The Fuzzball Program

and

Black Hole Complementarity

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KITP “Complementarity, Fuzz or Fire?” Workshop

21 Aug 2013

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1202.6421 S. D. Mathur, DT
1208.1770 O. Lunin, S. D. Mathur, DT
1208.2005 S. D. Mathur, DT
1211.0306 S. Giusto, O. Lunin, S. D. Mathur, DT
1306.5488 S. D. Mathur, DT

Outline

Part I

1. Momentum-carrying waves on D1-D5 microstates

2. CFT duals of all two-center Bena-Warner solutions

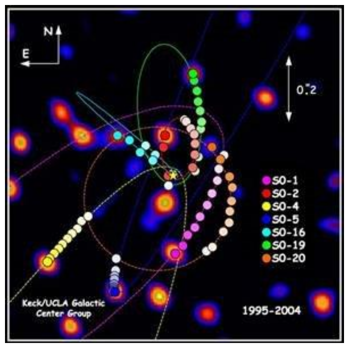
Part II

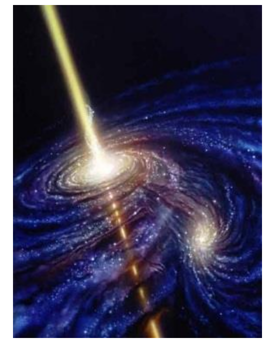
1. Fuzzball Complementarity

2. Fuzzballs and Firewalls

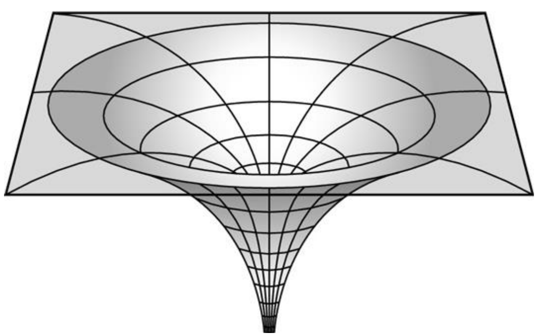
Black Holes

Physical: dark, heavy, compact bound state of matter

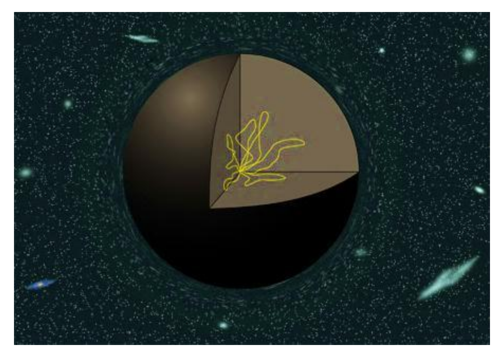




(Semi-)classical: geometry with horizon



Quantum: bound state in quantum gravity theory



Black hole hair

- Bekenstein-Hawking entropy $S \;\to\; e^S$ microstates
- Can physics of individual microstates modify Hawking's calculation?
- Many searches for Black hole 'hair': d.o.f. at the horizon.
- In classical gravity, many 'no-hair' theorems resulted.

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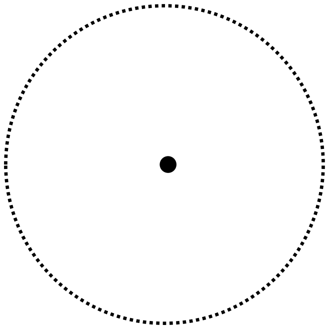
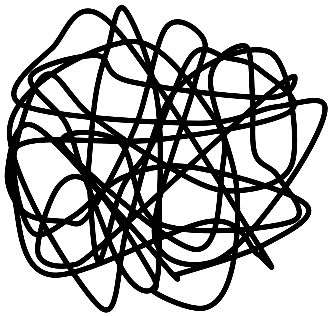
In String theory, we find examples of hair. Results suggests that

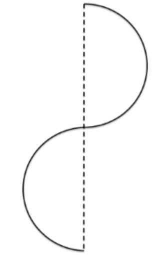
- Quantum effects important at would-be-horizon (fuzz)
- Bound states have non-trivial size (ball)





"Fuzzball"

Two-charge Black hole



- Multiwound fundamental string + momentum
- Entropy: exponential degeneracy of microscopic states

Sen ’94

- For classical profiles, string sources good supergravity background

  Classical profiles $\leftrightarrow$ coherent states

Callan, Maldacena, Peet ’95
Dabholkar, Gauntlett, Harvey, Waldram ’95
Lunin, Mathur ’01

- No horizons; string source
- Transverse vibrations only $\rightarrow$ non-trivial size

- F1-P is U-dual to D1-D5 bound state

Lunin, Mathur ’01

- Configurations are everywhere smooth in D1-D5 frame

Lunin, Maldacena, Maoz ’02

- Caveat: two-charge Black hole is string-scale sized.

D1-D5-P: three charges

- Add momentum to D1-D5 $\rightarrow$ macroscopic BPS black hole in 5D
- Entropy reproduced from microscopic degrees of freedom

Strominger, Vafa '96

Breckenridge, Myers, Peet, Vafa '96

- Large classes of three-charge microstate geometries constructed

Mathur, Saxena, Srivastava, Giusto, Ford,…
Bena, Warner, Bobev, Wang, Niehoff, Vasilakis,…
Berglund, Gimon, Levi,…
de Boer, El Showk, Shigemori,…
Giusto, Russo, DT '11,…

- Questions:

  - Do microstates have structure at the scale of the would-be horizon?
    (So far, all results indicate 'yes')

  - How large a subset of the BH degrees of freedom can we describe?

