From Renormalization Group to Emergent Gravity :

holographic description of quantum many-body systems

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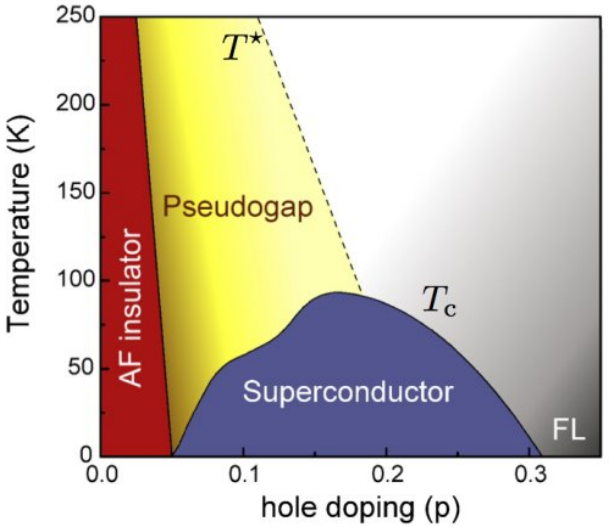
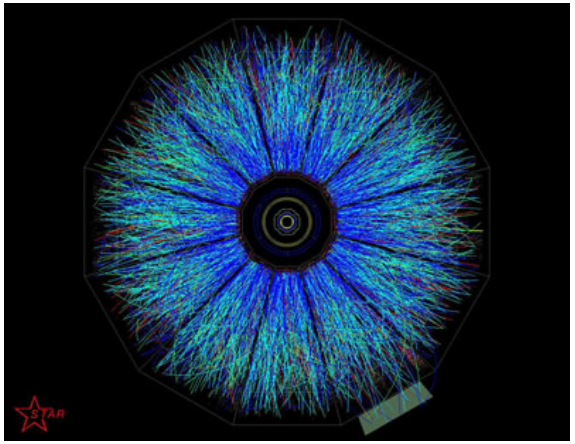
Strongly Correlated Gapless Phases

- Long distance physics is described by strongly interacting QFT
  - Quantum critical points/phases
  - Non-Fermi liquids
  - Quark-gluon plasma
  - …
- One may identify a QFT that captures the low energy physics, but it is in general hard to understand low energy dynamics
- Non-perturbative tools are desired!





AdS/CFT correspondence

[Maldacena; Gubser, Klebanov, Polyakov; Witten]

- D-dimensional quantum field theory is dual to (D +1)-dimensional gravity
  - Weak coupling description for strongly coupled QFT
  - Non-perturbative definition of string theory (quantum gravity)
- Best understood in the maximally supersymmetric gauge theory in 4D (no proof yet)
- Believed to be a general framework

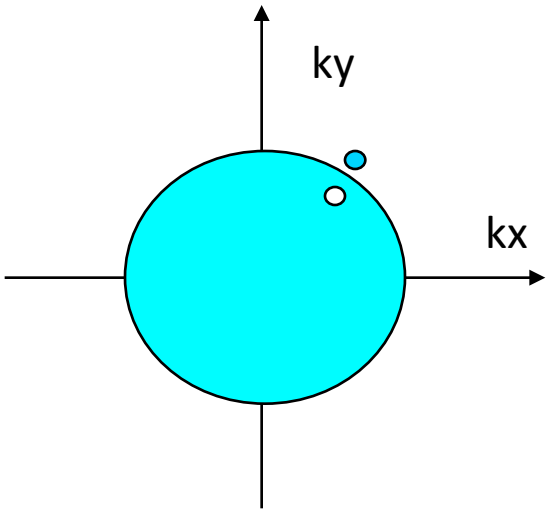
[Das, Jevicki; Gopakumar; Heemskerk, Penedones, Polchinski; Lee; Faulkner, Liu, Rangamani; Douglas, Mazzucato, Razamat,…]

States with FS is particularly challenging due to high entanglement:

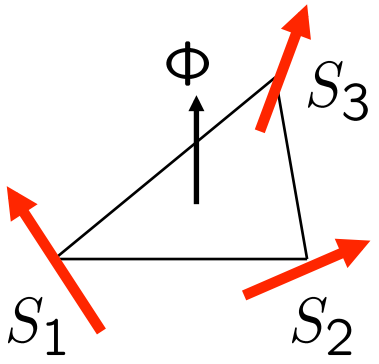
Spinon Fermi surface coupled with a U(1) gauge field

$$\vec{S}_r = f^\dagger_{r\alpha}\vec{\sigma}_{\alpha\beta}f_{r\beta}$$ Spinon : EM charge 0, spin 1/2

