KITP 2013

1) An alternative AMPS experiment which evades Harlow-Hayden (and produces flat mirrors)

2) Generalisations of Q.M. where information behaves differently (purifications, entropy bounds, hiding)

3) Fundamental destruction of information which respects conservation laws

J. Oppenheim (University College London)

Interdisciplinary Postdocs available!

1) An alternative AMPS experiment

Jan 2013

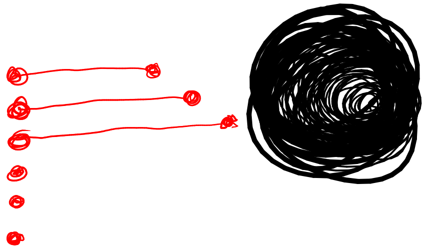
Recall Harlow-Hayden

$$|\Psi_{AMPS}\rangle = \frac{1}{\sqrt{|B||H|}} \sum_{b,h} |bh\rangle_{BH}\, U_R |bh0\rangle_R$$

$$U_R^\dagger |\Psi_{AMPS}\rangle = \underbrace{\left(\frac{1}{\sqrt{|B|}} \sum_b |b\rangle_B |b\rangle_{R_B}\right)}_{\text{woohoo!}} \otimes \left(\frac{1}{\sqrt{H}} \sum |h\rangle_H |h0\rangle_{R_H}\right)$$

takes too long

But an alternative experiment yields unscrambled, "paired" Hawking radiation

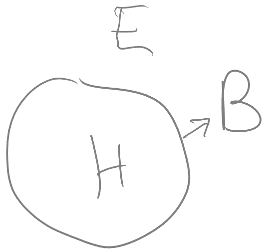


An entangled black hole

$$|\psi(0)\rangle_{MEBH} = \frac{1}{\sqrt{|B||H|}}\sum_{b,h} W_{BH}\,\underbrace{|bh\rangle_{BH}}_{\text{inside}}|0\rangle_E\,\underbrace{|b\rangle_{M_B}|h\rangle_{M_H}}_{\text{quantum memory}}$$

Let it evaporate a bit

$$|\psi(t)\rangle_{MEBH} = \frac{1}{\sqrt{|B||H|}}\sum_{b,h} U(t)_{BHE}\,W_{BH}\,|bh0\rangle_{BHE}|b\rangle_{M_B}|h\rangle_{M_H}$$



$$\rho_{HEM_B} \simeq \rho_{HE}\otimes\rho_{M_B} \rightarrow |\psi\rangle_{HE\bar{M}_H}\otimes|\psi\rangle_{\bar{B}M_B} = V_{BM_H}|\psi(t)\rangle_{MEBH}$$

Decoupling

B.H. as mirror

also takes too long ☹

But also

$$\rho_{HEB} \simeq \rho_{HE}\otimes\rho_{B} \rightarrow |\psi\rangle_{HE\bar{M}_H}\otimes|\psi\rangle_{B\bar{m}_B} = V_{M_BM_H}|\psi(t)\rangle_{MEBH}$$

Pre-computing the decoding

Since $V_{m_B M_H}$ acts only on the memory we can act it on the memory before we create the black-hole!

maximally entangled memory

$$\frac{1}{\sqrt{|B||H|}}\sum_{b,h}|bh\rangle_{m_B m_H}\,|bh\rangle_{N_B N_H}$$

pre compute decoding map

$$\frac{1}{\sqrt{|B||H|}}\sum_{b,h}V_{m_B m_H}|bh\rangle_{m_B m_H}\,|bh\rangle_{N_B N_H}$$

transfer 2nd register to heavy stuff

$$\frac{1}{\sqrt{|B||H|}}\sum_{b,h}V_{m_B m_H}|bh\rangle_{m_B m_H}\,|bh0\rangle_{\text{stuff}}$$

form black hole w/ stuff

$$\frac{1}{\sqrt{|B||H|}}\sum_{b,h}V_{m_B m_H}|bh\rangle_{m_B m_H}\,W_{BHE}|bh0\rangle_{BHE}$$

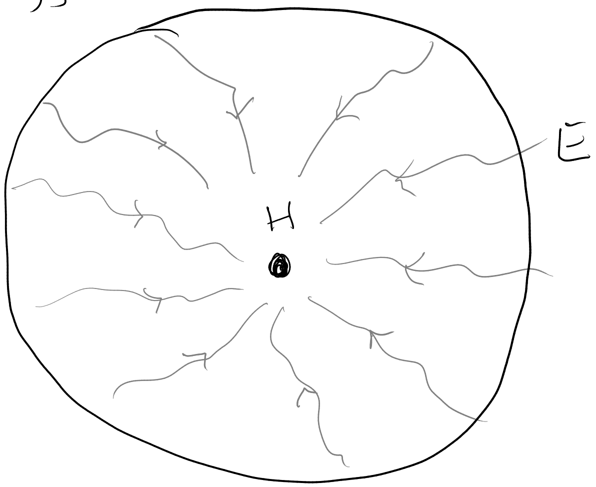
black hole evaporation is "paired" with memory

$$\left(\frac{1}{\sqrt{|B|}}\sum_{b}|b\rangle_{m_B}|b\rangle_{B}\right)\otimes\left(\frac{1}{\sqrt{|H|}}\sum|h\rangle_{m_H}|\Psi_h\rangle_{HE}\right)$$

Forming the entangled black hole

$$|bh0\rangle_{stuff} \longrightarrow |bh0\rangle_{BHE} \quad \text{or} \quad |bh0\rangle_{stuff} \longrightarrow |\Psi^{e}_{bh}\rangle_{BHE}$$

①

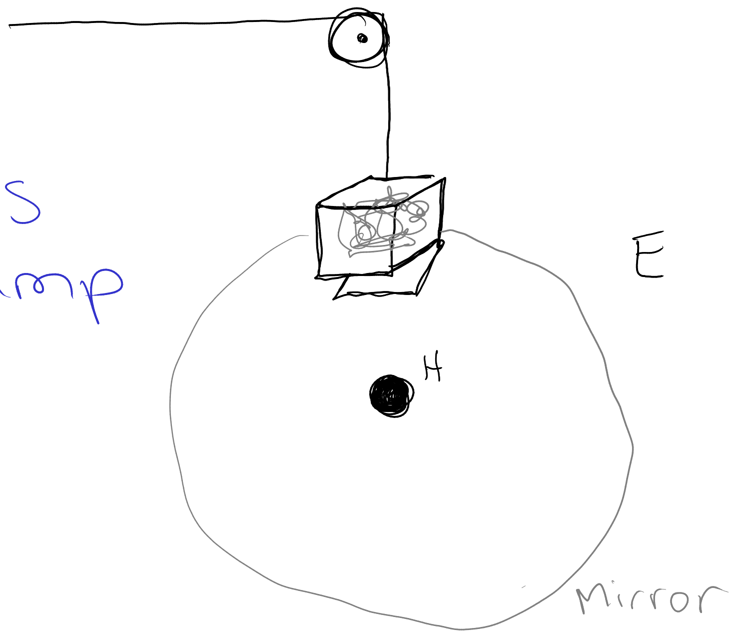


its okay if some stuff gets lost

$$S^{Bekenstein}_{H_0} = S^{Entanglement}_{H_0}$$

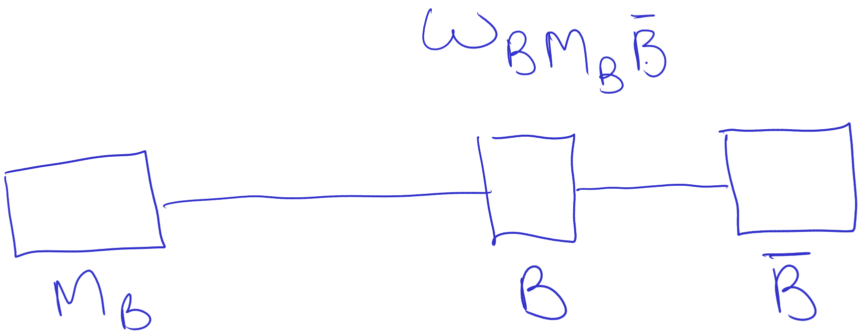
②

Bill and Bob's Garbage dump

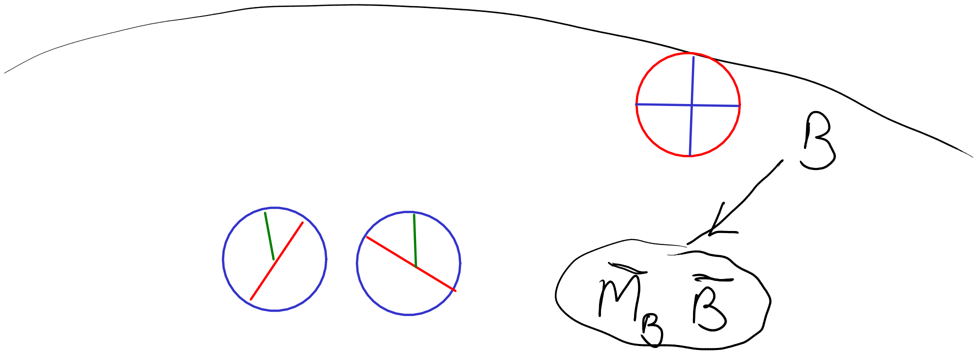


What should Alice measure?

