



Sarah Pearson  
Yavetz



Adrian Price-Whelan



Tomer

~~Monica Valluri + Andreas Kuepper~~

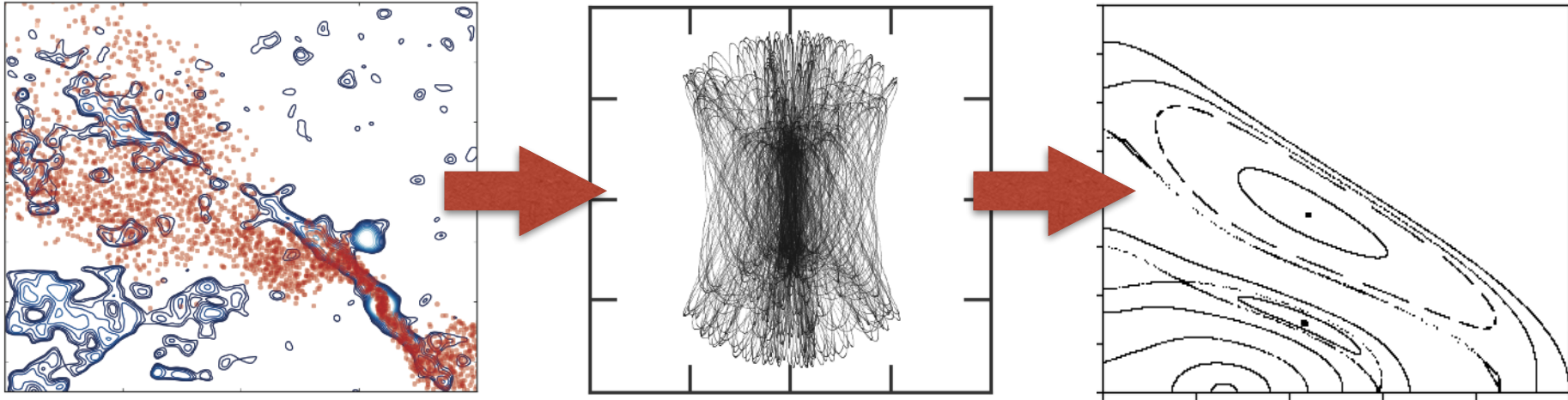
# Physical Manifestations of Chaos, Resonance and Regularity



Kathryn V Johnston  
Columbia University



Supported by the National Science Foundation and  
NASA

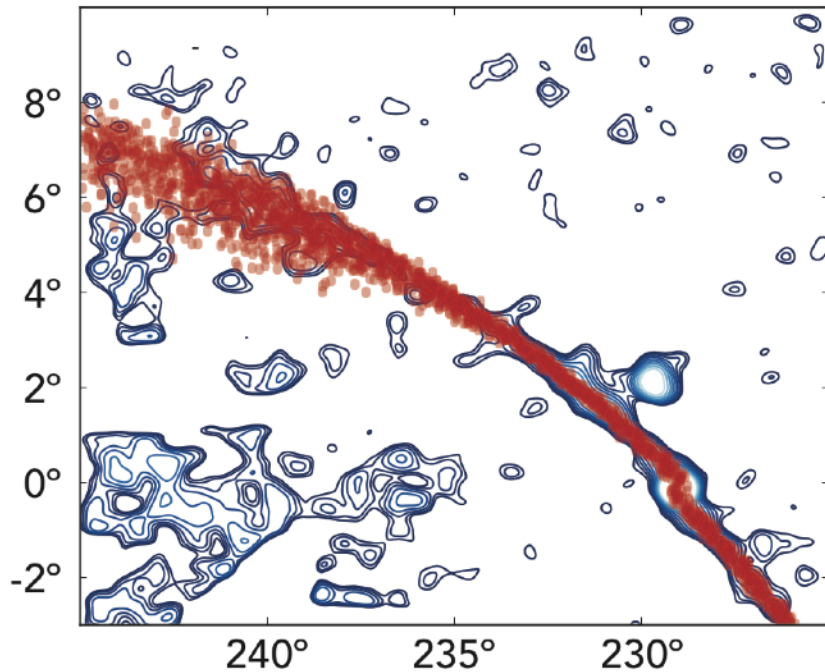


# Physical Manifestations of Chaos, Resonance and Regularity

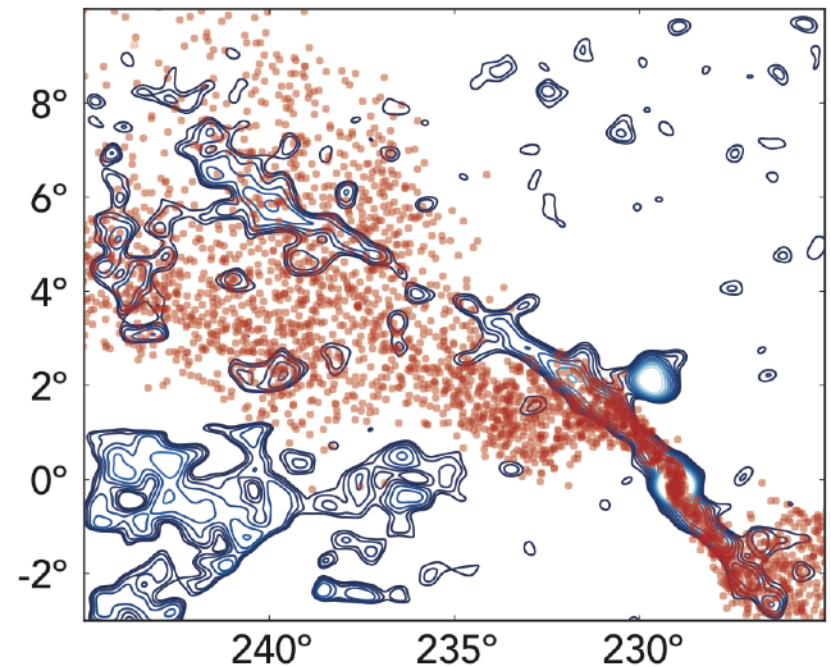


# “Problems” with Pal 5 simulations

Oblate potential



Triaxial potential



Pearson, Kuepper, Johnston & Price-Whelan (2015)

unavoidable “fanning” of stream

in Law & Majewski (2010) triaxial potential

???? Chaos ????



# Aside: Defining Regularity/Chaos

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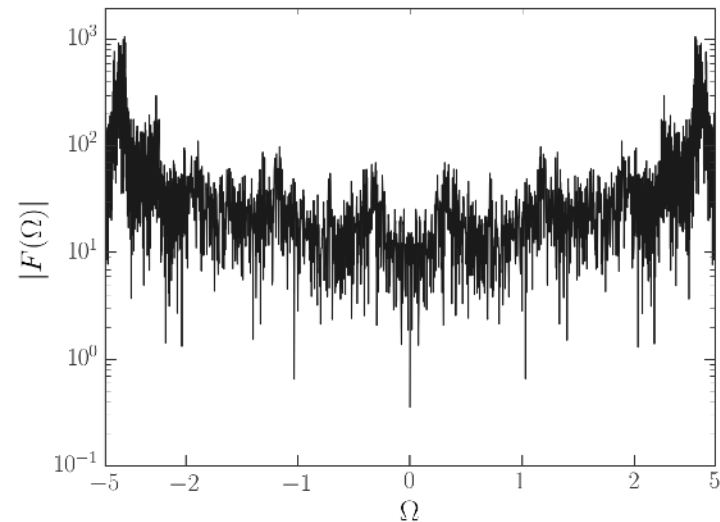
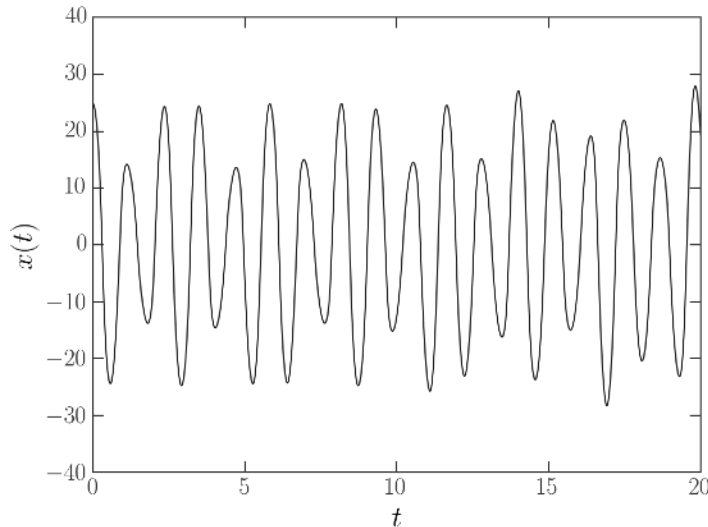
- Regular orbits:
  - existence of fixed orbital properties;
  - predictable path
  - fourier transform  $\Rightarrow$  3 clear fundamental frequencies ( $\Omega$ 's), time-independent
- Chaotic orbits:
  - changes in orbital properties
  - unpredictable path
  - $\Omega$ 's drift with time

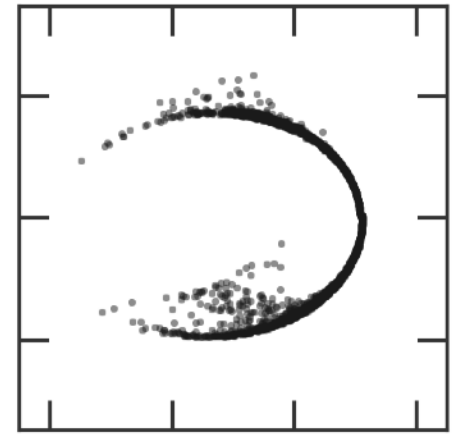
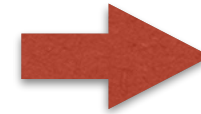
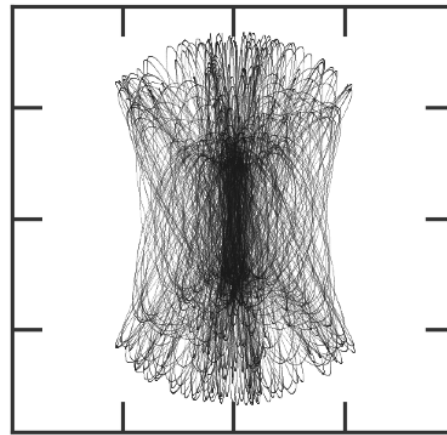
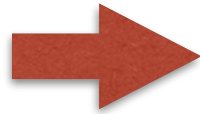
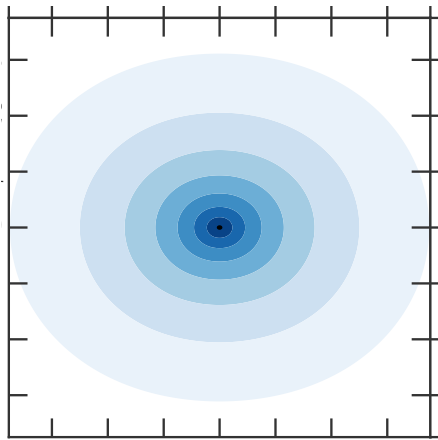
# Aside: Defining Regularity/Chaos

$\Omega$ 's in arbitrary potentials?

- numerically integrate orbits
- fourier transform  $(x+iv_y)$  in time
- $\Omega$ 's from peaks in power spectrum

