

"van der Waals" "Hamaker"  
"Casimir" "Lifshitz"  
"fluctuation" "electrodynamic"  
**Where now? What next?**

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and many friends

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<http://lpsb.nichd.nih.gov>



# Desiderata

Solve for more geometries and for real materials (pure Casimir is not questionable as an idea, but does it ever really exist?)

Piece apart the “zero-frequency” term

(dominates at long distances, especially at room temperature)

(important with polar materials, always with water)

Control conditions of measurements

-- to see if we are measuring what we think we are measuring

(high vacuum for clean surfaces)

(optical data, on samples themselves, to go with measurement)

Design shapes and materials so as to modify forces

Talk with each other! Nicely!!

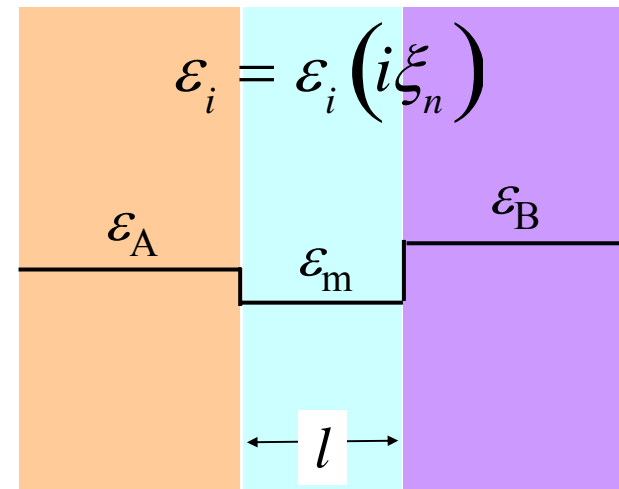
# Lifshitz et al.: half-spaces A and B across medium m

$$G_{\text{AmB}}(l) = -\frac{A(l)}{12\pi l^2} \quad A(l) = \frac{3kT}{2} \sum_{\text{Matsubara sampling frequencies } \xi_n} \left( \frac{\epsilon_A - \epsilon_m}{\epsilon_A + \epsilon_m} \right) \left( \frac{\epsilon_B - \epsilon_m}{\epsilon_B + \epsilon_m} \right) \times R_n(l)$$

