}|1\rangle^{k}\right)$$



The final state can be regarded as linear Superposition of 2^N N-GHZ states:

$$|\psi_F\rangle = \sum_{\{s_k\}}\left(\prod_{k=1}^{N}{}^{(k)}\langle\phi^{s_k}|\psi\rangle^{(k)}\right)\left(\prod_{k=1}^{N}|\phi^{s_k}\rangle\right)$$

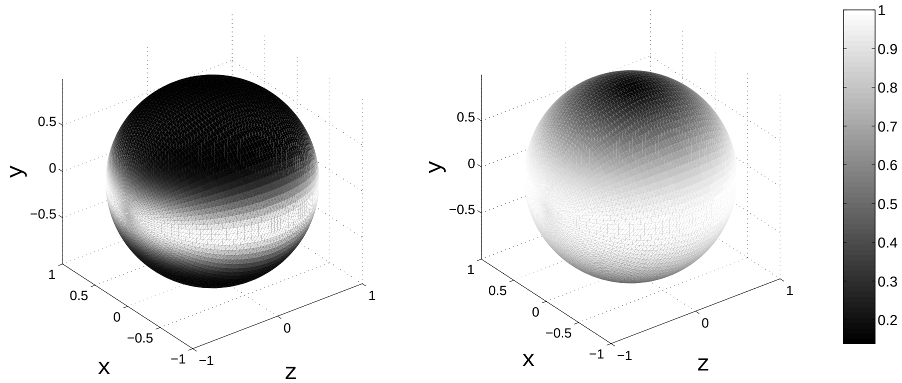
Where the basis

$$|\phi^{+}\rangle^{k} = \cos\frac{\eta_k}{2}|0\rangle^{(k)}+\sin\frac{\eta_k}{2}|1\rangle^{(k)}$$

$$|\phi^{-}\rangle^{k} = -\sin\frac{\eta_k}{2}|0\rangle^{(k)}+\cos\frac{\eta_k}{2}|1\rangle^{(k)}$$

Maximum distribution

$$p^{\max}(\eta_k) = \frac{1}{2}\left(1 + \sqrt{1 - \sin^2\theta_k \sin^2\phi_k}\right) \rightarrow \frac{1}{2^N}\left(1 + \sqrt{1 - \sin^2\theta \sin^2\phi}\right)^N$$



$N = 30$ $N = 3$

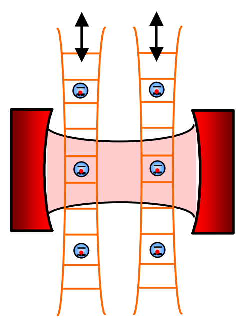
Summary

- To generating a N-qubit GHZ state, the initial separate state is not necessarily to be the **same** single-orbit state, only requires to lie in the Z-X plane.
- For an arbitrary initial separate state, the final state can be regarded as a superposition of 2^N N-GHZ states. There is an **optimal projection** decomposition which maximizes one of the GHZ state amplitude.

Encoding a physical qubit into a logical qubit

L. You

D. L. Zhou,
B. Zeng, Z. Xu (Tsinghua Univ.)
C. P. Sun (ITP)



Errors (single qubit)

any 1-bit error:

$$E_i = e_{i0} I + e_{i1} X_1 + e_{i2} Z_1 + e_{i3} X_1 Z_1$$

bit-flip: $X_1: \quad \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$

phase-error: $Z_1: \quad \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$

Error correcting codes

bit-flip code

$$|0\rangle \rightarrow |0_L\rangle \equiv |000\rangle$$
$$|1\rangle \rightarrow |1_L\rangle \equiv |111\rangle$$

phase-flip code

$$|0\rangle \rightarrow |0_L\rangle \equiv |+++\rangle$$
$$|1\rangle \rightarrow |1_L\rangle \equiv |---\rangle$$

Initial state

$$H = uJ_x^2$$

$$|\psi(0)\rangle = \left(\alpha|0\rangle + \beta|1\rangle\right) \otimes \underbrace{|0,\cdots,0\rangle}_{N}$$