1) neutral spinon
(Fermi surface)

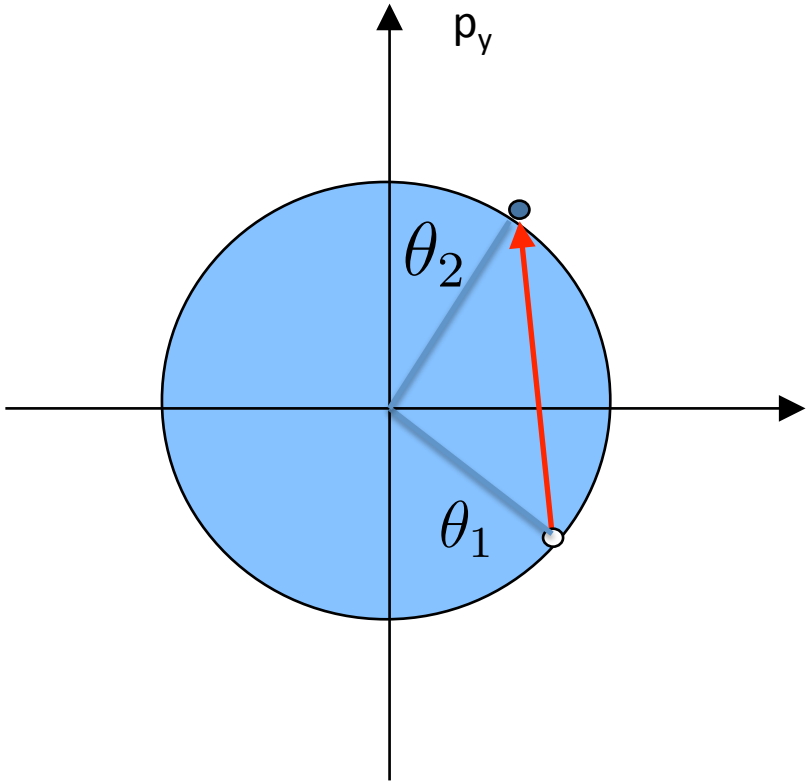


2) spin chirality :
compact U(1) gauge field



$$\nabla \times a \sim \sin\Phi \sim S_1 \cdot (S_2 \times S_3)$$

Low energy effective theory of FS is a matrix model



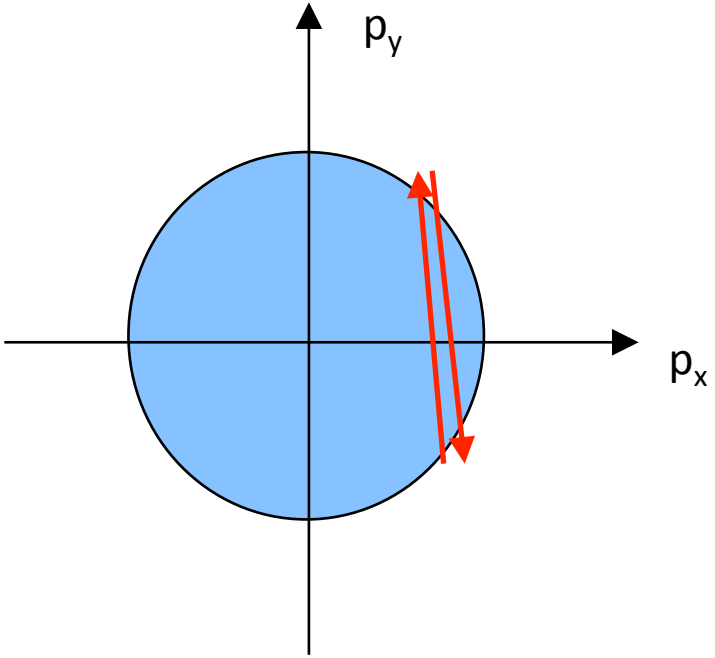
Low energy
Spin singlet/triplet
Particle-hole excitation