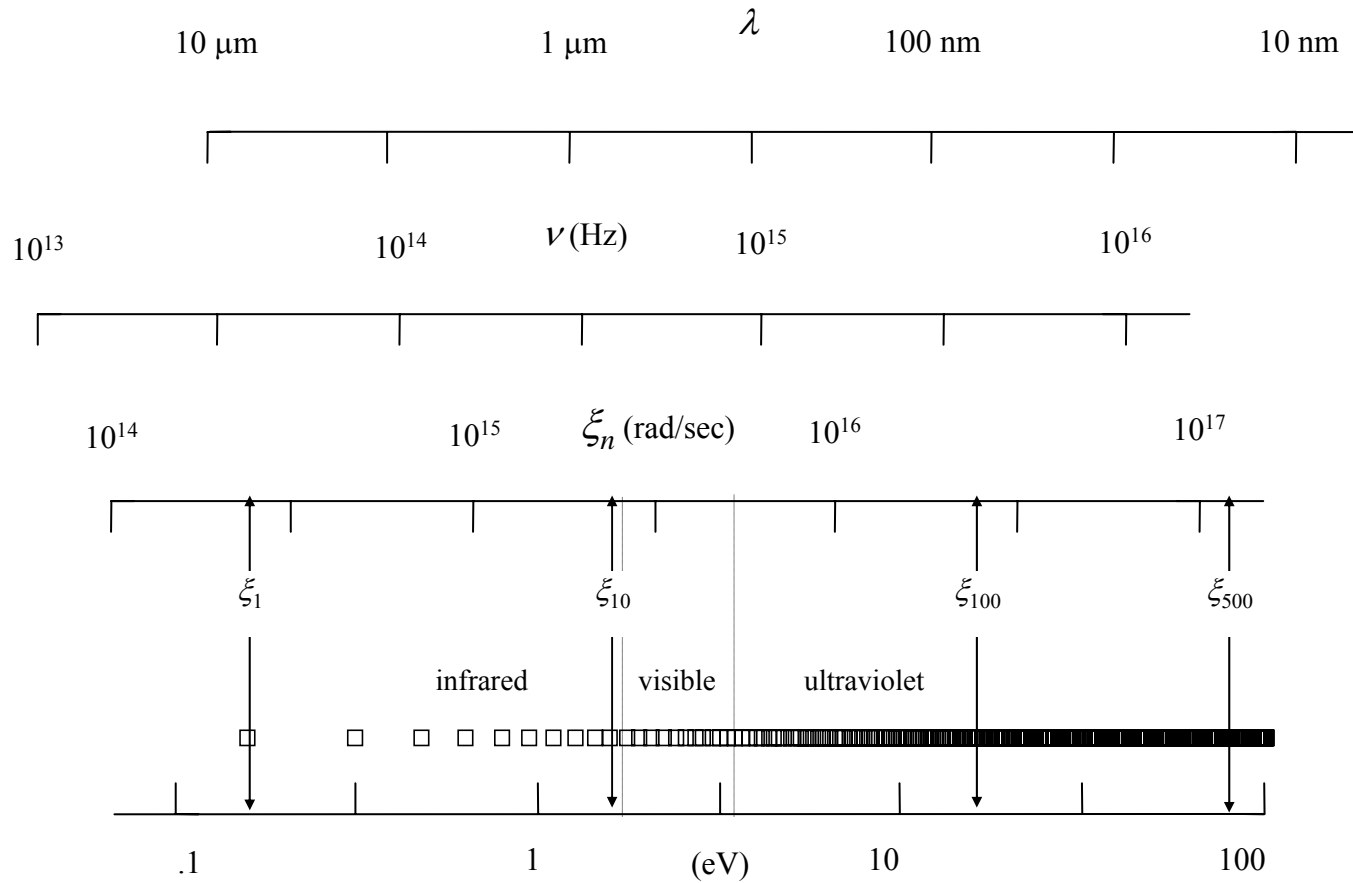
$$h\xi_n \equiv 2\pi kTn \quad R_n \approx (1 + r_n)e^{-r_n}$$

$$r_n = \left( \frac{2l}{c} \right) / \left( \frac{1}{\xi_n} \right) = \frac{2l}{\lambda_T} n; \quad \lambda_T \equiv \frac{hc}{2\pi k_B T_{\text{room}}} = 1.2 \mu\text{m}$$

$l =$	100 nm	1 $\mu\text{m}$	10 $\mu\text{m}$
$n: r_n = 1$	$n = 6$	$n = .6$	$n \ll 1$
$r_1 =$	.16	1.6	16
$R_1 =$	$\sim 1$	$< 1/2$	zero!



# Matsubara frequencies at $T_{\text{room}}$ , as $\lambda$ , $\nu$ , and $\xi_n \propto \pi \nu$



The first sampling frequency after  $\xi_0 = 0$ , corresponds to IR vibrations.