D1-D5 system: setup

Work in type IIB string theory on

$$\begin{array}{ccccc} \mathbb{R}^{1,4} & \times & S^1 & \times & T^4 \\ t, x^\mu & & y & & z^i \end{array}$$

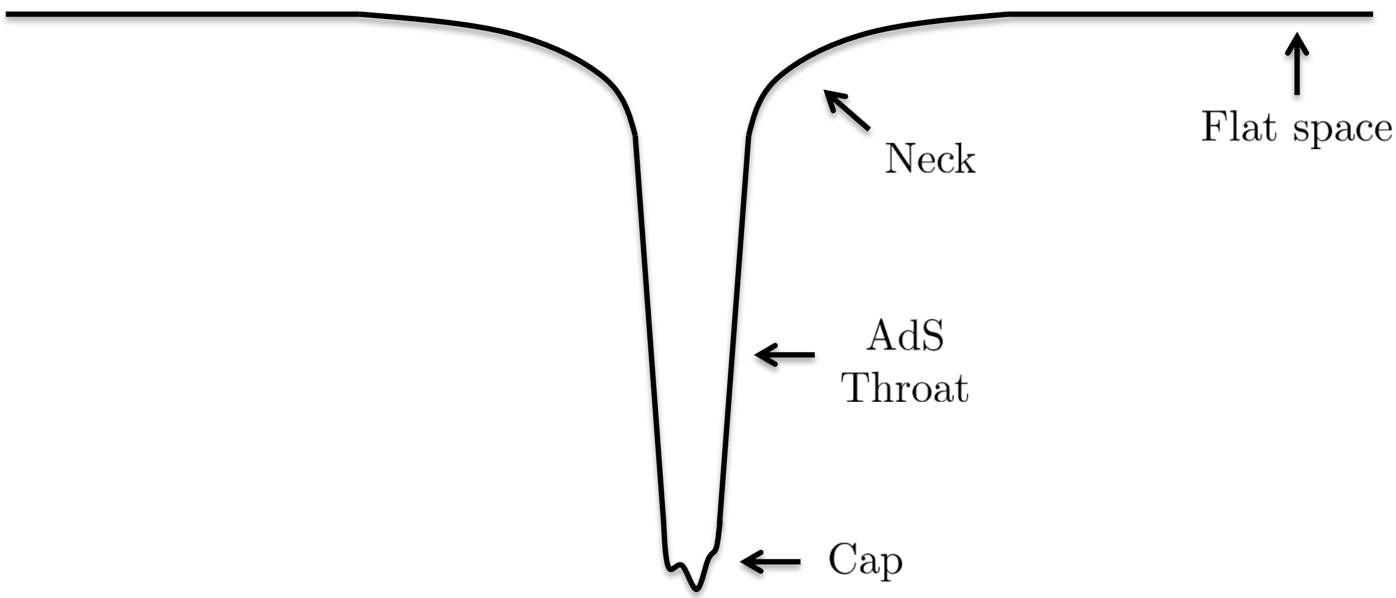
- Radius of S^1 : R_y
- Wrap n_1 D1 branes on S^1
- Wrap n_5 D5 branes on $S^1 \times T^4$

The bound state creates a geometry with D1 and D5 charges

$$Q_1 = \frac{1}{V}(2\pi)^4 g\alpha'^3 \, n_1 \,, \qquad Q_5 = g\alpha' \, n_5$$

For simplicity, set $Q_1 = Q_5 = Q$.

To get a large AdS throat, take $\sqrt{Q} \ll R_y$. Structure of geometry is then:



The throat is locally $AdS_3 \times S^3 \times T^4$.

D1-D5 CFT & AdS/CFT

- Worldvolume gauge theory on D1-D5 bound state flows in IR to a (4,4) SCFT.
- Orbifold point in moduli space: Free SCFT on $(T^4)^N/S_N$, $N = n_1 n_5$.

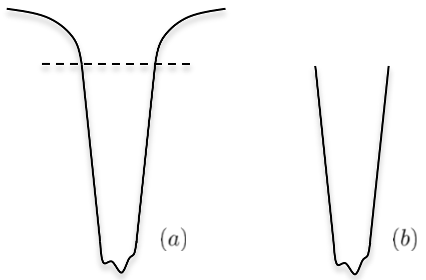
Vafa '95, Douglas '95

Symmetry generators:

- $\mathrm{Virasoro}_\mathrm{L} \times \mathrm{Virasoro}_\mathrm{R}$ $\qquad L_{-n} \longleftrightarrow AdS_3$
- R-symmetries $SU(2)_\mathrm{L} \times SU(2)_\mathrm{R}$ $\qquad J^a_{-n} \longleftrightarrow S^3$
- U(1) currents of T^4 translations $\qquad J^i_{-n} \longleftrightarrow T^4$

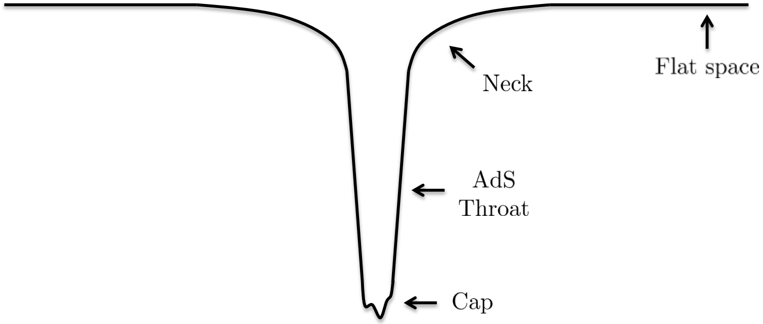
Maldacena '97

& Susy generators



2. Results

- Proposed classification of CFT states into ‘neck’ d.o.f. and ‘cap’ d.o.f.
- Constructed explicit gravitational solutions describing neck d.o.f.



Mathur, DT 1112.6413

Mathur, DT 1202.6421

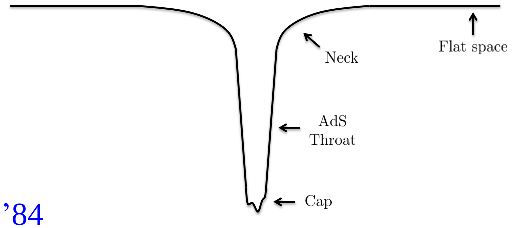
Lunin, Mathur, DT 1208.1770

- Identified CFT duals of known two-center Bena-Warner solutions describing ‘cap’ d.o.f.

Giusto, Lunin, Mathur, DT 1211.0306

Neck degrees of freedom

- Asymptotic symmetry group (ASG):
  symmetries preserving asymptotics of the space



- ASG of AdS_3 : $\mathrm{Virasoro}_{\mathrm{L}} \times \mathrm{Virasoro}_{\mathrm{R}}$.

