When superfluids are a drag

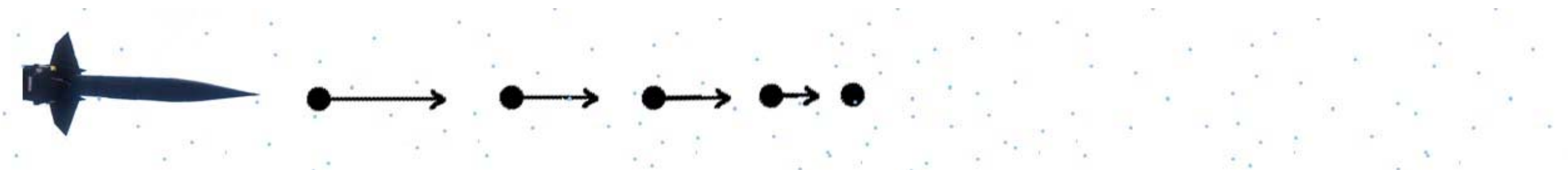
KITP
October 2008

David Roberts
Los Alamos National Laboratory

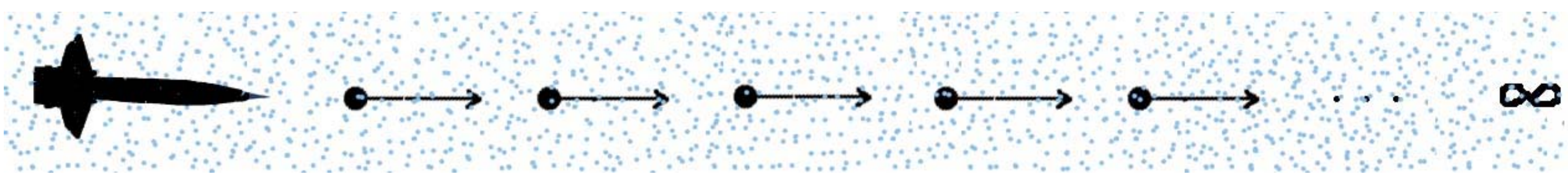
In collaboration with Yves Pomeau (ENS),
Andrew Sykes (Queensland),
Matt Davis (Queensland), …

What makes superfluids super?

Earth's atmosphere

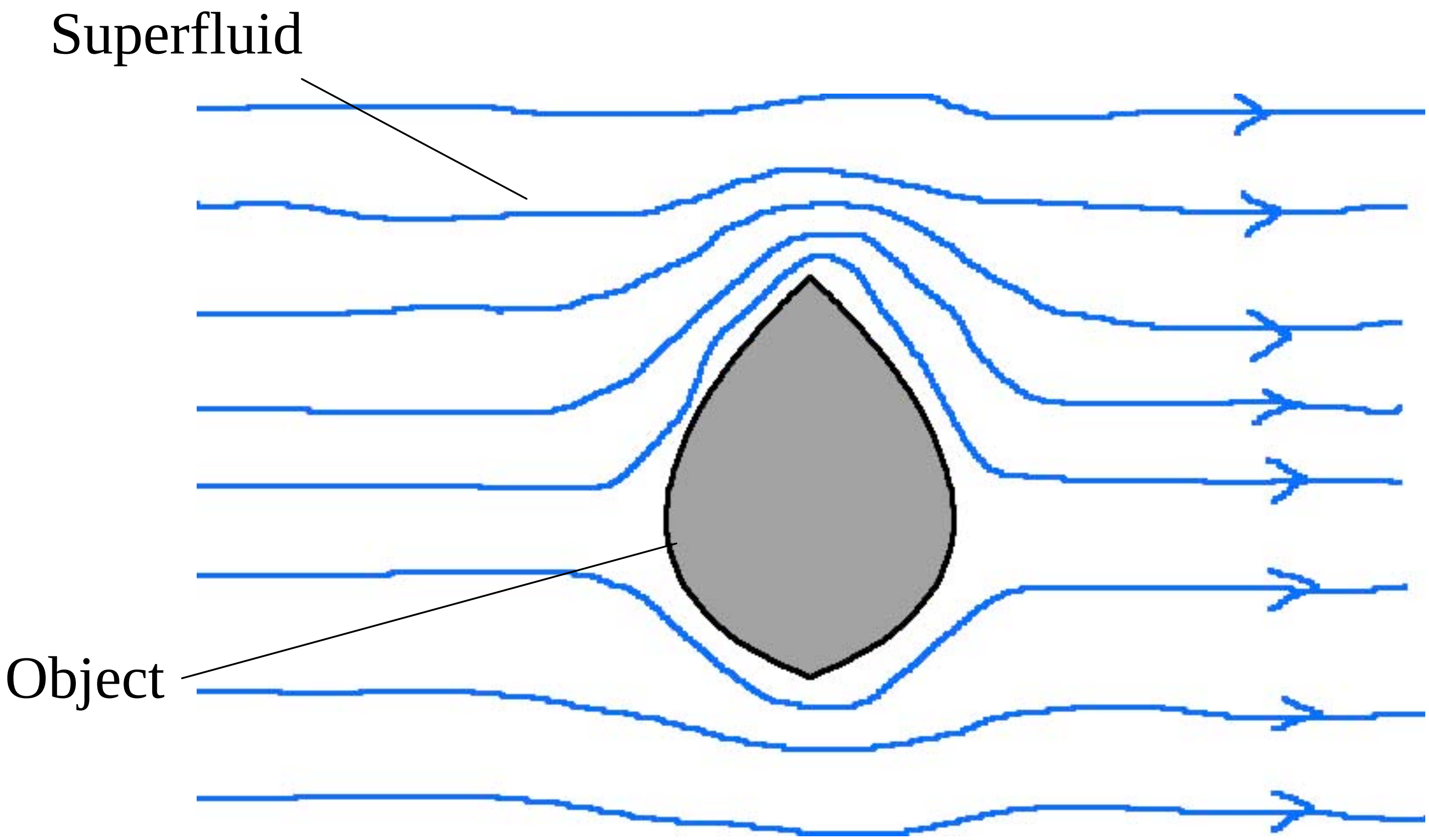


Dilute deep space



Dense superfluid

The Situation



$$v < v_c$$

Does the object experience dissipation (a.k.a. drag force)?

Rest of talk

- **Why do we care?**
  Superfluidity

- **What has been done?**
  Mean field results verify orthodox view

- **What is missing?**
  Quantum fluctuations

Why do we care?

Superfluidity

- Quantum mechanics at work on large scale
- "Helium II" discovered in 1938 by Kapitza;
- Interested many great minds, e.g. Feynman, Onsager, Landau, etc.
- Numerous Nobel prizes
  - 1962 (Landau), 1973 (Josephson), 1978 (Kapitsa), 1996 (Lee, Osheroff, Richardson), 2003 (Leggett), ...
- Relevant in applications:
  - **atom lasers**
  - superconductivity
  - gyroscopes

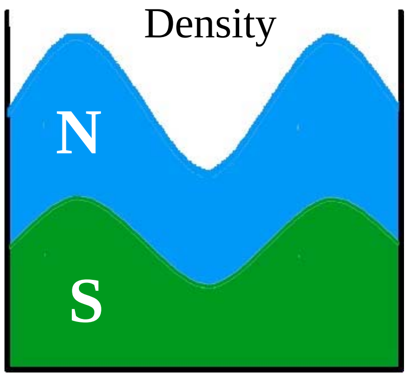
  ...
- Coldest part of the universe -- relevant in several areas:
  - cryogenics
  - astrophysics

  ...