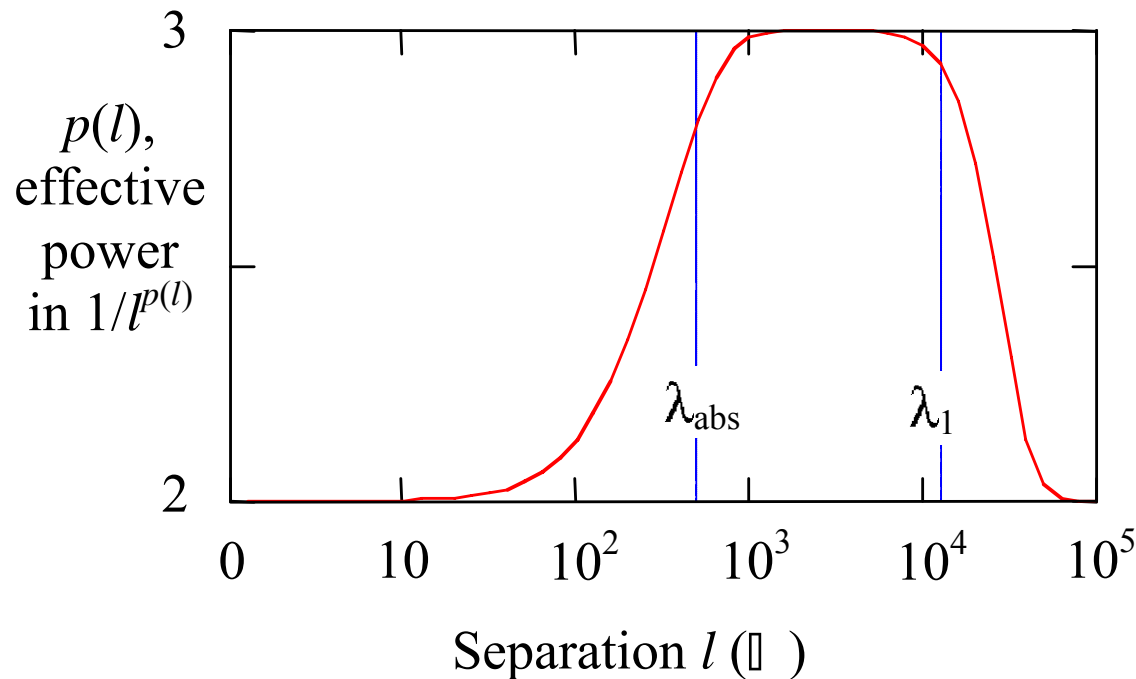
$$\lambda_1 = 2\pi c / \xi_1 = 7.82 \times 10^{-4} \text{ cm} = 7.82 \mu\text{m} = 7.82 \times 10^4 \text{ Angstroms.}$$

“Power law” from a single-oscillator,

$$\lambda_{\text{absorption}} = 500 \text{ Angstrom},$$

$$\epsilon_{\text{left}} = \epsilon_{\text{right}} \text{ across vacuum}$$

$$G_{\text{AmB}}(l) = -\frac{A_{\text{Am/Bm}}(l)}{12\pi l^2} \quad p(l) = -\frac{d \ln(G_{\text{AmB}}(l))}{d \ln(l)}$$



# Two metals at $T_{\text{room}}$ $l \sim 10 \mu\text{m}$ in vacuum, never Casimir

$$A(l) = \frac{3kT}{2} \sum_{\text{Matsubara sampling frequencies } \xi_n} \left( \frac{\epsilon_A - \epsilon_m}{\epsilon_A + \epsilon_m} \right) \left( \frac{\epsilon_B - \epsilon_m}{\epsilon_B + \epsilon_m} \right) \times R_n(l) \rightarrow \frac{3kT}{4}$$

$$G(l) \approx -\frac{A(l)}{12\pi l^2} = -\frac{3kT}{48\pi l^2} = -\frac{kT}{16\pi l^2} \quad \text{without magnetic terms}$$

$$G(l) \approx -\frac{3kT}{24\pi l^2} \zeta(3) = -\frac{kT}{8\pi l^2} \zeta(3) \quad \text{with magnetic terms, and proper summation}$$

vs. Casimir ( $T = 0 \text{ K}$ )

$$G_{\text{Casimir}}(l) \approx -\frac{\hbar c \pi^2}{720 l^3}; P_{\text{Casimir}}(l) \approx -\frac{\hbar c \pi^2}{240 l^4}$$

Skinny cylinder parallel to plane,  
separation  $z$ , radius  $R$ , thermal de Broglie wavelength  $\lambda_T$   
traditional pairwise sum vs. Collective fluctuations

Pairwise  
summation

$$E(z) : -kT \frac{R^2}{z^3} L$$

Metallic wire

Emig Jaffe  
Kardar,  
Scardicchio

$$E(z) : \frac{kT}{z \ln(R / \lambda_T)} L; \quad R = \lambda_T = z$$

PRL 96:  
080403  
(2006)