P.O.B.S.

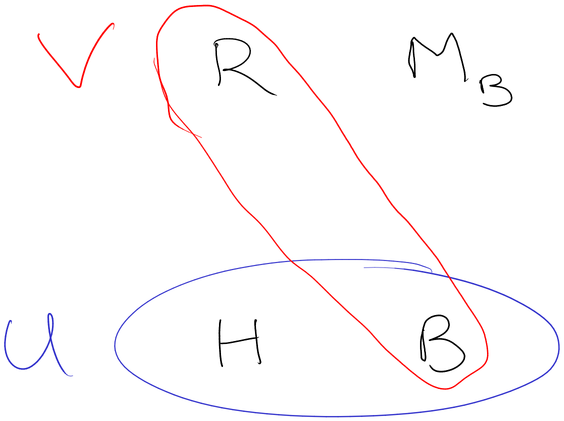


A Bell violation of $B\bar{B}$ and BM_B implies superluminal signalling (eg monogomy of CHSH)



The funhouse mirror

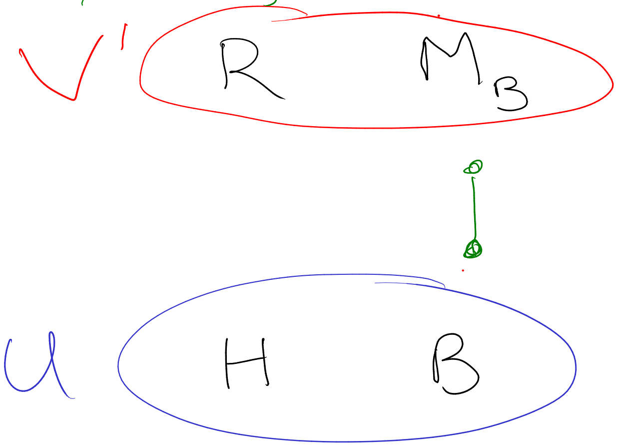
(Hayden Preskill)



$$\rho_{HM_B} \simeq \rho_H \otimes \rho_{M_B}$$

$$V_{RB}\Psi = \Psi_{H\tilde{R}} \otimes \Psi_{M_B\bar{B}}$$

A flat mirror



$$\rho_{HB} \simeq \rho_H \otimes \rho_B$$

$$V_{RM_B}\Psi = \Psi_{H\tilde{R}} \otimes \Psi_{\overline{M_B}B}$$

Parts (2) and (3)

Modifications of Q.M. motivated by AMPS and the black hole information problem

1206.5030
JHEP 9, 16 (2012)

Generalising the state space
w/ M. Mueller and O. Dahlsten

0902.2361

Fundamental information Destruction w/ B. Reznik

Black holes are a lens for quantum gravity, quantum mechanics

But if this is to be the case, we ought to consider theories beyond quantum theory

What types of evolution laws and states are allowed by nature?

Summary of Results

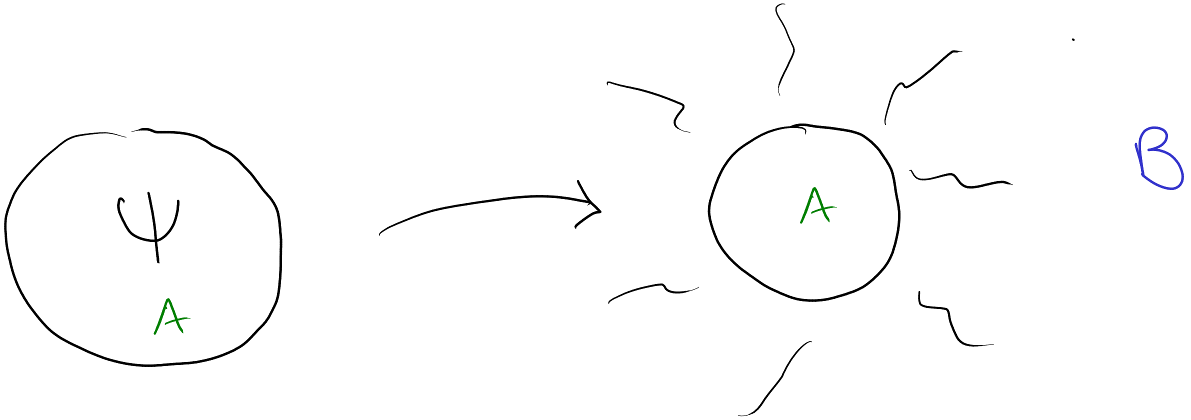
Black holes are a lens

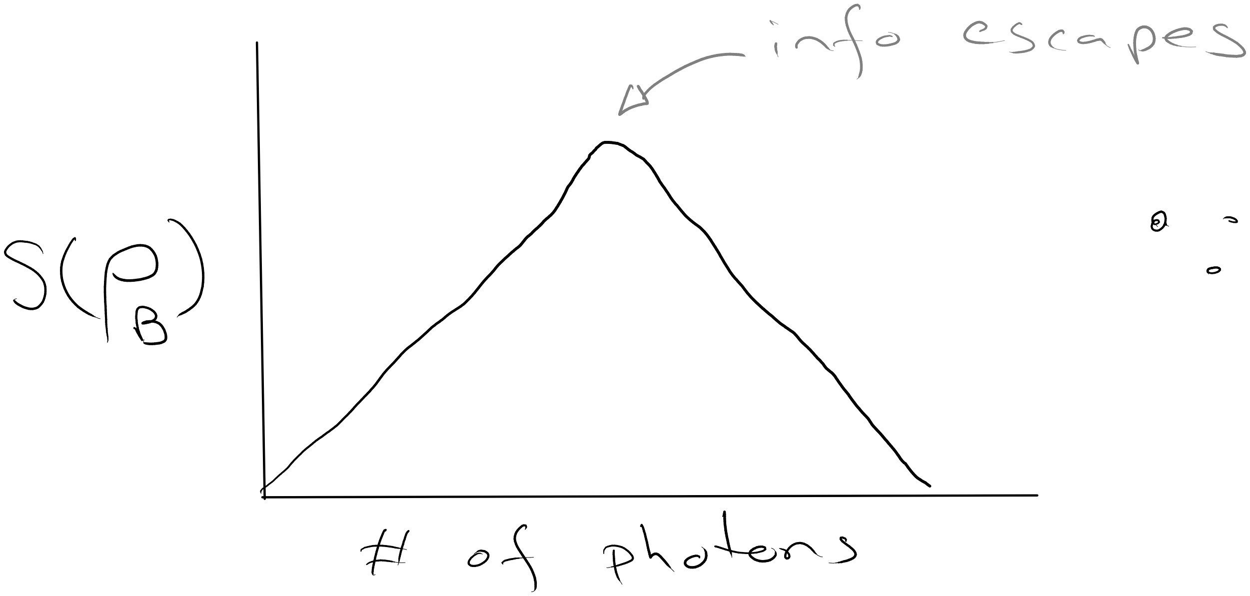
- some generalisations of Q.M. allow us to evade AMPS through information hiding. Perhaps even some versions of monogamy?
- a decoupling theorem which determines how fast info can come out of a black hole under the assumption of reversibility
- we can have generalised evolution laws which solve BH info problem through information destruction while still respecting conservation laws

Part 2: Lets assume information is preserved

How fast does info come out the b.h.?