chaotic timescale =  $\Omega$ 's drift by order-unity



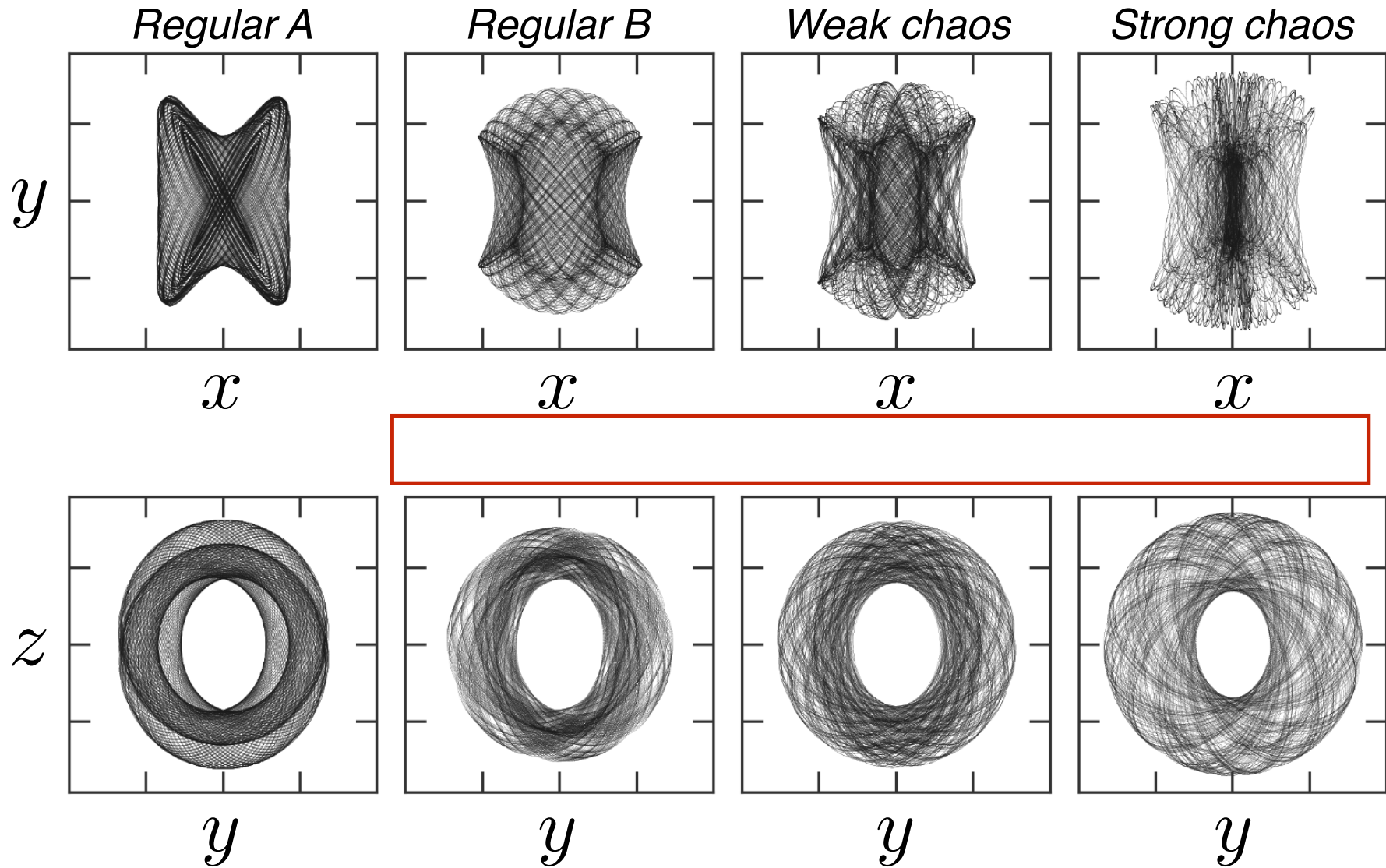


???? CHAOS ????  
???? at  $R > 15\text{kpc}$  ????  
???? within a Hubble

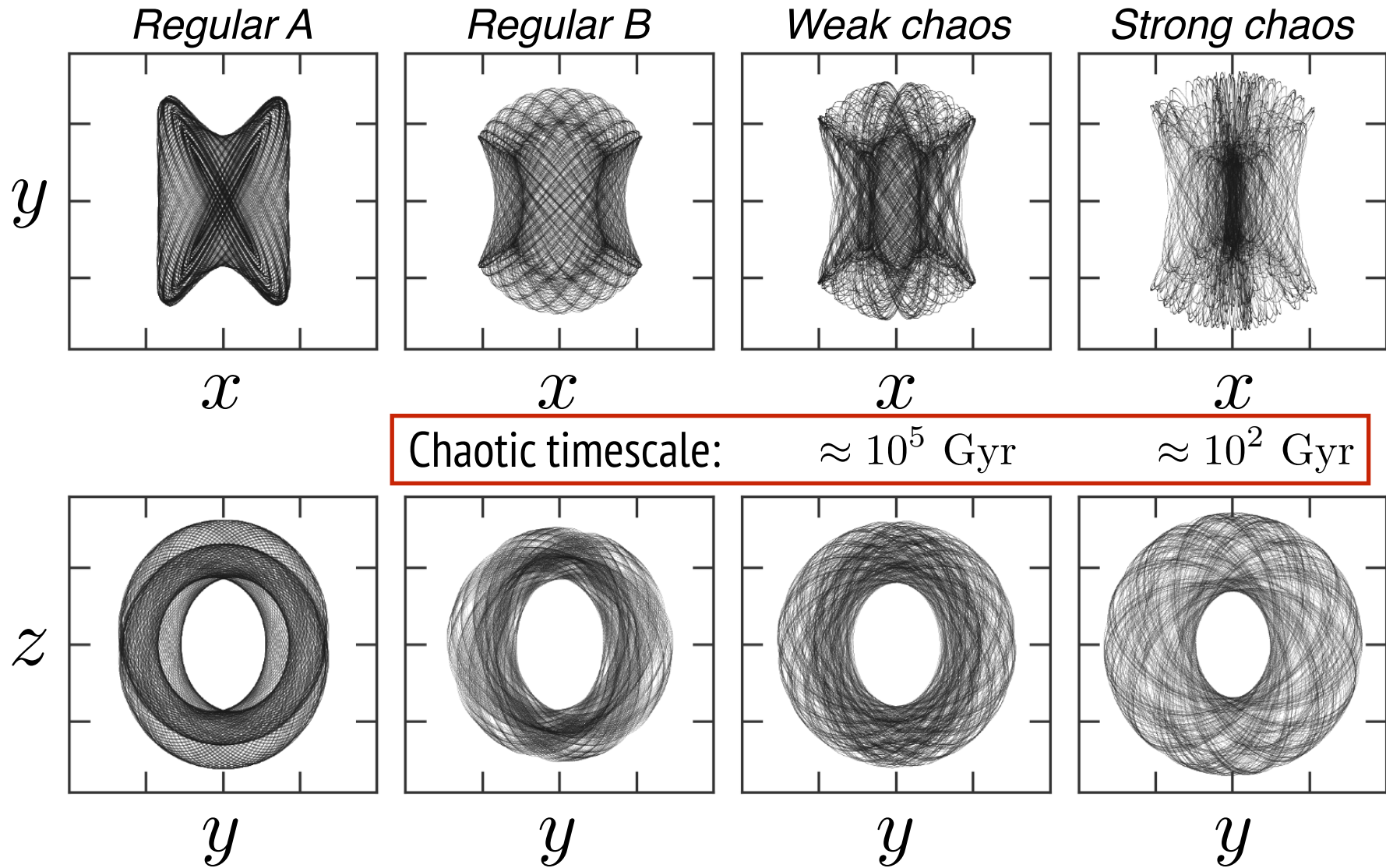


Price-Whelan, Johnston, Valluri,  
Pearson & Kuepper, 2016

# Representative orbits



# Representative orbits





# Cluster Evolution - morphologies

64 orbital periods  $\sim$  20 Gyr

timescale

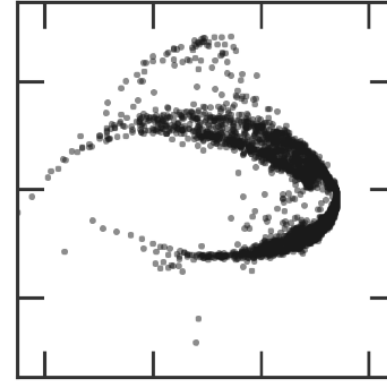
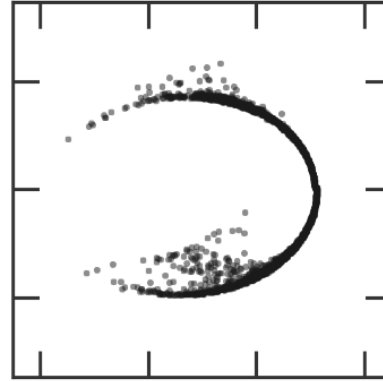
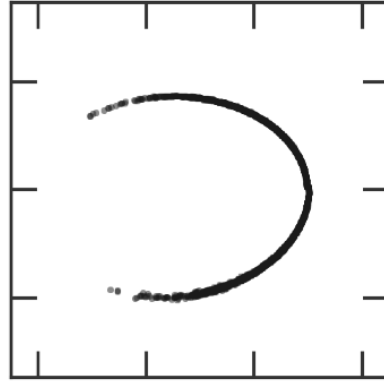
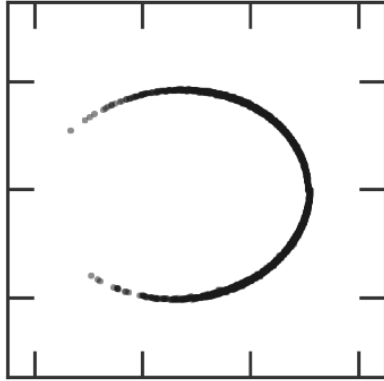
for chaos: infinite!

infinite!

$10^5$  Gyrs

$10^2$  Gyrs

*x/y  
plane*



# Cluster Evolution - morphologies + frequencies

64 orbital periods  $\sim 20$  Gyr

timescale

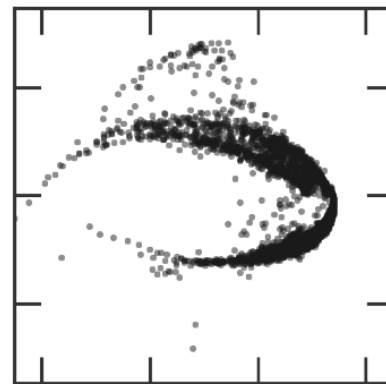
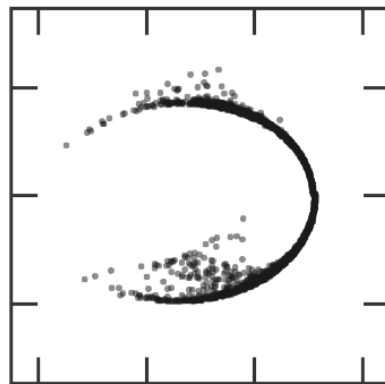
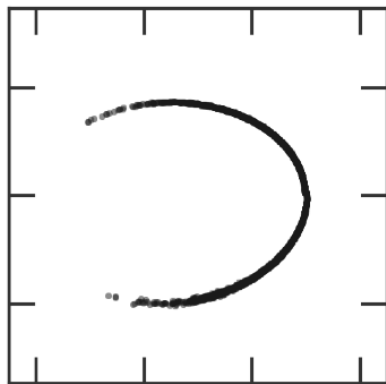
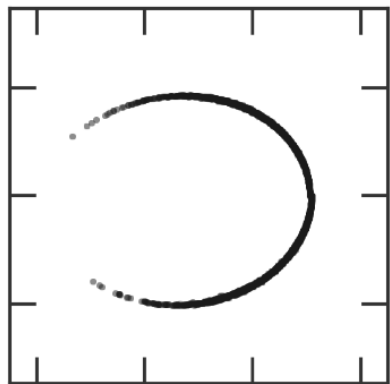
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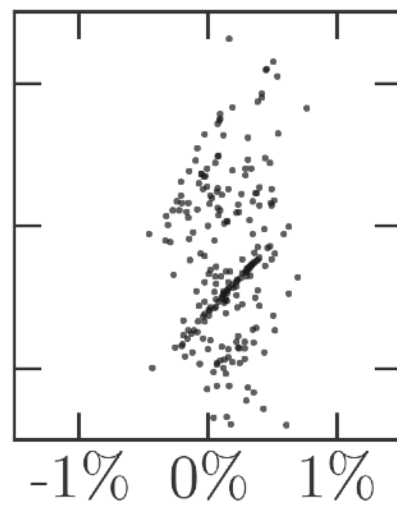
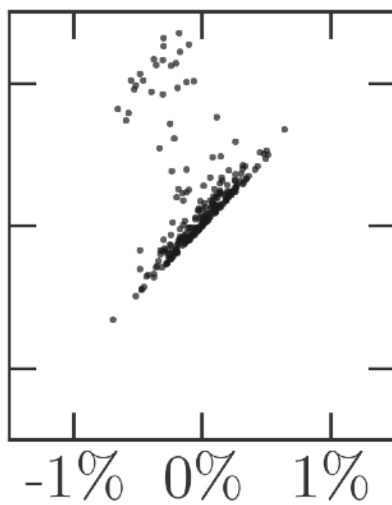
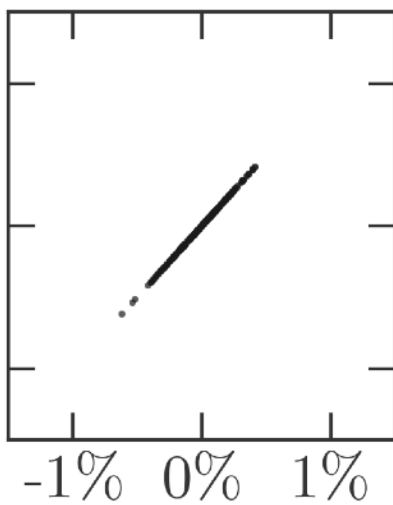
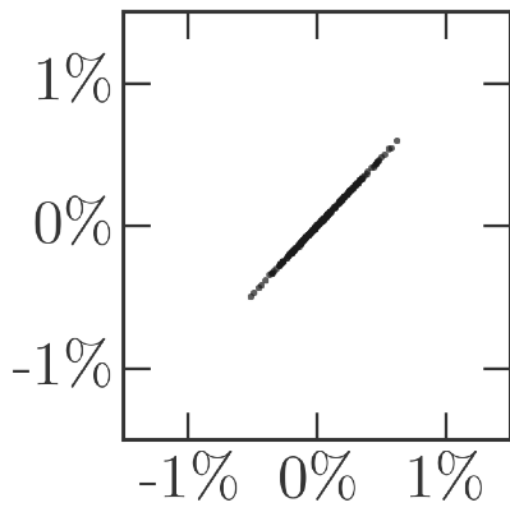
$10^5$  Gyr

$10^2$  Gyr

*x/y*  
plane



$\delta\Omega_{3,i}$

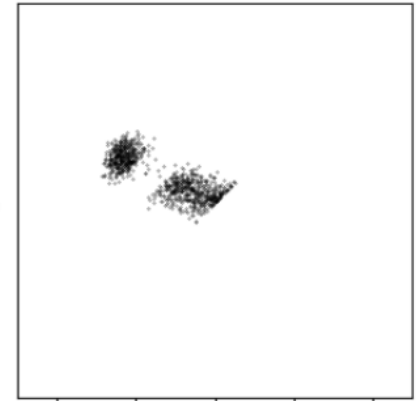
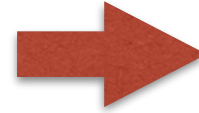
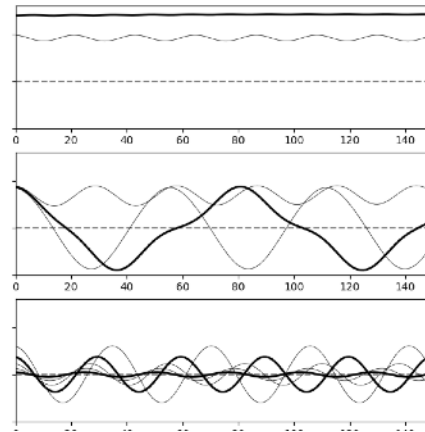
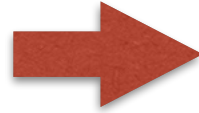
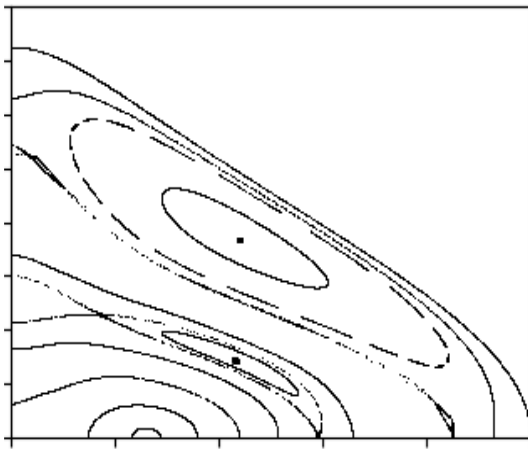