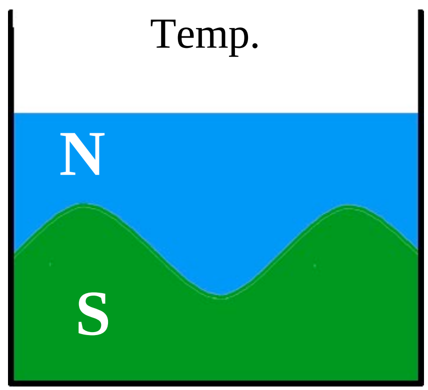
The phenomenon of superfluidity

In addition to dissipationless flow - A collection of strange behavior

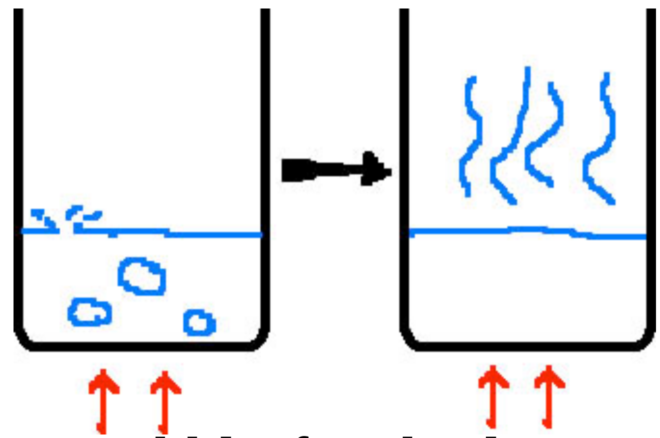
Examples from superfluid Helium:



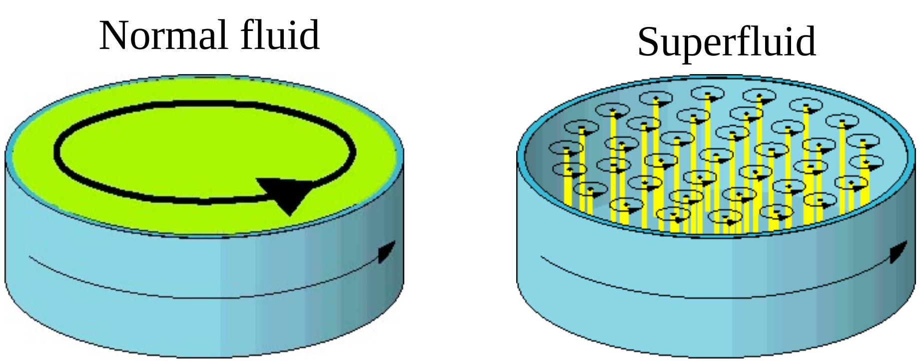
First sound



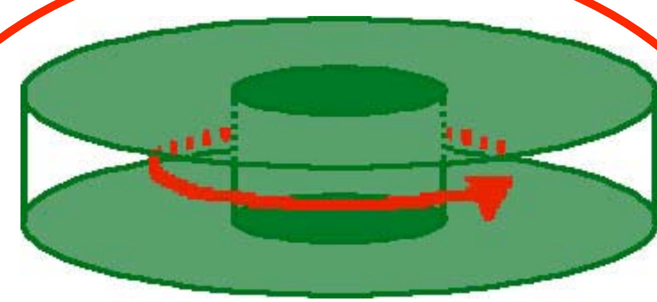
Second sound



Bubble-free boiling



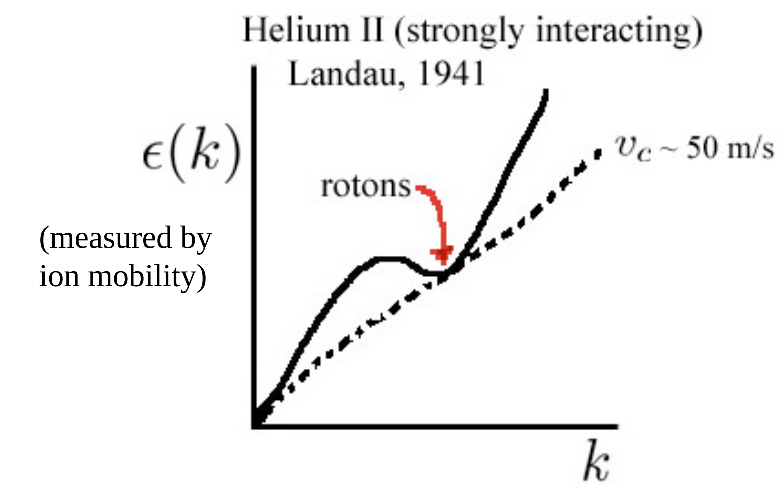
Irrotational flow and
quantized vortices

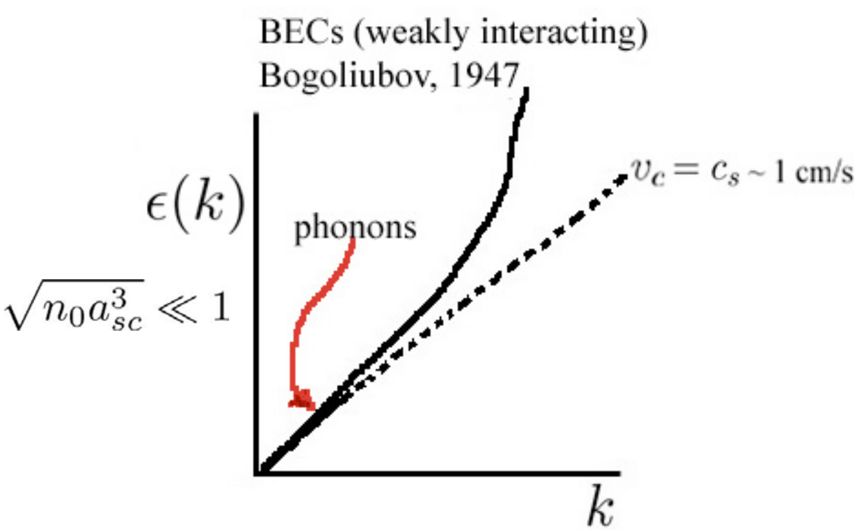


Persistent currents

Landau theory predicts dissipationless flow if $v < v_L$

Landau criterion: $v_L = \min(\epsilon(k)/\hbar k)$





Excitation spectrum

↓

Non-zero critical velocity for dilute BEC

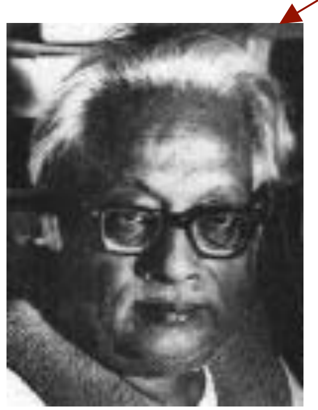
↓

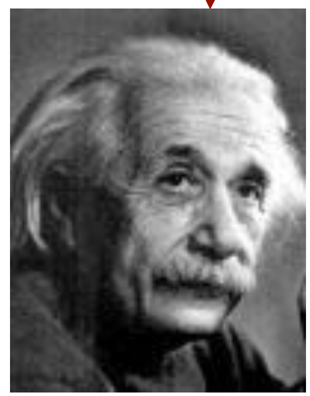
Dissipationless flow at T = 0 ??

