Quantum Glass

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Many body physics

- Everything on Earth is governed by known laws of physics
- World is complicated − 10^{23} interacting degrees of freedom in macroscopic object
- Two body problem: solved in undergraduate mechanics
- Three body problem − unsolved
- 10^{23} body problem?

Enter statistical physics

- 10^{23} degrees of freedom are *easier* to describe than 3
- Abandon attempt to describe what individual `particles' are doing – develop *statistical description*
- Statistical physics and condensed matter physics

On the centrality of emergence

- `More is different' – PW Anderson, (1972)

- Emergent phenomenon: `a collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart.' – Laughlin, Nobel lecture (1998)

- Examples: superconductors, magnets, crystals…

The ergodic assumption

- 20th century statistical physics assumes *ergodicity* − everything that *can* happen, *does* happen (and is equally likely to happen)

- Characterize system not be detailed configuration (microstate) but by small number of macroscopic properties (e.g. temperature) - `macrostate'

- Notion of *thermal equilibrium* as detailed balance over microstates

- Equilibrium statistical mechanics has a century worth of triumphs to it's credit − but does it describe everything?

Dynamics as the next frontier

- Care not just about what equilibrium state is, but also about whether, how, and how fast we approach it
- A *much* harder problem! (and the key focus of this workshop)
- This talk: a subset of questions pertaining to dynamics

Glass

- Are there systems that *violate* ergodic hypothesis? Which never come to equilibrium

- Yes: e.g. window glass

- This is not a talk about window glass

- This is a talk about its quantum analogs - `quantum glass’

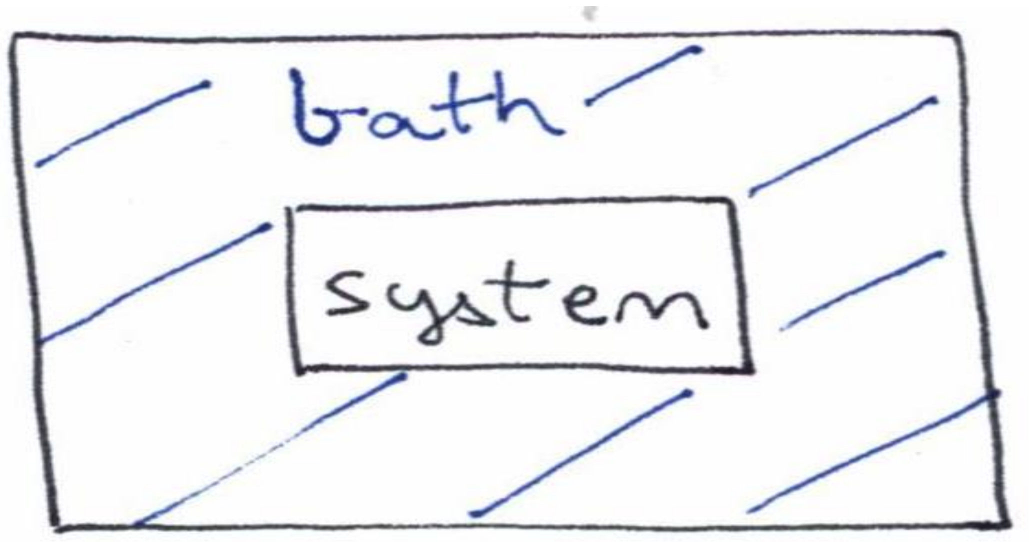
Quantum glass

- The problem: a *many body quantum system* which *violates* ergodic hypothesis and does not come to equilibrium even at infinite times
- Two examples (discussed in this talk)
- Many body localization (glassiness from randomness)
- Fractons (glassiness from constraints)

Many body localization

(subject of KITP program, 2015)

Foundations of quantum statistical mechanics



Equilibrates at long times

Memory of initial conditions is `lost’

Statistical mechanics of isolated quantum many particle systems?