Page (PRL, 1980)



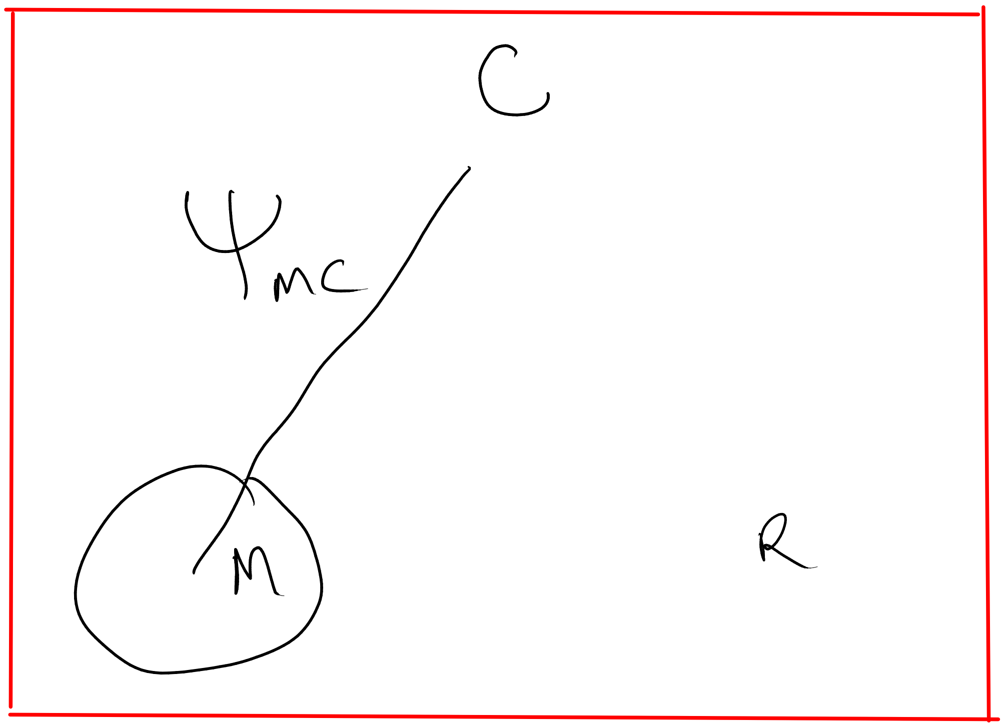


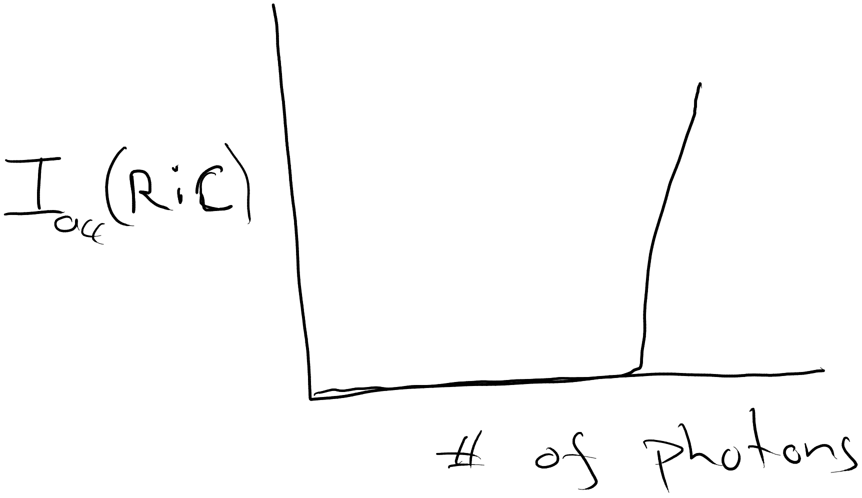
$$\therefore S(\rho_A) = S(\rho_B)$$

$$S = -\mathrm{tr}\,\rho \log \rho$$

3 insights from Quantum Information

1) Classical information generically comes out only at the very end

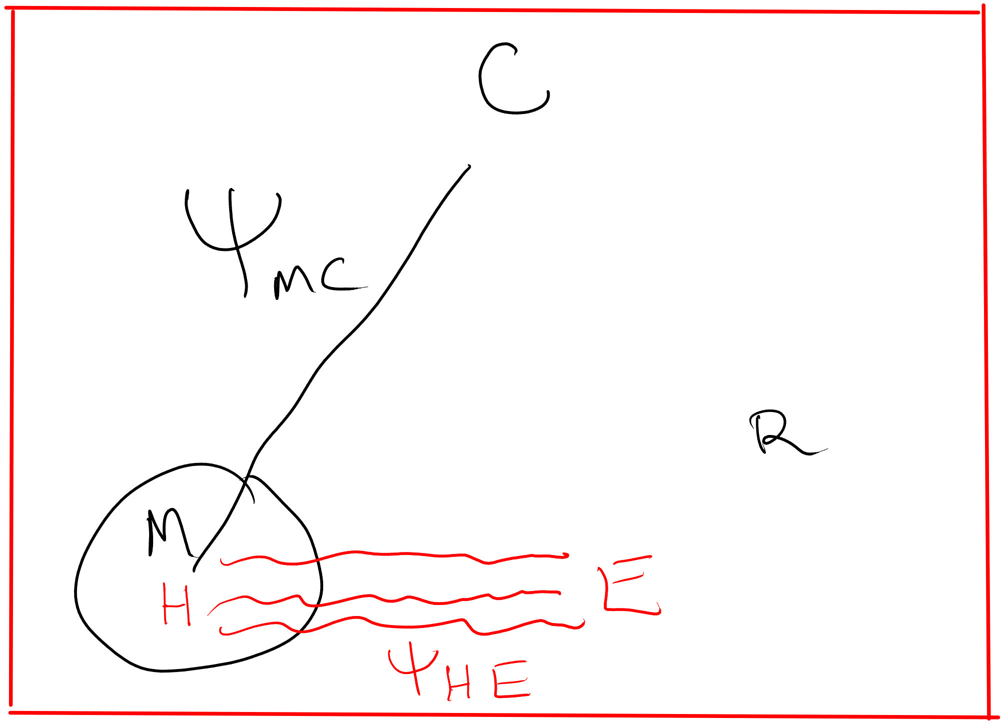


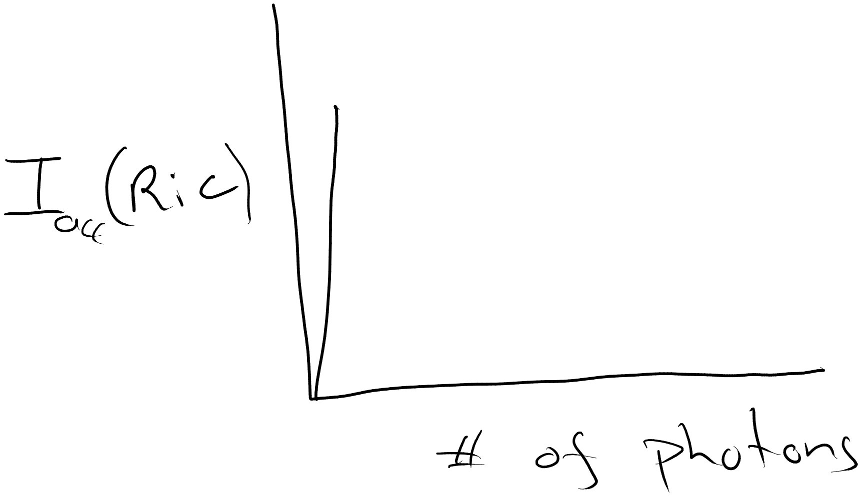


$$\Psi_{MC} = \sum |x\rangle\langle x|_M \otimes |x\rangle\langle x|_C$$

Smolin, J.O.
(PRL, 2005)