N ancilla bits

Cavity QED
Josephson junctions
Atoms in an optical lattice

Three qubit bit-flip code

$$|0\rangle \rightarrow |0_L\rangle \equiv |000\rangle$$
$$|1\rangle \rightarrow |1_L\rangle \equiv |111\rangle$$

$1\, bit\ error\ rate\ p<1,$

$(3\, bit)\ error\ rate\quad p^3+3p^2\,(<p),$

error-detection (quantum measurement):

$P_0 \equiv |000\rangle\langle 000| + |111\rangle\langle 111|$ no error

$P_1 \equiv |100\rangle\langle 100| + |011\rangle\langle 011|$ bit flip on qubit one

$P_2 \equiv |010\rangle\langle 010| + |101\rangle\langle 101|$ bit flip on qubit two

$P_3 \equiv |001\rangle\langle 001| + |110\rangle\langle 110|$ bit flip on qubit three

Recovery: flip the flipped qubit again

Three qubit phase-flip code

$$|0\rangle \to |0_L\rangle \equiv |+++\rangle$$
$$|1\rangle \to |1_L\rangle \equiv |---\rangle$$

error-detection (quantum measurement):

$P_0 \equiv |+++\rangle\langle+++| + |---\rangle\langle---|$ no error

$P_1 \equiv |-++\rangle\langle-++| + |+--\rangle\langle+--|$ phase flip on qubit one

$P_2 \equiv |+-+\rangle\langle+-+| + |-+-\rangle\langle-+-|$ qubit two

$P_3 \equiv |++-\rangle\langle++-| + |--+\rangle\langle--+|$ qubit three

Recovery: flip the flipped qubit again

The Shor code

$$|0\rangle \to |0_L\rangle \equiv \left(\frac{|000\rangle + |111\rangle}{\sqrt{2}}\right)^{\otimes 3}$$

$$|1\rangle \to |1_L\rangle \equiv \left(\frac{|000\rangle - |111\rangle}{\sqrt{2}}\right)^{\otimes 3}$$

P. Shor, PRA **52,** 2493 (1995).

Our encoding scheme

$$\alpha|0\rangle+\beta|1\rangle \rightarrow \alpha\underbrace{|0,\cdots,0\rangle}_{N+1}+\beta\underbrace{|1,\cdots,1\rangle}_{N+1}$$

Initial state

$$|\psi(0)\rangle=(\alpha|0\rangle+\beta|1\rangle)\otimes\underbrace{|0,\cdots,0\rangle}_{N}$$

N ancilla bits

First protocol

$$(\alpha|0\rangle+\beta|1\rangle)\otimes\underbrace{|0,\cdots,0\rangle}$$

$$H=uJ_x^2$$

$$\underbrace{\qquad\qquad}$$

$$\sigma_z^{(1)}\sigma_z^{(2)}$$

$$\underbrace{\qquad\qquad}$$

$$H=uJ_x^2$$

Alternative protocol

$$\left(\alpha|0\rangle \underbrace{+\beta|1\rangle\right)\otimes|0,\cdots,0\rangle}$$

$$H = uJ_x^2$$

Measure 1st qubit

Experimental systems

Ion traps

$$\left(\alpha|0\rangle+\beta|1\rangle\right)\underbrace{|0,\cdots,0\rangle}_{N=8}$$

$$\to \alpha|0,0,0\rangle|0,0,0\rangle|0,0,0\rangle+\beta|1,1,1\rangle|1,1,1\rangle|1,1,1\rangle$$

$$\to \alpha\left[\frac{1}{\sqrt{2}}\left(|0,0,0\rangle-|1,1,1\rangle\right)\right]^{\otimes 3}+\beta\left[\frac{1}{\sqrt{2}}\left(|0,0,0\rangle+|1,1,1\rangle\right)\right]^{\otimes 3}$$