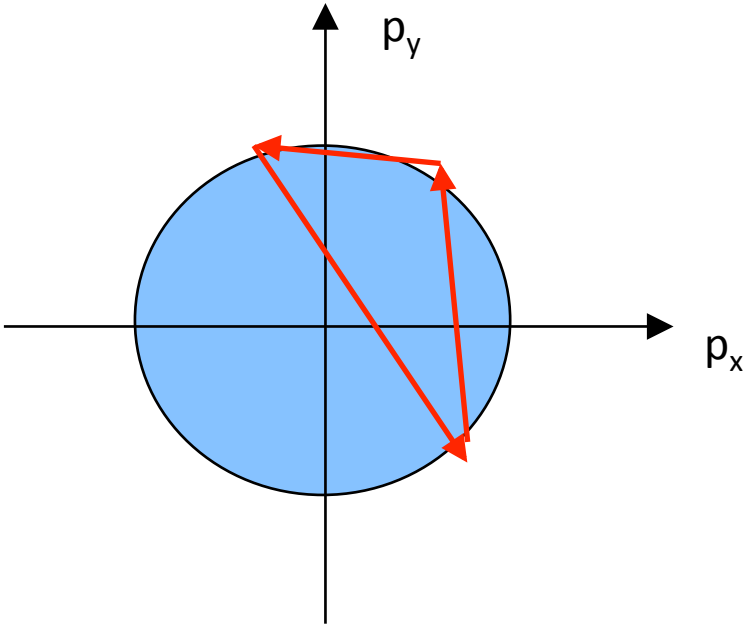
$$\phi^i_{\theta_2\theta_1} = f^*_{\alpha\theta_2}\sigma^i_{\alpha\beta}f_{\beta\theta_1}$$

$$\sigma^i = (I, \vec{\sigma})$$

Low energy effective theory of FS is a matrix model

- Low energy effective theory can be constructed in terms of traces of the matrix fields

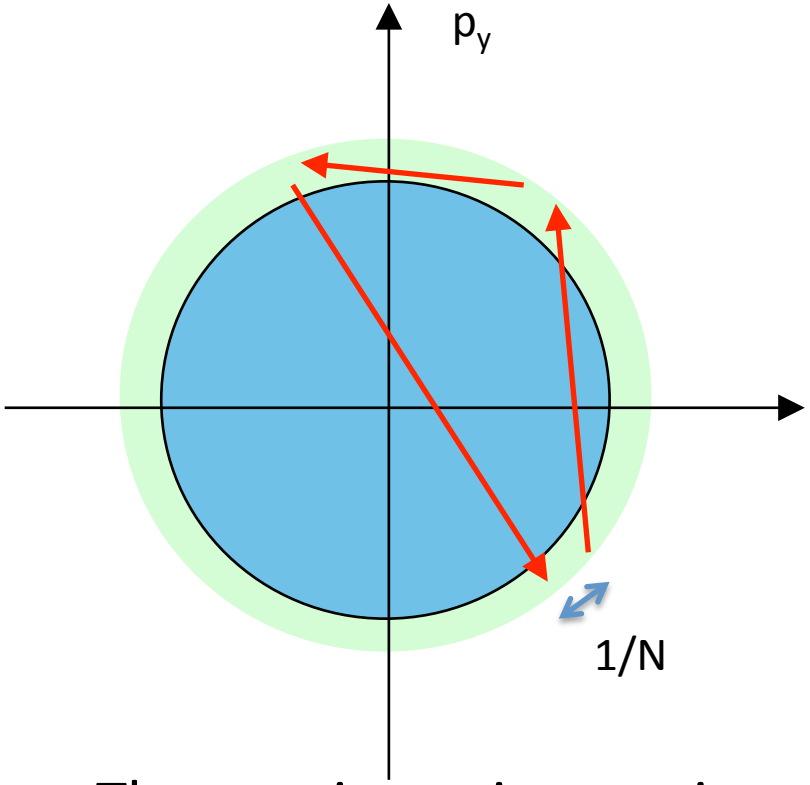


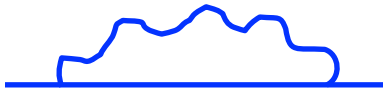


$$\phi_{\theta_1\theta_2}\phi_{\theta_2\theta_1} = tr[\phi^2]$$

$$\phi_{\theta_1\theta_3}\phi_{\theta_3\theta_2}\phi_{\theta_2\theta_1} = tr[\phi^3]$$