$\delta\Omega_{1,i}$

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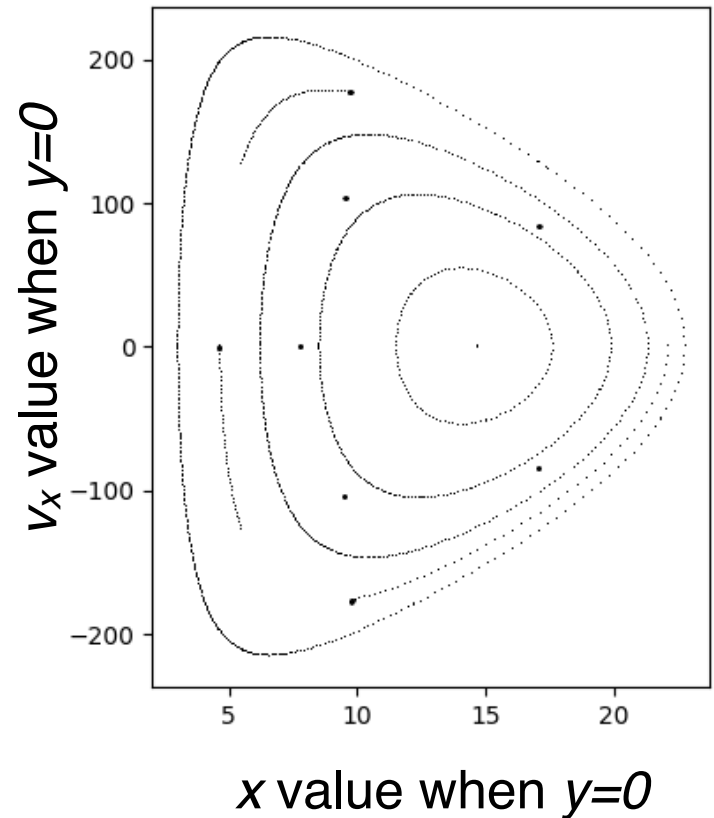
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???? at  $R > 15\text{kpc}$  ????  
???? within a Hubble



Yavetz, Johnston, Pearson & Price-Whelan 2019, *in prep*

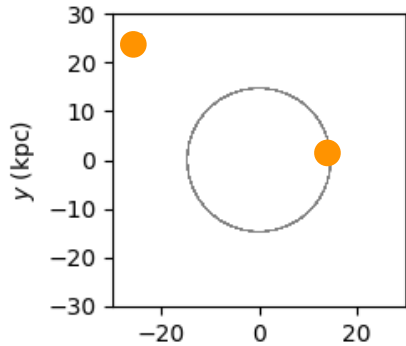
# Aside: Poincaré Maps (“surface of section”) ..... and resonant orbits

plots orbits of a single energy e.g. in spherical potential

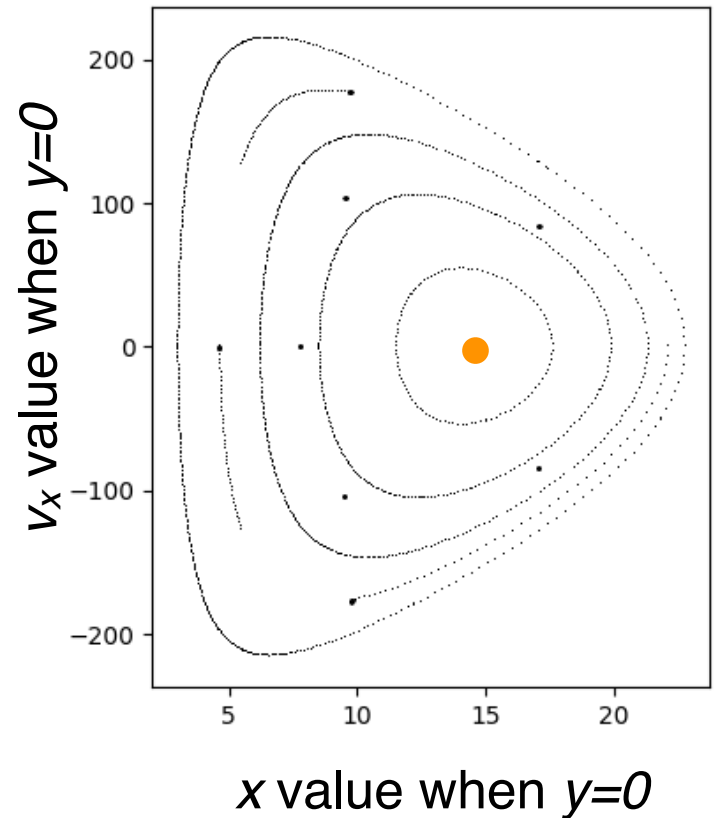


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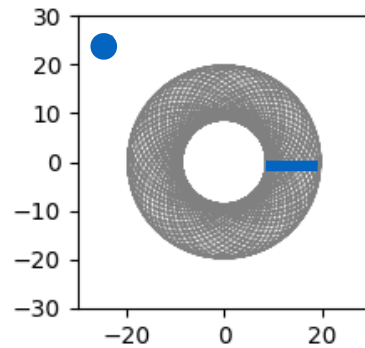
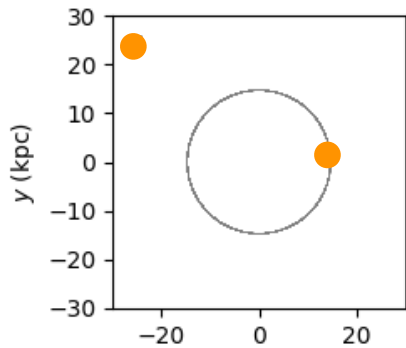
**circular**



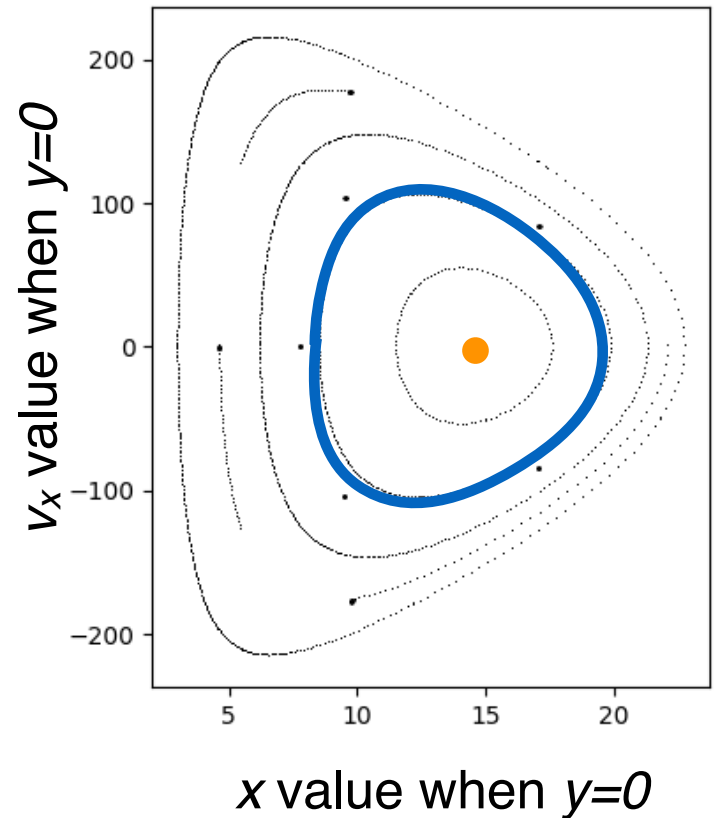
# Aside: Poincaré Maps (“surface of section”)

## ..... and resonant orbits

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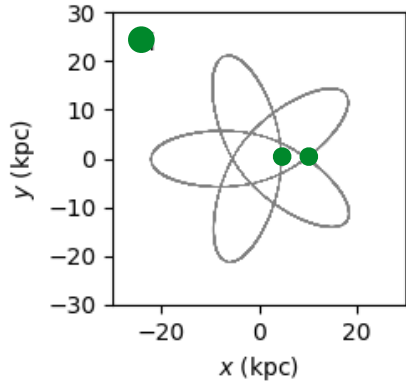
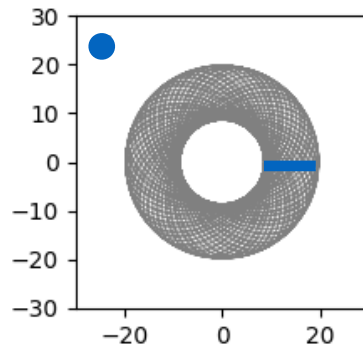
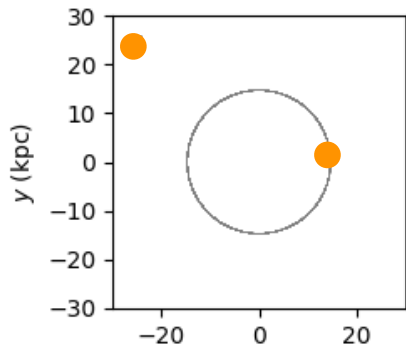
**circular**  
**regular/eccentric**



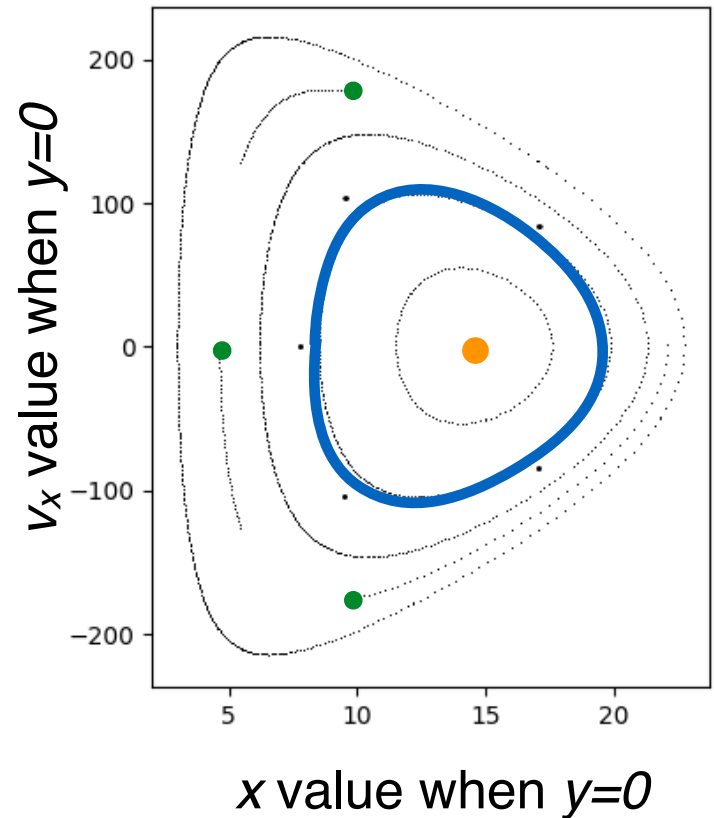
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## ..... and resonant orbits

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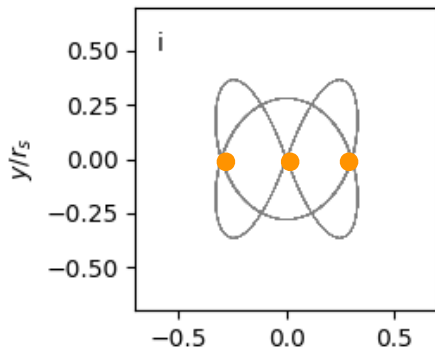
**circular**  
**regular/eccentric**  
**resonant,  $n.\Omega = 0$**



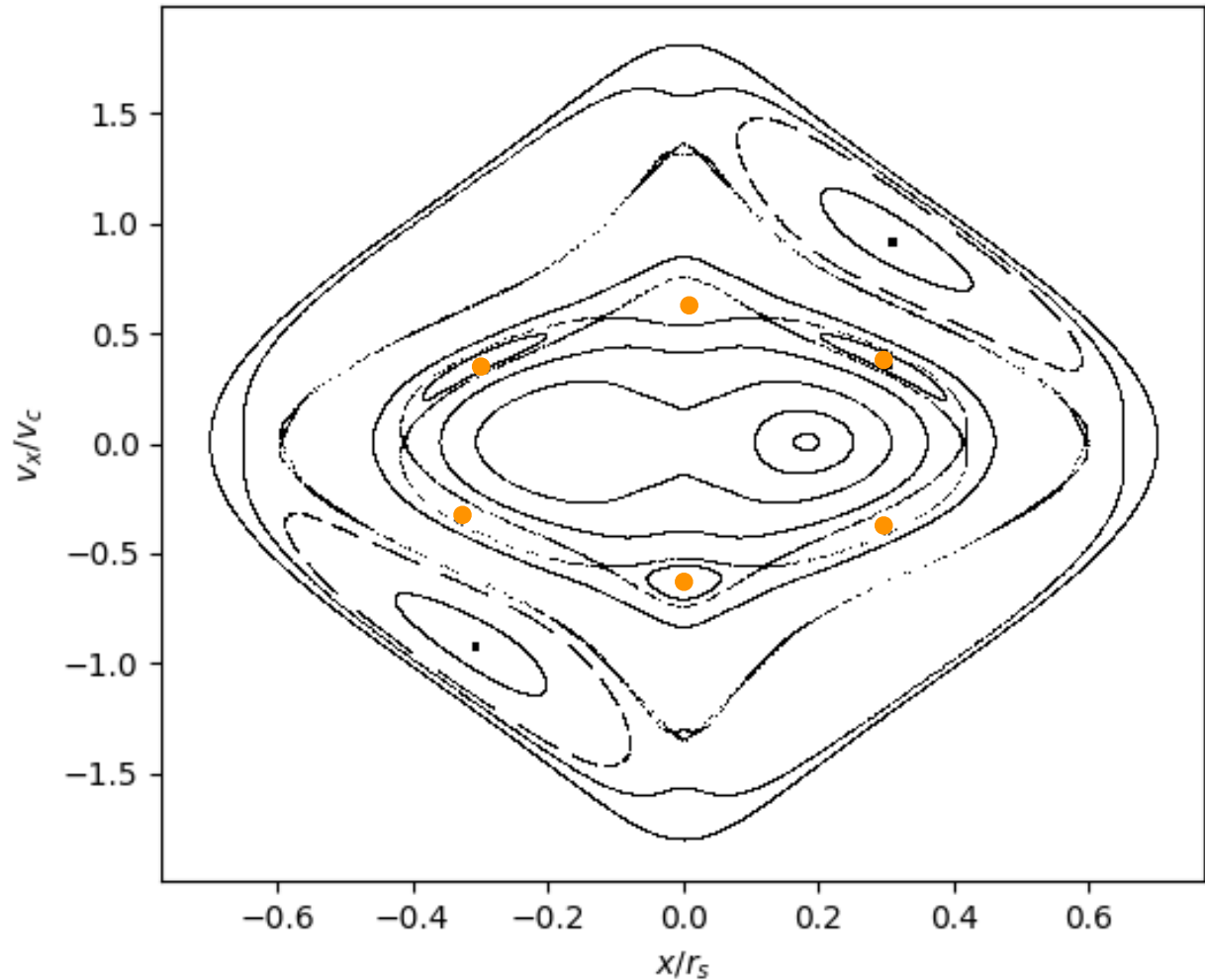
# Poincaré Map for squashed potential

2D logarithmic potential: axis ratio=0.6;  $r_s = 0.14$ ;  $v_c = 1.0$ ;  $E_{\text{tot}} = -0.337$