Crucial Point:
Assuming dissipation is caused by the creation of excitations at T = 0

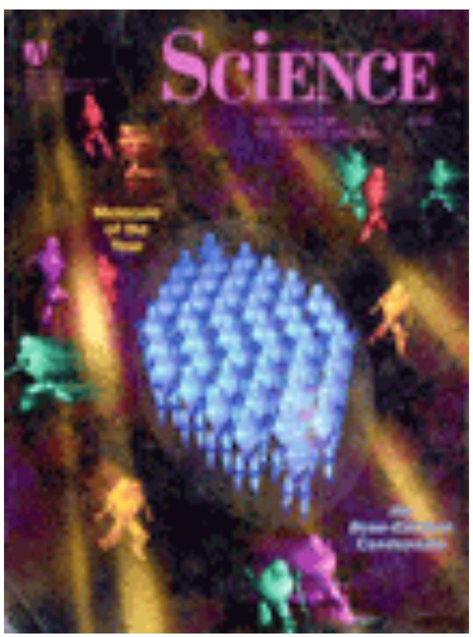
Theoretical underpinnings of superfluidity

What's going on at the atomic level: **Bose-Einstein condensates (BEC)**





BEC predicted by Einstein in 1924

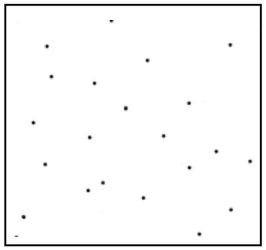


Superfluids are characterized by a significant number of bosons being in the ground state

F. London (1938)

2 main bosonic superfluids

Ultracold trapped alkali atomic gas



diluteness parameter $\sqrt{n_0 a_{sc}^3} << 1$

effective range of interaction (a_{sc})

<u>Harder to see superfluid properties</u>

Associated phenomena:

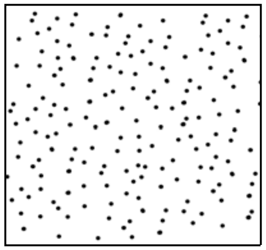
- Non-classical moment of inertia (e.g. scissor modes) (*Oxford, 1999*)
- Quantized vortices (*ENS*, *Boulder*, *etc.*)
- **Dissipationless flow (*MIT, 1999, 2000*)**

⋮

<u>BEC more apparent (1995)</u>

- Almost all atoms in BEC at T=0

Helium II (liquid)



diluteness parameter $\sqrt{n_0 a_{sc}^3} \sim 1$

<u>Easier to see superfluid properties (1938)</u>

Associated phenomena:

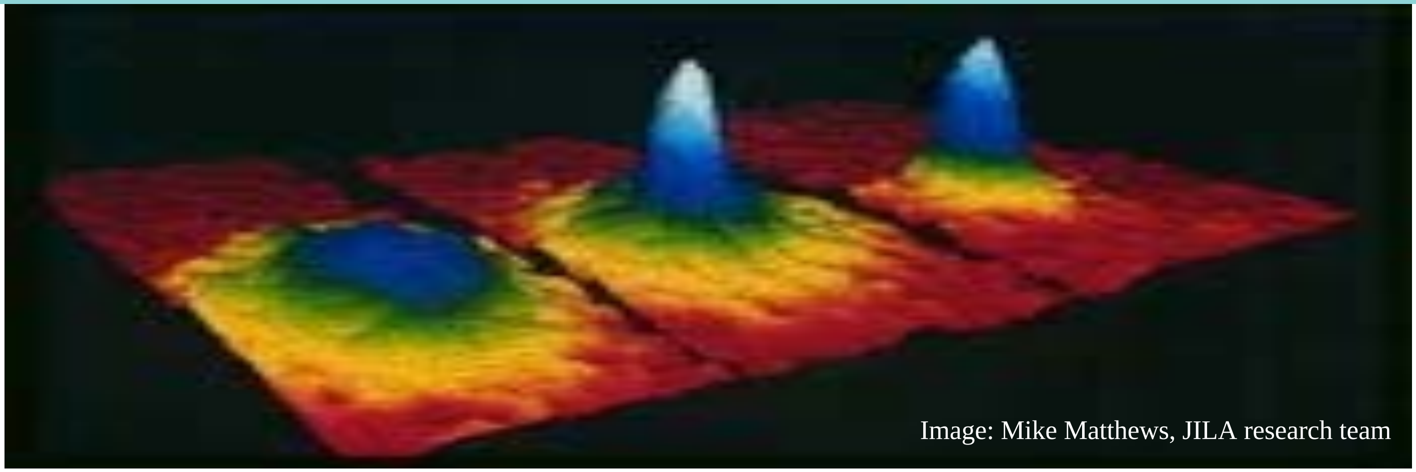
- Dissipationless flow (Kapitza, 1938)
- Persistent currents, bubble-free boiling, etc.