- An important fundamental question
- Relevant for ongoing experiments
  - Cold atoms
  - Trapped ions
  - Cold circuits
  - Solid state
  - …

system

$$i\hbar \frac{d\rho}{dt} = [H, \rho]$$

- Quantum information storage and quantum computation

Quantum mechanics never forgets

- Does the many body dynamics bring the system to equilibrium in the limit of long times?
  - Unitarity: Schrodinger equation preserves all information about initial conditions at all times.

- Information can be rendered inaccessible by `hiding' it in global operators.

- Do accessible (`local') measurements see thermal equilibrium at long times?

Quantum thermalization

- Quantum thermalization is thermalization of subsystems

- Can B act as a reservoir for A?
- Strong numerical evidence that SOME isolated quantum systems do thermalize in this way.

Do ALL systems quantum thermalize?

- No!

- Localized systems have no internal reservoir (Anderson 1958 – Nobel Prize 1977)

- Failure of quantum thermalization

- Memory of the initial conditions preserved in local observables for infinite times.

Many body localization: basic idea

- Particles live in a very non-uniform energy landscape (randomness)
- Hopping particles changes the energy of the system
- Energy must be conserved − so hopping cannot happen
- Particles must remain forever `stuck’ near where they began.

Many body localization

- Conjecture and one body proof: Anderson (1958) (Nobel Prize 1977)
- Lowest order perturbation theory: Fleishman and Anderson (1980)
- All orders in perturbation theory: Gornyi Mirlin Polyakov (2005) and Basko Aleiner Altshuler (2006)

    - Zero conductivity at non-zero energy density (`finite temperature’)

- Numerical verification Huse, Oganesyan, Pal (2007, 2010) (and many others thereafter) – spin chains

$$H = \sum_{i=1}^{L} [h_i \hat{S}_i^z + J \hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1}]$$

- Mathematical Proof (Imbrie 2014)

Thermal phase	Single-particle localized	Many-body localized
Memory of initial conditions 'hidden' in global operators at long times	Some memory of local initial conditions preserved in local observables at long times	Some memory of local initial conditions preserved in local observables at long times.
ETH true	ETH false	ETH false
Non-zero DC conductivity Transport diffusive (or faster)	Zero DC transport	Zero DC transport
Continuous local spectrum	Discrete local spectrum	Discrete local spectrum
Eigenstates with volume-law entanglement	Eigenstates with area-law entanglement	Eigenstates with area-law entanglement
Ballistic spreading of entanglement from non-entangled initial condition	No spreading of entanglement	Logarithmic spreading of entanglement from non-entangled initial condition
Dephasing and dissipation	No dephasing, no dissipation	Dephasing but no dissipation

Table from: RN and Huse, Annual Reviews of Condensed Matter Physics (2015)

A new frontier for quantum stat mech

- Ergodic hypothesis fails

- `Traditional' tools of equilibrium statistical mechanics inapplicable

- Traditional theorems of statistical mechanics inapplicable

- In this regime there can exist entirely new phases of quantum matter (e.g. gapless topological orders, time crystals)…

Many body localization

- Was the subject of a KITP program in 2015
- Anushya Chandran gave chalk talk
- So far, everything I have said could also have been in her talk
- What’s new?

Many body localization (MBL) with long range interactions

- `Traditional’ analysis was for idealized systems where the interactions between degrees of freedom were short ranged in real space.
- `Real’ physics contains long range interactions
  - E.g. Gravitational and electrical interactions follow `inverse square law’ (Gauss law interactions)

  - Can long range interacting systems be MBL?

Conventional wisdom: No

- Start with a system of *non-interacting* particles (one body problem)
- Turn on a *weak* interaction and ask what changes − is the new state of the system `closely connected' to the state in the non-interacting limit?
- This is the strategy that was successfully used to establish localization for short range interacting systems
- For long range interacting systems, this strategy *necessarily* fails, even for infinitesimally weak interactions
- Burin 2006, Yao et al 2014
- Conclusion: no MBL with long range interactions

Problems with conventional wisdom

- Conventional wisdom simply demonstrates a ***breakdown in perturbation theory*** – i.e. we cannot calculate what happens with standard techniques
- Specifically, it demonstrates that interacting system is not in any way `close’ to glassy state that obtains in non-interacting system
- But this does not mean localized system is not glassy!