Low energy effective theory of FS is a matrix model

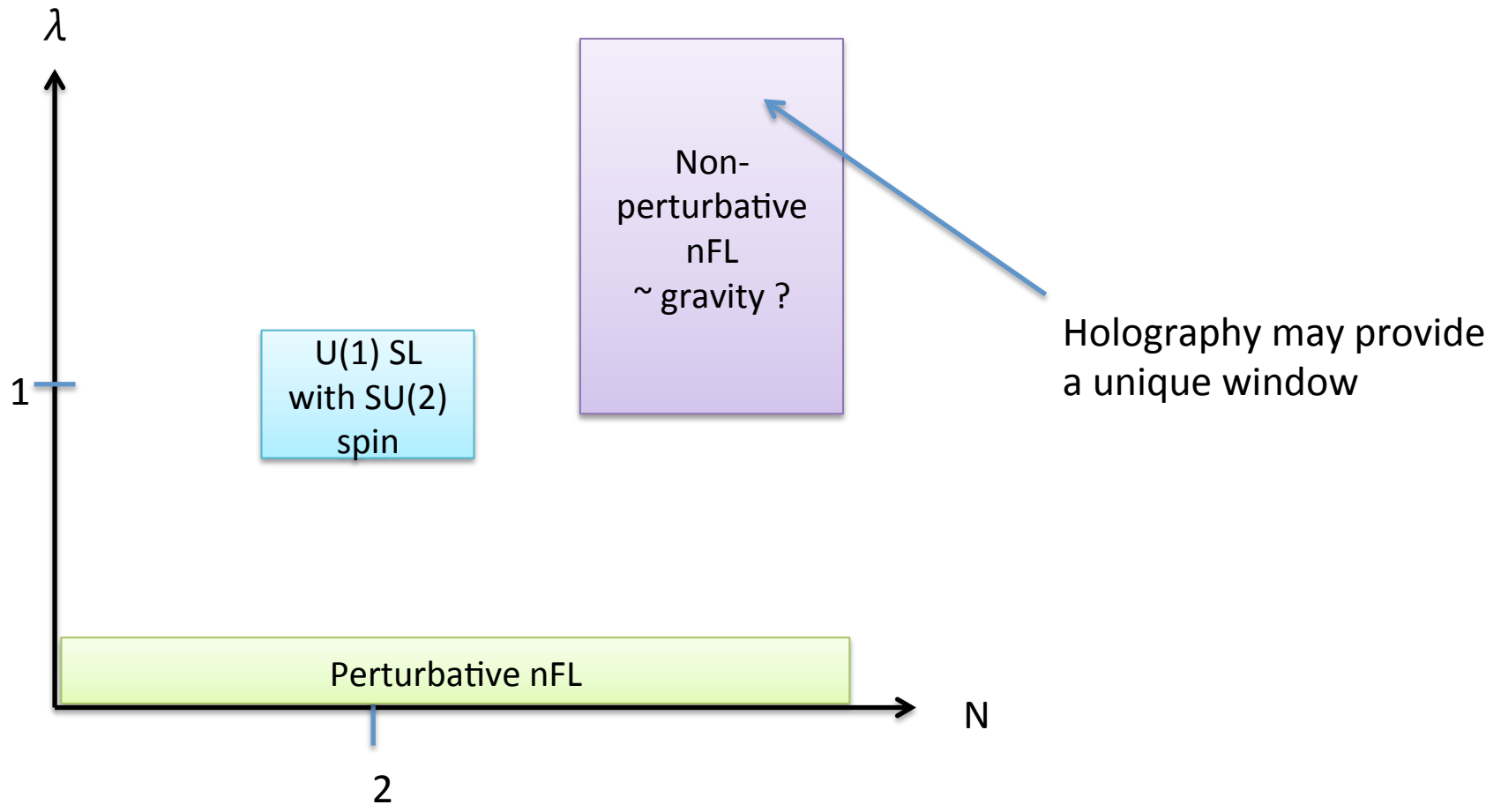




$$G^{-1}(p) = i\frac{\omega^{2/3}}{N} + \frac{k^2}{2m} - \mu$$

- The matrix carries continuous flavour
- At a finite energy energy, there is a finite uncertainty in the transverse momentum ~ 1/N $\omega^{2/3}$, and the continuous flavour effectively becomes N discrete indices

Parameter space



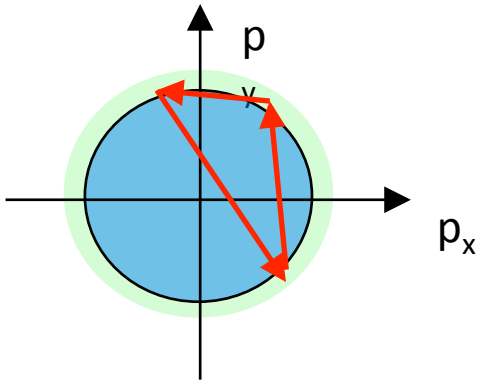
[Halperin, Lee, Read (93), Polchinski(93); Althsuler,Ioffe,Millis(94); Kim, Furusaki, Wen, Lee(94); Nayak, Wilczek(94); SL(09); Metlitski and Sachdev (10); Mross, McGreevy, Liu, Senthil(10)]

