Majorana Returns!

- Majorana/Fermi (1937): “Real version” of Dirac Theory
- Majorana disappears (1938)
- Neutrino as Majorana (??): Double beta decay (lepton number!)
- Neutralino of SUSY is a Majorana fermion
- Read and Green (2000): $5/2$ FQHE = Chiral $p+ip$ SC
- Kitaev (2001): Majorana 1D chain
- Das Sarma, Freedman, Nayak (2005): $5/2$ FQH ‘Majorana’ Qubit
- Das Sarma, Nayak, Tewari (2006): $SrRuO_4$ Majorana (half-vortex)
- Fu, Kane (2008): Topological Insulator + SC
- Alicea (2010): $SO/SM + SC$
- Lutchyn, Sau, Das Sarma (2010): Majorana Nanowire
- Freedman, Kitaev: Topological Quantum Computation (~1995---)
Vortices in 2D spinless \((p_x + ip_y)\) Superconductor

CHIRAL, \(p\)-WAVE, SPINLESS; ZERO ENERGY MODE AT THE CORE

Order parameter phase rotates by \(2\pi n\) around the core

Order parameter amplitude suppressed at the core

Majorana is an anyon
Not a regular fermion because it is zero-energy!

Bound states in vortex cores are e-h symmetric

Low energy normal bound states in the core

ZERO-ENERGY CORE STATE IS MAJORANA
PROTECTED BY INDEX THEOREM (e-h symmetry)
Missing operations in TQC using Majorana (Ising anyons)


Ising anyons are *almost* universal: need $\vartheta = \pi/4$ phase gate and CNOT gate.

- CNOT gate can be implemented by measuring the total parity of two topological qubits
- One idea for $\vartheta = \pi/4$ phase gate is Dynamical Topology Change (DTC) (Bravyi, Kitaev 2000; Bonderson, Das Sarma, Freedman, Nayak 2010)

Hard to implement in FQHE systems, but perhaps feasible in other systems such as Superconductor/Semiconductor/Magnetic Insulator heterostructures

ISH (Ising Semiconductor Heterostructure)
A Blueprint for a Topologically Fault-tolerant Quantum Computer

Parsa Bonderson, Sankar Das Sarma, Michael Freedman, and Chetan Nayak

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3Department of Physics, University of California Santa Barbara CA 93106

arXiv 2010
The proposal for FQH topological qubit, quantum memory, and\ NOT gate using non-Abelian Ising ($\text{SU}_2$)\ TQFT: Fabry-Perot interferometry using the quasiparticle current paths along edges encircling anti-dots

Interference is the only definitive way of establishing non-Abelian-ness
Look for a many-body quantum state sensitive only to the topology

Quasiparticles in \((\nu = 5/2)\) FQH system

Quasiparticles in vortex state of 2D p-wave superconductor
QUASI-PARTICLE INTERFEROMETRY FOR LOGICAL GATES

Inventors: Michael H. Freedman, Redmond, WA (US); Chetan V. Nayak, Santa Monica, CA (US); Sankar Das Sarma, Potomac, MD (US)

Assignee: Microsoft Corporation, Redmond, WA (US)

Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days

Qubit decoherence due to anyon tunneling:
The limit to topological protection

Splitting of Majorana-Fermion Modes due to Intervortex Tunneling
in a $p_x + i p_y$ Superconductor

Meng Cheng, Roman M. Lutchyn, Victor Galitski, and S. Das Sarma

Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland,

New results:
- Tunneling energy splitting calculated:
  $$E_+ - E_- \approx \frac{\Delta_0}{\pi^{\frac{3}{2}}} \frac{\cos(k_F R + \frac{\pi}{4})}{\sqrt{k_F R}} e^{-\frac{R}{\xi}}$$
- New discovery is that the sign of the energy splitting oscillates! A behavior never seen before. It indicates that the additional effect of fluctuations is important for dephasing.
Enrico Fermi apparently said: “There are many categories of scientists, people of second and third rank, who do their best, but do not go very far. There are also people of first class, who make great discoveries, which are of capital importance for the development of science. But then there are the geniuses, like Galileo and Newton. Well, Ettore was one of these.”

Ettore Majorana

born in Catania, Sicily, 1906
rose rapidly through academic ranks
friend and scientific collaborator of Fermi, Heisenberg etc
stream of high quality papers

1933 problems... gastritis, reclusive,
no publications for several years

1937 Fermi was allowed to write-up and submit under
Majorana’s name his last and most profound paper which
Majorana had derived some years before.