Brown, Henneaux '84

- D1-D5 AdS/CFT generalization:

Maldacena '97

$$L_{-n} \quad \longleftrightarrow \quad AdS_3$$

$$J^a_{-n} \quad \longleftrightarrow \quad S^3$$

$$J^i_{-n} \quad \longleftrightarrow \quad T^4 \quad \longleftarrow$$

$$J^i = \sum_{\text{copies}} \partial X^i$$

$$\Rightarrow \quad J^i_{-n} = \sum_{\text{copies}} \alpha^i_{-n} \, .$$

- Proposal: Neck d.o.f. correspond to action of chiral algebra generators

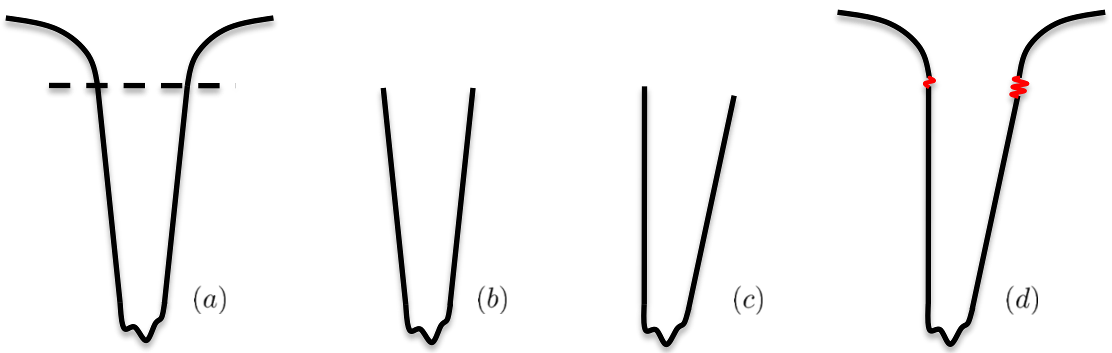
Mathur, DT 1112.6413

Neck degrees of freedom

- Take the D1-D5 ground state $|0\rangle_R$ and seek dual of $J^z_{-n}|0\rangle_R$ $\quad (z = z^1)$
- In the $AdS_3 \times S^3 \times T^4$ throat, J^z_{-n} is diffeomorphism along T^4
- However $J^z_{-n}|0\rangle_R$ has higher energy than $|0\rangle_R$

  $\rightarrow$ perturbation can't be a diffeomorphism everywhere

- We find a solution which is non-trivial in the neck region between throat and flat asymptotics.

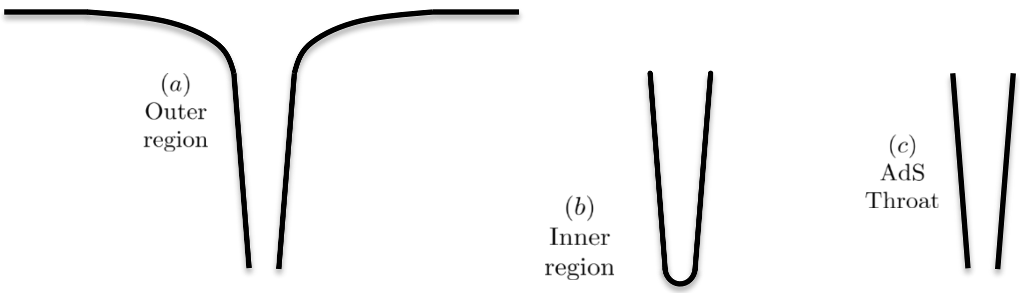


Method

1. Solve e.o.m. in 'outer' and 'inner' regions separately,

   & match solutions in the 'throat'

Mathur, DT 1112.6413



2. Extend to closed-form perturbation on full background

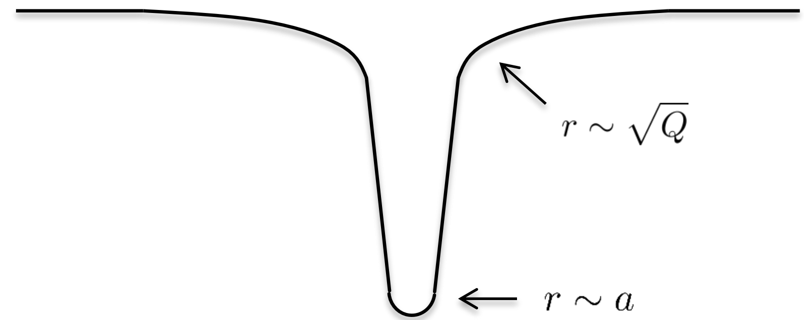
Mathur, DT 1202.6421

3. Generalize to full nonlinear solution & to other backgrounds

Lunin, Mathur, DT 1208.1770

Comment: Horowitz, Marolf '96, Kaloper, Myers, Roussel '96, Horowitz, Yang '97

Full non-linear solution for $J^{z}_{-n}|0\rangle_R$:



$$
\begin{aligned}
ds^2 &= -\frac{1}{H}\,[du+A]\,[dv+B] + H\,f\left[\frac{dr^2}{r^2+a^2} + d\theta^2\right] + H\left[r^2 c_\theta^2 d\psi^2 + (r^2+a^2) s_\theta^2 d\phi^2\right] \\
&\qquad + dz^i dz^i\,, \\
C^{(2)} &= \frac{1}{2H}[dv+B]\wedge[du+A] + Q\,c_\theta^2\,d\phi\wedge d\psi\,,
\end{aligned}
$$

$$
A = \frac{aQ}{f}\{s_\theta^2 d\phi - c_\theta^2 d\psi\} + \Phi\,dz_1\,, \quad B = \frac{aQ}{f}\{s_\theta^2 d\phi + c_\theta^2 d\psi\}\,, \quad f = r^2 + a^2 c_\theta^2\,, \quad H = 1 + \frac{Q}{f}\,,
$$

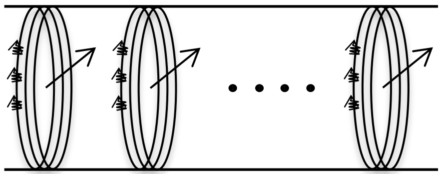
$$
\Phi = \sum_{n=-\infty}^{\infty} c_n e^{-in\frac{v}{R_y}} \left(\frac{r^2}{r^2+a^2}\right)^{|n|/2} .
$$

where

$$
a = \frac{Q}{R_y}, \qquad c_\theta = \cos\theta, \qquad s_\theta = \sin\theta\,.
$$