Recent realization: Long range interacting systems can be MBL

- Basic idea: Start from *strong* interactions, treat them non-perturbatively
- Interactions drive system into *correlated* phase best described in terms of emergent excitations
- If emergent excitations are short range interacting, can MBL
- Can *implement* this program for `Gauss law' interactions (i.e. systems of electric charges) in d=1,2,3, using *confinement* and/or Higgs mechanism
- RN + Sondhi, Phys Rev X 2017

Example 1: Schwinger model (QED_{1+1})

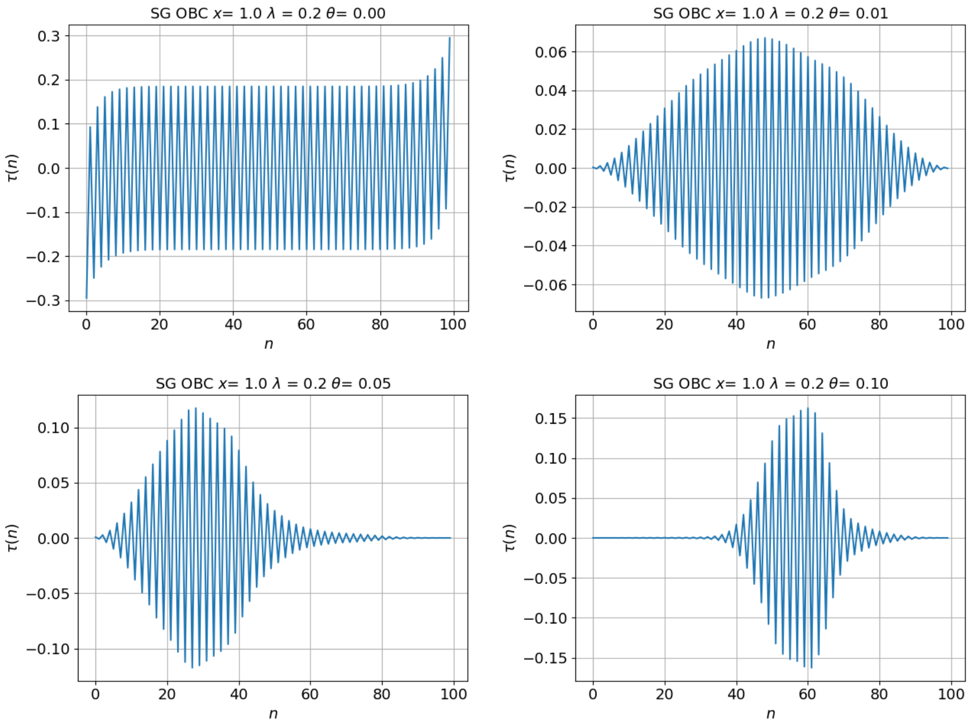
- Electric charges in an imaginary one dimensional world have a `constant force’ interaction (instead of inverse square)
- This problem can be solved *exactly* by a method known as `bosonization’
- With disorder, obtain a localized phase…which can be shown to be stable to perturbations

Intuition for localization in Schwinger model

- Free charge excitations costs infinite energy (*confinement)*
- Finite energy excitations must be charge neutral
- Charge neutral excitations do not have long range interaction, and so must be localized.

Numerical verification

- Akhtar, **RN**, Sondhi, Phys Rev B 2018



2: Localized superconductors in three dimensions

- Need a way to `kill' long range interaction
- Use *superconductivity* − perfectly screens charge
- There can exist `localized superconductors'
- Demonstration relies on description of superconductor as a `Higgs' phase of matter
- Superconductors for electric charge − but insulators for heat, and preserve forever a memory of the initial condition in local observables.
- RN and Sondhi Phys Rev X 2017, Pretko and RN Phys Rev B 2018
- (Weaker versions of these results were also obtained in 1990s by Matthew Fisher and collaborators…at KITP).