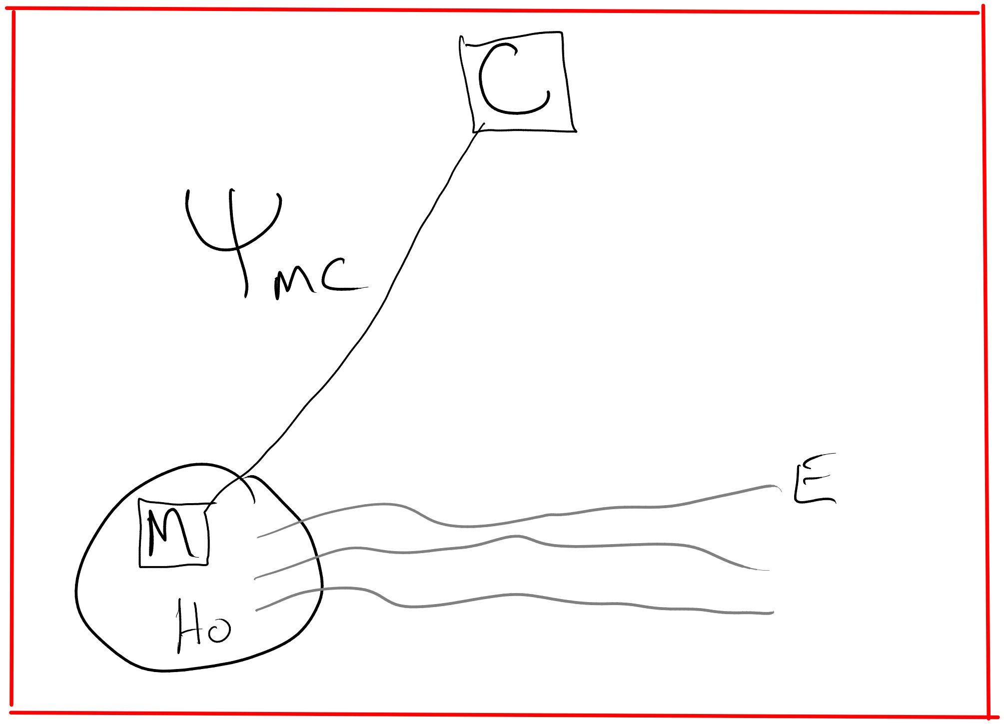
2) For entangled black holes information comes out almost instantly

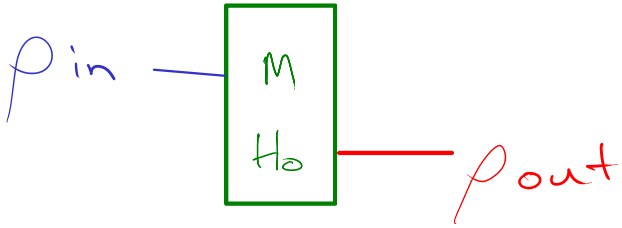




Hayden, Preskill
(JHEP, 2007)

3) No hiding theorem (decoupling)





if $\rho_{in,c} \simeq \rho_{in} \otimes \rho_c$ i.e. no info in B.H.

then $\rho_{EC,out} \simeq \mu_{out'} \otimes \Psi_{MC}$ i.e. all info is outside

Braunstein, Pati (PRL, 2007)

Horodecki, J.O., Winter (Nature, 2005)

Generalisations of quantum theory

1) **transitivity**: for every state φ, there exists a reversible transformation T such that $\varphi = T\omega$ (ω a standard state)

2) **Classical states** are contained within our state space

3) If K is the number of parameters needed to describe a system, then for two systems A, B $K_{AB} \le K_A K_B$

4) **Convexity**: if ω_1, ω_2 are states then so is $p\omega_1 + (1-p)\omega_2$ $0 \le p \le 1$ (same w/ measurements)

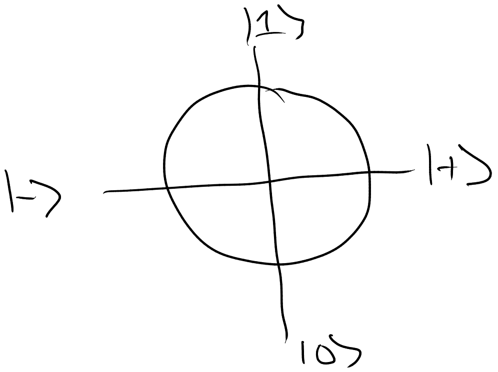
N.B. N is the number of states we can distinguish with a single measurement

$K = N^r$ (Hardy - Wootters)

Examples

qubit

$$\rho = \frac{1}{2}\mathbb{1} + \vec{n}\cdot\vec{\sigma}$$



$K = 4$

$N = 2$

$K = N^2$

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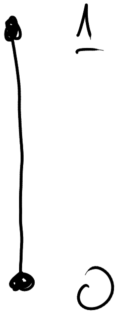
bit