At Fermi’s urging, Majorana applied and got Chair in Naples (1938)

March 1938: trip to Palermo, arrived, boarded a ship straight back to Napoli
DISAPPEARED without a trace.

a) retired to monastery,
   to escape spiritual crisis and to embrace his deep Catholic faith
b) jumped overboard in suicide
Experimental status: NOT observed

Majorana fermion - electrically neutral particle which is its own antiparticle \( \gamma = \gamma^\dagger \)

E. Majorana (1937)

Relevance: particle physics (neutrinos) (neutralinos)

Experimental status: NOT observed

EMERGENT MAJORANA?

F. Wilczek, Nature Physics’09
‘Unusual’ recent popularity of emergent Majorana modes in solids

Majorana returns
Frank Wilczek  Nature Physics 2009 September

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of ‘Majorana fermions’ — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

Non-Abelian states of matter
Ady Stern¹  Nature 2010 March

Barbara Goss Levi Search and Discovery  Physics Today 2011 March
The expanding search for Majorana particles

Search for Majorana Fermions Nearing Success at Last?

Researchers think they are on the verge of discovering weird new particles that borrow a trick from superconductors and could give a big boost to quantum computers
Non-abelian statistics and topological qubits increasing simplicity

- Fractional Quantum Hall
- Chiral p-wave superconductors
- Topological insulator/S-wave superconductor
- Spin-orbit coupled Semiconductor/S-wave superconductor

Majorana for the Elite (5/2 FQHE)

- Bringing Majorana to the Masses (ISH)
Generic New Platform for Topological Quantum Computation Using Semiconductor Heterostructures

Jay D. Sau, Roman M. Lutchyn, Sumanta Tewari, and S. Das Sarma

Majorana Fermions and a Topological Phase Transition in Semiconductor-Superconductor Heterostructures

Roman M. Lutchyn, Jay D. Sau, and S. Das Sarma

Search for Majorana Fermions in Multiband Semiconducting Nanowires

Roman M. Lutchyn, Tudor D. Stanescu, and S. Das Sarma
Majorana fermions, topological superconductivity, spin-orbit coupling...
Majorana bound states in SM/SC heterostructure

Bogoliubov-de-Gennes equations

\[ H_{\text{BdG}} \Psi = E \Psi \]

\[ \Psi = (\psi_\uparrow, \psi_\downarrow, \psi_\downarrow^\dagger, -\psi_\uparrow^\dagger)^T \]

Non-degenerate zero-energy solution exists when \( \sqrt{\mu^2 + \Delta_0^2} < |V_z| \)

Majorana number \( \mathcal{M} \)

\[ \mathcal{M} = e^{i\pi C_1} = \pm 1 \]

Sau, Lutchyn, Tewari, Das Sarma, PRL’10
Band structure of spin-split spin-orbit coupled semiconductor

Rashba induced chiral spin-texture allows singlet pairing

Need single non-degenerate Fermi-surface

$H_{Sm} = k^2 + V_z \sigma_z$

Rashba + Zeeman break inversion and time-reversal for chiral edge mode

Sau, Lutchyn, Tewari, Das Sarma  PRL(2010)
Superconductors are natural hosts for Majorana

Bogoliubov quasiparticle \( \gamma = u\psi + v\psi^\dagger \)

\[ u = v^* \]

Majorana fermion \( \gamma = \gamma^\dagger \)

equal superposition of a particle and a hole

\[ \text{electrons} = \text{holes} + \text{Cooper pair} \]

Look for ZERO energy states!

Bound states in vortices

Midgap states at the interfaces

\[ E = 0 \quad \Delta_0 \]

empty

occupied
Topological protection of zero-energy mode

Bogoliubov-de-Gennes equations

$$\begin{pmatrix} h_0 & \Delta \\ \Delta^\dagger & -h_0^T \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}$$

Particle-hole symmetry:

If \( \begin{pmatrix} u \\ v \end{pmatrix} \) is a solution with \( E \)
If \( \begin{pmatrix} v^* \\ u^* \end{pmatrix} \) is a solution with \( -E \)

For spinless fermions particle-hole symmetry guarantees Majorana mode at \( E = 0 \)

Two topological classes of BdG Hamiltonians

2N + 1 solutions

Bound states in the vortex core

2N solutions

Topological reconstruction of the spectrum requires closing of the bulk gap

Read and Green, PRB’00

E

E = 0

-E
Example: 2D chiral p-wave superconductors

Zero-energy states appear in chiral superfluids

He-3: Kopnin and Salomaa PRB’91;
Chiral superfluids/superconductors, Volovik (1999), Read & Green (2000)
Chiral p-wave superconductor SrRuO_4: Das Sarma, Nayak, Tewari (2006)