Conclusions to this part

- Long range interacting systems can be MBL
- Key ingredient: *strong* interactions, which drive system into correlated phase
- Demonstrated for two examples
  - QED_{1+1} − a confining MBL phase (also a Mott glass)
  - Superconductors (more generally Higgsed phases of gauge theories)
    - SC is compatible with MBL; MBL could even stabilize SC to energy densities where it would not arise in thermal eqm

Many body localization summary

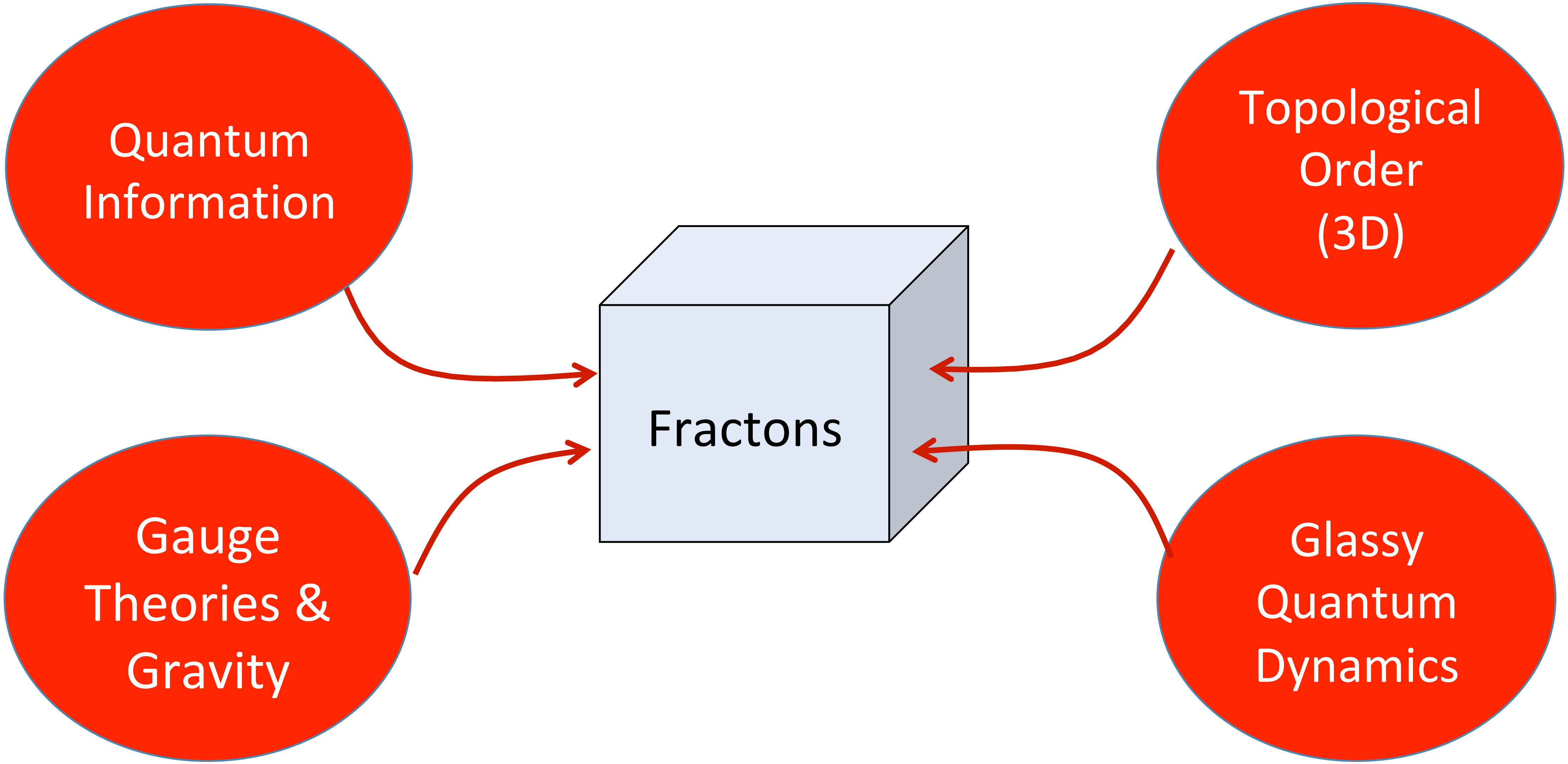
- Interplay of **randomness** and **energy conservation** can give rise to phases which violate ergodic hypothesis, never equilibrate − **quantum glass**
- **Proven** for systems where interactions between degrees of freedom are short range in real space
- Now believe it can occur even for long range interacting systems (e.g. systems of electrical charges).
- Is many body localization the only (generic) way to get **quantum glass**?