⋮

<u>Difficult to see BEC (1980s)</u>

- ~10% of atoms in BEC at T=0

Trapped atomic condensates: easier superfluids

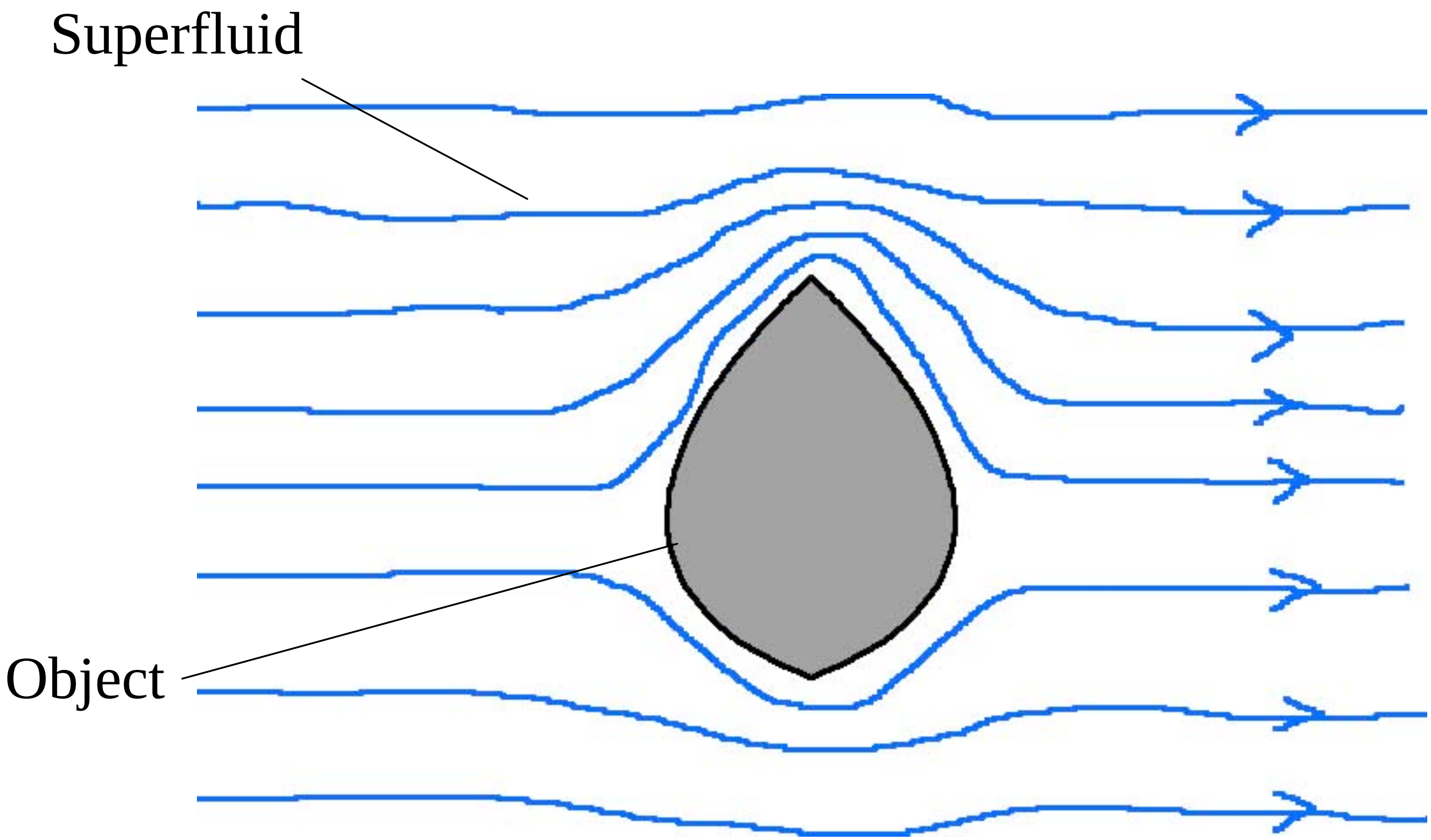


- Achieved in 1995 – Nobel Prizes so far 1997, 2001
- Applications - atom lasers, quantum computing, better clocks, etc.
- Testing ground

Ideal medium to study superfluidity

- Better understood theoretically – because of $\sqrt{n_0 a_{sc}^3}$
- More precise control experimentally
  - control atomic interactions using magnetic fields, confining potential

Recap



$$v < v_c$$

Does the object experience dissipation (a.k.a. drag force)?

Theory

Force on object:

describes BEC flow

$$\vec{F} = -\int d^3r \langle \hat{\psi}^\dagger(r)[\vec{\nabla}\Phi(r)]\hat{\psi}(r)\rangle_{T=0}$$

External potential describes stationary object → breaks translational symmetry

zero population of q.p's

All 'r's understood to be vector quantities

Mean field calculation

$\hat{\psi}(r) = \Psi^{(0)}(r)$ → order parameter

Symmetric about x=0

$$\vec{F}_{GPE} = -\int d^3r |\Psi^{(0)}(r)|^2 \vec{\nabla}\Phi(r)$$

=> density asymmetry in $|\Psi^{(0)}|^2$
→ leads to drag force

Assuming a steady state,

GPE: $(\hat{T} + \Phi(r) - \mu)\Psi^{(0)}(r) + |\Psi^{(0)}(r)|^2\Psi^{(0)}(r) = 0$

scaled by healing length $\xi = \dfrac{1}{\sqrt{8\pi n_0 a_{sc}}}$

scattering length
(repulsive interactions)

where $\hat{T} \equiv -\vec{\nabla}^2 + \sqrt{2}i\bar{v}\dfrac{\partial}{\partial x} + \dfrac{\bar{v}^2}{2}$

Galilean transformation
for moving flow

Note: $\bar{v}_s = 1$

Notes: - Flow is in x-direction
- $\bar{v}$ is flow velocity far from potential

What is known theoretically?
Some drag results from GPE

Note: all velocities at infinity

- Drag on weak repulsive impurity (linear analysis)
  3-d, 2-d; $v_c = v_L = c_s$ (*Astrakharchik and Pitaevskii, 2004*)

- Repulsive potential
  1-d; $0 < v_c \leq v_L$ (creation of gray solitons – *Hakim, 1997; Pavloff, 2002)*

<u>Numerical simulations of macroscopic objects</u>

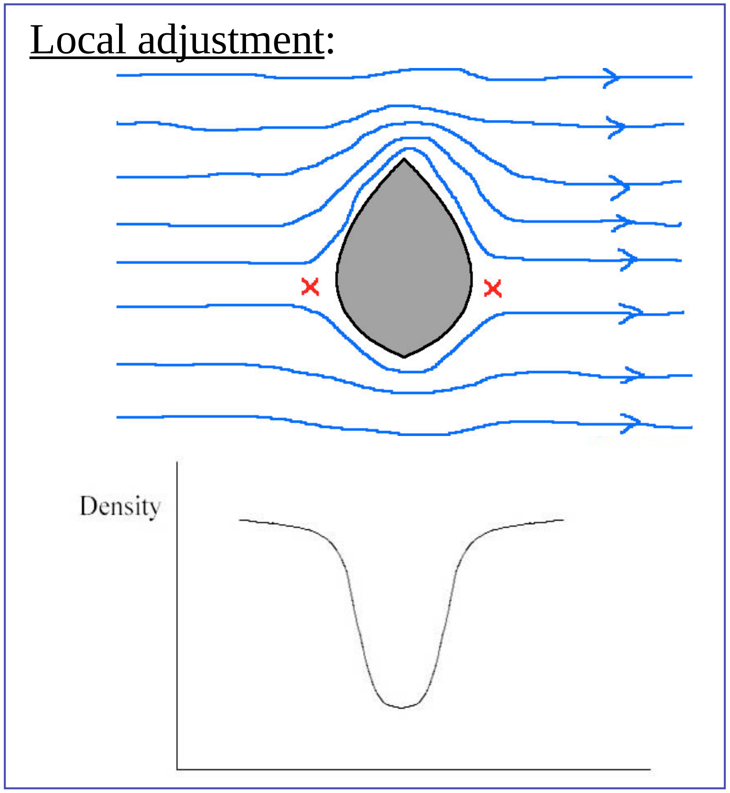
- Vortex shedding
  –in 2-d, $v_c/c_s \approx 0.4$ (*Frisch, Pomeau, Rica, 1992; Winiecki, McCann, Adams, 1999; Huepe, Brachet, 1997*)

  –in 3-d, $v_c/c_s \approx 0.1$ (*Adams et al*)

  –Complications:
  - edge effects (*Fedichev & Shlyapnikov*)
  - vortex stretching (*Brachet et al*)
  - etc.

Lesson from GPE

If max. local fluid velocity > v_L → dissipation/drag
(non-linear effects: vortex shedding, etc.)



GPE (mean field)

↓

Non-zero modified critical velocity

↓

Dissipationless flow exists
(at level of mean field approx.)

Recall d'Alembert's paradox in a potential flow

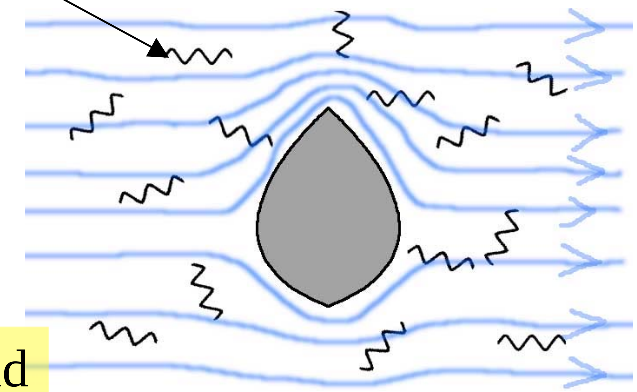
What is missing?
GPE (mean field) ignores quantum fluctuations

$$\hat{\psi}(r) = \Psi^{(1)}(r) + \hat{\phi}(r)$$

quantum field

condensate (mean field)

quantum fluctuations



until now viewed as negligible corrections to mean field

q.f's scale as $\sqrt{n_0 a_{sc}^3}$ << 1 for (typical) alkali condensates]

à can be ignored for many purposes

e.g. density profile, collective oscillations, interference, quantized vortices, etc.