Chirality may originate from the order parameter or band structure

Chiral superconductors:
- strontium ruthenate
  Rice & Sigrist, 1995

Carolli-de Gennes-Matricon bound states

\[ E_n = \omega_0 \left( n + \frac{1}{2} \right) \]

Heterostructures:
- topological insulator/s-wave superconductor
- semiconductor/s-wave superconductor
... among others
Engineering spinless $p+ip$ superconductor

Rather than looking for $p_x + ip_y$ SC in nature, we could try to engineer suitable Hamiltonians via proximity effect.

Chirality has to come from the bandstructure

Strong spin-orbit interaction is necessary to avoid fermion doubling

Superconducting heterostructures

2D: Majoranas “live” in vortices
1D: Majoranas “live” at the ends of wires

Sau, Lutchyn, Tewari, Das Sarma, PRL’10

Sau et al. PRB (2010)
Sau, Tewari, Das Sarma Ann Phys’10

Ordinary S-wave SC
+ 2D (or 1D) Semiconductor
with Strong SO Coupling

1D Lutchyn, Sau, Das Sarma, PRL(2010)

Q1D Lutchyn, Stanescu, Das Sarma, PRL’11
Semiconductor with spin-orbit interaction

$H_0 = \begin{pmatrix} \frac{p^2}{2m} - \mu & \alpha i (p_x - ip_y) \\ -\alpha i (p_x + ip_y) & \frac{p^2}{2m} - \mu \end{pmatrix}$

Fermi sea

Spin orientation changes around Fermi surface

Sau, Lutchyn, Tewari, Das Sarma, PRL’10;

Single Fermi surface!
Practical route to spinless p+ip superconductivity

Generic New Platform for Topological Quantum Computation Using Semiconductor Heterostructures

Jay D. Sau, Roman M. Lutchyn, Sumanta Tewari, and S. Das Sarma

Proximity-induced $\Delta_{\text{ind}}$
Proximity-induced $V_z$

**Challenge:** creating two interfaces

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Majorana fermions in a tunable semiconductor device

Jason Alicea

Department of Physics, California Institute of Technology, Pasadena, California 91125, USA

Proximity-induced $\Delta_{\text{ind}}$

In-plane magnetic field

**Challenge:** low electron density, effects of disorder
1D wires with spin-orbit: helical state

\[
H_0 = \int_{-L}^{L} dx \psi^\dagger_\sigma(x) \left( -\frac{\partial^2}{2m^*} - \mu + i\alpha \sigma_y \partial_x + V_x \sigma_x \right) \psi_{\sigma'}(x)
\]

- Single channel nanowire
- Spin-orbit coupling
- Zeeman splitting

\[ \varepsilon(p_x) \]

Magnetic field \( B_x \) opens up gap in the spectrum at \( p_x = 0 \)

\[
V_x \quad \mu
\]

InAs, InSb nanowires

- Large spin-orbit (\( \alpha \sim 0.1eV\AA \))
- Large g-factor (\( g \sim 10 - 50 \))
- Good contacts with metals
Majorana quantum wires

\[ H_{\text{MW}} = \int_{-L}^{L} dx \left[ \psi_\sigma^\dagger \left( -\frac{\partial^2_x}{2m^*} - \mu + i\alpha_\sigma y \partial_x + V_x \sigma_x \right) \psi_{\sigma'} + \Delta_0^* \psi_\uparrow \psi_\downarrow + \Delta_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \right] \]

Rashba spin-orbit + in-plane field

Proximity-induced superconductivity

Diagonalize \( H_0 \)

\[ \begin{align*}
\Delta_{-+}(p) \Psi_-(p) \Psi_+(p) \\
\Delta_{++}(p) \Psi_+(p) \Psi_+(p)
\end{align*} \]

Lutchyn, Sau, Das Sarma PRL 2010

Drive topological phase transition by changing \( V_x \) or \( \mu \)
Density of states across phase transition