Cavity QED
Josephson junctions
Atoms in an optical lattice

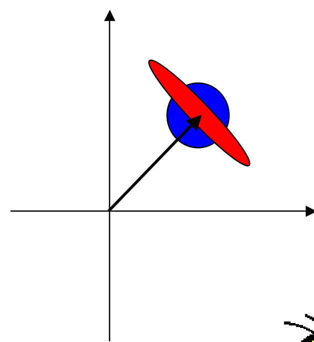
Georgia Institute of Technology

Atomic Spin Squeezed States

$$S(\xi) = \exp\left[(g a^{\dagger 2} - g^{*} a^{2}) t\right]$$

$$[x, p_x] = i\hbar$$

$$\Delta x \Delta p_x \geq \frac{1}{2}\hbar$$



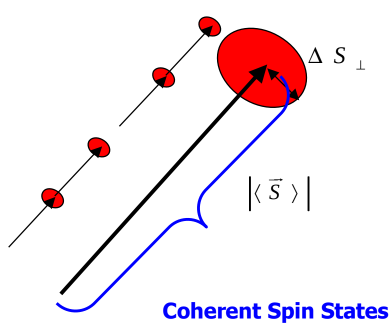
Spin squeezing

$$[S_i, S_j] = i\hbar S_k$$

$$\Delta S_x \Delta S_y \geq \frac{1}{4}\hbar^2 \langle S_k \rangle^2$$

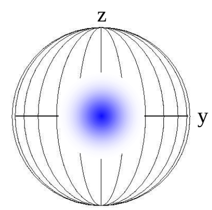
System of fixed particles

$$S = \frac{N}{2}$$



SU(2) coherent spin state

Coordinate rotation “squeezes” spin?



Squeezing:
area preserving deformations

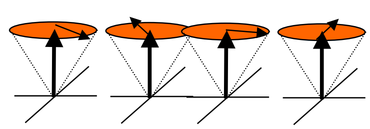
$$\frac{\Delta S_{\perp}}{\left|\langle \vec{S} \rangle\right|}$$

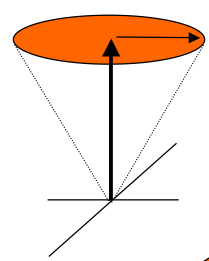
M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993)

D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, Phys. Rev. A. **50**, 67 (1994)

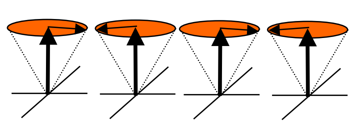
Spin-spin correlation

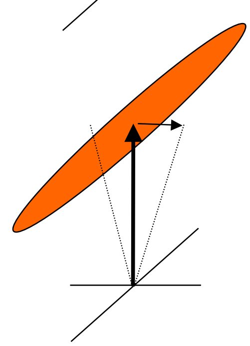
random





correlated





M. Kitagawa and M. Ueda,
PRA **47**, 5138 (1993)

Spin squeezing

$$x := \frac{2}{S}\left\langle \left(\Delta S_{\perp}\right)^2 \right\rangle_{\min}$$

M. Kitagawa, M. Ueda, Phys. Rev. A **47**, 5138(1993).
D. Wineland, et al, Phys. Rev. A **50**, 67(1994).

Entanglement & spin squeezing

$$\xi^2 \equiv \frac{(2S)(\Delta \vec{S}\cdot\vec{n}_1)^2}{(\vec{S}\cdot\vec{n}_2)^2 + (\vec{S}\cdot\vec{n}_3)^2} < 1$$

$\vec{n}_1, \vec{n}_2, \vec{n}_3$ Mutually orthogonal unit vectors

Inseparable N-particle state

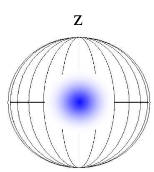
$$\rho_{\otimes N} \neq \sum_k p_k \rho_k^{(1)} \otimes \rho_k^{(2)} \otimes \rho_k^{(3)} \cdots \otimes \rho_k^{(N)}$$

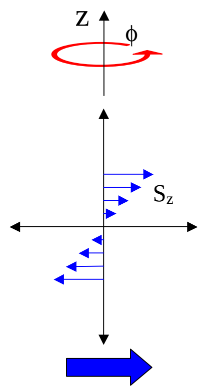
A. Sorensen, L-M. Duan, J. I. Cirac, and P. Zoller, Nature **409**, 63 (2001).