Matrix field theory

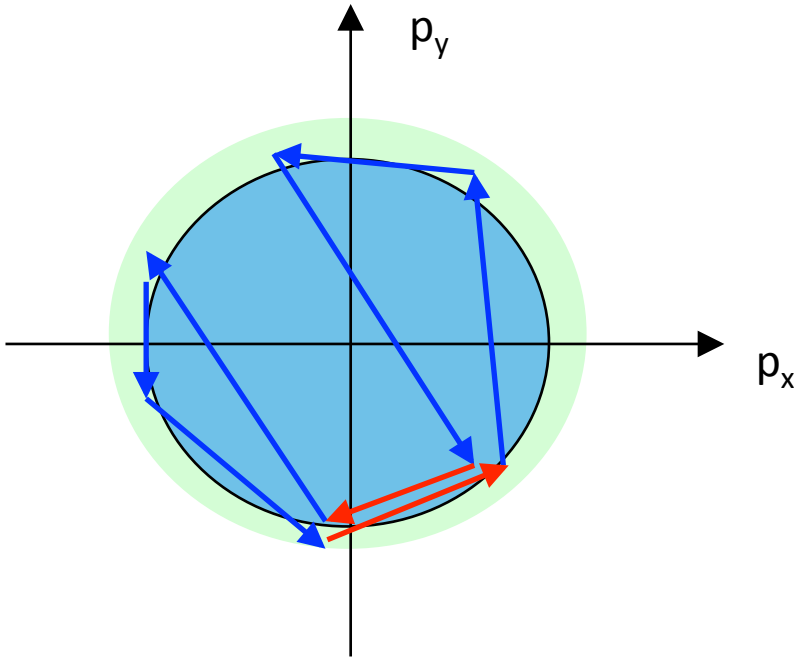
$$Z[J(x)] = \int D\phi \; e^{i \int dx J_n(x) O_n}$$

- O_n : the complete set of single-trace operators

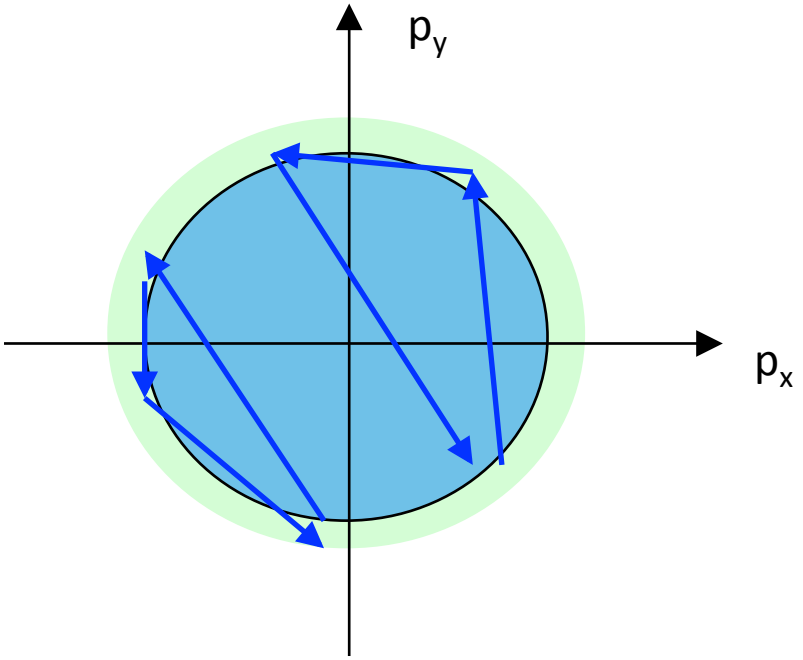
e.g. $tr[\phi^n], \quad tr[\phi \partial_\mu \partial_\nu \phi], \quad \ldots$