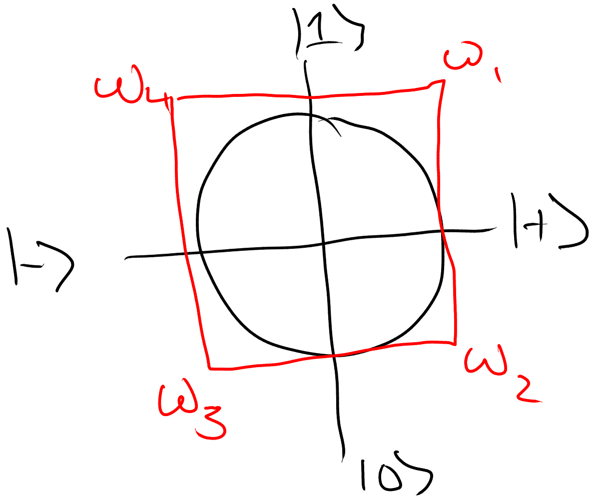


$K = 2$

$N = 2$

$K = N$

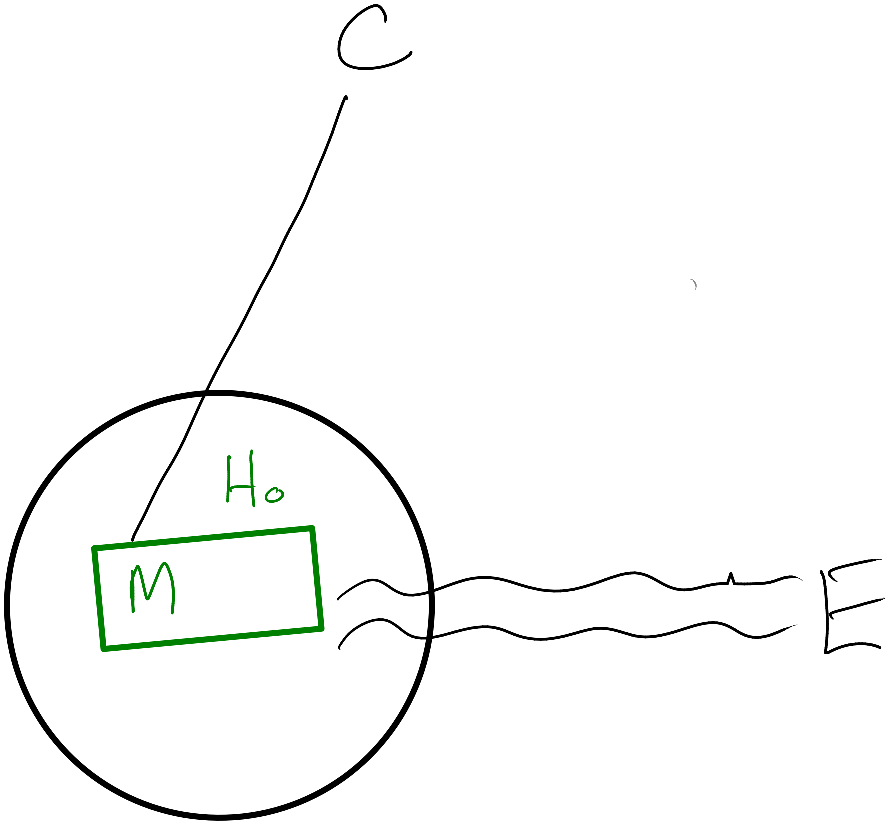
Eg. gbit

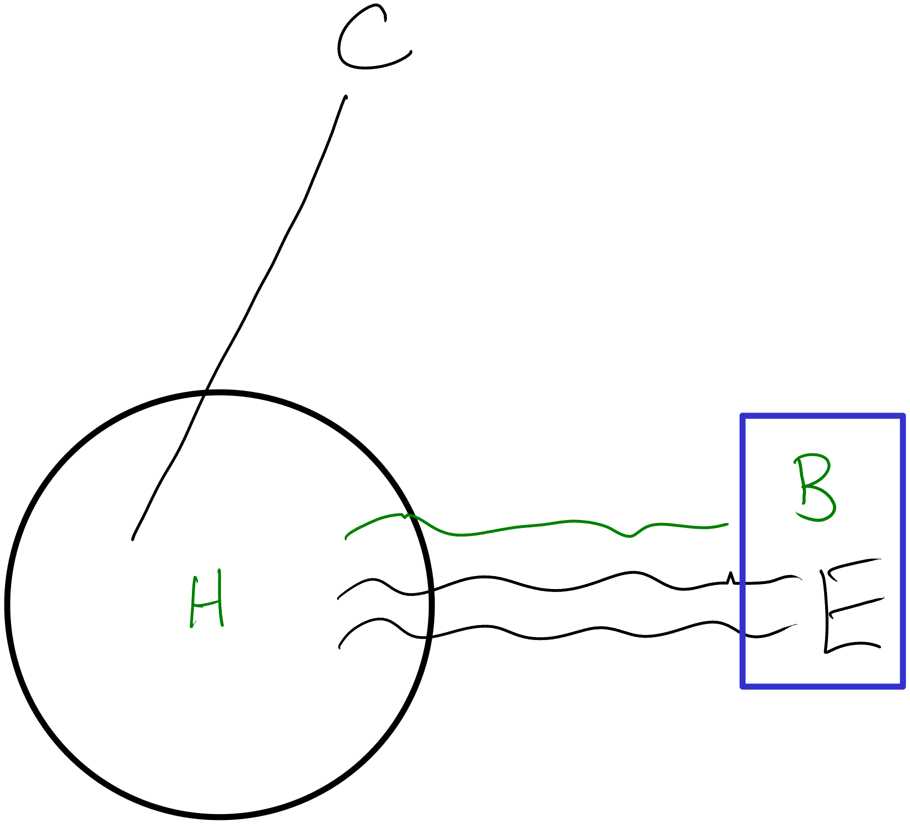


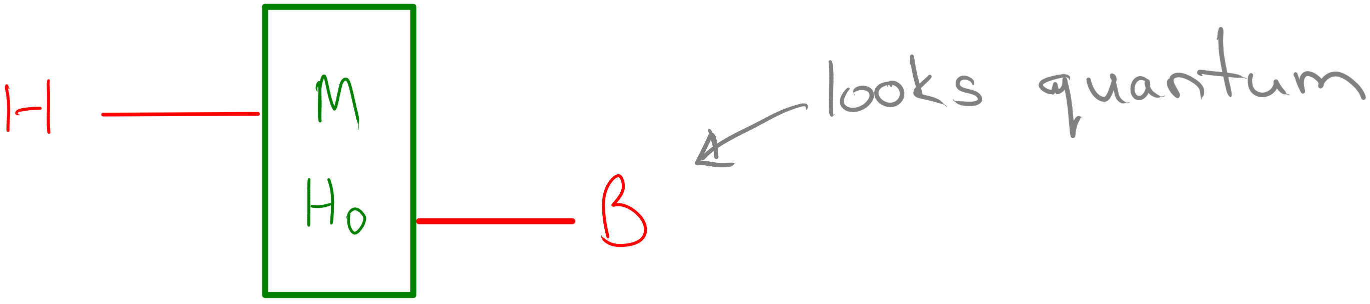
$K = 3$
$N = 2$

2 gbits (PR box)
can maximally violate CHSH

Physical Set-up







looks quantum

General Decoupling Theorem

The rate at which information leaves a black hole depends on N, K.

$$\int_{U\in G} \left\| \sigma(U)^{CH} - \psi^{C} \otimes \mu^{H} \right\|_1^2 \le P\Big(\underset{\underset{\text{purity}}{\uparrow}}{\psi}^{CH_0M}\Big) \frac{N_C N_{H_0M}}{K_H}$$

We recover the original Hayden-Preskill result

$$\boxed{\log N_B \gg \frac{2}{r} \log N_M}$$

in Q.M. $r=2$, and take M to be k qubits

so B just needs to be $k+c$ qubits, w/ c small

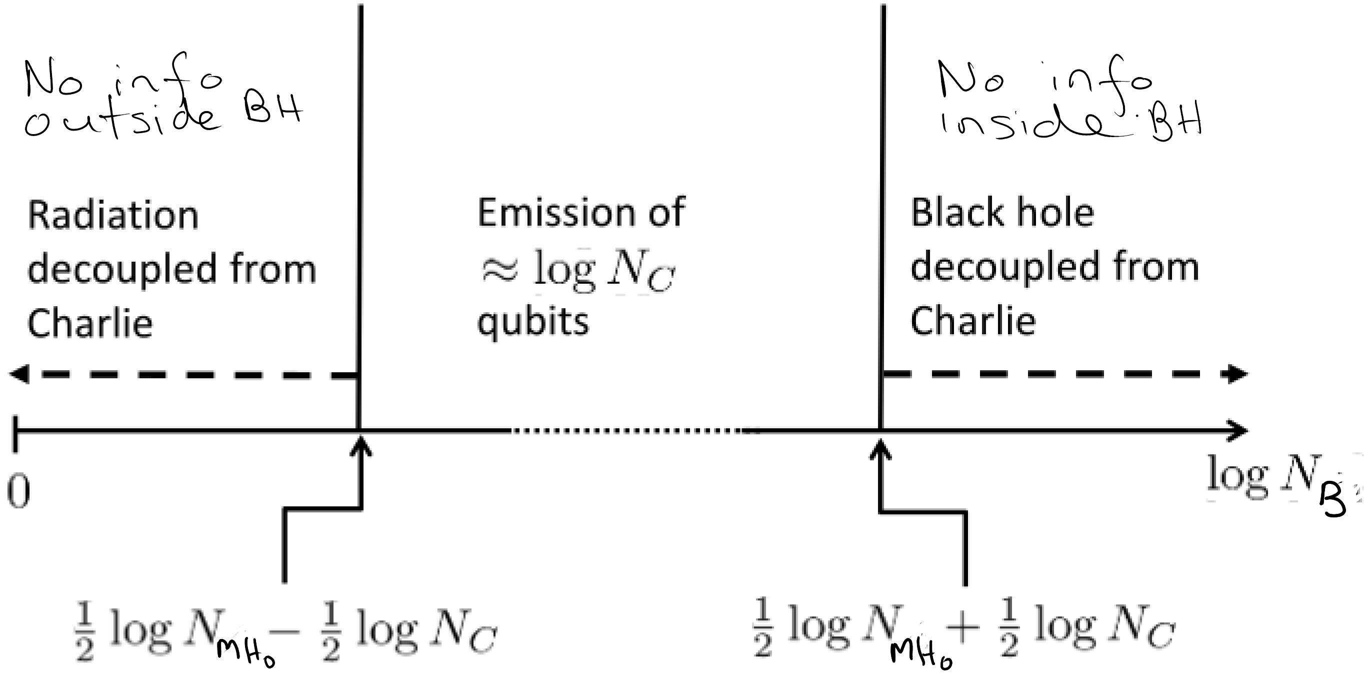
$$2^{k+c} \gg 2^k$$

For theories with r large, the information can leave the black hole even if the number of radiated bits is less than the amount of information inside!!

How can this happen?

Just as we used our decoupling theorem to tell when H was decoupled from Charlie, we can use it to tell when B (radiation) is.