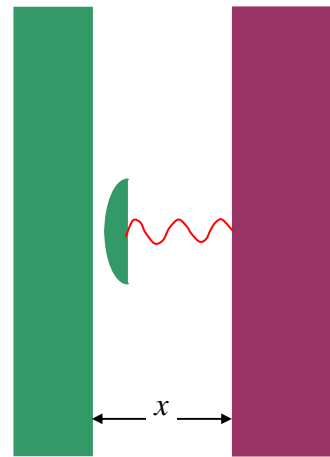
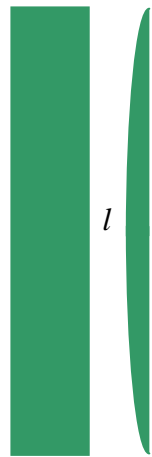
Measurements not often noticed



## Force balances (room temperature)

Glass (Derjaguin, Lifshitz, Abrikosova, 1950's)

Mica (Tabor, Winterton, Israelachvili, 1970's)

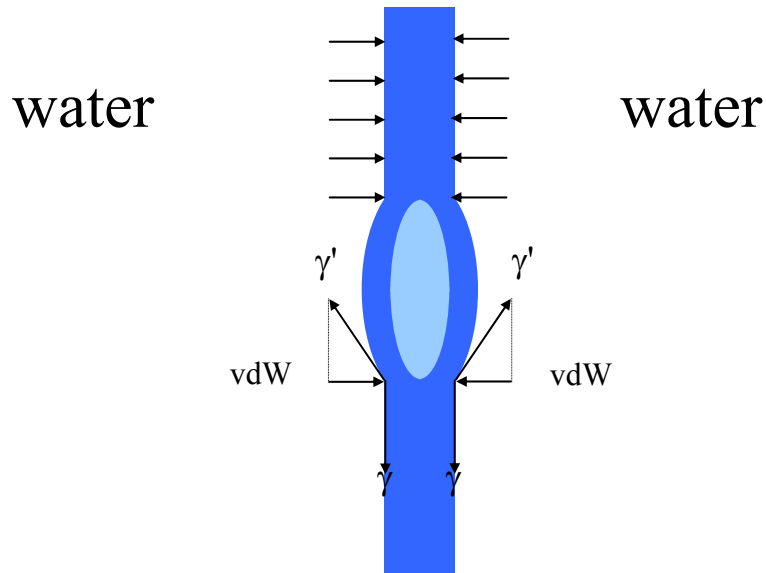


B. V. Derjaguin, "The force between molecules" *Scientific American*, 203:47 - 53 (1960);

B. V. Derjaguin, I. I. Abrikosova & E. M. Lifshitz, "Direct measurement of molecular attraction between solids separated by a narrow gap" *Quarterly Reviews (London)*, 10: 295 - 329 (1956).

J. N. Israelachvili & D. Tabor, *Van der Waals Forces: Theory and Experiment*, vol. 7, pp. 1-55, *Progress in Surface and Membrane Science*, 1973

# Forces across bilayers (Haydon & Taylor, 1968)



Hydrocarbon  
membrane

~ 4 nm thick layer  
with trapped hc drop

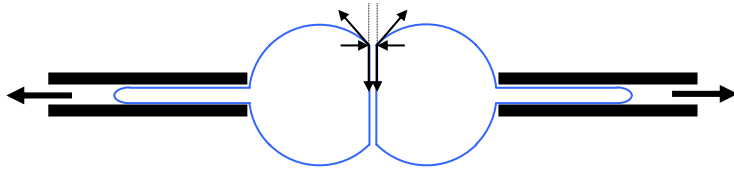
By the strength with which they flatten against each other, two juxtaposed bilayers create a measurable contact angle.