Difficulty



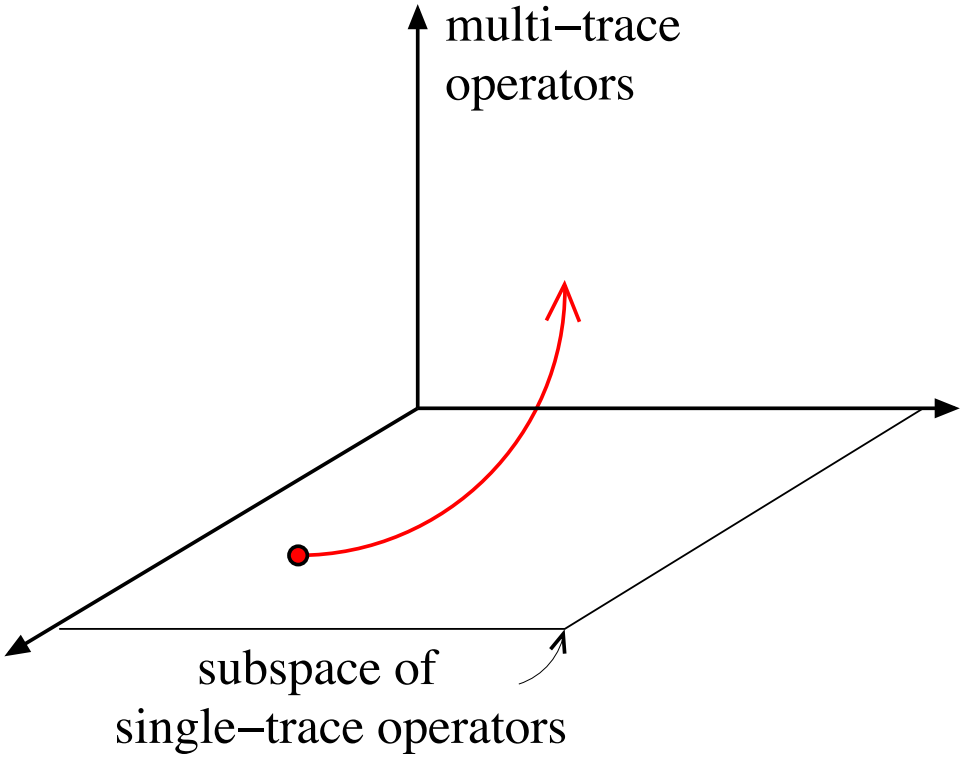
$$tr[\phi\phi\phi\phi\phi\phi\phi\phi]$$



$$tr[\phi\phi\phi]tr[\phi\phi\phi]$$

- Even though one starts with the single-trace operators at a given scale, multi-trace scatterings are generated at low energies

Conventional RG



$$\frac{dJ_O}{dl} = \beta_O[J_O, J_{OO}, ...],$$

$$\frac{dJ_{OO}}{dl} = \beta_{OO}[J_O, J_{OO}, ...],$$

$$...$$

- Along the RG flow, one has to keep track of an infinite number of multi-trace operators

An alternative : at each step of RG, one can remove multi-trace operators at the expense of making the sources for single-trace operators dynamical

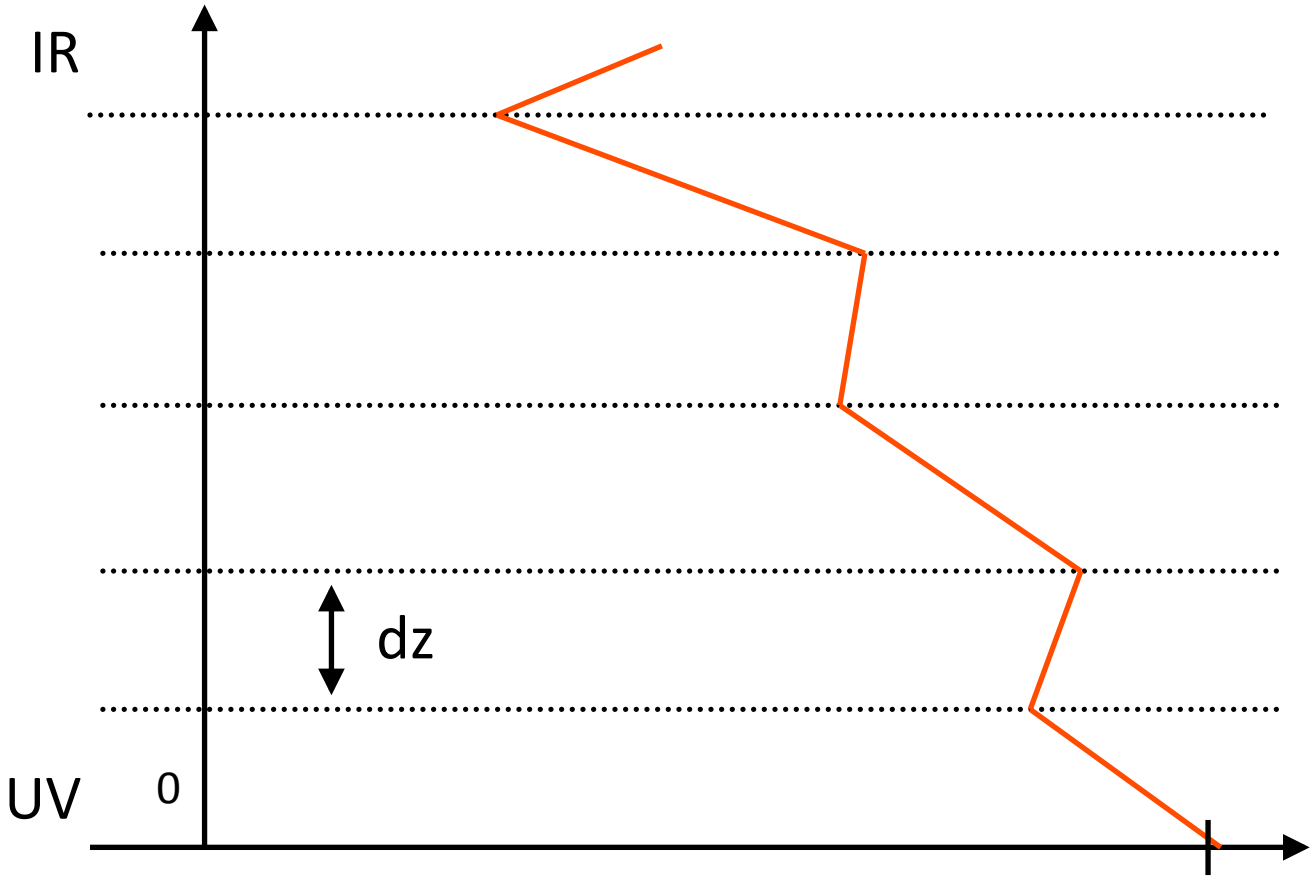
$$Z[J(x)] = \int D\phi \; e^{i\int dx[-J_n(x)O_n + J_{mn}(x)O_m O_n + ..]}$$

