Is there a normal fluid in the lowest Landau level?

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Motivation

- Traps allow exploration of rotating quantum bose fluids well beyond limitations of ^{4}He
  Schweikhard *et al* PRL 040404 (2004)
- There has been a vigorous recent investigation of the equilibrium states in the mean field Quantum Hall regime
  Butts & Rokhsar Nature **397** 327 (1999),
  Ho PRL 060403 (2001)
  Kavoulakis, Mottelson & Pethick PRA 62 063605 (2000)
  Cooper, Komineas, Read cond-mat 0404112
- There has been less investigation of the dynamics
- Linn &Fetter PRA 063603 (2000), PRA 4910 (1999)
  Mueller and Ho PRL 063602 (2003) , Baym PRA 69 043618 (2004)
  Sinova, Hanna & Macdonald PRL 030403 (2002)

Key results

- Will show that the LLL hydrodynamics is rather different to conventional (TF) hydrodynamics
- ‘Only’ vortices in the system (and no density modes)
- *But* the most convenient description is not in terms of the vortex co-ordinates
- Vortices interact weakly at short distances

Vortices in the LLL

LLL: NKW, Gunn & Smith PRL **80** 2265 (1998)

- Can represent any wavefunction in the LLL by

$$\psi(z,t) = \sum_{m=0} a_m(t) z^m e^{-|z|^2/2}$$

- Where z=x+i y
- Polynomial factorises

$$|\zeta\rangle = \psi(z,t) = C \prod_{\alpha=1}^{\infty} (z - \zeta_\alpha(t)) e^{-|z|^2/2}$$

- And the nodes $\{\zeta_\alpha\}$ are the positions of the vortices.

Properties of the Wavefunction

- Mean field wavefunction (for all N atoms)

$$\Phi = \prod_{i=1}^{N} \psi(z_i, t) = C^N \prod_{i=1}^{N} \prod_{\alpha=1}^{\infty} (z_i - \zeta_\alpha) e^{-|z|^2/2}$$

- Once $\{\zeta_\alpha\}$ is specified the whole state is specified
- Each Ψ in LLL $\Leftrightarrow \{\zeta_\alpha\}$

(not all the vortices need be inside the trap, some may be at infinity)

Contrast with Superfluid Case

- In a conventional superfluid the vortices and normal fluid are distinct

(*e.g.* Bogoliubov, phonons…)

- In this case there is apparently

No conventional normal fluid

Hamiltonian in vortex representation

- We use $E = \frac{\langle\zeta|\mathcal{H}|\zeta\rangle}{\langle\zeta|\zeta\rangle}$ as a variational (fully condensed) trial function. The $\{\zeta_\alpha\}$ become variational parameters.

$$\mathcal{H} = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}r_i^2 + \frac{1}{2}\eta \sum_{j=1,\neq i}^{N} \delta(\mathbf{r}_i - \mathbf{r}_j) - \omega.\mathbf{L} \right)$$

- Express energy in terms of the Symmetric Polynomials

$$P_n(\zeta) = \sum_{i_1 < i_2 < \cdots < i_n} \zeta_{i_1}\zeta_{i_2}\cdots\zeta_{i_n}$$

$$E = \pi N S^{-1} \sum_{m}^{M} |P_{M-m}(\zeta)|^2 (m+1)! - \pi N \omega S^{-1} \sum_{m}^{M} |P_{M-m}(\zeta)|^2 m m!$$

$$+\frac{\lambda}{4} N(N-1) S^{-2} \sum_{m,n,p,q=0}^{M} P^*_{M-m}(\zeta) P^*_{M-n}(\zeta) P_{M-p}(\zeta) P_{M-q}(\zeta)(p+q)! 2^{-(p+q)} \delta_{m+n,p+q}$$

$$S = (\pi \sum_{n=0}^{N_v} P^*_{Nv-m}(\zeta) P^*_{Nv-m}(\eta) m!)$$

- Compare with the incompressible case in a container of radius R (neglecting images)

$$\mathcal{H} = -\frac{1}{2}\Gamma^2 \rho \sum_{i<j} \ln |\zeta_i - \zeta_j| - \omega \Gamma \rho \sum_i (R^2 - |\zeta_i|^2)$$

•Multivortex interaction is analytic in the vortex co-ordinates

•The rotation terms couple to collective variables

Which variables to use?

$$\begin{aligned}\psi(z,t) &= c\prod_{\alpha}^{M}(z-\zeta_\alpha(t))e^{-|z|^2/2} \\ &= \sum_m (-1)^m P_{N-m}(\zeta) z^m e^{-|z|^2/2} \\ &= \sum_m a_m(t) z^m e^{-|z|^2/2}\end{aligned}$$

- The Hamiltonian indicates that the $P_{M\text{-}m}(\{\zeta\})$ or rather their numerical values, a_m, are more natural than $\{\zeta\}$ for calculations – and this is true dynamically as well
- Descriptions are equivalent and uniquely related:

$\{a_m\} \Rightarrow$ unique polynomial $\Rightarrow$ unique roots are ζ

A few examples of what can be studied

- Surface waves (linear & non-linear)

  Explicitly connected to vortex motion within the trap
- Two-vortex motion at small separation
- Molten small `blob' of vortex matter in the trap.