Generalization

- Local form of all supersymmetric solutions of minimal supergravity in 6D: 'GMR' form
  Gutowski, Martelli, Reall '03
- Above solution generalizes to non-linear deformation of any solution in GMR form.
- Solution is given in terms of a function which obeys wave equation on the background metric.
- GMR class includes all 2 charge D1-D5 solutions & some 3 charge D1-D5-P solutions



- Solve wave equation on D1-D5-P solutions from spectral flow:

$$\Phi_i \;=\; \sum_n c_n^i \left(\frac{r^2}{r^2[1+2a^2 n_L(n_L+1)]+a^2} \right)^{|n/2|} e^{inv/R_y}$$

- Metric:

$$
\begin{aligned}
ds^2 \;=\;& -\frac{1}{h}\left[du+\Phi_i dz^i\right]dv+\frac{Q_p}{h\,f}\,dv^2+h\,f\left(\frac{dr^2}{r^2+a^2\,(\gamma_1+\gamma_2)^2\,\eta}+d\theta^2\right)\\
&+h\left(r^2+a^2\,\gamma_1\,(\gamma_1+\gamma_2)\,\eta-\frac{Q_1Q_5\,a^2\,(\gamma_1^2-\gamma_2^2)\,\eta\,\cos^2\theta}{h^2\,f^2}\right)\cos^2\theta\,d\psi^2\\
&+h\left(r^2+a^2\,\gamma_2\,(\gamma_1+\gamma_2)\,\eta+\frac{Q_1Q_5\,a^2\,(\gamma_1^2-\gamma_2^2)\,\eta\,\sin^2\theta}{h^2\,f^2}\right)\sin^2\theta\,d\phi^2\\
&+\frac{Q_p\,a^2\,(\gamma_1+\gamma_2)^2\,\eta^2}{h\,f}\left(\cos^2\theta\,d\psi+\sin^2\theta\,d\phi\right)^2\\
&-\frac{2\sqrt{Q_1Q_5}\,a}{h\,f}\left(\gamma_1\,\cos^2\theta\,d\psi+\gamma_2\,\sin^2\theta\,d\phi\right)dv\\
&-\frac{2\sqrt{Q_1Q_5}\,a\,(\gamma_1+\gamma_2)\,\eta}{h\,f}\left(\cos^2\theta\,d\psi+\sin^2\theta\,d\phi\right)\left(\left[du+\Phi_i dz^i\right]-dv\right)\,,
\end{aligned}
$$

$$
a=\frac{\sqrt{Q_1\,Q_5}}{R}\,,\quad Q_p=-a^2\,\gamma_1\,\gamma_2\quad \eta=\frac{Q_1\,Q_5}{Q_1\,Q_5+Q_1\,Q_p+Q_5\,Q_p}\,,
$$

$$
f=r^2+a^2\,(\gamma_1+\gamma_2)\,\eta\,(\gamma_1\,\sin^2\theta+\gamma_2\,\cos^2\theta)\,,
$$

$$
H_1=1+\frac{Q_1}{f}\,,\quad H_5=1+\frac{Q_5}{f}\,,\quad h=\sqrt{H_1\,H_5}\,.
$$

$$
\boxed{
\begin{aligned}
\gamma_1 &= -n_L\,, \qquad \gamma_2 = n_L+1\,,\\
\Phi_i &= \sum_n c_n^i\left(\frac{r^2}{r^2[1+2a^2 n_L(n_L+1)]+a^2}\right)^{|n/2|} e^{inv/R_y}
\end{aligned}
}
$$

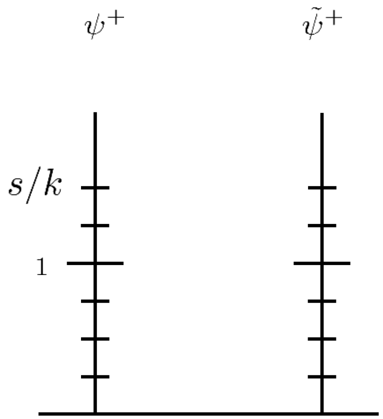
3. Microstates at the Cap

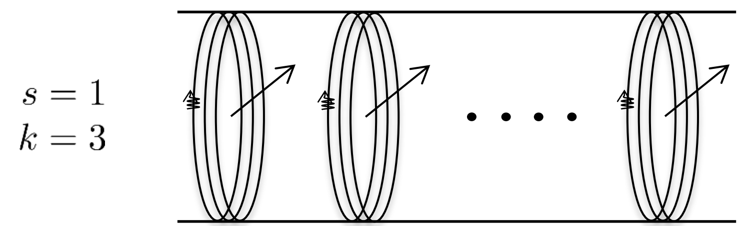
- Look for states which cannot be written in terms of symmetry algebra generators acting on a ground state.

- These should correspond to bulk solutions with non-trivial cap structure.

States with fractional filling

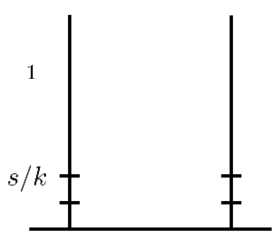
- Consider Fermi seas filled to fractional level s/k :





~ Spectral flow on cover

- For $s = nk + s'\,, \qquad 0 < s' < k-1$
these states cannot be obtained by acting with symmetry algebra generators on R ground states.



$$J^+_{-n} \;=\; \sum_{\substack{\text{copies} \\ m}} \psi^+_{-m}\tilde{\psi}^+_{-(n-m)}$$

Gravitational descriptions

- Gravitational descriptions of the states: previously known

- Generalizations of the gravitational solutions describing states with integer spectral flow parameter n_L to fractional values s/k .