D. A. Haydon & J. L. Taylor,  
"Contact angles for thin lipid films  
and the determination of London-van  
der Waals forces" *Nature*, 217: 739 -  
740 (1968)

Parsegian, V.A, Ninham B.W.:  
Application of the Lifshitz theory to  
the calculation of van der Waals  
forces across thin lipid films. *Nature*  
224: 1197-1198 (1969)

# Forces between bilayer vesicles (Evans, Rand)

Water inside and outside

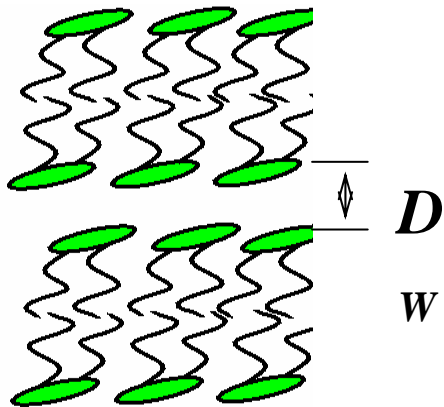
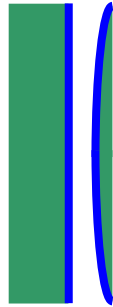


Suck on vesicles with pipettes

In practice, van der Waals forces appear mixed with lamellar motions as well as with repulsive hydration forces.

E. A. Evans, "Entropy-driven tension in vesicle membranes and unbinding of adherent vesicles" *Langmuir*, 7:1900-1908 (1991)

Between bilayers (Rand, Fuller, VAP, 1970's -90's)  
between bilayers on mica (Marra, Israelachvili)



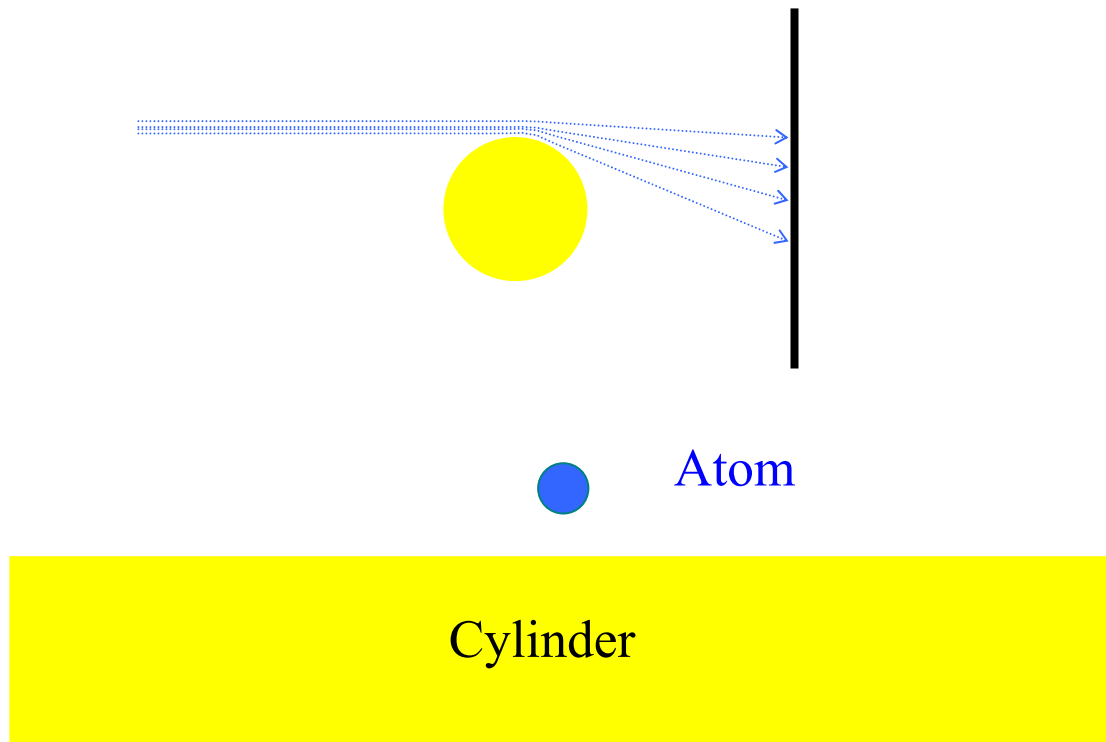
Between bilayers immobilized onto  
substrates

J. Marra & J. N. Israelachvili, "Direct  
measurements of forces between  
phosphatidylcholine and  
phosphatidylethanolamine bilayers in  
aqueous electrolyte solutions",  
Biochemistry, 24:4608-4618 (1985)