**resonant**



$$n \cdot \Omega = 0$$

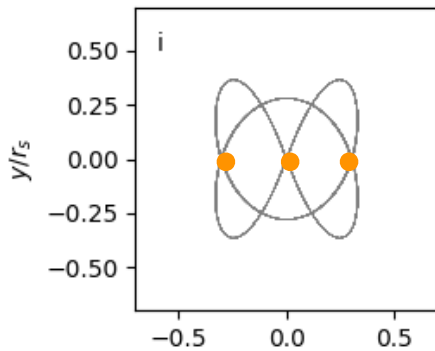




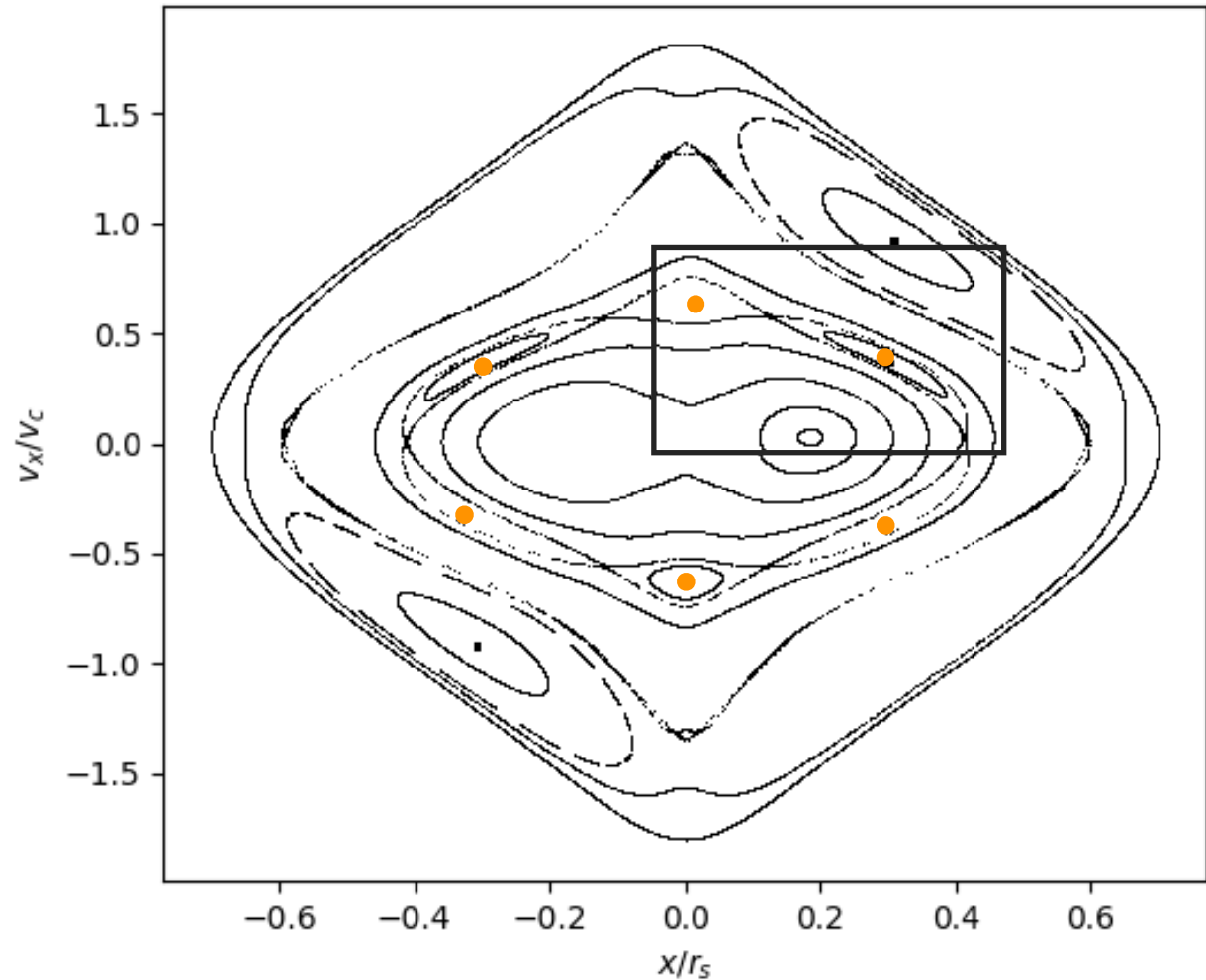
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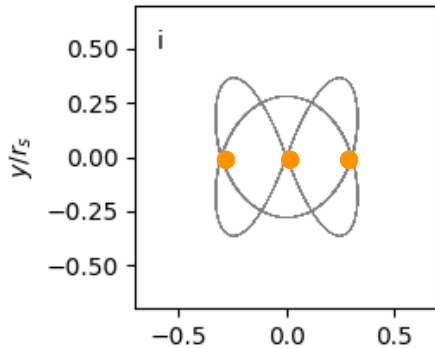
$$n \cdot \Omega = 0$$



# Poincaré Map for squashed potential

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**resonant**

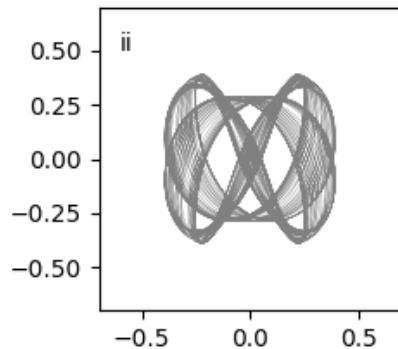


**$n \cdot \Omega = 0$**

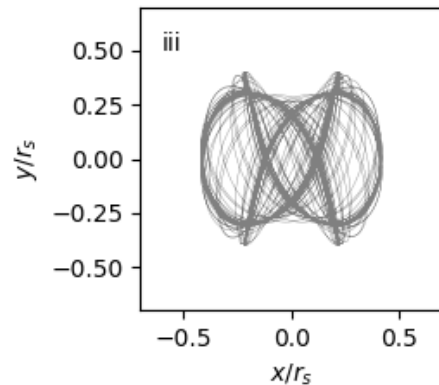


*?? chaotic ??*

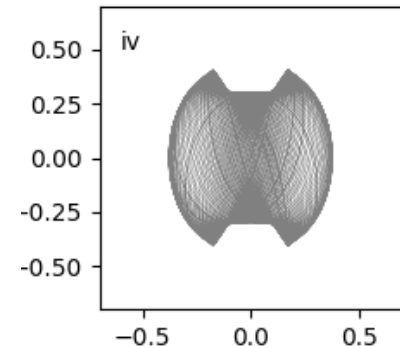
**“trapped”**



**“separatrix”**



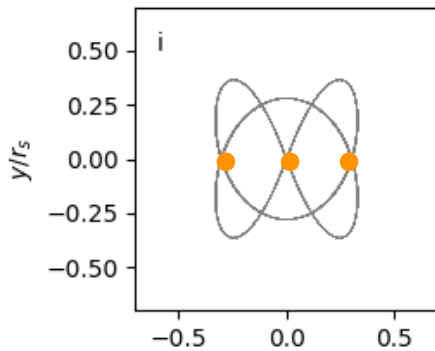
**“regular”**



# Poincaré Map for squashed potential

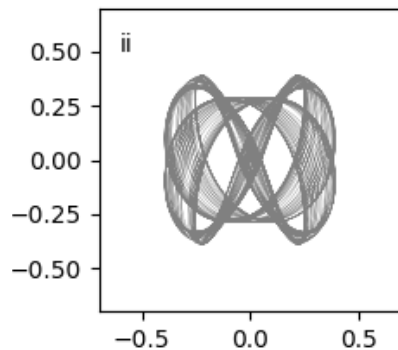
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**resonant**



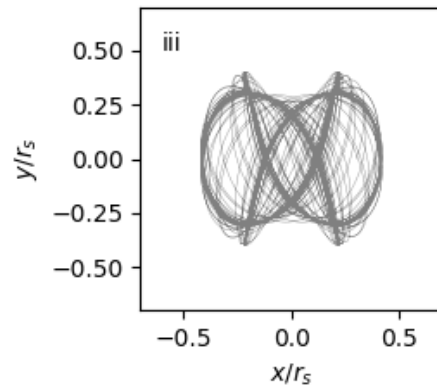
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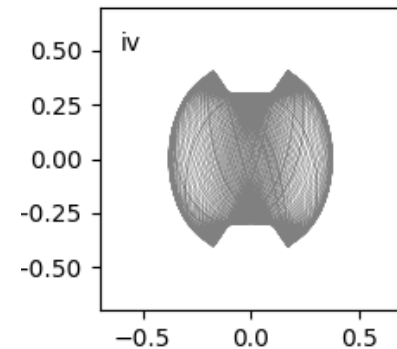


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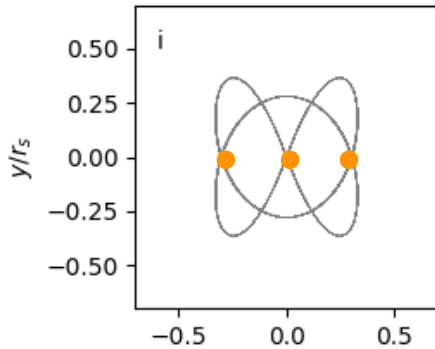
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# Poincaré Map for squashed potential

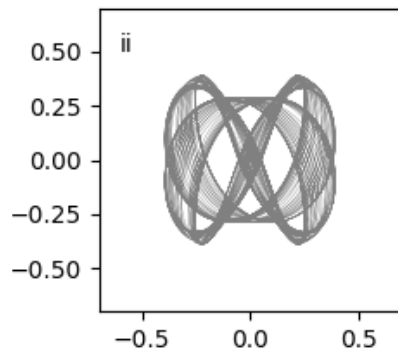
2D logarithmic potential: axis ratio=0.6;  $r_s = 0.14$ ;  $v_c = 1.0$ ;  $E_{\text{tot}} = -0.337$

**resonant**



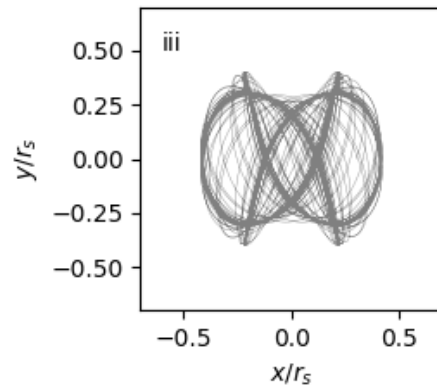
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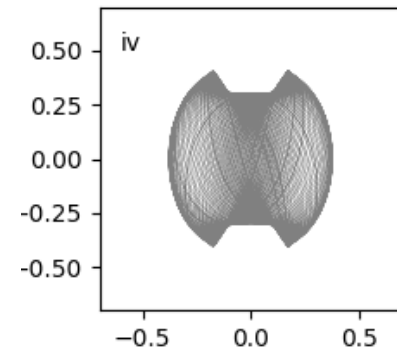


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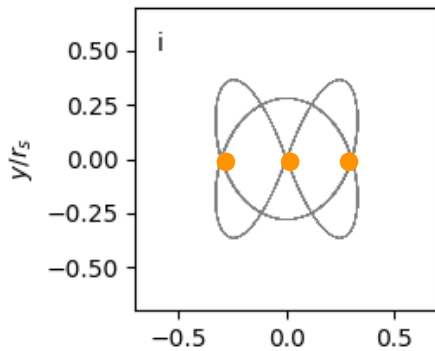
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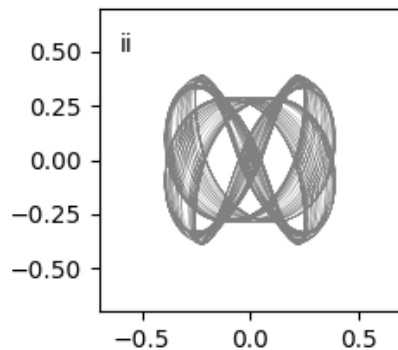
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**resonant**



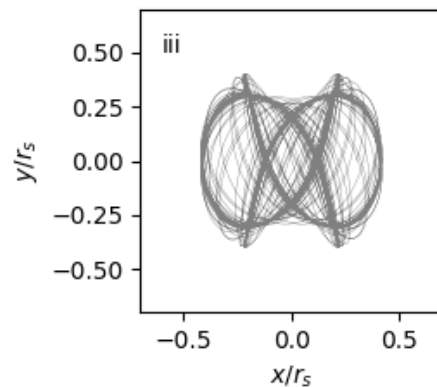
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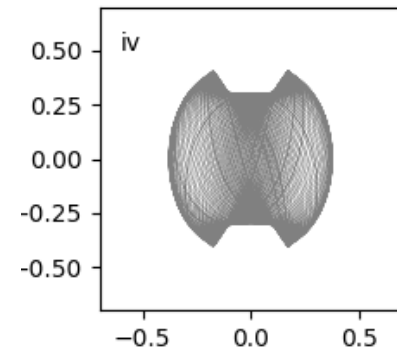