$$= \int Dj_n Dp_n D\phi \; e^{i\int dx \mathcal{L}'}$$

$$\mathcal{L}' = j_n(p_n - O_n) - J_n p_n + J_{nm} p_n p_m + \ldots$$

- J_n : Lagrangian multiplier that plays the role of dynamical source that enforces the constraint $p_n = O_n$
- P_n : dynamical operator

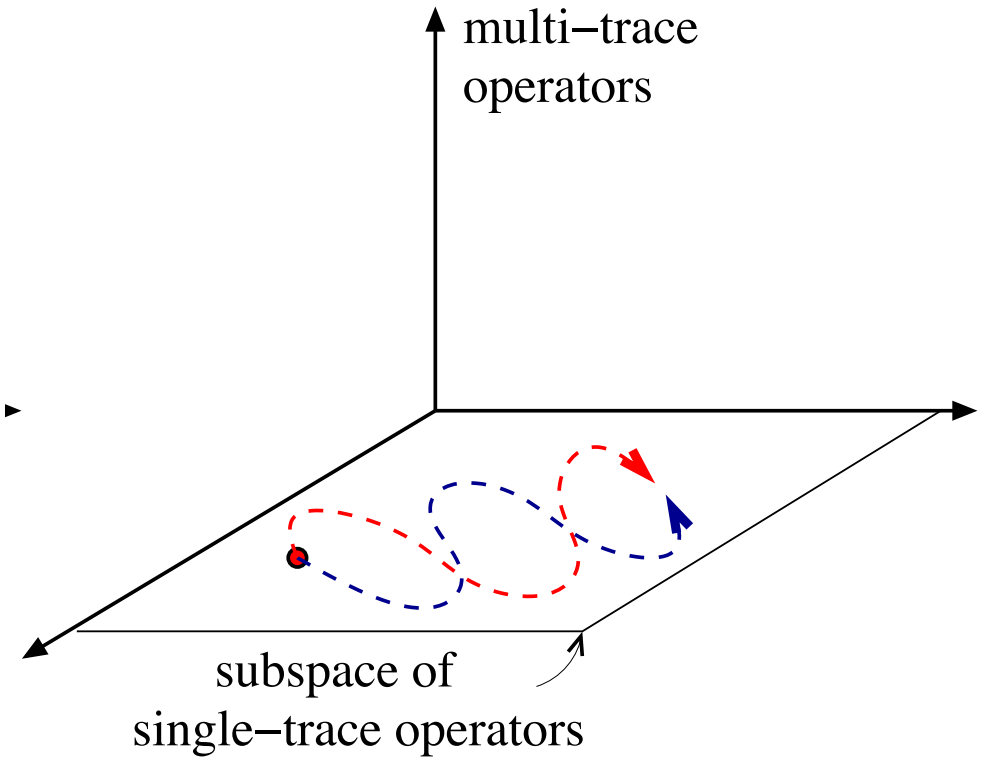
Extra dimension as a length scale



$$j_n^{(l)},\ p_n^{(l)}$$

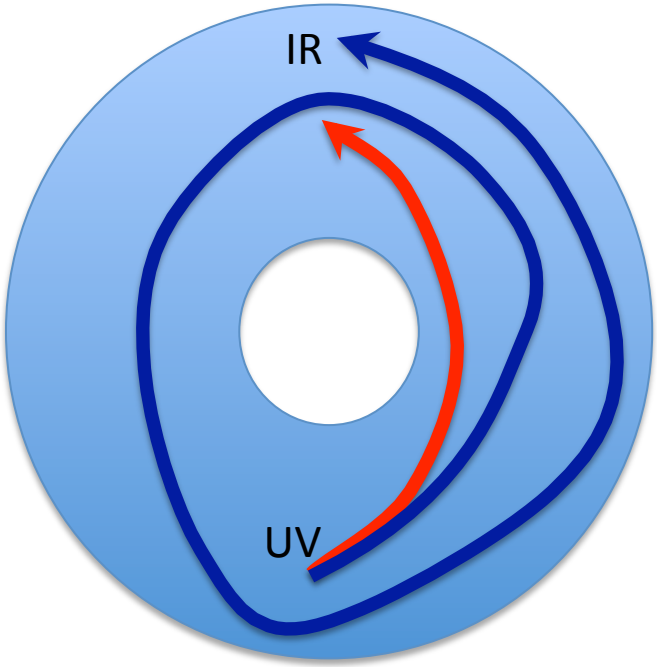
Boundary condition $j_n^{(0)}(x) = J_n(x)$

Quantum fluctuations in RG path



- Only single-trace operators appear
- Quantum fluctuations in the RG trajectory : sources become operators!
- RG flow is governed by a quantum `Hamiltonian'
- In the large N limit, saddle-point approximation works

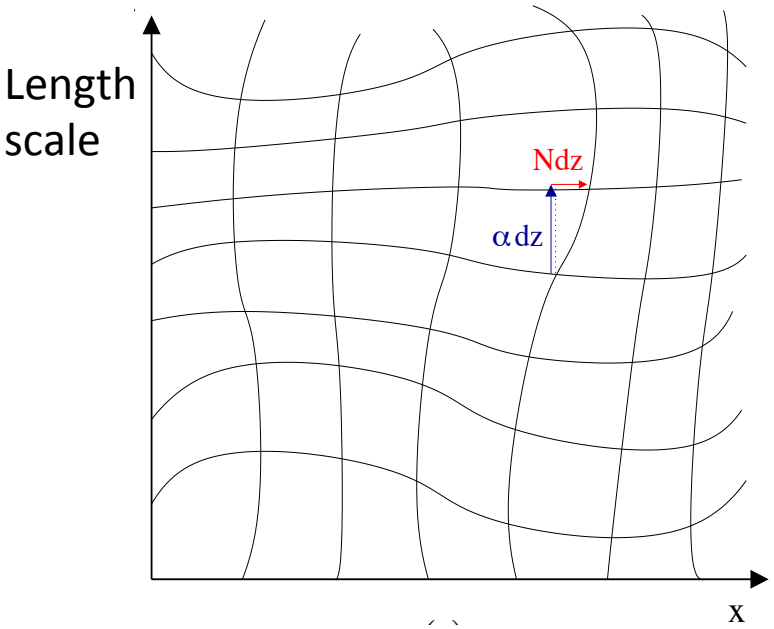
A new type of quantum order



- Space of sources is not in general simply connected : there are topologically distinct RG paths
- For sufficiently large N, `topological excitation' in RG path is suppressed
- Toplogical order associated with the emergence of the extra dimension
- Protected scaling dimension

[SL (11)]

Why gravity ?

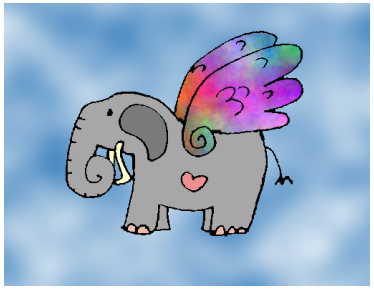


- Energy momentum tensor $T_{\mu\nu}$ couples to a source $g^{\mu\nu}$ which becomes dynamical metric in the bulk
- Freedom to choose different local RG schemes = Diffeomorphism in the bulk

[SL (12)]

Summary

- D-dimensional QFT can be explicitly mapped into a (D+1)-dimensional quantum theory of gravity based on a local RG
- Quantum beta function
- Example of emergent gravity



- Proof of AdS/CFT conjecture
- Concrete solvable model
- Characterization of new quantum order in terms of entanglement structure