Decoupling when $K = N^2$ (Quantum case)



No info outside BH

Radiation decoupled from Charlie

Emission of $\approx \log N_C$ qubits

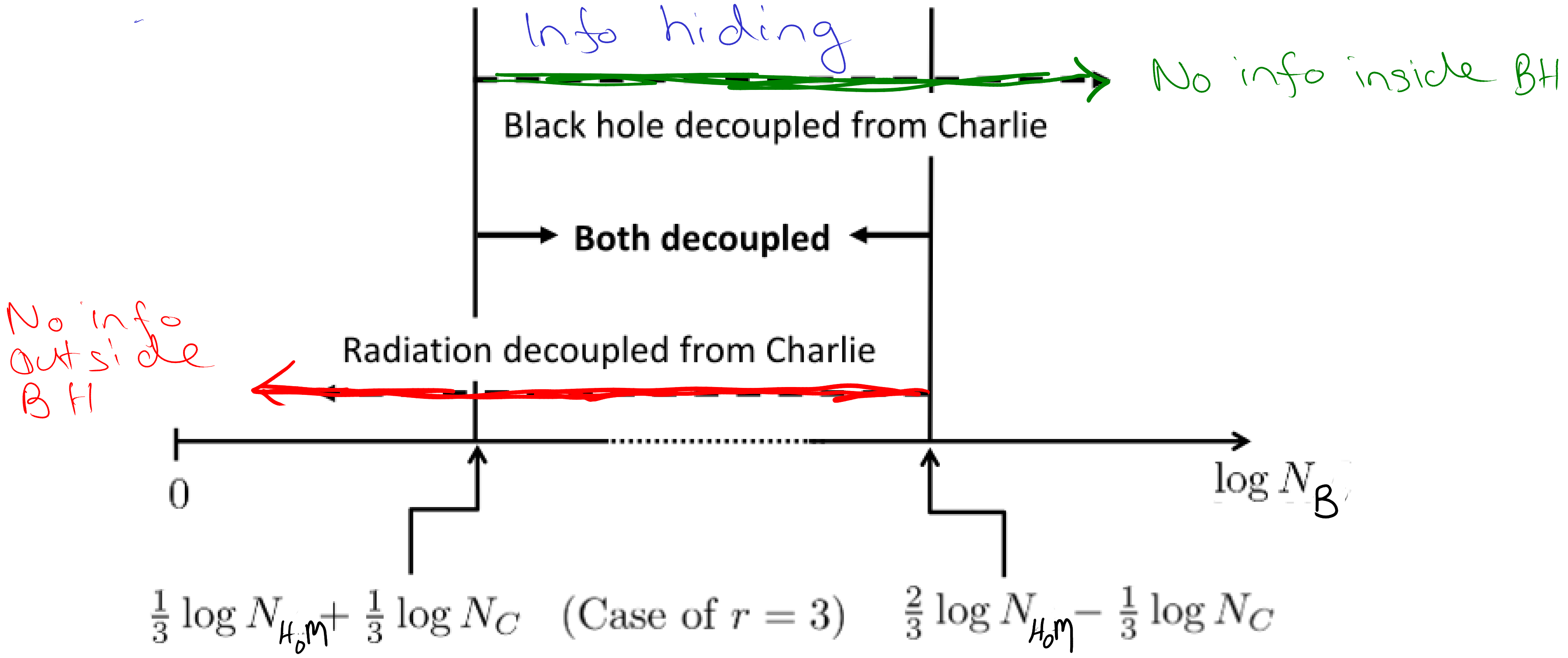
No info inside BH

Black hole decoupled from Charlie

(no hiding)

But more generally, we can have hiding

Both decoupled for $K = N^r$, $r \geq 3$



Info hiding

No info inside BH

Black hole decoupled from Charlie

Both decoupled

No info outside BH

Radiation decoupled from Charlie

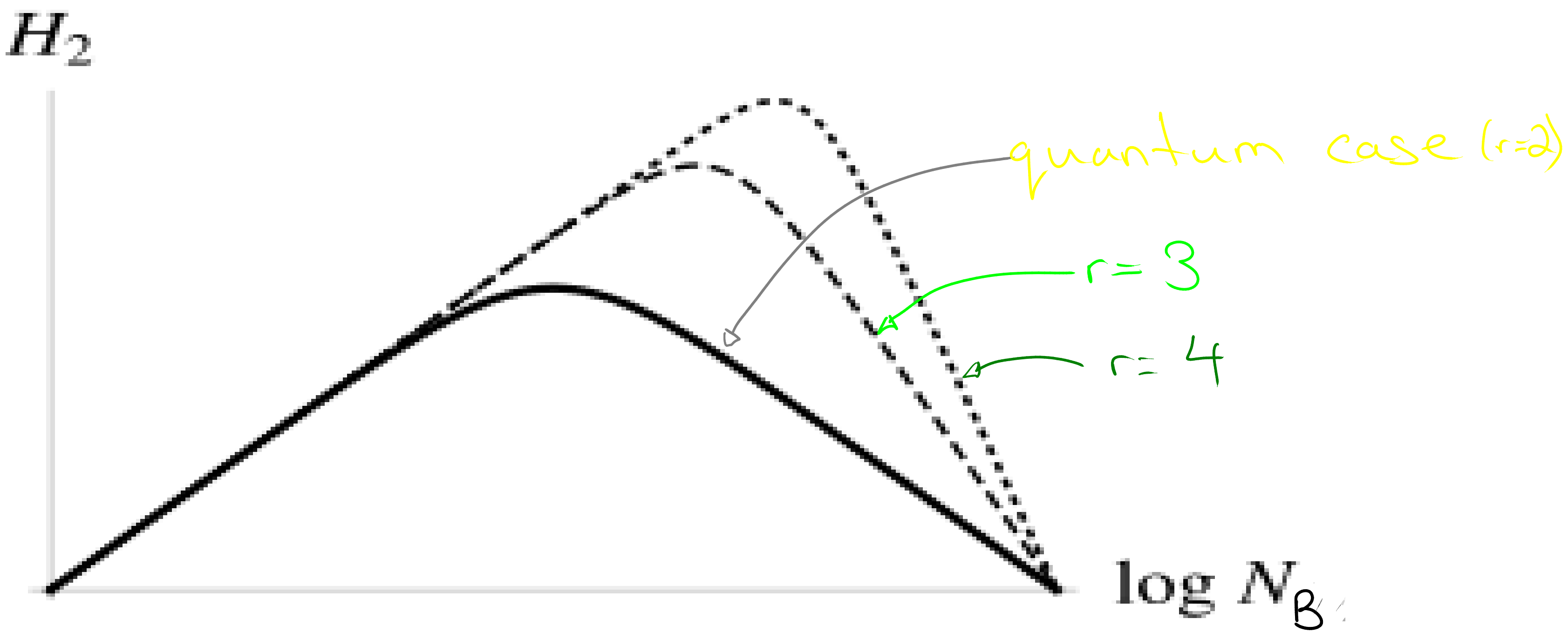
0

$\log N_B$

$\frac{1}{3}\log N_{H_oM} + \frac{1}{3}\log N_C$ (Case of $r = 3$) $\frac{2}{3}\log N_{H_oM} - \frac{1}{3}\log N_C$

Page Scenario

(black hole initially a pure state)



So by making r large enough we can satisfy both Hawkings calculation until the Plank scale, and "unitarity".

Summary (if we assume reversibility)

- generelazations of QM, with different relations between N, K
- information flows differently
- small systems can purify large ones and respect entropy bounds

apparent contradiction

- information leaves the black hole faster
- but takes a longer time to arrive outside the black hole

- Can we have polyamourous entanglement?

Not in the sense of Bell violations but perhaps in the AMPS sense

Now for Part 3

Information destruction respecting conservation laws through a "relational" theory

0902.2361

Quantum mechanics and QFT are unitary. Pure states evolve into pure states.

$$\dot{\rho}(t) = -i[H, \rho]$$

But does it have to be this way?
Are there more general evolution laws?

① That which is not forbidden is required
② Black hole evaporation
③ Quantum measurement: "which branch"
④ Open quantum systems

Can evolution be non-unitary?

Yes!

① That which is not forbidden is required
② Black hole evaporation
③ Quantum measurement: "which branch"

No!

① God is not a gambler.
② AdS/CFT or Boundary Unitarity
③ Banks, Peskin and Susskind (84)

You must either violate locality or conservation laws.