Finite-size numerical studies $L_x = 10\mu m$

DoS in topologically non-trivial phase

$$|V_x| > \sqrt{\mu^2 + \Delta_0^2}$$

DoS in topologically trivial phase

$$|V_x| < \sqrt{\mu^2 + \Delta_0^2}$$

s-wave superconductor (Al or Nb)
Tunneling experiments

- probing Majorana bound states using tunneling experiments

Resonant Andreev reflection

\[
g = \frac{2e^2}{h} \quad |V_x| > \sqrt{\mu^2 + \Delta_0^2}
\]

\[
t = 0
\]

\[
g = 0 \quad |V_x| < \sqrt{\mu^2 + \Delta_0^2}
\]

Sau, Tewari, Lutchyn, Satansecu, Das Sarma, PRB’10
Fractional ac Josephson effect

Andreev bound states

\[ \Delta_L = \Delta e^{i\varphi}, \quad \Delta_R = \Delta_0 \]

Short junction limit \((L \ll \xi)\)

particle-hole symmetry protects true level crossing at \(\varphi = \pi\)

Lutchyn, Sau, Das Sarma, PRL’10

Topologically non-trivial phase transition: \(V_x \geq \sqrt{\mu^2 + \Delta_0^2}\)
Josephson current through heterostructure

Josephson current through heterostructure

\[ I_{\pm}(\varphi) \propto \frac{\partial E_{\pm}(\varphi)}{\partial \varphi} \]

Fractional ac Josephson effect is a robust signature of topological SC
Experimental proposal: nanowire embedded into SQUID

\[ Z(\omega) \text{ is a function of the inductance of the SQUID!} \]

Measurement of Josephson inductance

\[ I_c \sim 10\text{nA} \quad L_J^{\text{min}} \sim 10 - 100\text{nH} \]

Lutchyn, Sau, Das Sarma, PRL’10
Experimental considerations

Tunable Supercurrent Through Semiconductor Nanowires
Yong-Joo Doh,¹* Jorden A. van Dam,¹* Aarnoud L. Roest,¹,² Erik P. A. M. Bakkers,² Leo P. Kouwenhoven,¹ Silvano De Franceschi¹†

Science 309, 272 (2005)

Supercurrent reversal in quantum dots
Jorden A. van Dam¹, Yuli V. Nazarov¹, Erik P. A. M. Bakkers², Silvano De Franceschi¹,² & Leo P. Kouwenhoven¹


Al/InAs/Al heterostructure

Experimental efforts: Delft, Harvard, McGill, UCSB, Weizmann ...
Multi-band semiconductor nanowires

Weak coupling analysis $\Delta \to 0$
$\mathcal{M} = (-1)^{\nu(0) - \nu(\Lambda)}$  Kitaev, arXiv’00

Topological phase exists when
Second band  $|V_x| > \sqrt{\left(\mu - E_{sb}\right)^2 + \Delta_0^2}$

First band  $|V_x| > \sqrt{\mu^2 + \Delta_0^2}$
Lutchyn, Stanescu, Das Sarma, arXiv’10
Topological periodic superconductor-nanowire structures

Jay D. Sau¹, Chien Hung Lin¹, Hoi-Yin Hui¹, and S. Das Sarma¹

¹Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA

FIG. 2: (a) Junction in a ring topological superconductor structure with chemical potential control over entire structure (such that $\mu \sim 0$ meV $< \mu_c$) shows a fractional Josephson effect. Flux dependence of ABS energies and the corresponding Josephson current in junction show $2\Phi_0$ periodicity. (b) Experimental adaptations modify geometry so that $\mu_g > \mu_c$ in superconductor (length $L_g = 1.5 \mu$m) with gate-induced chemical potential control only in junction ($\mu_g < \mu_c$) (length $L_g = 600$ nm). ABS spectrum show a conventional Josephson effect in this case despite tunneling signature of MFs in Fig. 1.
Super-current in a semiconducting nanowire
Jay Sau and Sankar Das Sarma, unpublished

\[ J(x) = \int dx' \psi^\dagger(x') \frac{\delta H_0(A)}{\delta A(x)} \psi(x'). \]

\[ \Phi_L = -\Phi/2 \quad \Phi_R = \Phi/2 \]

**Figure 1:** Nanowire geometry for measuring phase transition. All quantities \( V(\omega), I(\omega) \) and \( \Phi(\omega) \) are frequency-dependent. The josephson phase \( \Phi(\omega) = -\frac{i}{\omega} V(\omega) \).
The search for the Majorana ‘fermion’ may finally be coming to an end.

The Majorana mode may soon be observed in table-top experiments as an emergent zero-energy mode in solid state semiconductor-superconductor sandwich structures.

‘Majorana’ may return after a 75-year hiatus – thanks to Michael Freedman.

Happy Birthday Michael.