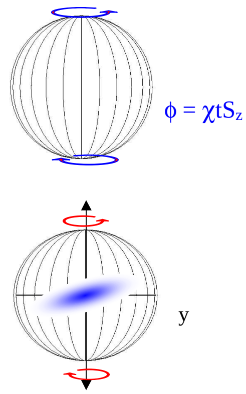
One axis twisting

$U(t) = \exp(-i\phi S_z)$

$U(t) = \exp(-i\chi t S_z^2)$







Minimum variance reduced from ½S to ½(S/3)$^{1/3}$

Two coupled condensates

$$H_{eff} = \frac{\tilde{u}}{2}\left(a^\dagger a^\dagger aa + b^\dagger b^\dagger bb\right) - \mu\left(a^\dagger a + b^\dagger b\right) - \lambda\left(a^\dagger b + b^\dagger a\right)$$

$$= \frac{\tilde{u}}{2} S_z^2 - \lambda S_x$$

A. Imamoglu, M. Lewenstein, and L. You, Phys. Rev. Lett. **77**, 2511 (1997)

$$S_+ = a b^\dagger$$

$$S_- = S_+^\dagger$$

$$S_z = \frac{1}{2}\left(b^\dagger b - a^\dagger a\right)$$

Number/phase squeezed state

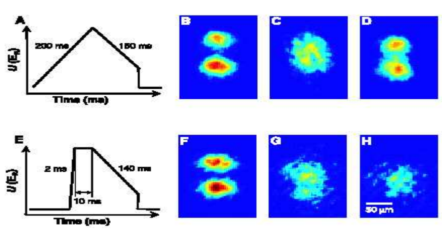
$$\langle G | b^{\dagger}(t) a(t) | G \rangle \propto N \exp\left[-2\tilde{u}^2 \sigma_{a/b}^2(N) t^2\right]$$

$$\tau_c = \frac{1}{2\tilde{u}\sigma_{a/b}}$$

TABLE I. Exponents of the atom number N dependence of various quantities for a condensate in the D-dimensional trap with $V_t(r) = ar^{\eta}$ and with normal atom number fluctuations.

	r_0	μ	$\tilde{u}$	τ_r	τ_c
N's exponent	$\frac{1}{\eta + D}$	$-\frac{D}{\eta + D}$	$\frac{\eta}{\eta + D}$	$\frac{D}{\eta + D}$	$-\frac{D - \eta}{2(\eta + D)}$

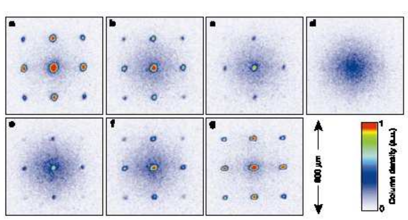
The relative phase



Squeezed States in a Bose-Einstein Condensate

C. Orzel, A. K. Tuchman, M. L. Fenselau, M. Yasuda,* M. A. Kasevich

Science **291**, 2386 (2001).



Collapse and revival of the matter wave field of a Bose–Einstein condensate

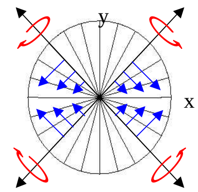
Markus Greiner, Olaf Mandel, Theodor W. Hänsch & Immanuel Bloch

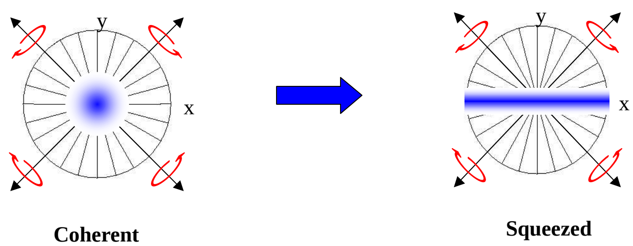
Nature **419**, 51 (2002).