Outline

- non-unitary theories
- the objection of BPS
- A way out?: relational theories
    - QM
    - QFT
    - Energy vs. others
    - Notions of locality
- Noether's Theorem
- Coupling to gravity
- Lorentz invariance
- Open questions

What evolution laws are permissable given the standard Hilbert space structur

① Density matrix should evolve to another density matrix

- trace preserving
- positive or completely positive

② Respect statistical interpretation of the density matrix

- linear

$$\because \quad \mathcal{L}(p\,\rho_1 + (1-p)\rho_2) = p\,\mathcal{L}(\rho_1) + (1-p)\,\mathcal{L}(\rho_2)$$

Lindblad Equation

Kossakowski, Gorini, Sudarshan 76

$$\dot{\rho} = \mathcal{L}\rho = -i[H,\rho] - \frac{1}{2}\sum_{k}\gamma_k\left(L_k^\dagger L_k \rho + \rho L_k^\dagger L_k - 2 L_k \rho L_k^\dagger\right)$$

$\mathcal{L}$ is the most general form of a semi-group generator if k is countable and $\mathcal{L}$ bounded.

Recall: A semi-group is a continuous, one parameter family of CPT maps $\Lambda(t)$ which are Markovian

$$\Lambda(t_1)\Lambda(t_2) = \Lambda(t_1 + t_2)$$

Pure Decoherence

Eg. $L_k = P_k$ a projector

$H = 0$ $\gamma_k = \gamma$

$$\boxed{\dot{\rho} = -\gamma\rho + \gamma\sum P_k \rho P_k}$$

$$\boxed{\rho(t) = \begin{pmatrix} p_1 & \sigma e^{-\gamma t} \\ \sigma e^{-\gamma t} & p_2 \end{pmatrix}}$$

$$\rho = \sum \sigma_{ij} |i\rangle\langle j|$$

$$\sigma_{ij}(t) = \begin{cases} \sigma_{ij}(0) & \text{for } i=j \\ e^{-\gamma t}\sigma_{ij}(0) & \text{for } i \neq j \end{cases}$$

Like a measurement in P_k basis

$$[|x_k\rangle\langle x_k|, P] \neq 0$$

Hawking 82

$$\dot{\rho} = -i\left[H_0,\rho\right] - \frac{a}{2m_p^4}\int d^3x\,\left[F^{\mu\nu}F_{\mu\nu}(x),\left[F^{\mu\nu}F_{\mu\nu}(x),\rho\right]\right]$$

- a local theory $L_x = F^{\mu\nu}(x)F_{\mu\nu}(x)$
- produces infinite momentum

Banks, Susskind, Peskin 84

- all theories suffer from this tension
- you can only decohere into boring observables (total momentum)

Possible routes

- Introduce an energy scale and hide violations of momentum/energy there

  Unruh and Wald 95

  Poulin and Preskill (in prep)

- Introduce memory effects, non-Markovian theory

  (non-locality in time)

Quantum Mechanics

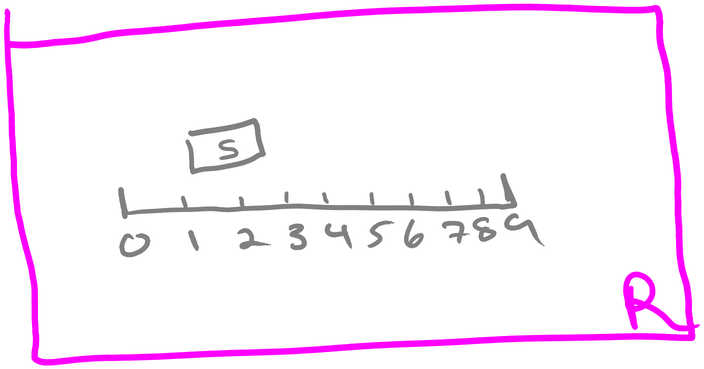
$$\dot{G} = i[H,G] - \frac{1}{2}\sum_k \gamma_k \left(L_k^\dagger L_k G + G L_k^\dagger L_k - 2 L_k^\dagger G L\right)$$

If $L_k = |x_k\rangle\langle x_k|$ (local) $\dot{P} \neq 0$

$[P, L_k] \neq 0$ momentum not conserved

Measuring X is impossible, it disturbs the momentum and breaks momentum conservation.

Measuring X is impossible, it disturbs the momentum and breaks momentum conservation.



$$[X_S, P_S] \neq 0$$

$$[X_R - X_S, P_R + P_S] = 0$$

(eg Aharonov, Susskind 67)

A relational theory

~~QM~~

$$L = |x\rangle\langle x| \longrightarrow Q = \int_{-\infty}^{\infty} |x\rangle_{1\,1}\langle x| \otimes |x\rangle_{2\,2}\langle x| \, dx$$

A coincidence detector

$$e^{-iPx} Q e^{iPx} = Q$$

$$[P_{total}, Q] = 0$$

$$Q = \int e^{-iPx} L e^{iPx} dx$$

$$\dot{\rho}_{12} = -i[H, \rho_{12}] - \frac{\gamma}{2}[Q, [Q, \rho_{12}]]$$

"local"

Hawking 82

$$\mathring{\rho} = -i[H_0,\rho] - \frac{a}{2m_p^4}\int d^3x \left[F^{\mu\nu}F_{\mu\nu}(x), [F^{\mu\nu}F_{\mu\nu}(x),\rho]\right]$$

$$L_x = F^{\mu\nu}F_{\mu\nu}(x)$$

Instead

$$Q = \int d^3x\, F^{\mu\nu}F_{\mu\nu}(x) \otimes |x\rangle\langle x|$$

- conserves momentum
- only acts at position $|x\rangle\langle x|$

QFT

- add extra fields to make the relational operators interesting

- take local Hermitian operators

$$[A(\bar{x})\ L_k(\bar{y})] \propto \delta(\bar{x}-\bar{y})$$

- "twirl": $Q_k = \int d\bar{x}\, L_k(\bar{x})$

momentum conserved

$$\begin{cases} [P_{tot}, Q_k] = 0 \\ e^{-iP_{tot}x} Q_k e^{iP_{tot}x} = Q_k \end{cases}$$

local:

$$\dot{A}(x) = i[H, A(x)] - \frac{1}{2}\sum_{ij} \gamma_{ij} [L_i(x), [L_j(x), A(x)]]$$

Examples

QM

$$Q = \int |x\rangle_1 {}_1\langle x| \otimes |x\rangle_2 {}_2\langle x| \, dx$$

QM, QFT

$$Q_k = \int L_k^{(1)}(x) \otimes |x\rangle_2 {}_2\langle x| \, dx$$

QFT

$$Q_k = \int L_k^{(1)}(x) \otimes L_k^{(2)}(x) \, dx$$

$$\psi(x) = \phi(x) + i\pi(x)$$

$$|1_x\rangle = \psi^\dagger(x)|0\rangle$$

$$L^{(1)}(x) = N(x) \equiv \psi^\dagger(x)\psi(x)$$

Locality in more detail

what do we mean by locality??

① Causality $[A(x), B(y)] = \Delta(x-y)$

$\Delta(x-y) = 0$ for $(x-y)^2 < 0$

② Non-local correlations

$\frac{dA(\bar{x})}{dt} = f(\bar{x})$ but...

$$\frac{d\,A(\bar{x})\,B(\bar{y})}{dt} \overset{?}{=} \frac{dA(\bar{x})}{dt} B(\bar{y}) + A(\bar{x}) \frac{dB(\bar{y})}{dt}$$

Causality

Causality is proven by adding a fictitious but relativistic environment and tracing it out to go from a unitary theory U_{SE} to a Lindblad equation $\mathcal{L}_S$

The environment

- infinite spatial correlalation
- infinitely many fields
- no transfer of energy/momentum etc
- gives Hermitian Q unless complex, in which case $Q, Q^\dagger$ occur in pairs

② Non-local creation/destruction of correlations

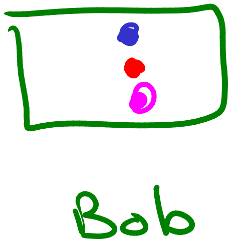
For $\frac{d}{dt}A(x)B(y) = i[H, A(x)B(y)]$ $\quad H = \int d\bar{x}\, \mathcal{H}(x)$

$$\frac{d}{dt}A(x)B(y) = \dot{A}B + A\dot{B}$$ but ....