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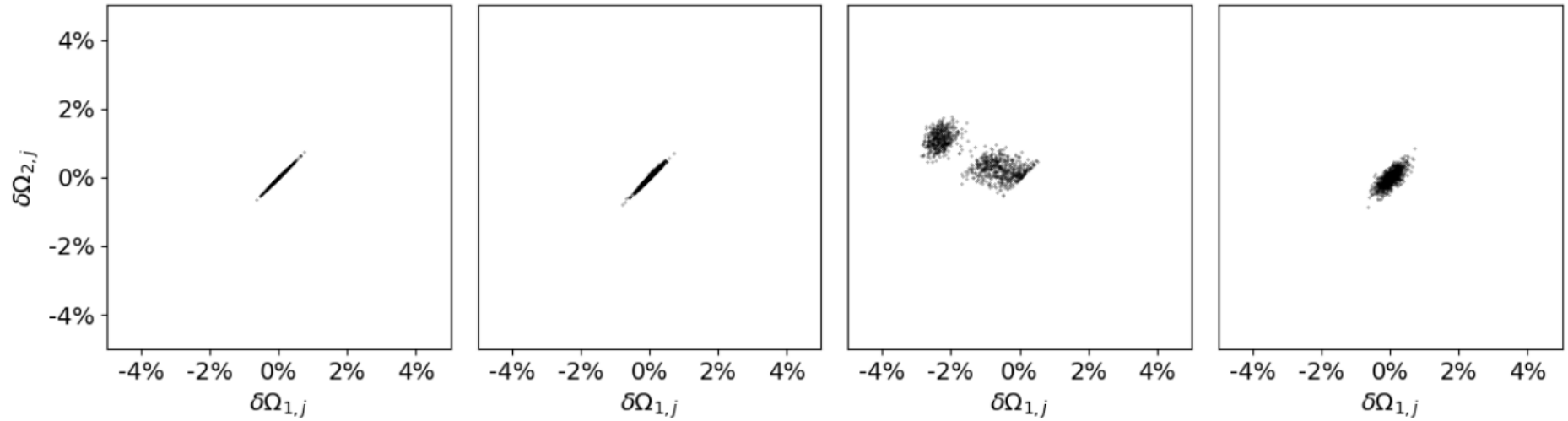
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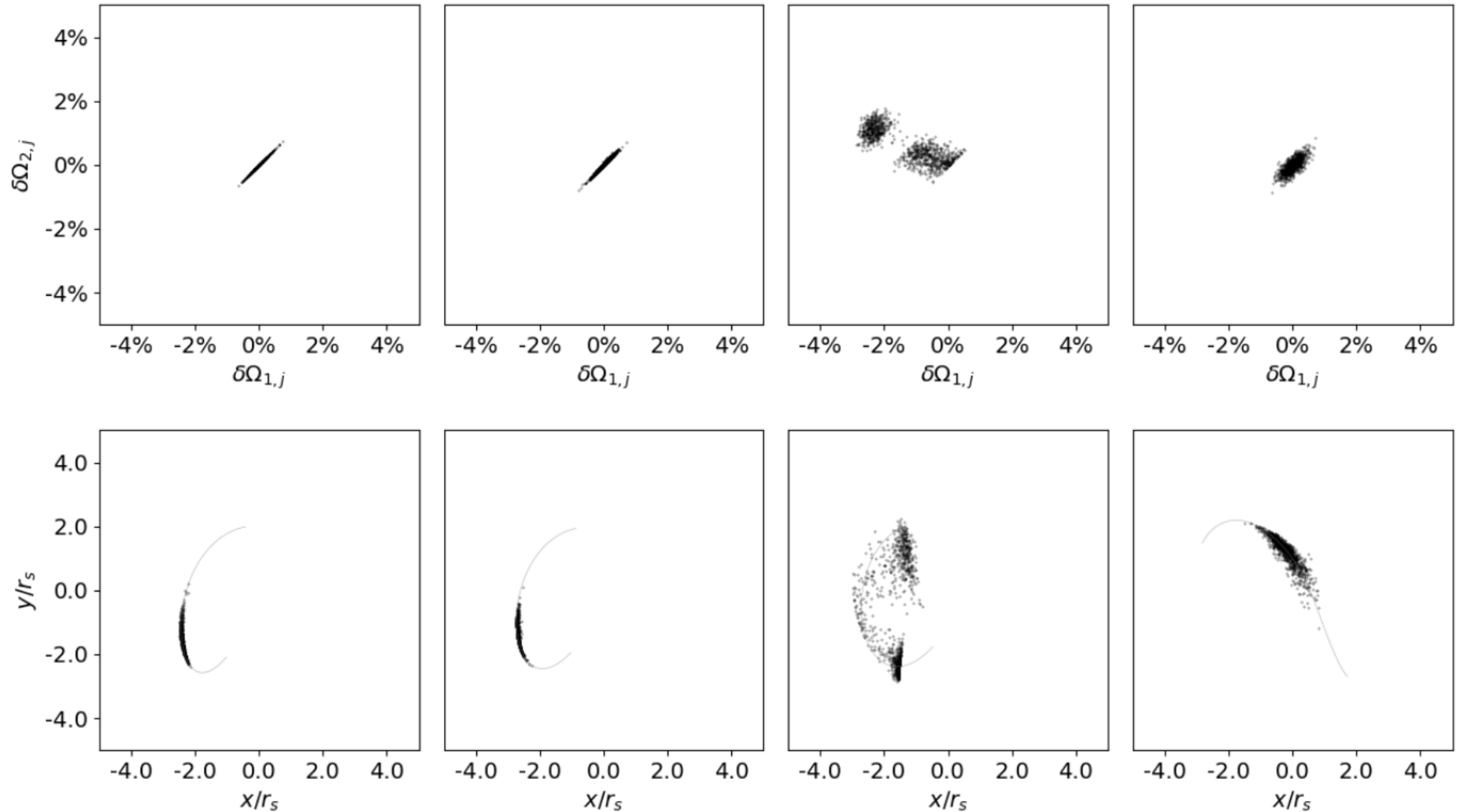
**“regular”**



# Cluster evolution - frequencies

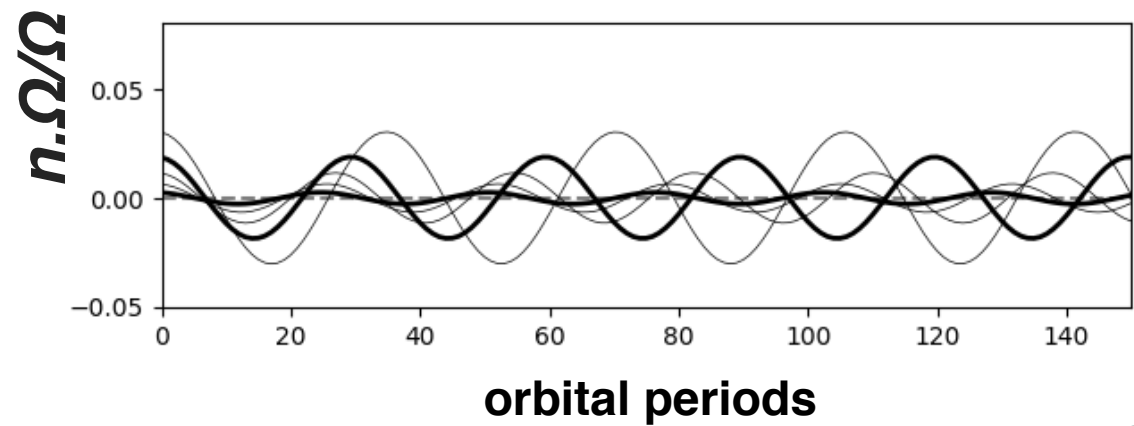


# Cluster evolution - frequencies and morphologies



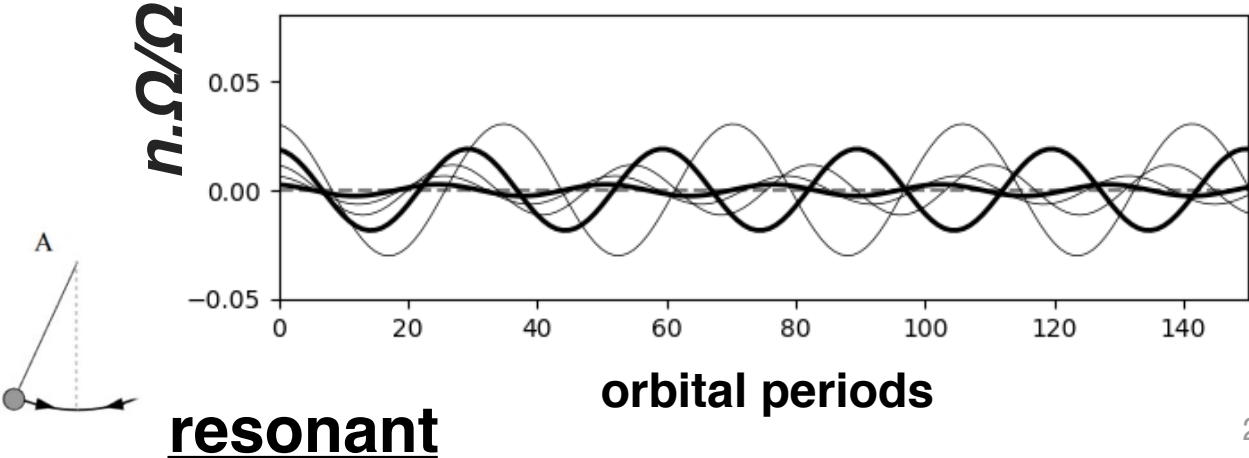
# Frequency evolution

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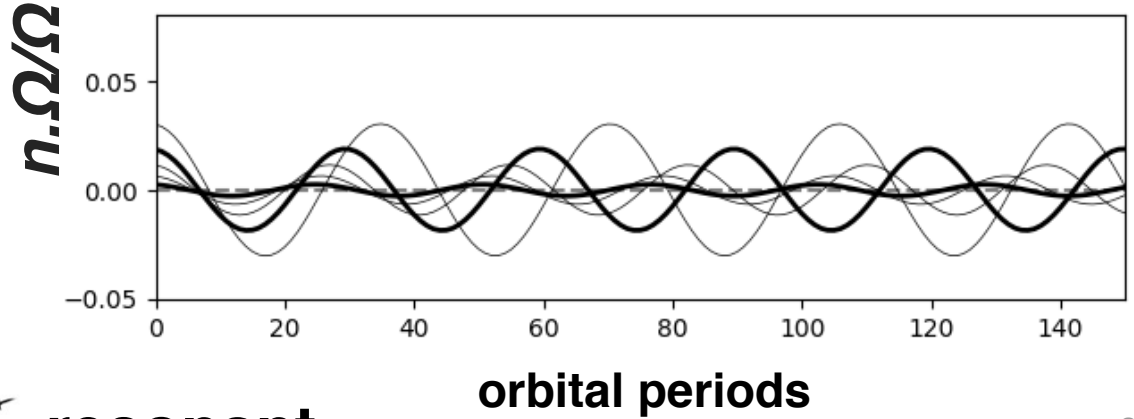
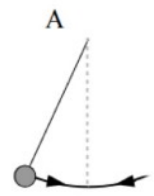
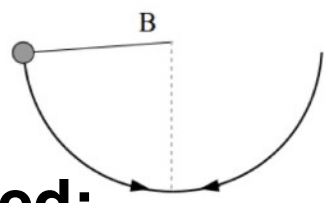


# Frequency evolution



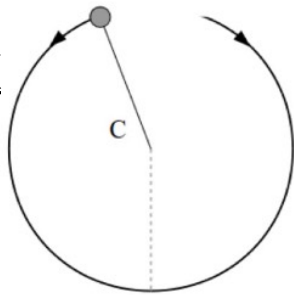
# Frequency evolution

**trapped:**  
*“librating” about resonance*

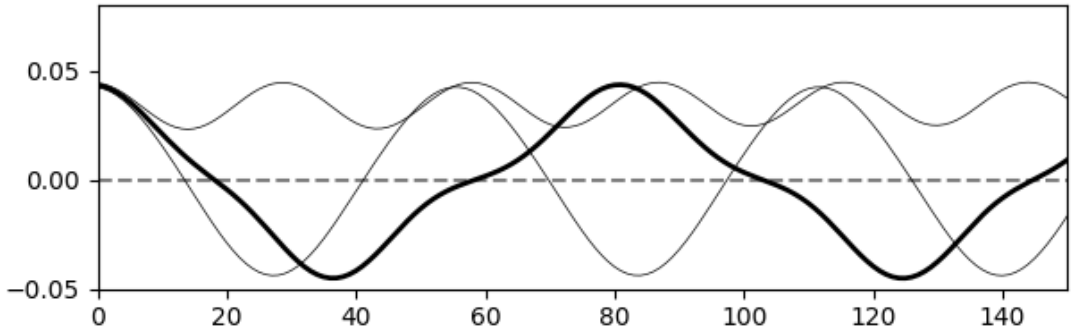


# Frequency evolution

**separatrix**

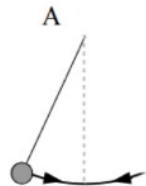
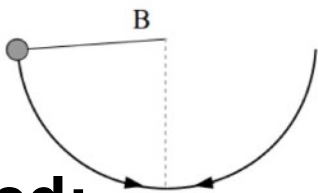


**$n \cdot \Omega / \Omega$**

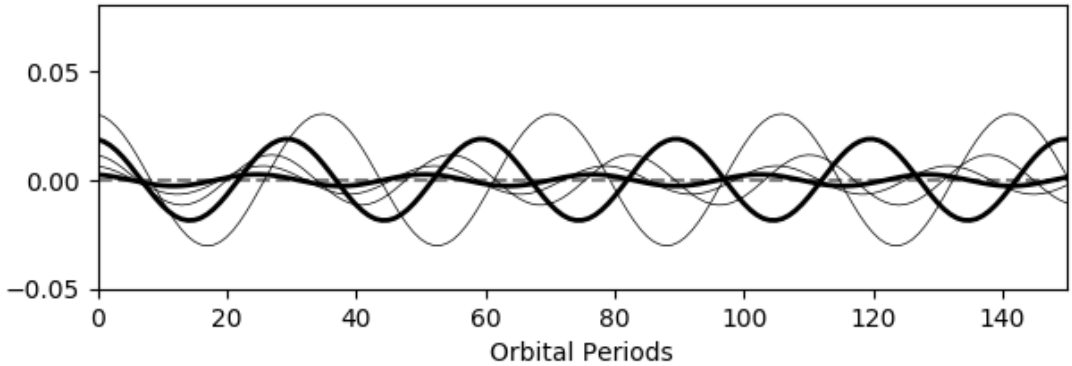


**trapped:**

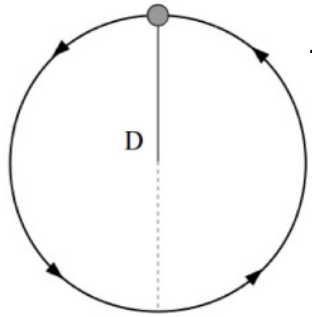
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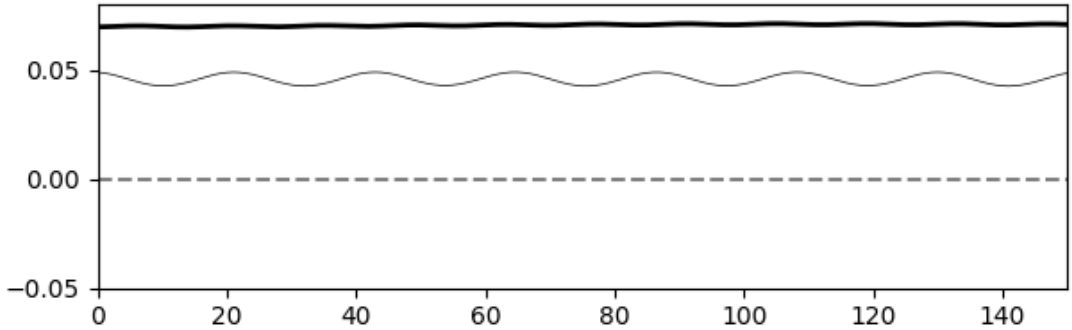
**resonant**