But q.fs have indirect experimental consequences

–shift of collective frequencies (Pitaevskii, Stringari, 1999)

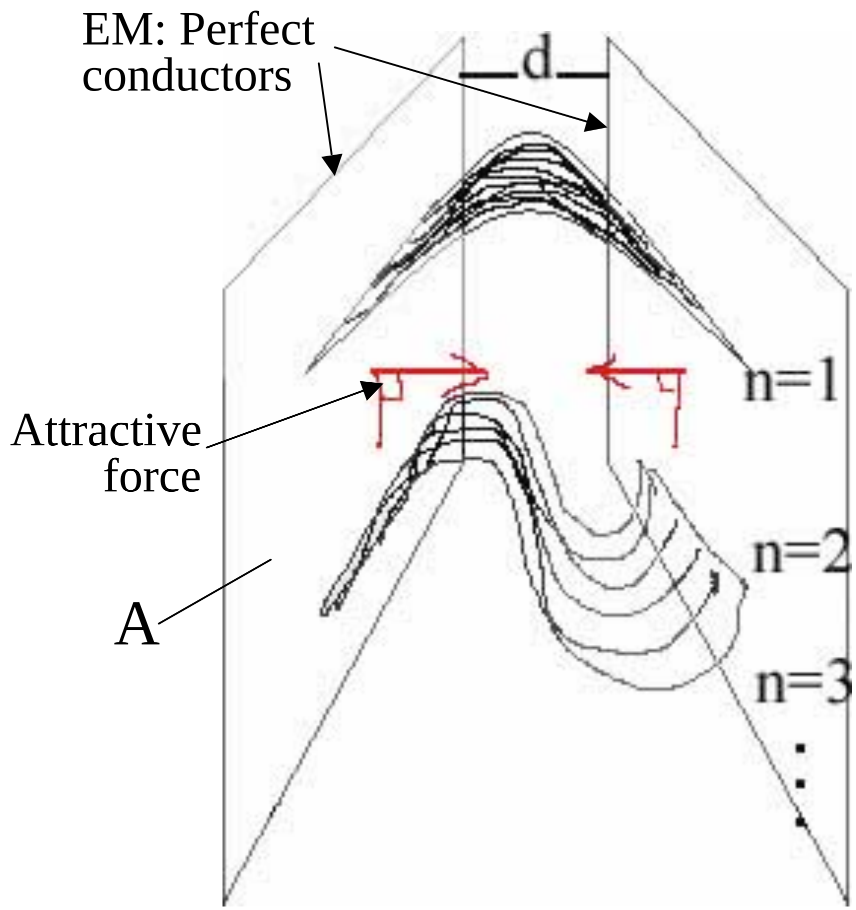
–suppression of density fluctuations in the phonon regime (Ketterle, 1999)

⋮

Aside:

EM vacuum
(Casimir, 1948)

$$F_{EM} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$

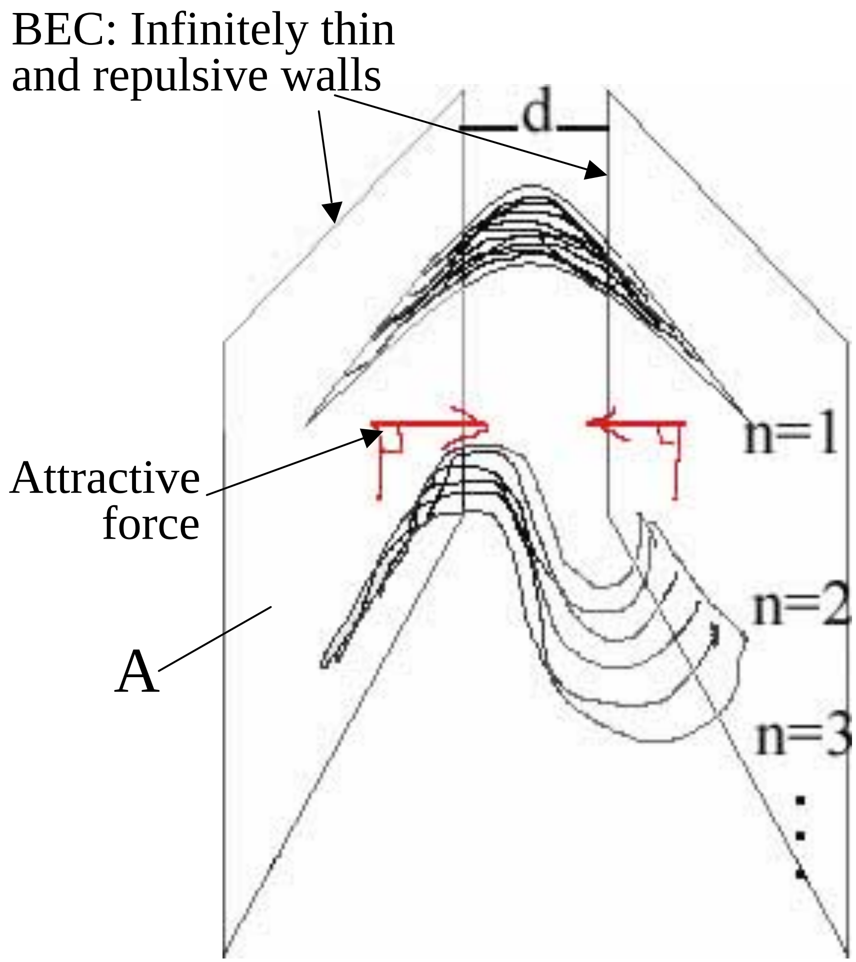


Boundary conditions on (static) EM vacuum => force

Static Casimir force in superfluid (BEC)

Excitation vacuum in BEC

$$F_{BEC} \approx -\frac{\pi^2}{480}\frac{\hbar c_s}{d^4}$$



Boundary conditions on (static) excitation vacuum => force

D.C.R. and Y. Pomeau. Modern Encyclopedia of Mathematical Physics MEMPhys (2008)