- Interesting orbifold structure in cap

Jejjala, Madden, Ross, Titchener '05

- Corresponds to most general Bena-Warner solution with 2 centers

Bena, Warner '05
Berglund, Gimon, Levi '05

- Four integers n_1, n_5, k, s specify the geometry

- Metric:

$$
\begin{aligned}
ds^2 \quad = \quad & -\frac{1}{h}\,(dt^2 - dy^2) + \frac{Q_p}{h\,f}\,(dt-dy)^2 + h\,f\left(\frac{dr^2}{r^2 + a^2\,(\gamma_1+\gamma_2)^2\,\eta} + d\theta^2\right) \\
& + h\left(r^2 + a^2\,\gamma_1\,(\gamma_1+\gamma_2)\,\eta - \frac{Q_1 Q_5\,a^2\,(\gamma_1^2 - \gamma_2^2)\,\eta\,\cos^2\theta}{h^2\,f^2}\right)\cos^2\theta\,d\psi^2 \\
& + h\left(r^2 + a^2\,\gamma_2\,(\gamma_1+\gamma_2)\,\eta + \frac{Q_1 Q_5\,a^2\,(\gamma_1^2 - \gamma_2^2)\,\eta\,\sin^2\theta}{h^2\,f^2}\right)\sin^2\theta\,d\phi^2 \\
& + \frac{Q_p\,a^2\,(\gamma_1+\gamma_2)^2\,\eta^2}{h\,f}\,(\cos^2\theta\,d\psi + \sin^2\theta\,d\phi)^2 \\
& - \frac{2\sqrt{Q_1 Q_5}\,a}{h\,f}\,(\gamma_1\,\cos^2\theta\,d\psi + \gamma_2\,\sin^2\theta\,d\phi)\,(dt-dy) \\
& - \frac{2\sqrt{Q_1 Q_5}\,a\,(\gamma_1+\gamma_2)\,\eta}{h\,f}\,(\cos^2\theta\,d\psi + \sin^2\theta\,d\phi)\,dy + \sqrt{\frac{H_1}{H_5}}\sum_{a=1}^{4} dz_a^2\,,
\end{aligned}
$$

$$
\begin{aligned}
& a = \frac{\sqrt{Q_1\,Q_5}}{R}\,, \quad Q_p = -a^2\,\gamma_1\,\gamma_2 \quad \eta = \frac{Q_1\,Q_5}{Q_1\,Q_5 + Q_1\,Q_p + Q_5\,Q_p}\,, \\
& f = r^2 + a^2\,(\gamma_1+\gamma_2)\,\eta\,(\gamma_1\,\sin^2\theta + \gamma_2\,\cos^2\theta)\,, \\
& H_1 = 1 + \frac{Q_1}{f}\,, \quad H_5 = 1 + \frac{Q_5}{f}\,, \quad h = \sqrt{H_1\,H_5}\,.
\end{aligned}
$$

$$
\gamma_1 = -\frac{s}{k}\,, \quad \gamma_2 = \frac{s+1}{k}\,.
$$

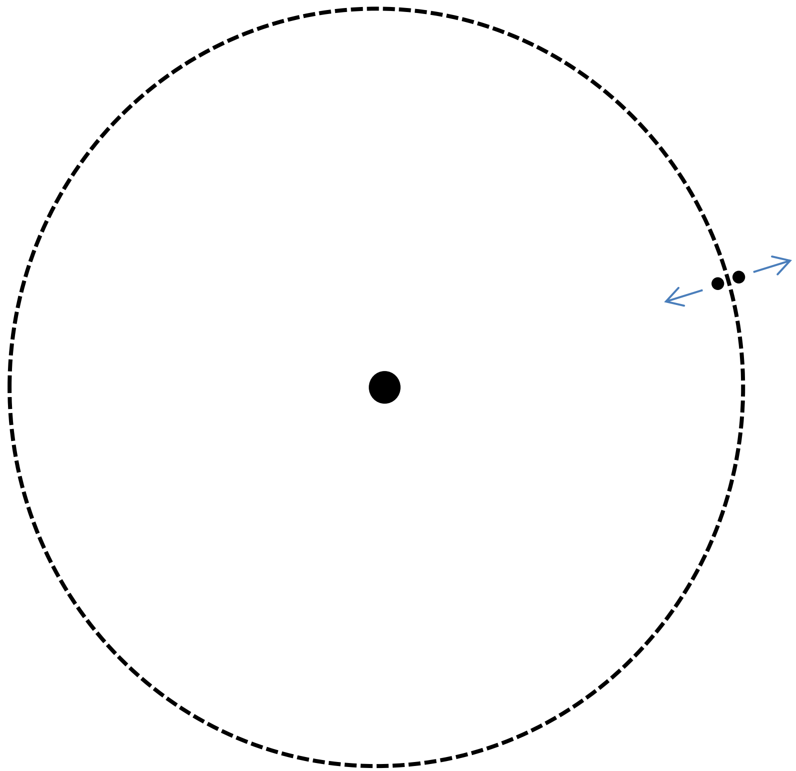
Part I Summary

- Examples of both cap states and neck states identified
- Supports proposal that neck d.o.f. correspond to symmetry algebra generators
- Cap states are step towards understanding generic state of black hole

Part II: Black Hole Complementarity

The Information Paradox

BH Horizon:
normal lab physics
(small curvature)



Hawking radiation:
pair creation

$\rightarrow$ entangled pair

- Endpoint of process: violation of unitarity or exotic remnants

Hawking '75

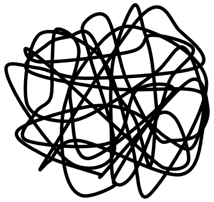
- Conclusions robust including small corrections (Mathur's theorem)

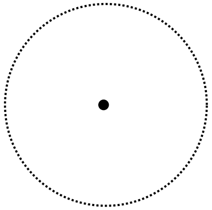
Mathur 0909.1038

The Fuzzball Proposal

- Conjecture: the generic state of a black hole is described by a solution of string theory which
  - resembles the traditional black hole far outside the would-be horizon
  - ends in a complicated structure involving sources of string theory just outside the would-be horizon.

- There is no horizon and no interior.





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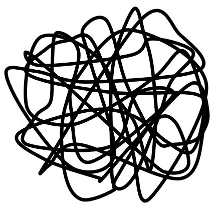
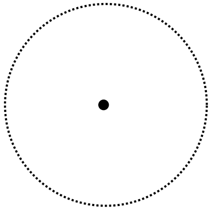




- A collapsing shell of matter should tunnel into a fuzzball configuration
  - tunneling time has been estimated to be << the Hawking evaporation time (may be expected to be of order the 'scrambling time').