Hydrodynamic Variables

- If we re-write

$$a_m(t) = \sqrt{\frac{\rho_m(t)}{\pi m!}} e^{-i\phi_m(t)}$$

- Then Lagrangian is

$$\mathcal{L} = N \left\{ S^{-1} \sum_{m=0}^{M} \rho_m [\dot{\phi}_m - (1 + m[1-\omega])] - \frac{\lambda N}{4\pi} \sum_{m,n,p,q=1}^{M} \delta_{p+q,m+n} \right.$$
$$\left. \sqrt{\frac{(m+n)!}{2^{m+n} m! n!}} \sqrt{\frac{(p+q)!}{2^{p+q} p! q!}} \sqrt{\rho_m \rho_n \rho_p \rho_q} \cos((\phi_m + \phi_n) - (\phi_p + \phi_q)) \right\}$$

Surface waves

- Consider the case of
  $\rho_0 \simeq 1, \rho_m \ll 1, \rho_n = 0$ (n≠ m and n≠ 0)

  This leads to

$$\dot{\phi}_m - \dot{\phi}_0 \quad = \quad m(1-\omega) + \frac{\lambda N}{2\pi}\left(1 - 2^{-(m-1)}\right)$$
$$+\rho_m \frac{\lambda N}{2\pi}(2^{-(m-2)} - 1 - \frac{(2m)!}{m!^2}2^{-2m})$$

linear: Kavoulakis, Mottelson & Pethick PRA **62** 063605 (2000)

contrast with TF result, where $\omega \propto \sqrt{m}$

Stringari PRL **77** 2360 (1996)

How are these surface waves?

- Need to interpret in terms of

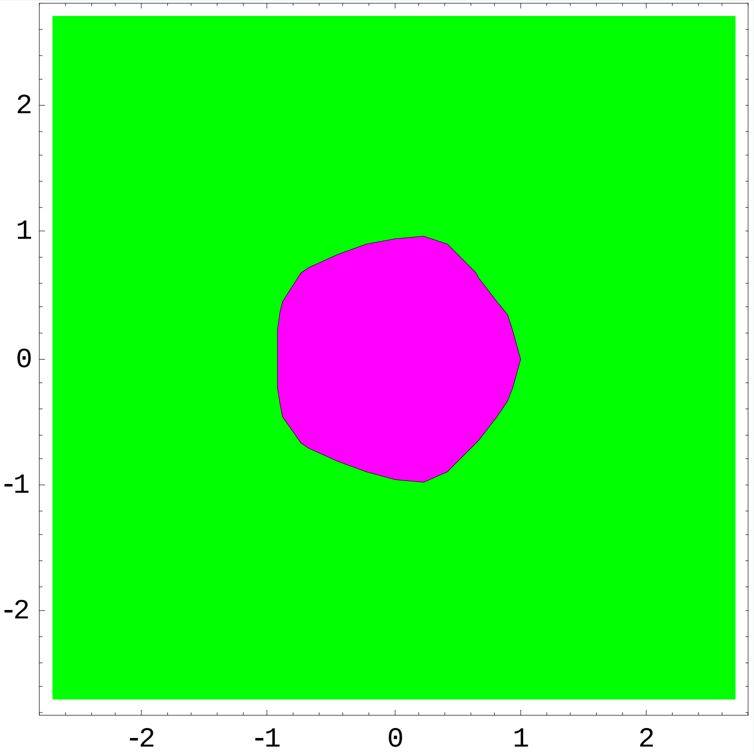
$$\psi(z,t) = \left(a_0(t) + a_m(t) z^m\right) \mathrm{e}^{-|\mathrm{z}|^2/2}$$