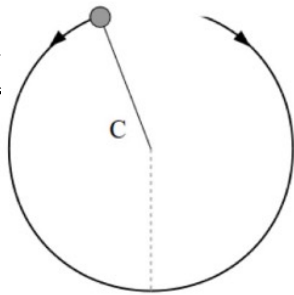
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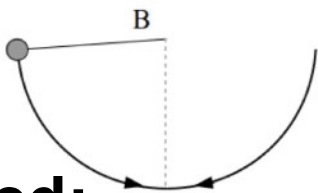
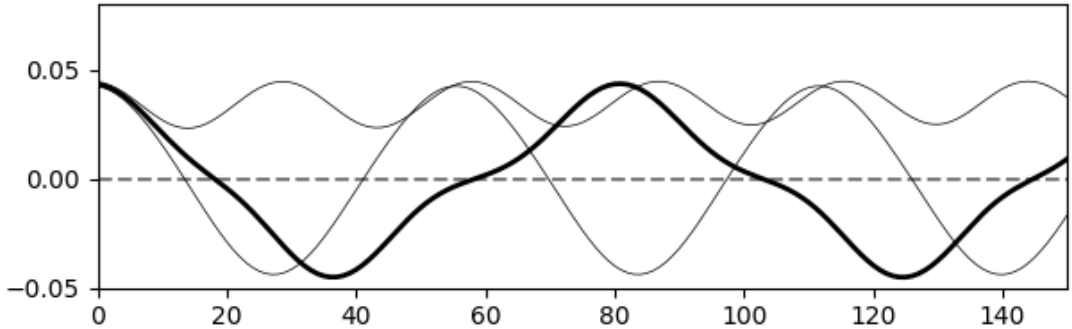
**regular:**  
"circulating"



**separatrix**

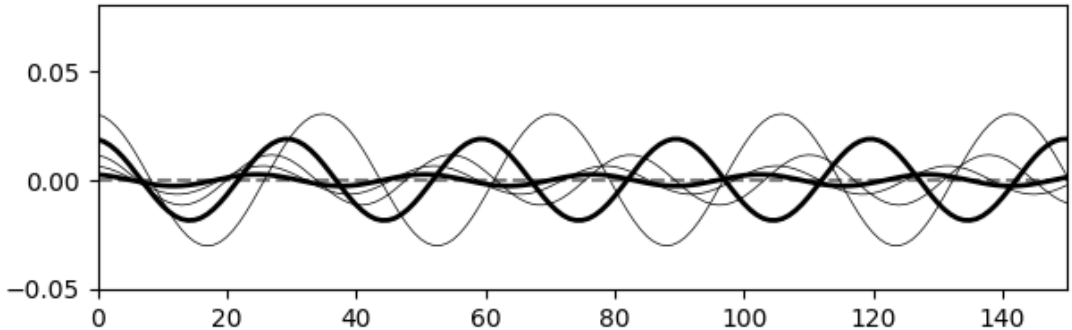


$n \cdot \Omega / \Omega$



**trapped:**

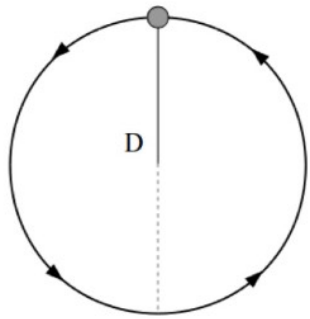
"librating" about  
resonance



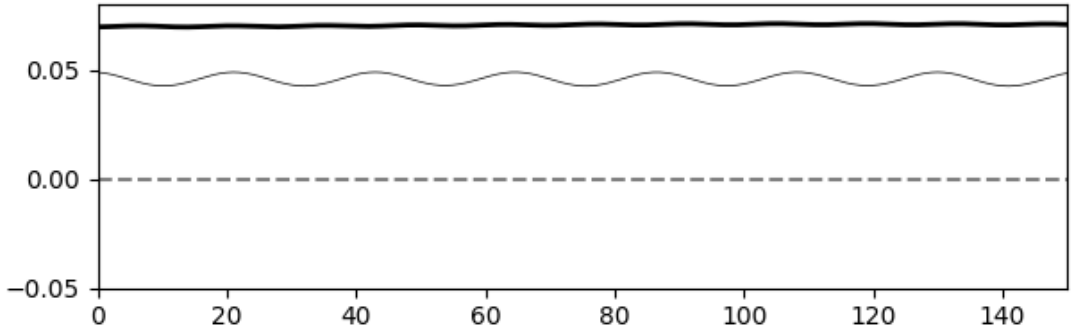
**resonant**

orbital periods

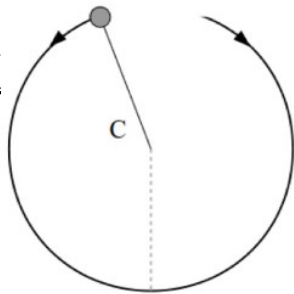
# Frequency evolution



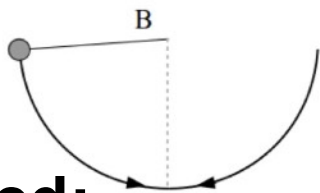
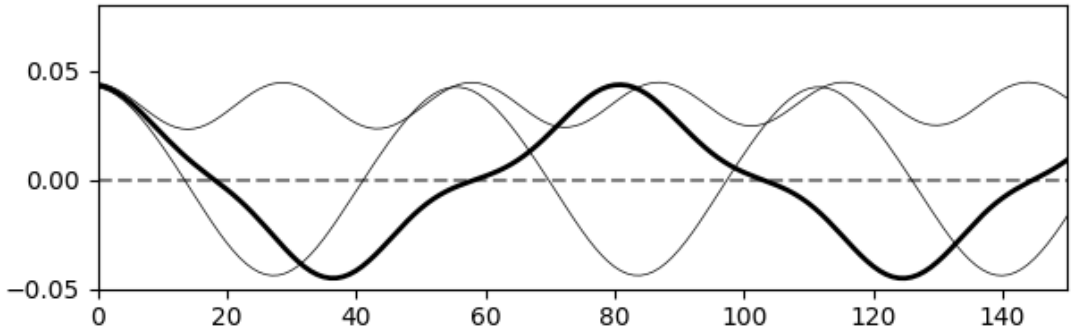
**regular:**  
"circulating"



**separatrix**

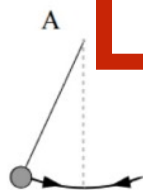


$n \cdot \Omega / \Omega$



**trapped:**

"librating" about  
resonance

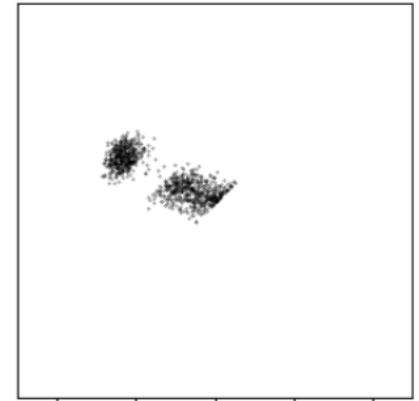
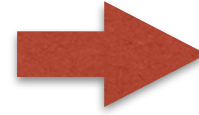
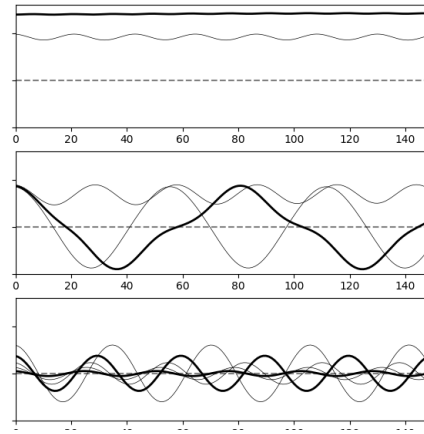
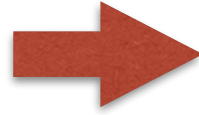
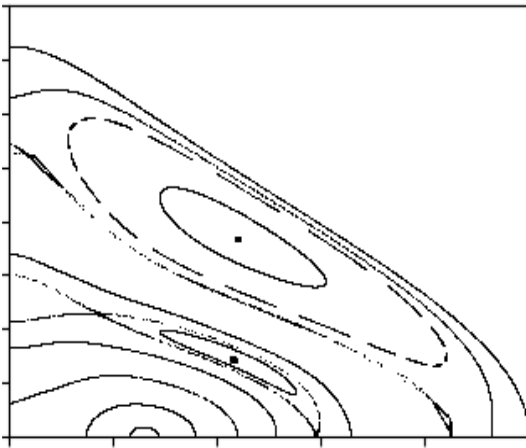


**resonant**

neighboring orbits at separatrix:  
 $\Delta\Omega \sim \text{few } \%$   
 timescale to diverge  $\sim 10$  orbital periods



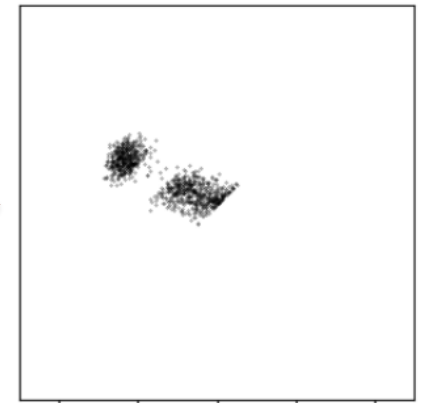
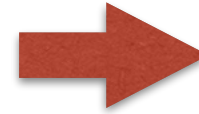
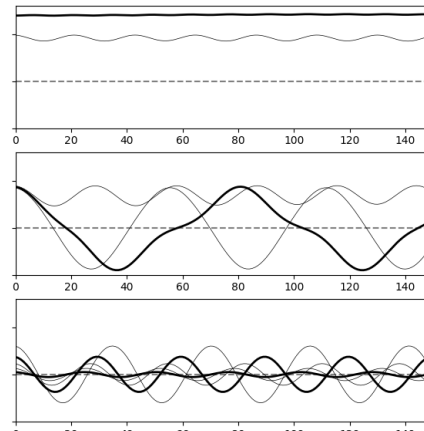
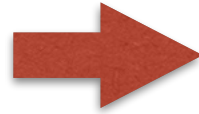
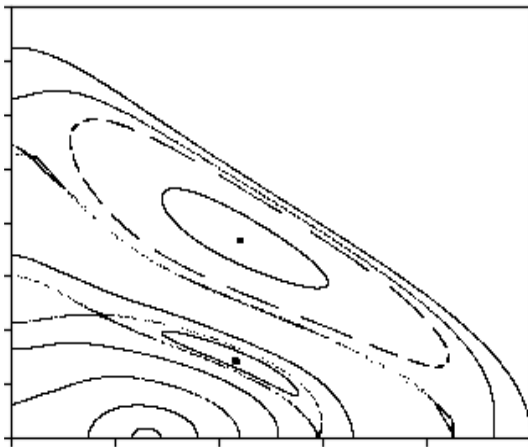
orbital periods



~~???? CHAOS ????  
“Separatrix Divergence”~~



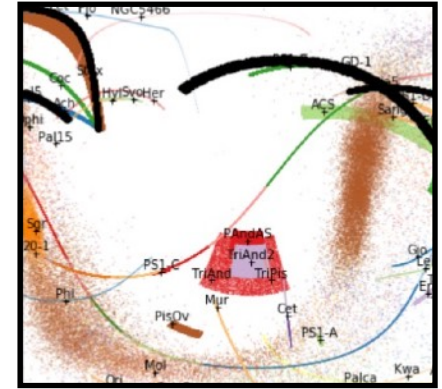
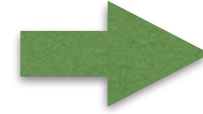
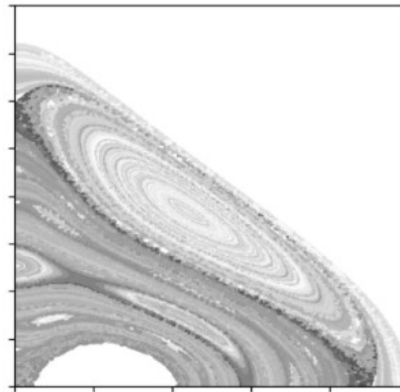
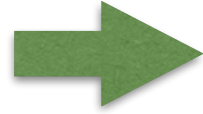
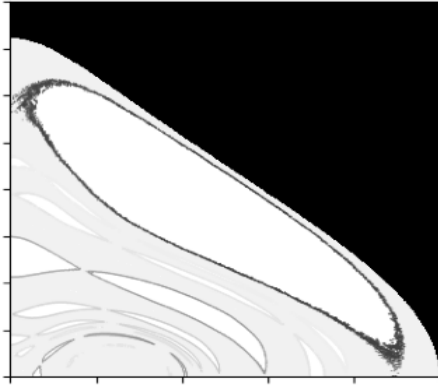
Yavetz, Johnston, Pearson & Price-Whelan 2019, in prep



*“Separatrix Divergence”*  
???? why care ?????