Fractons

- A different (generic) way to get ergodicity breaking

- Quantum descendants of classical models first proposed to describe window glass (!)

- Defining property: excitations exhibit *fractionalized mobility*, being either totally immobile, or able to move only in certain directions



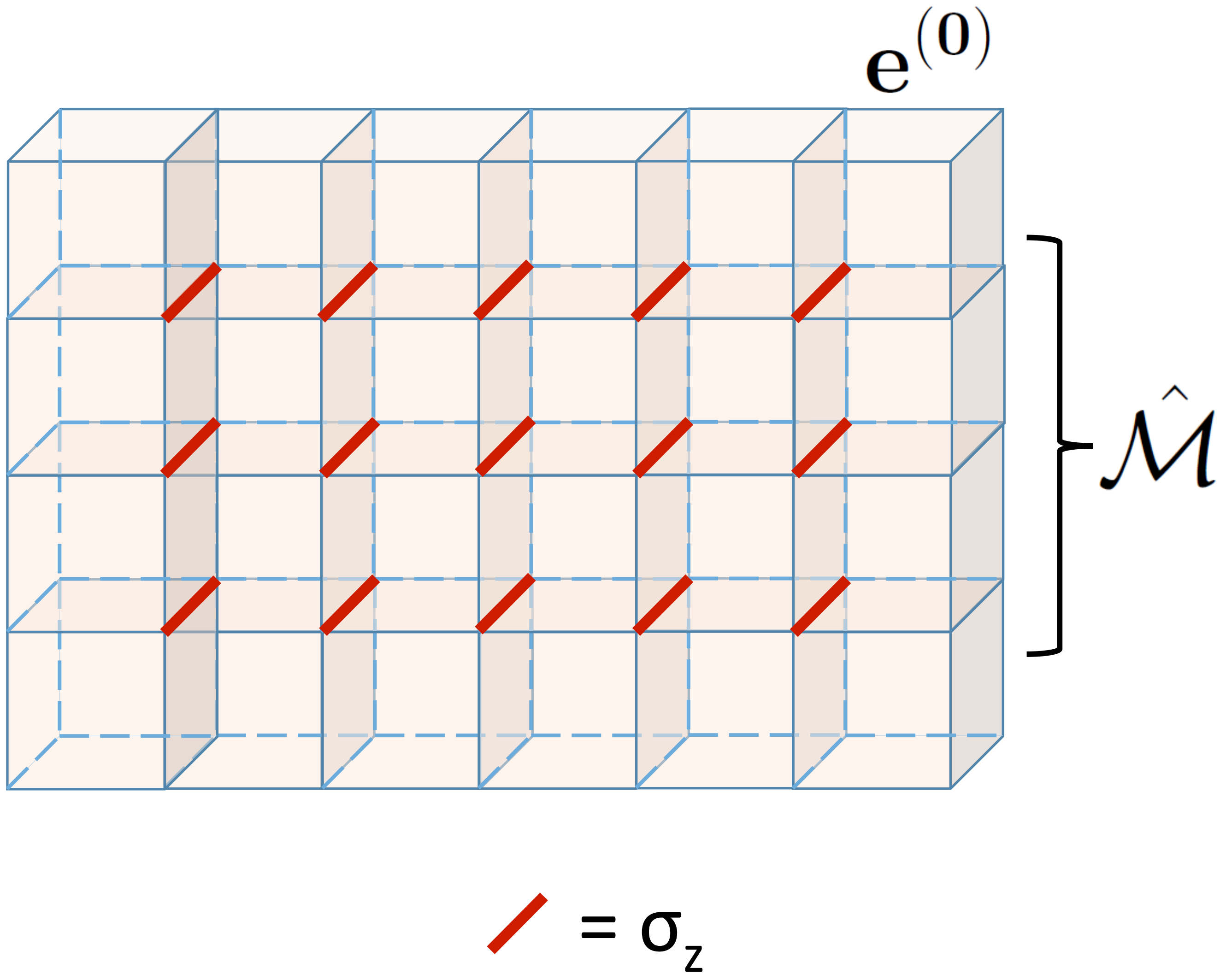
Ideas from all these fields inform study of fractons, and insight from fractons may inform all these fields