V. A. Parsegian, "Reconciliation of  
van der Waals force measurements  
between phosphatidylcholine bilayers  
in water and between bilayer-coated  
mica surfaces," Langmuir 9:3625-  
3628 (1993)

# Deflection of an atomic beam $T = T_{\text{room}}$

Shih, Raskin, Kusch (Columbia, NBS 1970's)



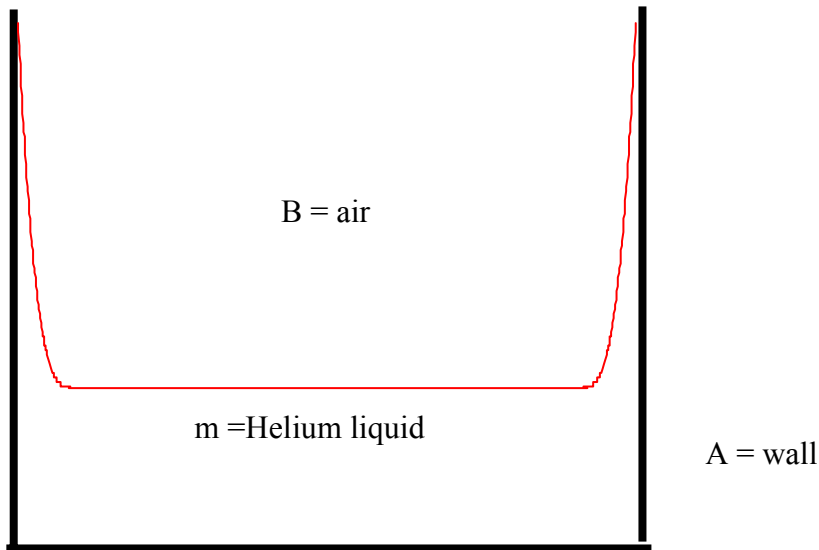
Arnold Shih & V. A. P.,  
"Van der Waals forces  
between heavy alkali  
atoms and gold surfaces:  
comparison of measured  
and predicted values",  
Phys Rev A, 12(3):835 -  
841 (1975)

Parsegian, V.A.: Formulae  
for the electrodynamic  
interaction of point  
particles with  
a substrate. Molecular  
Physics No. 6 27: 1503-  
1511 (1974)

Find a clean  $1/d^3$  power law (non-retarded point-particle to surface)

# Liquid helium crawling the walls

Sabisky & Anderson (1973)



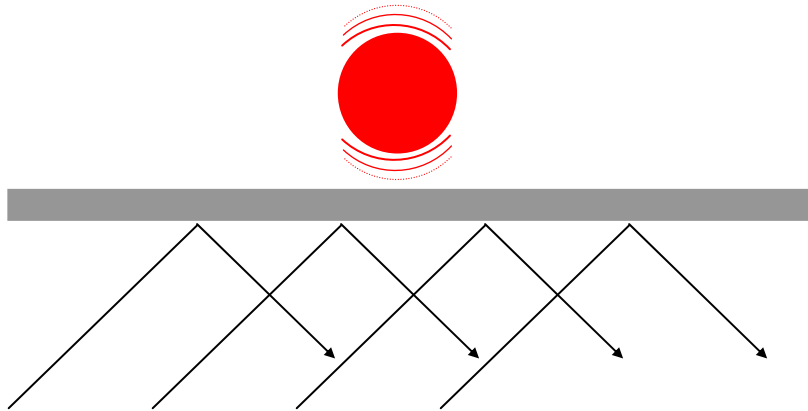
Put into a vessel, liquid helium will wet the walls, defying gravity with a layer of finite thickness

E. S. Sabisky and C. H. Anderson, "Verification of the Lifshitz Theory of the van der Waals Potential Using Liquid-Helium Films", *Physical Review A*, 7: 790-806, 1973

They measured the full spectrum of the substrate and used the full spectrum of the helium. This is one of the very few real (i.e., satisfactory) comparisons of measurement with theory!