Yavetz, Johnston, Pearson & Price-Whelan 2019, *in prep*



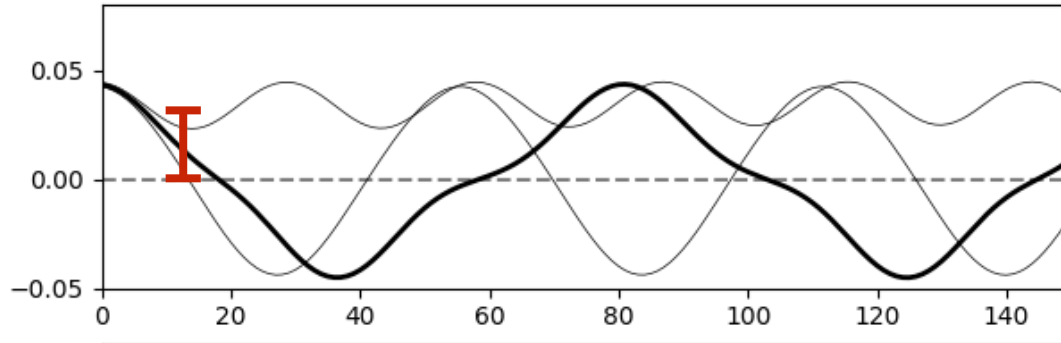
Observing manifestations of  
fundamental dynamics  
around galaxies



Yavetz, Johnston, Pearson & Price-Whelan 2019, in prep

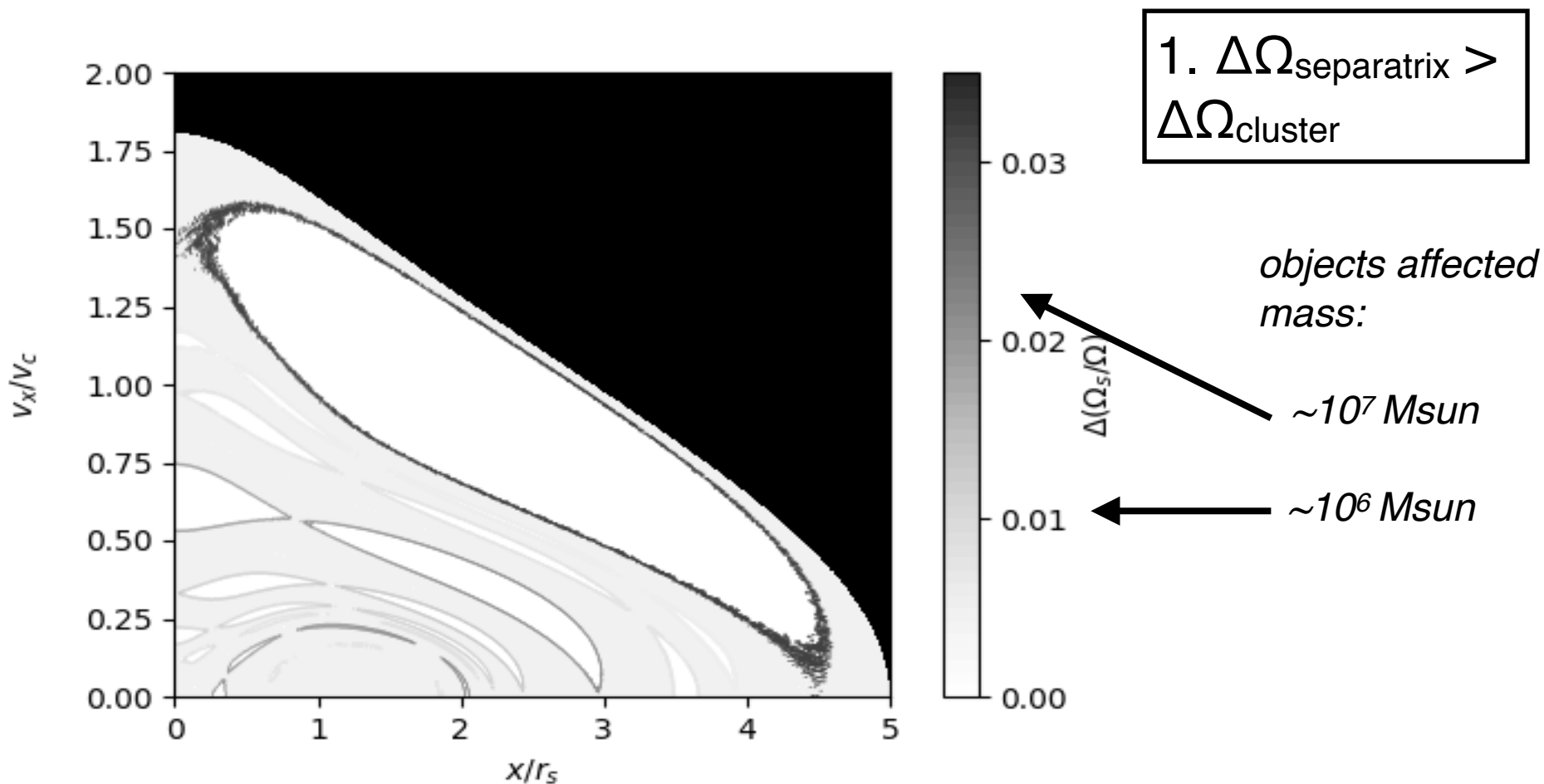


# Separatrix Divergence - quantification

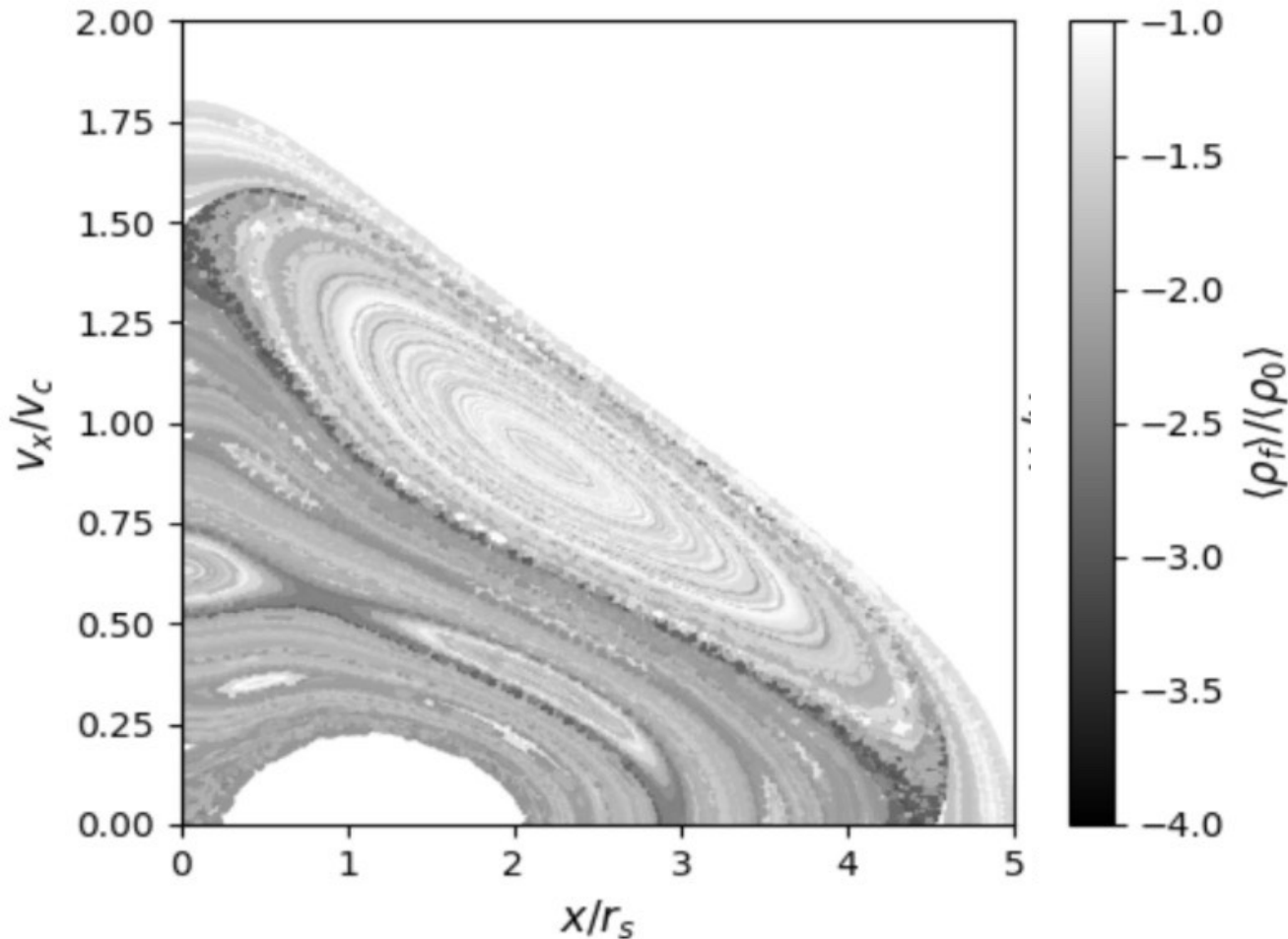


1.  $\Delta\Omega_{\text{separatrix}} >$   
 $\Delta\Omega_{\text{cluster}}$

# Separatrix Divergence - quantification

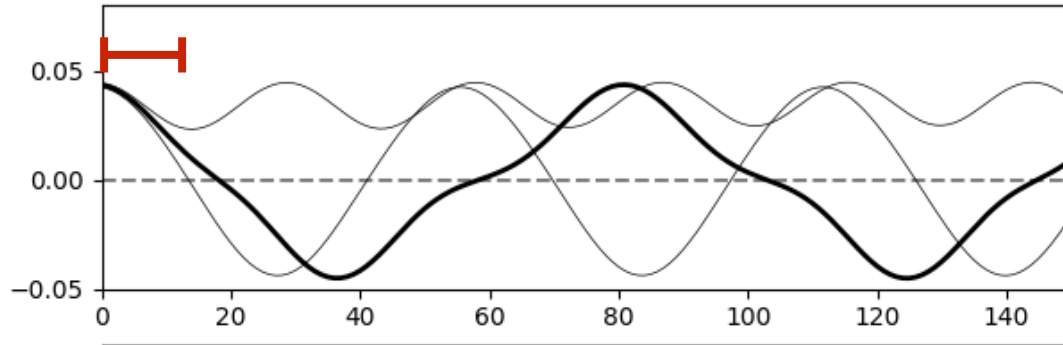


# Separatrix Divergence - quantification



1.  $\Delta\Omega_{\text{separatrix}} > \Delta\Omega_{\text{cluster}}$

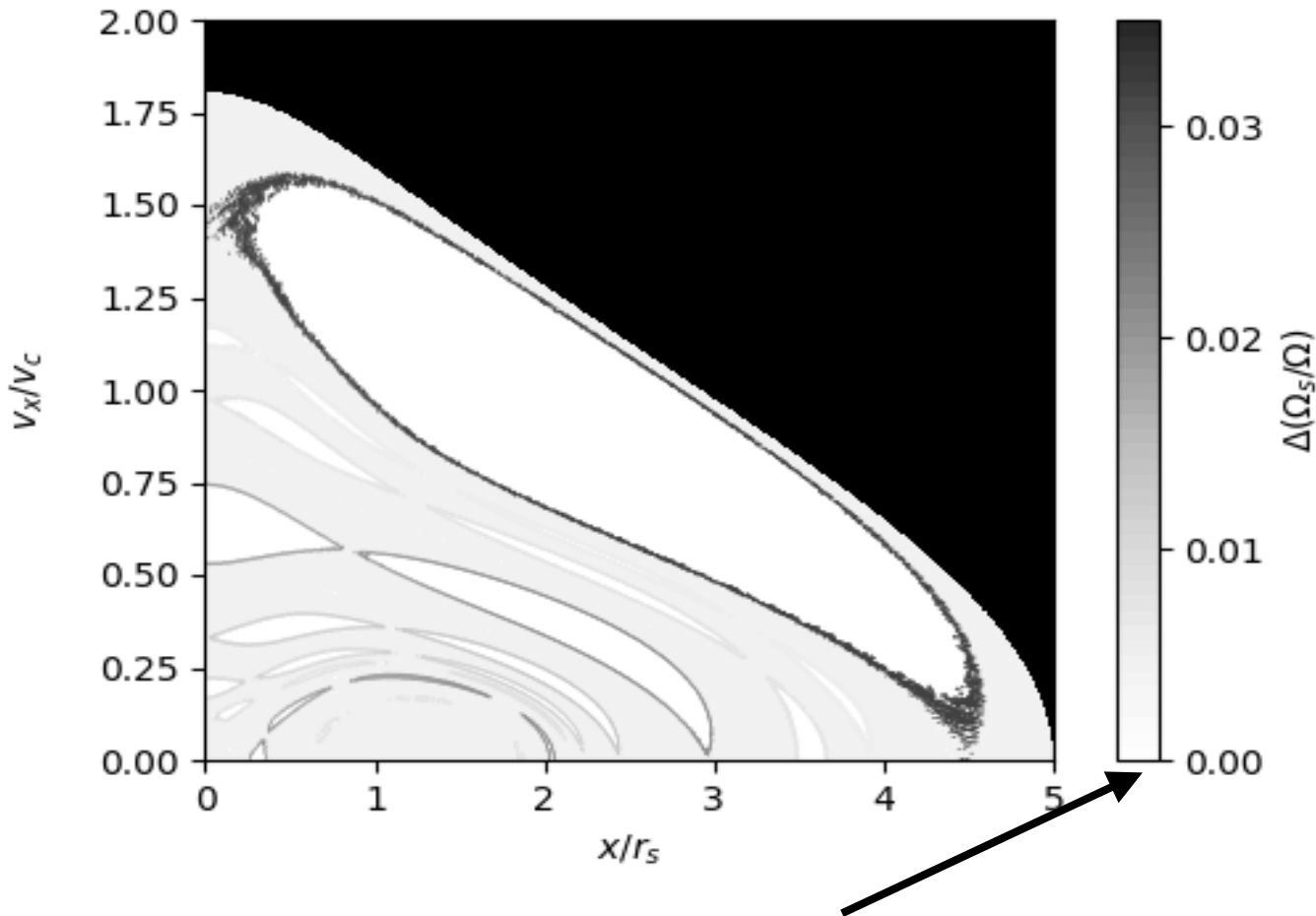
# Separatrix Divergence - quantification



1.  $\Delta\Omega_{\text{separatrix}} > \Delta\Omega_{\text{cluster}}$

2. timescale  
 $\sim n \cdot \Omega$  libration  
 $<$  Hubble time

# Separatrix Divergence - quantification



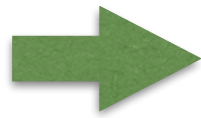
1.  $\Delta\Omega_{\text{separatrix}} > \Delta\Omega_{\text{cluster}}$