Two axis twisting

Swirling Effect Cancels Out

$U(t) = \exp[-\chi t(S_+^2 - S_-^2)]$



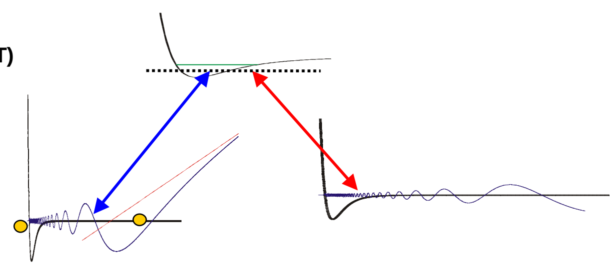


Minimum variance reduced to ½

Georgia Institute of Technology

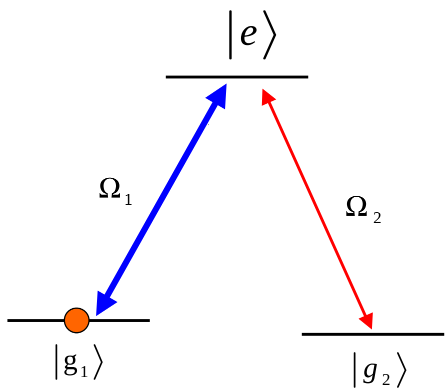
Creating optimal spin squeezing of Bose condensed atoms

Li You
K. Helmerson (NIST)
M. Zhang
B. Deb (PRL, India)



Kristian Helmerson and L. You,
Phys. Rev. Lett. **87**, 170402 (2001).

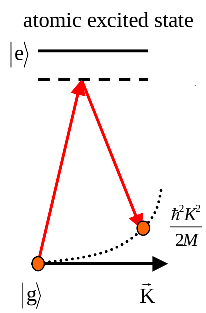
Atomic Raman

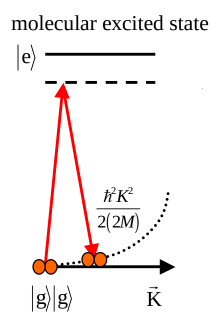


For each atom:

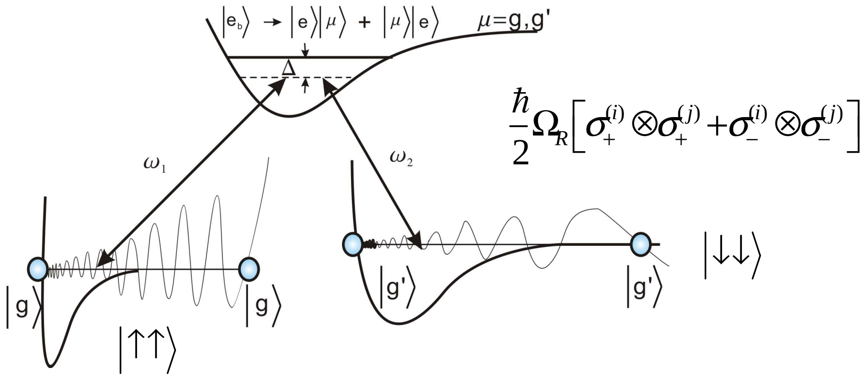
aborption of one photon
emission of one photon