X-Cube Model: Vijay Haah Fu PRB 2016

Cubes are always +1 or -1

Fracton = -1 e-value of Cubic term

- No local operator can create single fractons.
- Isolated fractons created at ends of membrane operator.
- Cannot move fractons by acting with any local operator (without creating additional excitations)
- Totally immobile excitations



Fracton lattice models

- Exactly solvable lattice models where you can create excitations which *cannot* be moved by any local operation, without creating additional excitations (costs energy)
- If energy is conserved, then these excitations must be forever stuck
- **Another route to quantum glass**
- Much more complicated (fractal) models with richer behavior also exist…

A complementary perspective: gauge theories

- Recall that in familiar gauge theories (e.g. electromagnetism) there is a Gauss law constraint $\nabla \cdot E = \rho$ `charges produce electric field'

- This constraint implies a conservation law for charge

$$\int dV \rho = \int dV \nabla \cdot E = \int dS E$$

- Normal gauge theories are written in terms of vector gauge fields. What if we wrote down a gauge theory in terms of higher rank (tensor) gauge fields?

Higher rank gauge theories

- Pretko (2016)
- Antisymmetric tensors give nothing new (dual to vectors)
- Consider a gauge theory involving *symmetric tensor* fields, of rank n > 1 (n=2 is sufficient). Work in the continuum for now. Let there be a U(1) symmetry, with a conserved charge coupled to the gauge field.
- Generalized Gauss Law constraint e.g. $\partial_i \partial_j E^{ij} = \rho,$
- Additional conservation law $$\int \vec{x}\rho = \int x^k \partial_i \partial_j E^{ij} = -\int \partial_j E^{kj} = 0$$
- Only processes that conserve dipole moment are allowed.
  - Charges are immobile (fractons).

Fracton gauge theories

- Offer an alternative perspective on the fracton phenomenon
- Where else in physics do symmetric tensors appear?
  - General relativity (metric tensor)
  - Elasticity theory (stress and strain)
- Tantalizing connections, still being explored
- But the study of quantum glass may also have some relevance for these venerable fields!
- Also ongoing − study of connections between fractons and MBL

Summary

- Complex systems of many interacting particles best described in *statistical* terms
- Existing statistical descriptions rely on notion of `equilibrium state' which the system will reach `in the long run'
- We want to understand whether, how, and how fast a system gets to equilibrium
- Also want to understand whether a system can forever evade equilibration

Quantum glasses

- Forever evade equilibration, and preserve a memory of their initial conditions in local observables for infinite times
- Two known (generic) routes
- Many body localization
  - Randomness plus energy conservation
  - Originally for short range interacting systems, but not also for systems of electric charges.
- Fractons
  - Constraints plus energy conservation
  - Tantalizing connections to gauge theory, elasticity theory, gravity…

Outlook

- This workshop is exploring quantum glass − and also various other problems pertaining to quantum dynamics
- Thanks to KITP for support!