- Open question: what happens if you fall onto a fuzzball configuration?
  - Conjecture: for coarse, high energy (E >> T) processes, there should be a 'complementary' description involving free infall

Mathur 1012.2101

Mathur, Plumberg 1101.4899

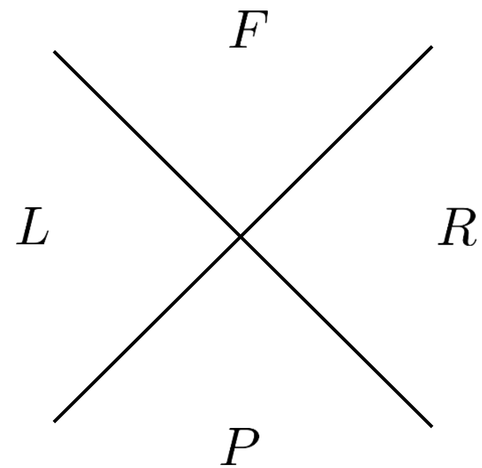
- In the Fuzzball scenario, what is the role of the classical BH metric?

- What quantities does the classical BH metric accurately describe?

Correlators in Rindler space

Rindler space:

- Accelerated observer in Minkowski space
- Near-horizon region of a black hole



- Minkowski space decomposes into four Rindler wedges

- Consider a free scalar field theory
- Minkowski vacuum restricted to right Rindler wedge is a thermal state

$$|0\rangle_M = C \sum_k e^{-\frac{E_k}{2}} |E_k\rangle_L |E_k\rangle_R, \qquad C = \Big(\sum_k e^{-E_k} \Big)^{-\frac{1}{2}}$$

Correlators in Rindler space

- Consider the right Rindler wedge, in a particular generic state.

Divide correlators into those which

1. Are well approximated by the canonical ensemble (coarse/non-fine-tuned)
2. Are not well approximated by the canonical ensemble (fine-tuned).
   (sensitive to some details of generic microstates)

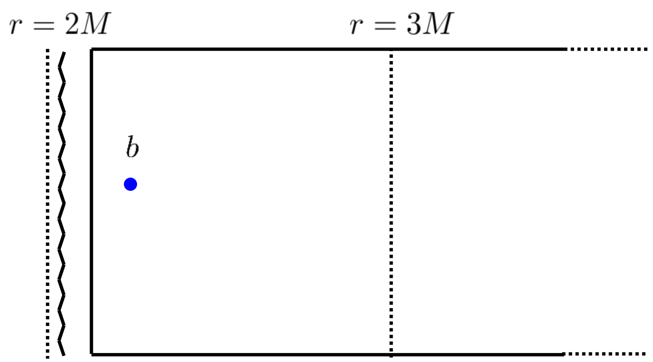
- Minkowski vacuum $\leftrightarrow$ canonical ensemble,
  so accurately describes coarse/non-fine-tuned correlators:

$$
{}_R\langle E_k|\hat{O}_R|E_k\rangle_R \;\approx\; \frac{1}{\sum_l e^{-E_l}} \sum_i e^{-E_i}\, {}_R\langle E_i|\hat{O}_R|E_i\rangle_R \;=\; {}_M\langle 0|\hat{O}_R|0\rangle_M
$$

- Suggests correct role of traditional black hole metric.

Fuzzball Complementarity: a conjecture

- **Picture 1**: space-time is cut off by the fuzzball surface: state is a solution of string theory.
  - This description is appropriate for **all physical processes** (and all observers).



Fuzzball Complementarity: a conjecture

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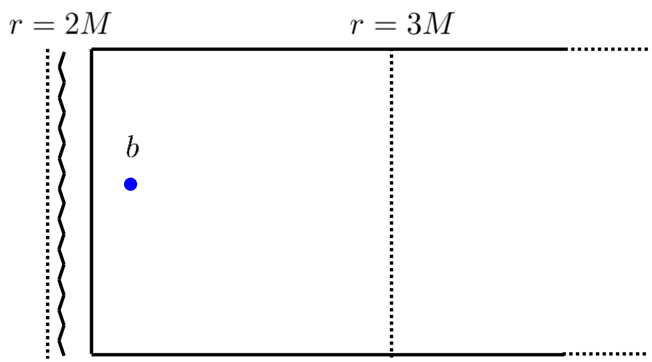
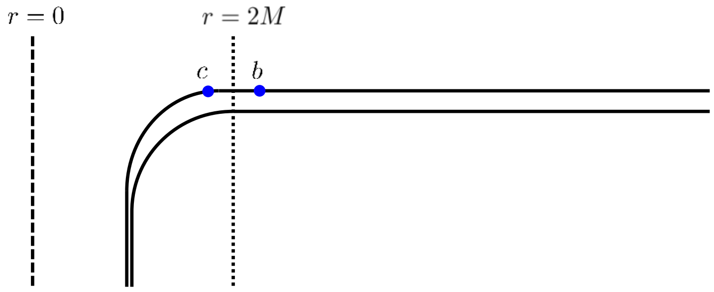




- **Picture 2**: Traditional black hole metric.
  - This description accurately describes many processes, including a set of coarse, high energy (E >> T) impact processes

Mathur, Plumberg 1101.4899

Mathur review 1205.0776

What Fuzzball Complementarity is not

- It is not an attempt to reconstruct the interior space-time.

- It does not postulate new observer-dependent physics in black holes.

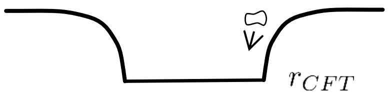
Rather, the conjecture is that in the fuzzball scenario,
the traditional black metric still accurately describes many processes,
including a set of coarse, high-energy (E >> T) impact processes.

Toy Model: AdS/CFT

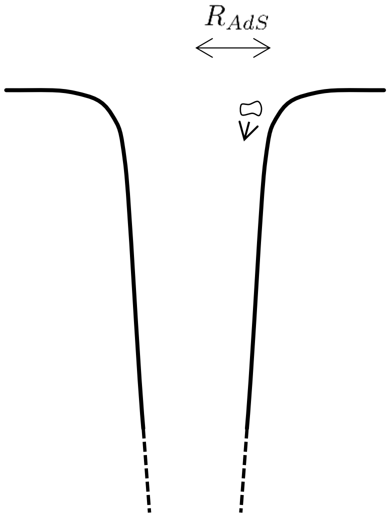
Analogy between Fuzzball Complementarity and AdS/CFT: Mathur, DT 1208.2005

- Consider D1-D5 system in regime where AdS description is weakly coupled

- In the CFT description, an incoming graviton hits the D1-D5 bound state and breaks up into excitations of the (strongly coupled) CFT.