Bragg/Raman





2-atom (molecular) Raman



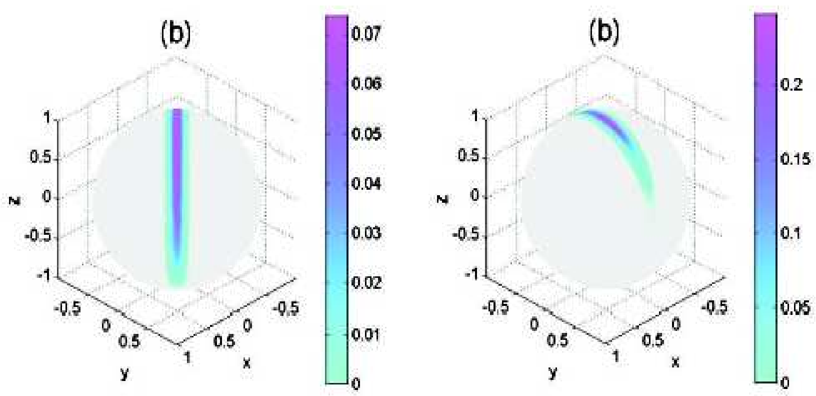
$$\frac{\hbar}{2}\Omega_R\left[\sigma_+^{(i)}\otimes\sigma_+^{(j)}+\sigma_-^{(i)}\otimes\sigma_-^{(j)}\right]$$

For every pair of atoms:

absorption of one photon

emission of one photon

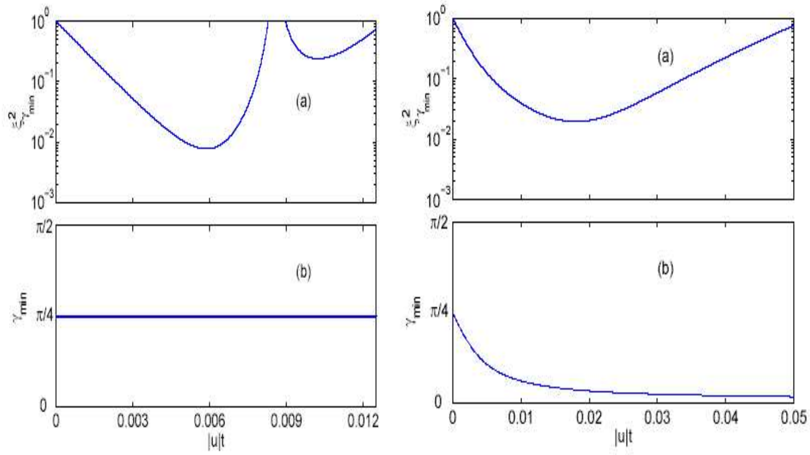
Numerical results (1)



$H_R = u(J_x{}^2 - J_y{}^2)$ $\quad$ $H_S = uJ_x{}^2$

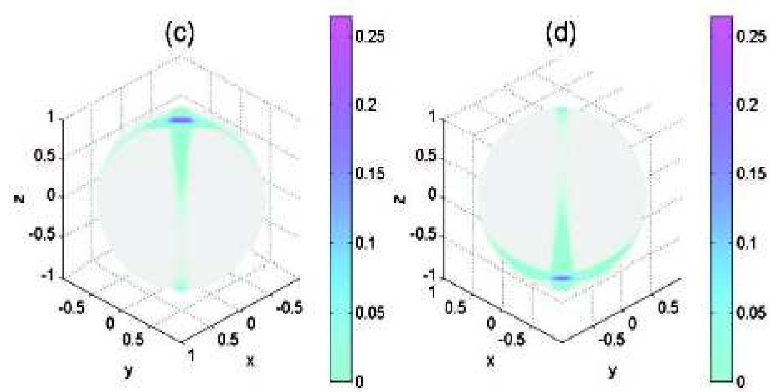
optimal spin squeezing snap shots

Numerical results (2), N=100



$H_R = u(J_x^{\,2} - J_y^{\,2})$ $\qquad$ $H_S = uJ_x^{\,2}$

Numerical results (3)



$H_R = u(J_x^{\,2} - J_y^{\,2})$

50% overlap with the maximally entangled state

Summary

- A model for optimal spin squeezing
- Signifincat overlap with N-GHZ
- Their generation and detection in Bose-Einstein condensates?