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 $<$  Hubble time

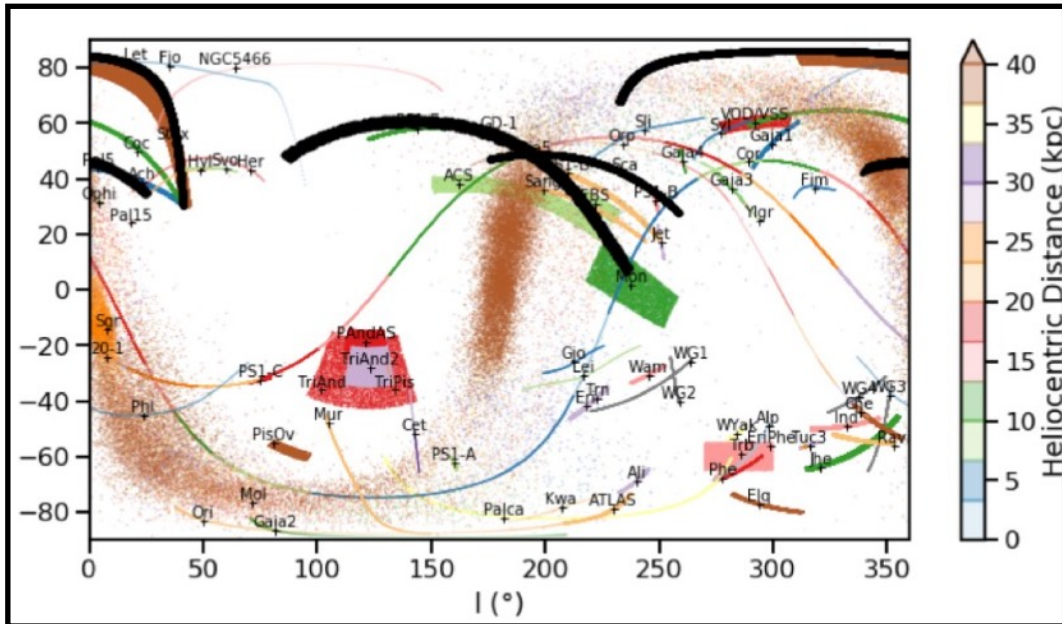
3. finite  
probability of  
occurrence

*$\sim$  maximum cross section of effect of separatrix*

# Thin streams

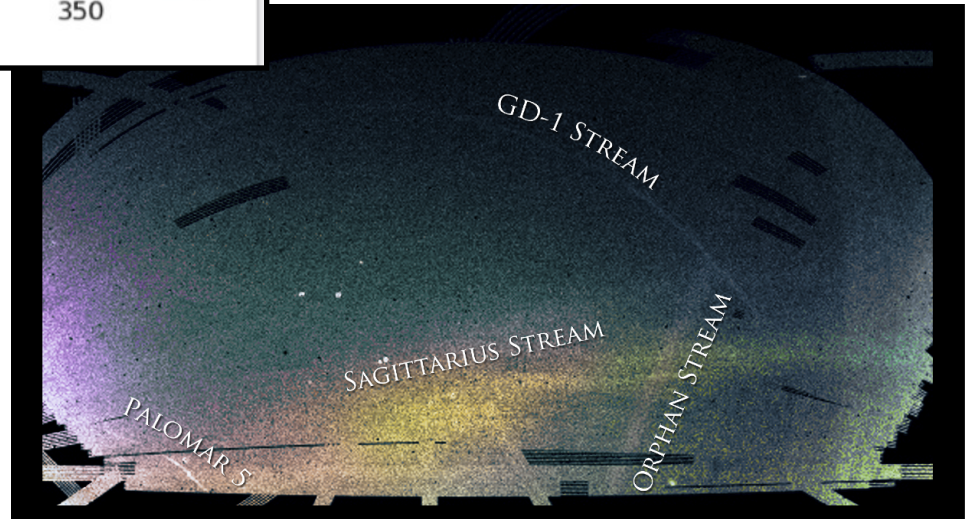


regularity/resonance

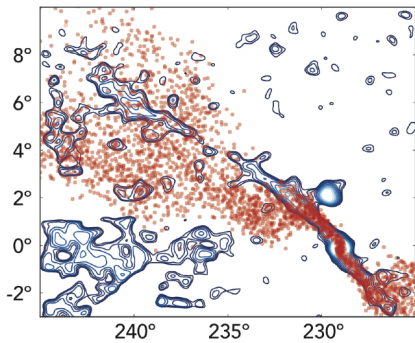
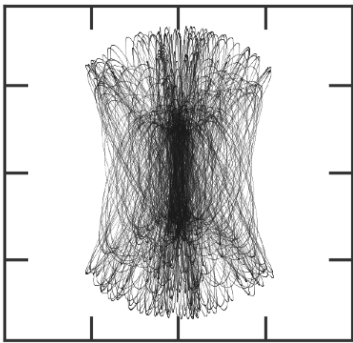
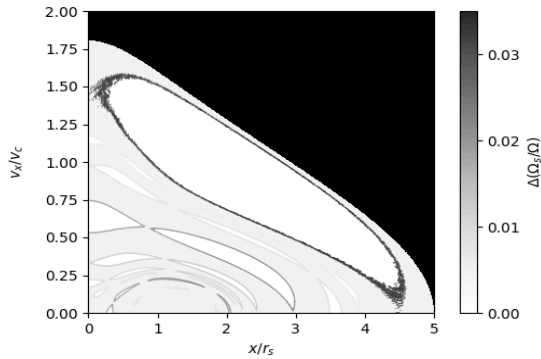


from Cecilia Mateu's  
*Galstreams* python package

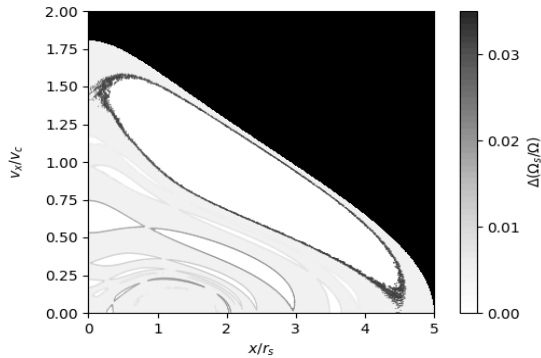
Ana Bonaca,  
following “field of  
streams” (Belokurov et al 2005)



# Conclusion - observing fundamental dynamics

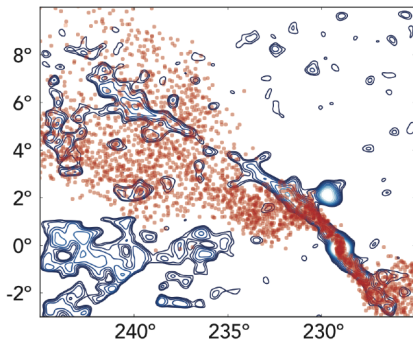
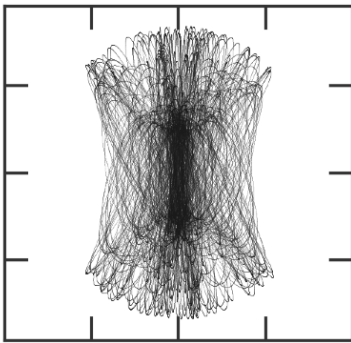


# Conclusion - observing fundamental dynamics



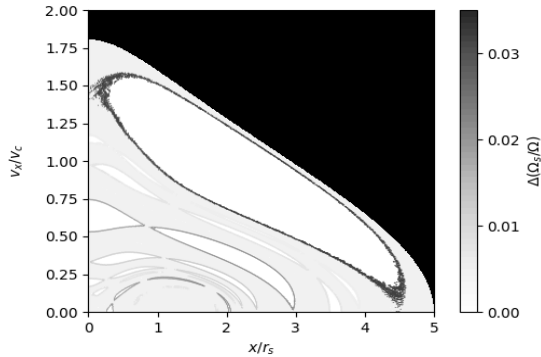
Morphology  
= cheap  
potential  
indicator

*mere existence of thin streams  
is constraining, for MW and  
other galaxies*



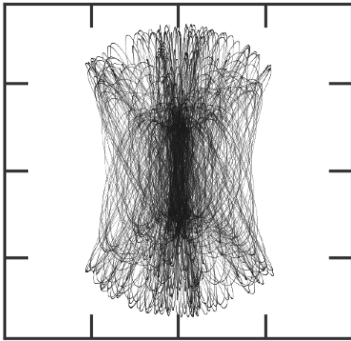


# Conclusion - observing fundamental dynamics



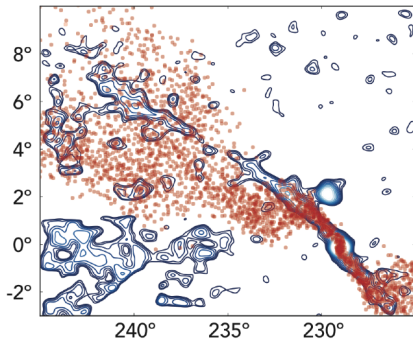
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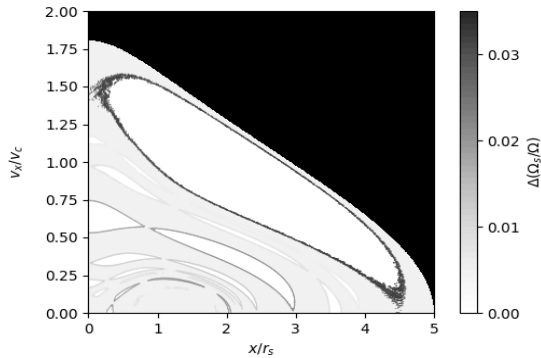


Novel  
dynamical  
regime

*few orbits, not phase-averaged,  
ensemble properties,  
projected to observables*

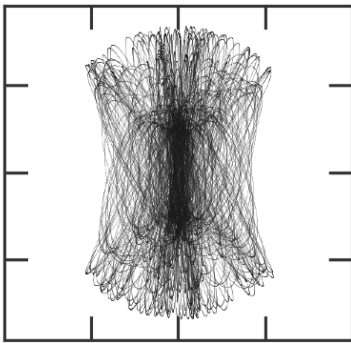


# Conclusion - observing fundamental dynamics



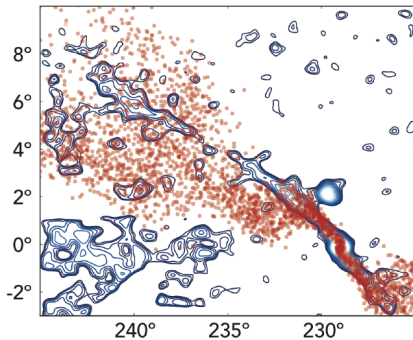
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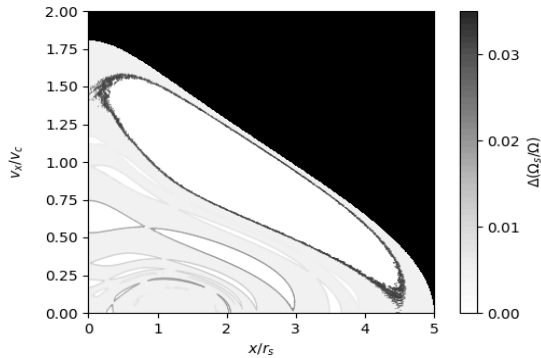
*few orbits, not phase-averaged,  
ensemble properties,  
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Varied  
applications

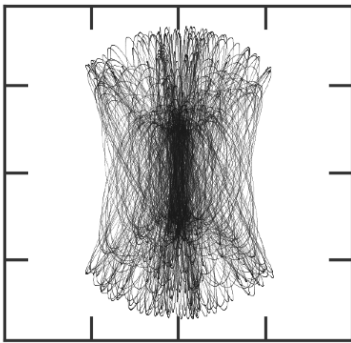
*unbound stellar ensembles,  
from binary stars (Spergel, Oh,  
Price-Whelan) to dwarf galaxies*

# Conclusion - observing fundamental dynamics



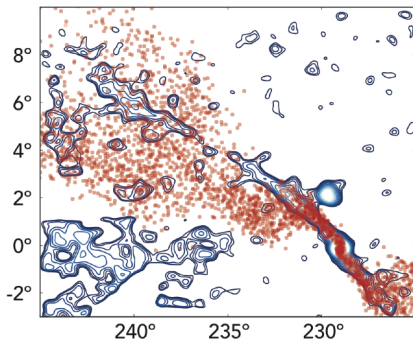
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Novel  
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regime

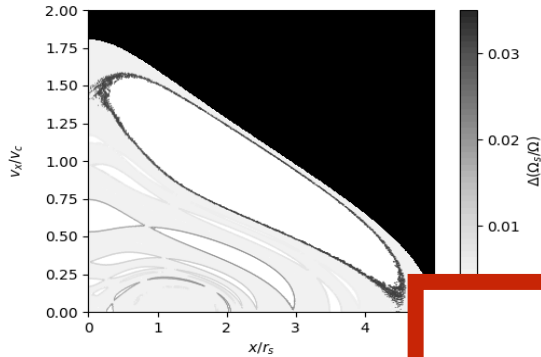
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Varied  
applications

*unbound stellar ensembles,  
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# Conclusion - observing fundamental dynamics

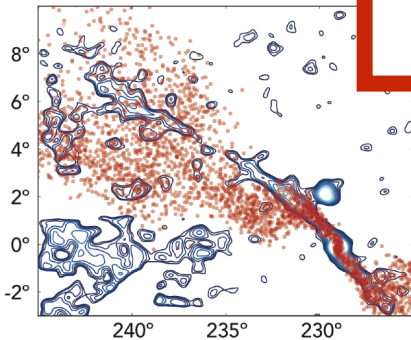
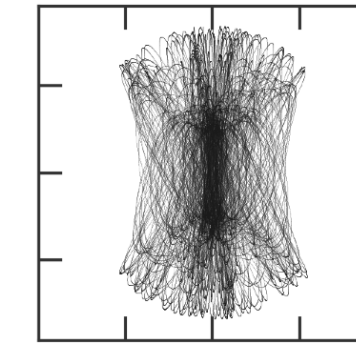


Morphology  
= cheap  
potential

*mere existence of thin streams  
is constraining, for MW and  
other galaxies*

***Let's observe  
the Poincaré Map  
of the  
Milky Way!***

*phase-averaged,  
properties,  
observables*



Varied  
applications

*unbound stellar ensembles,  
from binary stars (Spergel, Oh,  
Price-Whelan) to dwarf galaxies*