- In the dual gravity description, the graviton experiences a smooth space-time with low curvature through the vicinity of the CFT location.



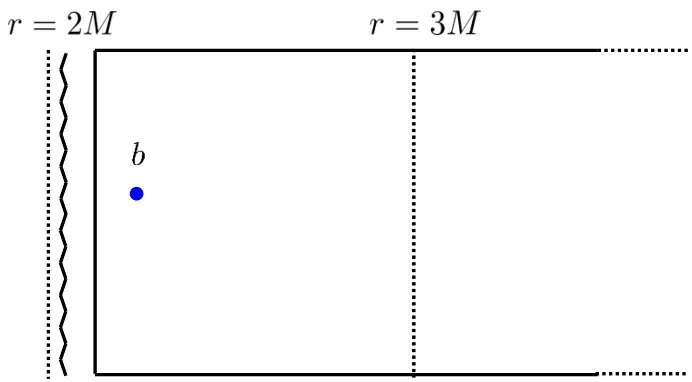
Black Hole Complementarity

- **Picture 1**: space-time is cut off by a 'stretched horizon': a Planck-temperature membrane.

  Susskind, Thorlacius, Uglum '93

  – This description is appropriate for an **observer at infinity**.

  – Postulate 1: Formation and evaporation of BH is unitary
  – Postulate 2: Semiclassical physics outside 'stretched horizon'
  – Postulate 3: Black hole has e^S states & discrete energy levels



- **Picture 2**: Traditional black hole metric.

  – This description is appropriate for an **infalling observer.**

  – Postulate 4: An observer falling into the black hole and crossing the horizon experiences semiclassical physics in the traditional black hole geometry

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- This requires sharpening. For example:

  - **Postulate 4A**: Semiclassical physics in the traditional black hole geometry describes all possible low-energy measurements made by an infalling observer, including **fine-tuned** experiments.

Black Hole Complementarity

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- This requires sharpening. For example:
  - **Postulate 4A**: Semiclassical physics in the traditional black hole geometry describes all possible low-energy measurements made by an infalling observer, including **fine-tuned** experiments.

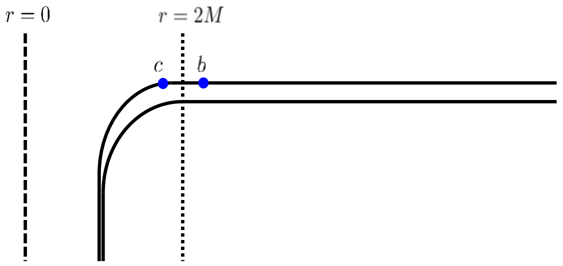
- With hindsight, this seems unphysical from the point of view of the fuzzball program; the traditional black hole metric is thought of as an ensemble average description, so should not describe fine-tuned measurements.

- By applying Mathur's theorem, it has recently been shown to be inconsistent.

Almheiri, Marolf, Polchinski, Sully 1207.3123

Firewall Argument 1

- **Postulate 4A** claims that Picture 2 describes fine-tuned experiments.
- After the Page time, a fine-tuned experiment shows that b is (almost) maximally entangled with the early radiation E.
- However, in Picture 2, b is entangled with its partner c, which is not compatible (by applying strong subadditivity, as in Mathur's theorem).



Conclusions:

- Require structure at the horizon (for all observers)
- BH metric does not describe fine-tuned experiments.

Conclusions:

- Require structure at the horizon
- BH metric does not describe fine-tuned experiments.

- Extrapolation: consider excited states (hair) at the horizon in Picture 2.
  (Alternatively, new physics at macroscopic distances outside horizon, e.g. Giddings et al.)

- One requires excitations all the way to the Planck scale – "Firewall".
  (Reconstruct Planck-temperature membrane/shock-wave in Picture 2.)

=> Fuzzball / Firewall

Firewall Argument 2

- Original claim: Firewall argument excludes fuzzball complementarity

Almheiri, Marolf, Polchinski, Sully 1207.3123

Almheiri, Marolf, Polchinski, Stanford, Sully 1304.5483**v1**

- This claim was later retracted

Almheiri, Marolf, Polchinski, Stanford, Sully 1304.5483**v2**

Argument is nevertheless interesting to consider:

- Firewall argument 2: If infalling quantum interacts significantly with Hawking quanta b outside the fuzzball surface, then after the Page time this interaction cannot be 'undone' by the fuzzball surface, since there is no local entanglement.

Some Important Subtleties

Mathur, DT 1306.5488

- Can an infalling quantum interact significantly with Hawking radiation before reaching the fuzzball surface?

- How should one define the fuzzball surface / stretched horizon?
  What are its properties?

Some Important Subtleties

Mathur, DT 1306.5488

- Can an infalling quantum interact significantly with Hawking radiation before reaching the fuzzball surface?

- How should one define the fuzzball surface / stretched horizon?
  What are its properties?

- Extrapolate model of stretched horizon (SH) from fuzzball results:

- Conservative assumption: generic fuzzball state has its d.o.f located at a Planck distance l_p from the would-be horizon
  (If incorrect, firewall argument 2 is even weaker)

- Stretched horizon: exp(S(M)) states of mass M at radius $r_0(M) + l_p$

Some Important Subtleties

- Appears inconsistent to allow infalling quantum to reach SH without SH first expanding

- Mechanism: tunneling into exp(S(M+E)) states, new radius r_0(M+E)

- Expect that SH should first expand into a local ‘bubble’ of size

$$s_{bubble} \sim \left(\frac{E}{T} \right)^{\frac{1}{(D-2)}} l_p$$

- Can an infalling quantum interact significantly with Hawking radiation before reaching this location?

Interaction cross section with Hawking quanta

- Consider free-fall from outside 2 r0 (Ignore fine-tuning from lowering to near the horizon)

- Take for concreteness graviton-graviton cross-section, which grows with energy (Other particles can be treated similarly).

- Naïve estimate: c.o.m. energy is transplanckian when interaction happens with order 1 probability.

- On general considerations, 3 regimes of impact parameter d .
  Let R_s be the Schwarzschild radius of a black hole with mass equal to the center-of-mass energy of the collision, E_c .

  i) smallest: BH formation $d \lesssim R_s$
  ii) intermediate: depends on theory $R_s \lesssim d \lesssim \alpha R_s$
  iii) largest: classical deflection $d \gg \alpha R_s$
  (for some constant α).