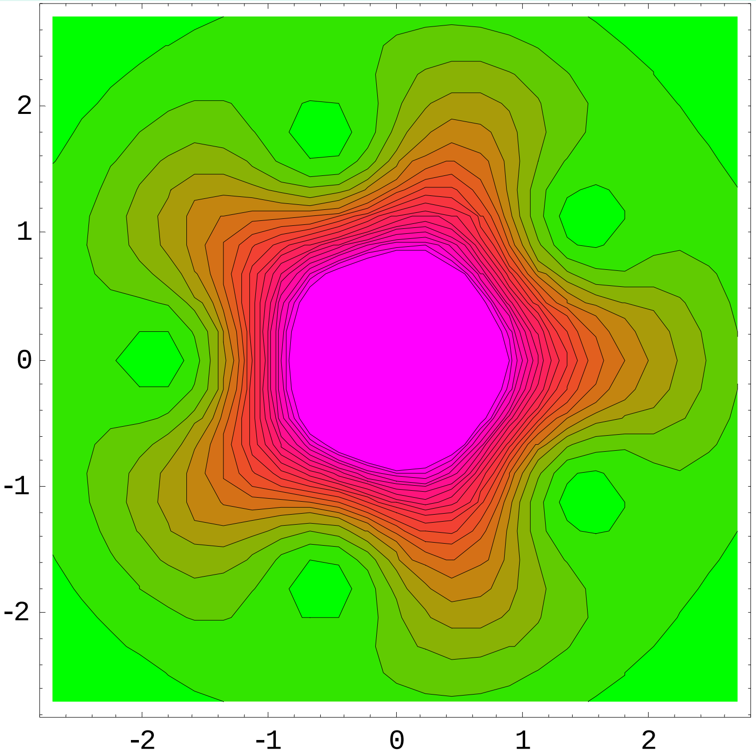
- There are *m* roots, so *m* vortices in a regular polygon

Relationships between surface waves and vortices in the TF case were realised by

Tsubota, Kasamatsu & Ueda PRA 023603 (2002)
Anglin PRL 87 240401 (2001)

m=5 a_5=0.001

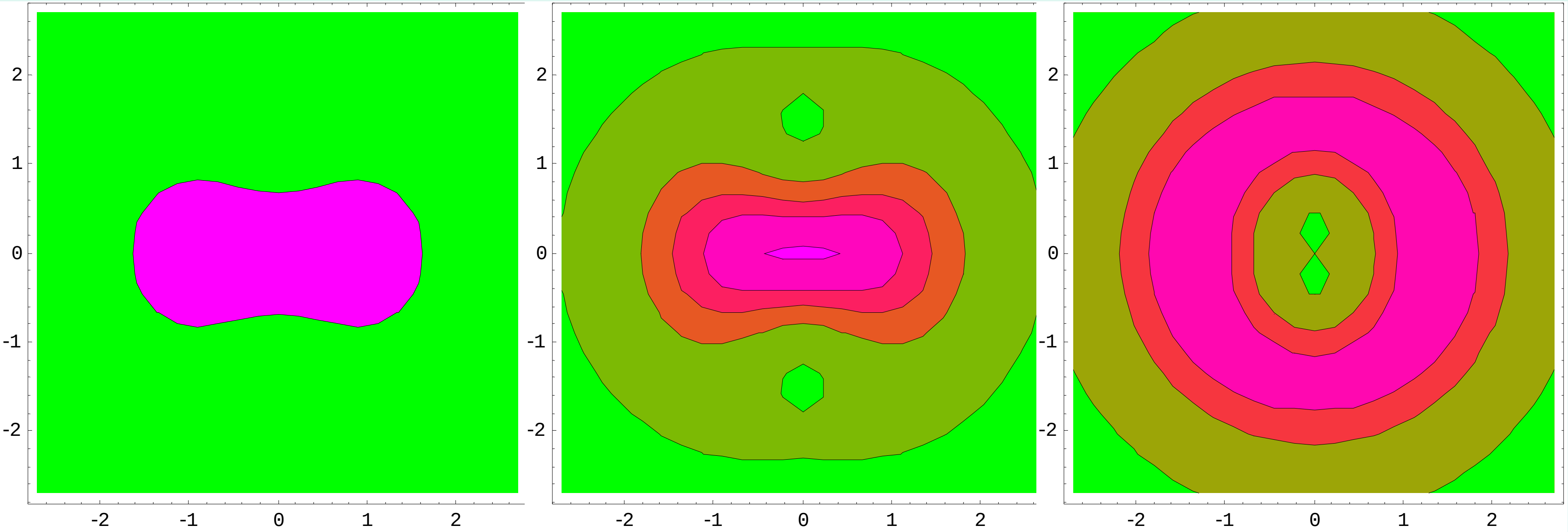




- If one neglects small scale detail – surface wave
- Look more closely see the vortices responsible
- As the vortices move in….

Two vortices

- Now we will examine the dynamics of the vortices as evolve from representing a surface wave to moving under each other’s influence at small separation within the trap – for simplicity consider 2 vortices.