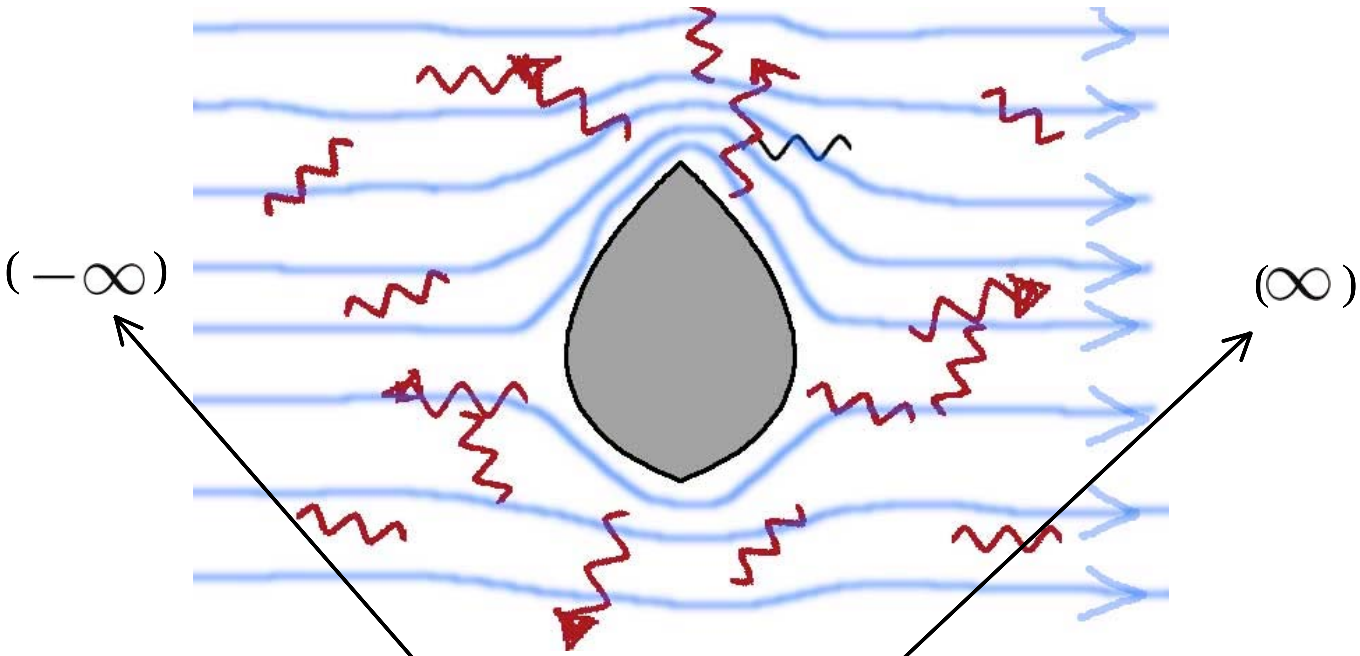
Casimir force in EM and BEC: Physical manifestation of q.fs

EM vacuum (Casimir, 1948)	Quasiparticle vacuum in BEC
polarizations (→ 2); speed of light (→ c) $F_{EM} = -2\frac{\pi^2}{480}\frac{\hbar c}{d^4}$	polarizations (→ 1); speed of sound (→ c_s) $F_{BEC} \approx -1\frac{\pi^2}{480}\frac{\hbar c_s}{d^4}$
$\epsilon_k^{EM} = c\sqrt{k_x^2 + k_y^2 + \left(\frac{n\pi}{d}\right)^2}$	$\epsilon_k^{BEC} \approx c_s\sqrt{k_x^2 + k_y^2 + \left(\frac{n\pi}{d}\right)^2}$ dominated by low k part of energy spectrum

Putting boundary conditions on (static) vacuum => force

Can quantum fluctuations produce a drag force at $v < v_L$?

Boundary conditions on excitation vacuum => drag force?



non-local perturbation

No EM analogy in moving case

Calculation of Casimir-like drag

Fluctuations with non-uniform medium (object)

measure of density asymmetry

symmetric

$$\vec{F} = -\int d^3r \left(|\Psi^{(1)}(r)|^2 + \langle \hat{\phi}^\dagger(r)\hat{\phi}(r)\rangle_{T=0} \right) \vec{\nabla}\Phi(r).$$

f_{cond}: Force from condensate modified by fluctuations

f_{fluc}: Force from fluctuations

Reminder: $\hat{\psi}(r) = \Psi^{(1)}(r) + \hat{\phi}(r)$

Expand $\hat{\phi}(r)$ in terms of q.p. operators

$$\hat{\phi}(r) = \sum_{k'} \left(u_k(r)\hat{\alpha}_k - v_k^*(r)\hat{\alpha}_k^\dagger \right)$$

Weakly interacting particles → Non-interacting q.p.:

Ignoring quasiparticle interactions (Beliaev/Landau terms)

$$\hat{H} = \sum_{k'} E_k \hat{\alpha}_k^\dagger \hat{\alpha}_k - \underbrace{\sum_{k'} E_k \int d^3r |v_k(r)|^2}_{} + E_0$$

Quantum depletion/fluctuations
$\propto E_0 \sqrt{n_0 a_{sc}^3}$ for uniform gases

$$E_k = \sqrt{2}\bar{v}k_x + k\sqrt{k^2+2}$$

Definition of T=0: $\langle \hat{\phi}^\dagger(r)\hat{\phi}(r) \rangle_{T=0} = \sum_{k'} |v_k(r)|^2$
(zero population of q.p.)

Equations governing quantum fluctuations (non-uniform medium)

Bogoliubov-de Gennes equations:

$$\hat{\mathcal{L}} u_k(r) - (\Psi^{(0)})^2 v_k(r) = E_k u_k(r)$$
$$\hat{\mathcal{L}}^* v_k(r) - (\Psi^{(0)*})^2 u_k(r) = -E_k v_k(r)$$

moving BEC

$$\hat{\mathcal{L}} = \hat{T} + \Phi(x) - \mu + 2|\Psi^{(1)}|^2 \qquad E_k = \sqrt{2}\bar{v}k_x + k\sqrt{k^2+2}$$

Normalization: $\int d^3r \left(|u_k(r)|^2 - |v_k(r)|^2\right) = 1$ → q.p's obey bosonic commutation relations

Generalized GPE (*Castin and Dum, 1998*):

(T=0)

Ensures orthogonality between excited states and condensate

$$GPE + \sum_{k'} \left[2|v_k(r)|^2 - u_k(r)v_k^*(r) - \sum_{k'} c_k v_k^*(r) \right] \Psi^{(1)}(r) = 0$$

UV divergent because of contact approximation (need to renormalize)

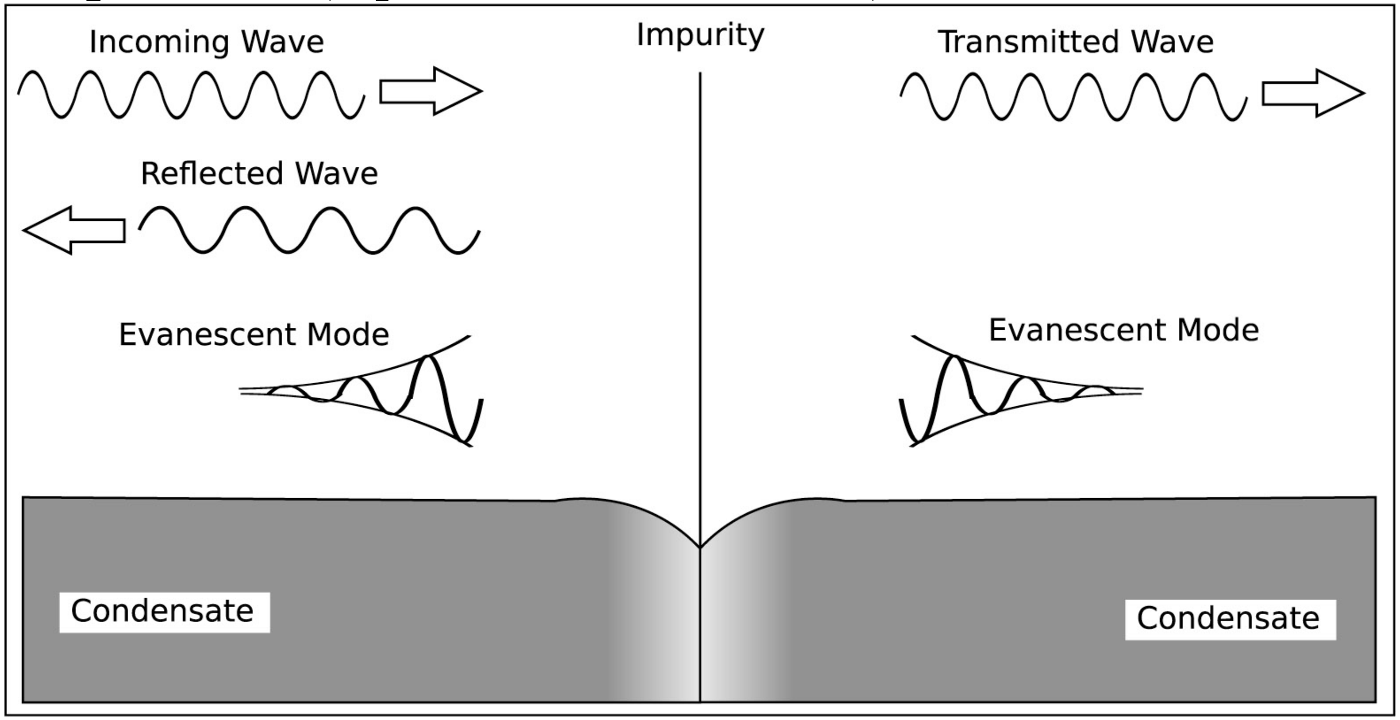
In general acts as effective complex potential
=> mass transfer between condensate and fluctuations