Banks, Fischler '99

Interaction cross section with Hawking quanta

Combine i) and ii) – Probability of 'significant' interaction becomes order 1 at

$$s_\alpha \;\sim\; \left(\frac{E}{T}\right)^{\frac{1}{2(D-2)}} l_p$$

So we find

$$s_\alpha \;\ll\; s_{bubble} \,.$$

In case iii), we show that the deflection is parametrically smaller than the Planck length.

For the case of a fixed cross-section which does not grow with energy, for sufficiently large E/T the same conclusions apply.

Mathur, DT 1306.5488

So it appears that for sufficiently large E/T , there is no significant interaction with Hawking quanta before the location that the fuzzball surface tunnels out to.

Open questions:

- How large should E/T be in any given physical situation?
- Firewall => Fuzzball ?

Summary: Part II

- Mathur’s theorem: horizon cannot be ‘information free’
- Suggests that a black hole in a generic state is a fuzzball
- Fuzzball complementarity is weaker than Black Hole Complementarity, and will be interesting to explore further.

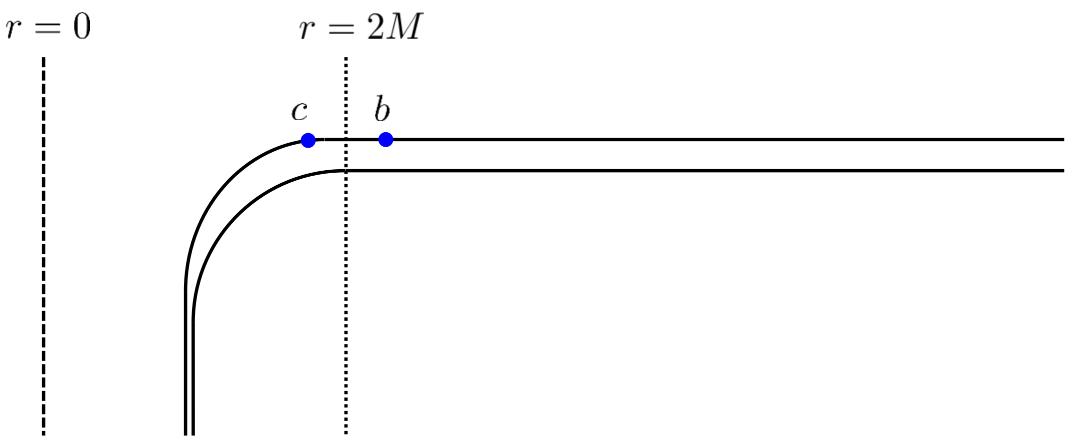
Thanks!

Bonus slides

Mathur's theorem

Mathur 0909.1038

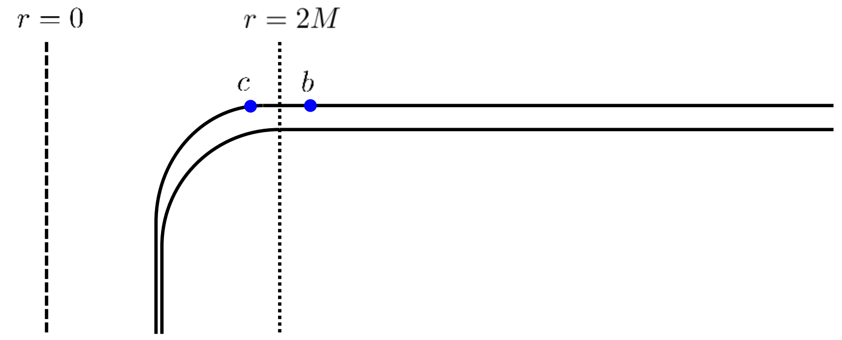
- A precise statement of the information paradox
- Assumptions: good semiclassical limit over 'nice slices' inside and outside the horizon



- Includes **small corrections** to leading order process

Leading order pair creation process

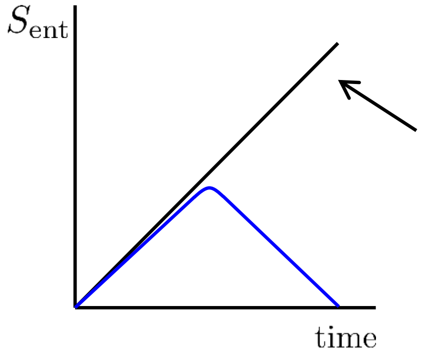
- Model evolution of state:
  - H: Interior
  - b, c: next Hawking pair
  - E: early radiation



$$
\begin{aligned}
|\Psi\rangle &= \sum_i C_i H_i E_i \\
&\to \sum_i C_i H_i \frac{1}{\sqrt{2}}(0_c 0_b + 1_c 1_b) E_i
\end{aligned}
$$

- Entanglement entropy of BH with surroundings increases by ln 2:

$$
S_{BE} = S_E + S_B = S_E + \ln 2
$$



Including small corrections

- Allow small corrections from entire matter inside the black hole
  - Small probability of emitting orthogonal state to Hawking state

$$
\begin{aligned}
S^1 &= \frac{1}{\sqrt{2}}(0_c 0_b + 1_c 1_b) \\
S^2 &= \frac{1}{\sqrt{2}}(0_c 0_b - 1_c 1_b)
\end{aligned}
$$

- Assume that Hawking quanta which have already left do not affect process
  - (similar to how a piece of paper burns).

$$H_i \quad \to \quad H_i^1 S^1 + H_i^2 S^2$$

$$
\begin{aligned}
|\Psi\rangle &= \sum_i C_i H_i E_i \\
&\to \sum_i C_i (H_i^1 S^1 + H_i^2 S^2) E_i \\
&= S^1 \Lambda^1 + S^2 \Lambda^2 \,,
\end{aligned}
\qquad ||\Lambda^2|| < \epsilon \ll 1
$$

- Use strong subadditivity inequality:

$$
\begin{aligned}
S_{BC} &< \epsilon \\
S_B \,, S_C &> \ln 2 - \epsilon \\
S_{BE} + S_{BC} &\geq S_E + S_C
\end{aligned}
$$

$$\boxed{S_{BE} \quad > \quad S_E + \ln 2 - 2\epsilon}$$