Two vortex system

$$\omega(2) = 2(1-\omega) - \frac{\lambda N}{4\pi}\left(1 - \frac{3}{4}\rho_2\right)$$

- As $\rho_2 \to 1$, vortex positions (ζ_1=-ζ_2 =$\zeta \to 0$)
- Relationship between vortex positions and a_0 and a_2 from

$$C\,(z-\zeta)(z+\zeta) = C\,(z^2-\zeta^2) = a_0 + a_2 z^2$$

$$|C|^2 = \frac{1}{\pi}\frac{1}{2+|\zeta|^2}$$

- so $\rho_2 = \dfrac{2}{2+\zeta^4}$

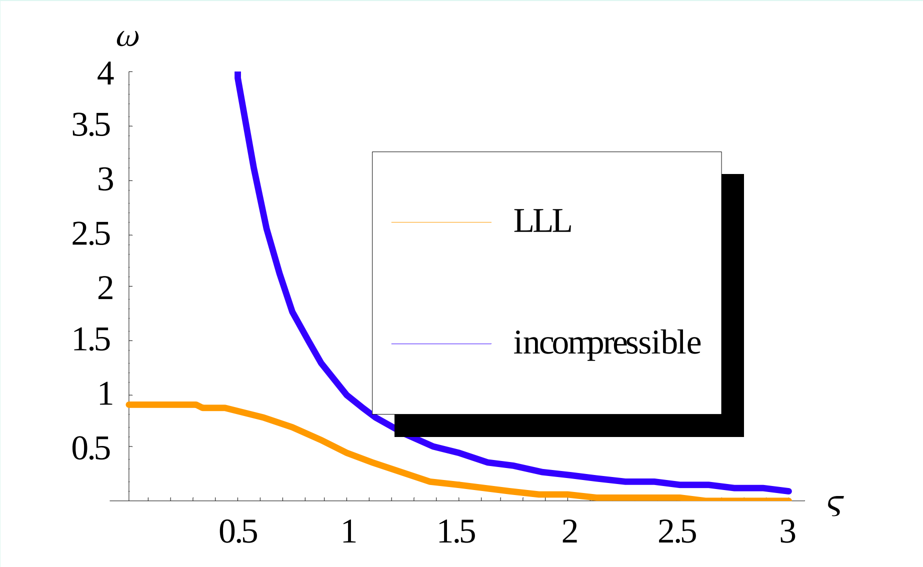
LLL versus incompressible vortex dynamics

Compare LLL frequency for the relative motion of two vortices at separation ζ

$$\omega(2) = 2(1-\omega) - \frac{\lambda N}{4\pi}\left(1 - \frac{3}{4}\frac{2}{(2+\zeta^4)}\right)$$

With the incompressible result:

$\omega=\Gamma/(2\,\pi\,\zeta^2)$



Vortex Patch

- The ‘soft’ nature of the vortex interaction at short distances suggests that small aggregates of ‘molten’ vortices will also behave differently to patches of incompressible molten vortices.
- Consider M vortices *randomly* arranged in a region of extent

$$|\zeta_\alpha| \leq \frac{1}{\sqrt{M}}$$

- ρ_M and ρ_{M-1} are largest so we find

$$\dot{\phi}_{M-1} \propto \frac{d}{dt} arg(\zeta_1 + \cdots + \zeta_M) = (M-1)(1-\omega) + \frac{3\lambda N}{4\pi}\sqrt{\frac{1}{\pi M}} + O(\rho_{M-1})$$

- This contrasts strongly with vortices in the incompressible case, where there would be very high frequency motion due to the close pairs $\sim \frac{1}{r_{ij}}$

Normal Fluid ?

- Although LLL wavefunctions completely specified by vortices one could choose to divide them into those inside and outside the trap
- Outside: treated collectively as the surface waves- and treat them as the normal fluid.
- There is evidence of the the surface waves in the TF limit being in general an agent for allowing vortices to enter the system
- Kusamatsu, Tsubota & Ueda PRA 67 033610 (2003)
- Lobo, Sinatra & Castin PRL 92 020403 (2004)

How do the vortices do this?

- As vortex lattice “cools” must exchange energy and angular momentum with Tkachenko waves and hence with surface waves
- Turbulence of the surface waves in a LLL system may be the simplest form of turbulence one can imagine – what is the equilibrium power spectrum etc….

Key results

- Have shown that the LLL hydrodynamics is rather different to conventional (TF) hydrodynamics
- ‘Only’ vortices in the system (and no density modes) – no normal fluid in the conventional sense.
- *But* the most convenient description is not in terms of the vortex co-ordinates
- Vortices interact weakly at short distances
- Outer vortices may be thought of as surface waves and as a kind of normal fluid.