For $Q_k = \int dx\, L_k(x)$ Hermitian

$$\frac{d\,A(\bar{x})\,B(\bar{y})}{dt} = \frac{dA(\bar{x})}{dt}B(\bar{y}) + A(\bar{x})\frac{dB(\bar{y})}{dt} + V\left(A(\bar{x})\,B(\bar{y})\right)$$

$$V\left(A(\bar{x})B(\bar{y})\right) = -\frac{1}{2}\sum \gamma_{ij}\left[L_i(\bar{x}), A(\bar{x})\right]\left[L_j(\bar{y}), B(\bar{y})\right]$$



CMB?

Relationalism in the extreme

$V(A \otimes B)$ reflects the fact that there may be no way to distinguish x from y if all field values at x and y are the same

$Q = \int L(x)\, |x\rangle\langle x|\, dx$ doesn't create or destroy correlations at space-like seperated points.

In general, for all generators of a symmetry G_θ

$$Q_k = \int \prod_\theta d\alpha_\theta \, e^{-iG_\theta \alpha_\theta} L_k e^{iG_\theta \alpha_\theta}$$

But not for time translation.

① Time translation no longer generated by H. Evolution is via the Lindblad equation. No Noether's theorem

② $$[L(\bar{x},t), L(\bar{x}',t)] = 0$$

but $$[L(\bar{x},t), L(\bar{x},t')] \neq 0$$

$\therefore$ $Q = \int e^{-iHt} L e^{iHt}$ is not local

③ t appears on LHS of Lindblad eqn

$$\frac{d\rho}{dt} = -i[H,\rho] - \frac{1}{2}\sum \gamma_i [Q_i, [Q_i, \rho]]$$

Energy conservation

① choose a physical clock τ

$$H = H_0 + \Pi_\tau$$

② impose time translation invariance

$$\frac{d\rho}{dt} = 0$$

③ $\bar{Q}_k = \int e^{-iHt} Q_k \otimes |0\rangle\langle 0| e^{iHt} \, dt$

$$= \int dt \; Q_k(t) \otimes |t\rangle\langle t|$$

$[H, \bar{Q}_k] = 0$

$$i[\Pi_\tau, \rho] = -i[H_0, \rho] - \frac{1}{2}\sum \gamma_k [\bar{Q}_k, [\bar{Q}_k, \rho]]$$

$$\frac{d\rho}{d\tau} = -i[H_0, \rho] - \frac{1}{2}\sum \gamma_k [\bar{Q}_k, [\bar{Q}_k, \rho]]$$

Lorentz Invariant?

Space (∂_x) & time (∂_τ) not on equal footing

$$\partial_\mu \phi(x) = -i[\phi(x), P_\mu] + D_\mu(\phi)$$

$$D_\mu = -\frac{1}{2}\gamma_\mu[\bar{Q},[\bar{Q},\phi]]$$

then project along ∂_τ

$$\partial_\tau \to \partial_\tau - D_\tau$$

while

$$\partial_x \phi(x) = -i[\phi, P_x]$$

(cf. Srednicki, Alicki et. al.)

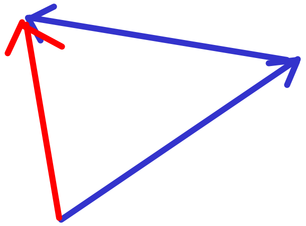
- $\partial_x \phi(x) = -i[\phi, P_x]$ is kinematic identity

$$P_i = \int d_x\, \pi \frac{\partial \phi(x)}{\partial x_i}$$

- $\partial_\tau \phi(x) = -i[\phi(x), P_\tau]$ is dynamical

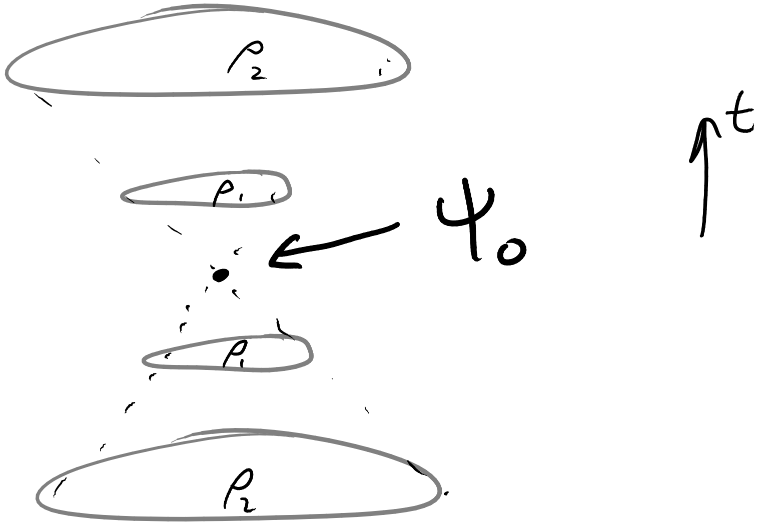
$\therefore$ $\partial_\tau \phi(x) = -i[\phi(x), P_\tau] + D_\tau(\phi)$

But this is funny:



Time Symmetric

Increasing entropy does not give a direction in time



$$\rho(t) = \rho(0) \begin{array}{ll} -\frac{1}{2}\int_0^t \left(L^\dagger L\rho + \rho L^\dagger L - 2L^\dagger \rho L\right) dt & t \geq 0 \\ +\frac{1}{2}\int_0^t \left(L^\dagger L \rho + \rho L^\dagger L - 2L^\dagger \rho L\right) dt & t < 0 \end{array}$$

Noether's Theorem generalised

Symmetry + unitary evolution → conservation

Gross (84) this breaks down in non-unitary theories

$$\mathcal{L}\left(e^{-iG\epsilon}\rho e^{iG\epsilon}\right) = e^{-iG\epsilon}\mathcal{L}(\rho)e^{iG\epsilon}$$

For the Lindblad equation with generators L_i, this implies:

① $[H, G] = 0$

② $[L_i, G] = \Delta_i L_i$

Either L_i is proportional to raising/lowering operators or commutes w/ G

Locality <u>seems</u> to imply either

- L_i are Hermitian : $\dot{G} = 0$

- or come in pairs $L_i, L_i^\dagger$: $\dot{G} = \sum_i \gamma_i \Delta_i [L_i, L_i^\dagger]$

In each case the fictitious environments are very different

Lorentz invariance is made simple through energy-momentum conservation since $Q = \int L(x)\, dx\, dt$ is a scalar

Continuity Equation

$$G = \int dx\, g(x)$$

$$\frac{dg(x)}{dt} = i[H, g(x)]$$

$$\therefore \quad [H, G] = 0 \qquad\qquad i[H, g] = \nabla f^i$$

$$\boxed{\partial_\mu f^\mu = 0} \qquad w/ \quad f^0 \equiv g$$

For Lindblad evolution

$$K(x) \equiv -\frac{1}{2}\sum \gamma_i \left(L_i^\dagger L_i g + g L_i^\dagger L_i - 2 L_i^\dagger g L_i\right)$$

$$\partial_\mu f^\mu = K(x) \qquad\qquad \text{eg. } g^\mu = T^{\mu 0}$$

$$\boxed{T^{\mu\nu}{}_{;\nu}(x) = K^\mu(x)}$$

→ couple to gravity??

"Coupling" to gravity (toys)

- couple to curvature (more decoherence at high curvature)
- "couple to singularity"

$$Q = \int L(x)\, |\bar{x}\rangle\langle\bar{x}|\, d\bar{x}$$

- decoherence terms act at high energy Black holes as microscopes

Conclusions

- Relational model forced on us to resolve conflict between locality and conservation laws
- Clarify locality: causality vs correlations
- time/energy vs other symmetries/conserved quantities
- generalized Noether's theorem
- Lorentz invariant (weak vs strong)
- Time Symmetric
- resolves many of the problems in collapse models

Some open questions

- couple to gravity?
- modifications to gravity (eg Gauss's law)
- more realistic theories
- are the set of Lindblad operators, observables rich enough?
- local relational theories and indistinguishable particles
- experimental tests eg correlation destruction
- additional constraints
  eg. full information destruction: 2d projectors
- Non-markovian theories.