Drag on an impurity in a moving quasi-1D condensate

Impurity $\Phi(x) = \eta\delta(x)$

Any size

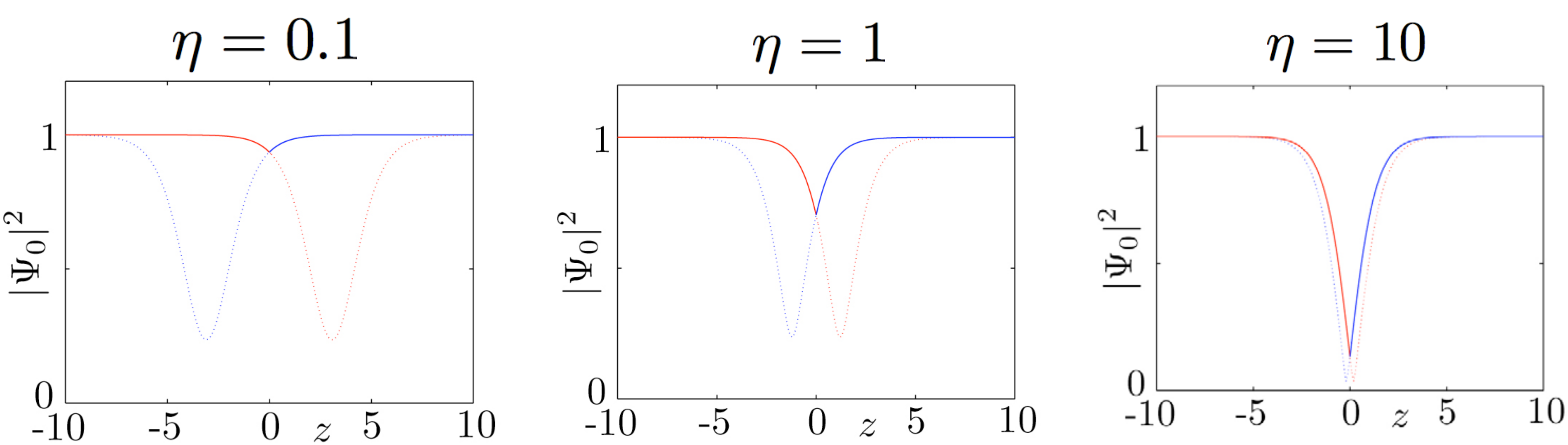
Exact solution for the Bogoliubov-de Gennes equations are possible (squared Jost solution)



In collaboration with Andrew Sykes, Matt Davis, University of Queensland

Mean field solution (Hakim 1998)

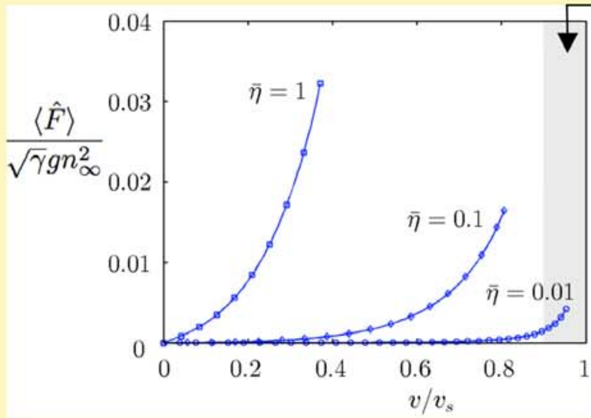
Amplitude of mean field solution for $v = v_c/2$:



Solid lines indicate the full solutions
Dotted lines indicate the soliton solutions

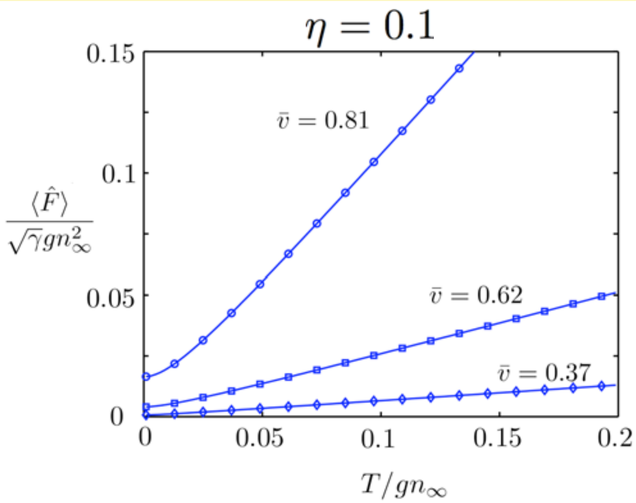
Drag force in quasi-1D condensate
(without GGPE contribution)

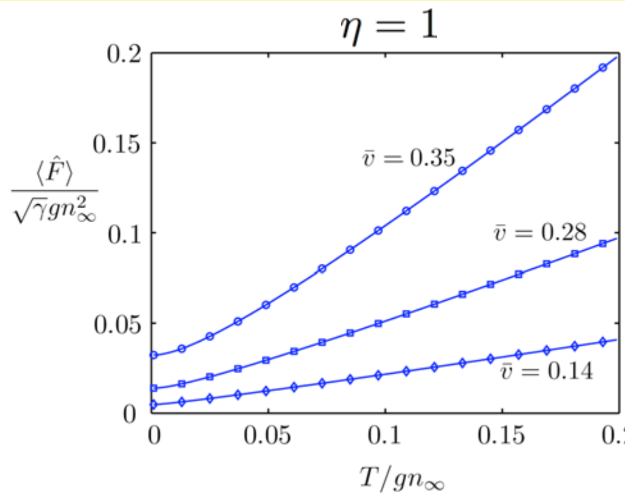
At T=0,
force vs. impurity velocity
for $0 \leq v \leq v_c$



Breakdown of Bogoliubov approximation

Low temp. dependence of drag force





$$\langle \hat{\phi}^\dagger(r)\hat{\phi}(r)\rangle_{T=0} = \sum_k f_k(|u_k(r)|^2 + |v_k(r)|^2) + |v_k(r)|^2$$

$$f_k \rightarrow \frac{1}{e^{\beta\epsilon}-1}$$ (Thermal distribution of quasiparticles, $T \ll T_c$)

$$\gamma = \frac{mg}{\hbar^2 n_\infty} \ll 1$$

Point impurity in a 3-D superfluid

scattering length characterizing particle-impurity interactions

$$\Phi(r) = \eta\delta^{(3)}(r) \quad \text{where} \quad \eta = \frac{b}{a_{sc}}\frac{1}{n_0\xi^3} \ll 1$$

uv cutoff: $\Lambda_{uv} \sim \frac{\xi}{a_{sc}} \gg 1$ due to

$$F_x = \frac{64\sqrt{2}}{3}\pi^{3/2}\eta^2 p_0 \xi^2 \sqrt{n_0 a_{sc}^3} \ln \Lambda_{uv} \bar{v}$$

zeroth order interaction pressure: (mean field density) $\frac{g n_0^2}{2}$

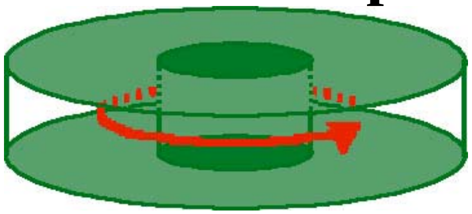
[D.C.R. PRA 74, 013613]

- Exists at all velocities and is consistent with a dissipative force
- Dominant effect at $v < v_c$
- $\Phi(r) = \Phi(x)$ - 1-D potential [D.C.R. and Y. Pomeau. PRL 95, 145303]

What effect experimentally?

Speculation:

Consistent with persistent current experiments in liquid Helium



The 'super' in superfluidity is a finite size effect!

New observables:

1) Time scale of effect $\propto$ Length of system / v_s
   - gives direct observable effect of quantum fluctuations
   - v_S very fast in Helium ~ 50 m/s so can understand why effect has not been seen
   - relatively slow in trapped gases ~ 1 cm/s so should be observable

2) Detect scattered fluctuations as small amount of heating
   - normal gas at zero temperature!

3) New boundary condition $\vec{n} \cdot (\vec{j}_n - \vec{j}_s) = \alpha(T_b - T) + \beta v_s^2$

Surface roughness

$\rightarrow$ in equilibrium, $T = T_b + \dfrac{\beta}{\alpha} v_s^2$

[Y. Pomeau, D.C.R., PRB 77 144508 (2008)]

Summary

Is BEC always dissipationless as $v \rightarrow 0$?
$\Rightarrow$NO, because of quantum fluctuations

1) For a 3D BEC, for all $v < v_c$, $F \propto v$ for a weak point impurity at T=0
2) Also solved for force at T=0 for impurity moving at any small v
3) Consistent with persistent currents but adds a new timescale where backscattering becomes relevant

Note :

- Consistent with experiments - semblance of v_c as the dominant mean field effect
- However, Casimir-like drag is the dominant term when $v < v_c$ and is potentially measurable

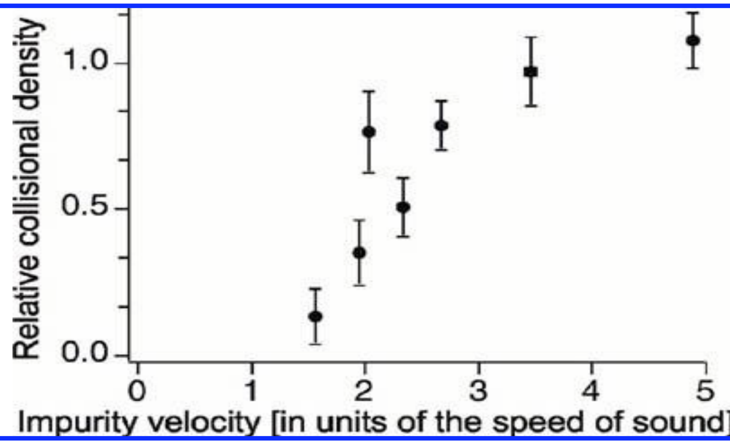
Outlook

- Way to detect superfluid-Mott insulator transition
- BCS fluids (Rishi Sharma, LANL)
- Toroidal trap experiments (LANL, Berkeley, NIST) with persistent currents
- Numerical simulations to show force is dissipative and to test backscattering hypothesis; surface roughness (dilute BECs with Matt Davis, University of Queensland)
- Iordanskii-like force on moving vortices at T=0

What is known experimentally?

Experimental evidence for dissipationless flow in dilute BECs

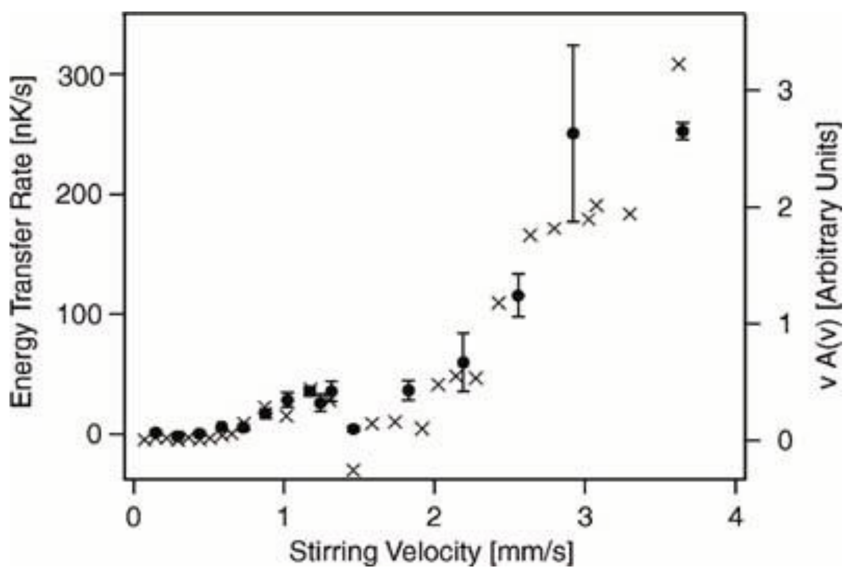
Impurity collisions:



A.P. Chikkatur *et. al.* PRL **85**, 483 (2000)

Moving macroscopic object (blue-detuned laser) through condensate:

heating rate



asymmetry (drag force)

$v_c \approx 0.1 c_s$

R. Onofrio *et. al.* PRL **85**, 2228 (2000)

Experimentally measured parameters

cross-sectional area

$$F_x = -\eta^2 p_0 A \sqrt{n_0 a_{sc}^3} f(\bar{v})$$

zeroth order interaction pressure: (mean field density) $\frac{g n_0^2}{2}$

all q.fs in 3-d

$$f(\bar{v}) \equiv \int d^3k \left(f_{cond}(k,\bar{v}) + f_{fluc}(k,\bar{v})\right)$$

$$F_x \approx -n_0^2 g A \frac{\Delta n_0}{n_0}$$

density asymmetry: $\sigma(\bar{v}) = \eta^2 \sqrt{n_0 a_{sc}^3} f(\bar{v})$

Heating rate (energy transfer per atom):

$$\frac{dE}{dt} = \frac{\vec{F}\cdot\vec{v}}{N} \approx n_0 g \frac{c_s}{L} \bar{v} \sigma(\bar{v})$$

typical condensate length

Typical experimental parameters:

$n_0 g \approx 100\,\mathrm{nK}$ $\quad c_s \approx 10\,\mathrm{cm}/s$ $\quad L \approx 10\,\mu\mathrm{m}$

Resolution achieved in MIT experiment is 10nK/s

→ $\sigma(\bar{v}) \gtrsim 10^{-4}$ to detect Casimir drag effect