# Colloids

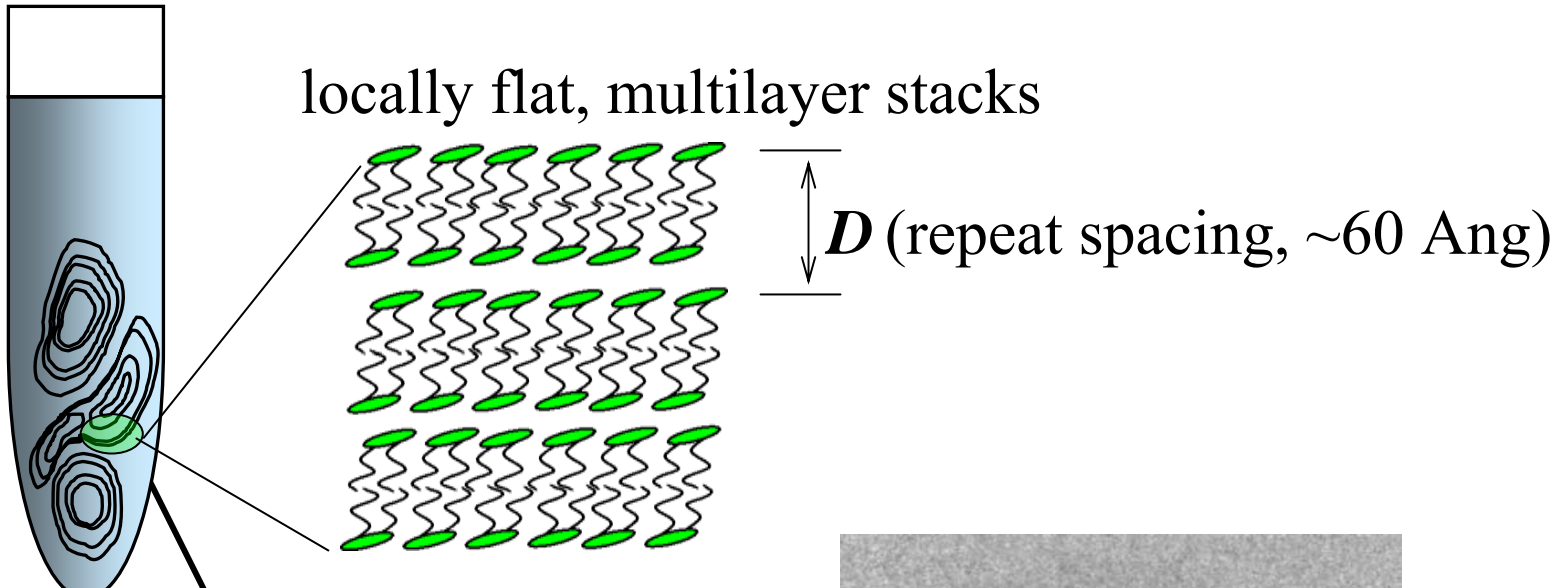
## D. Prieve



The bounce of particles, observed via reflected light, gives the force between sphere and flat.

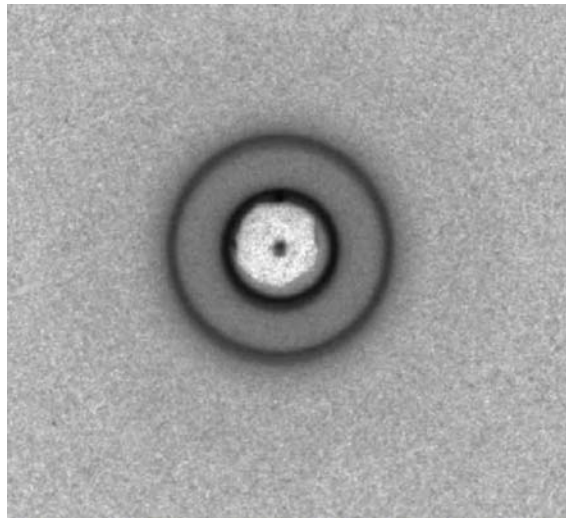
D. C. Prieve, "Measurement of colloidal forces with TIRM," *Advances in Colloid and Interface Science* 82:93-125 (1999)

# Multilayers: Neutral lipid bilayers in salt water



**Small-angle x-ray scattering**

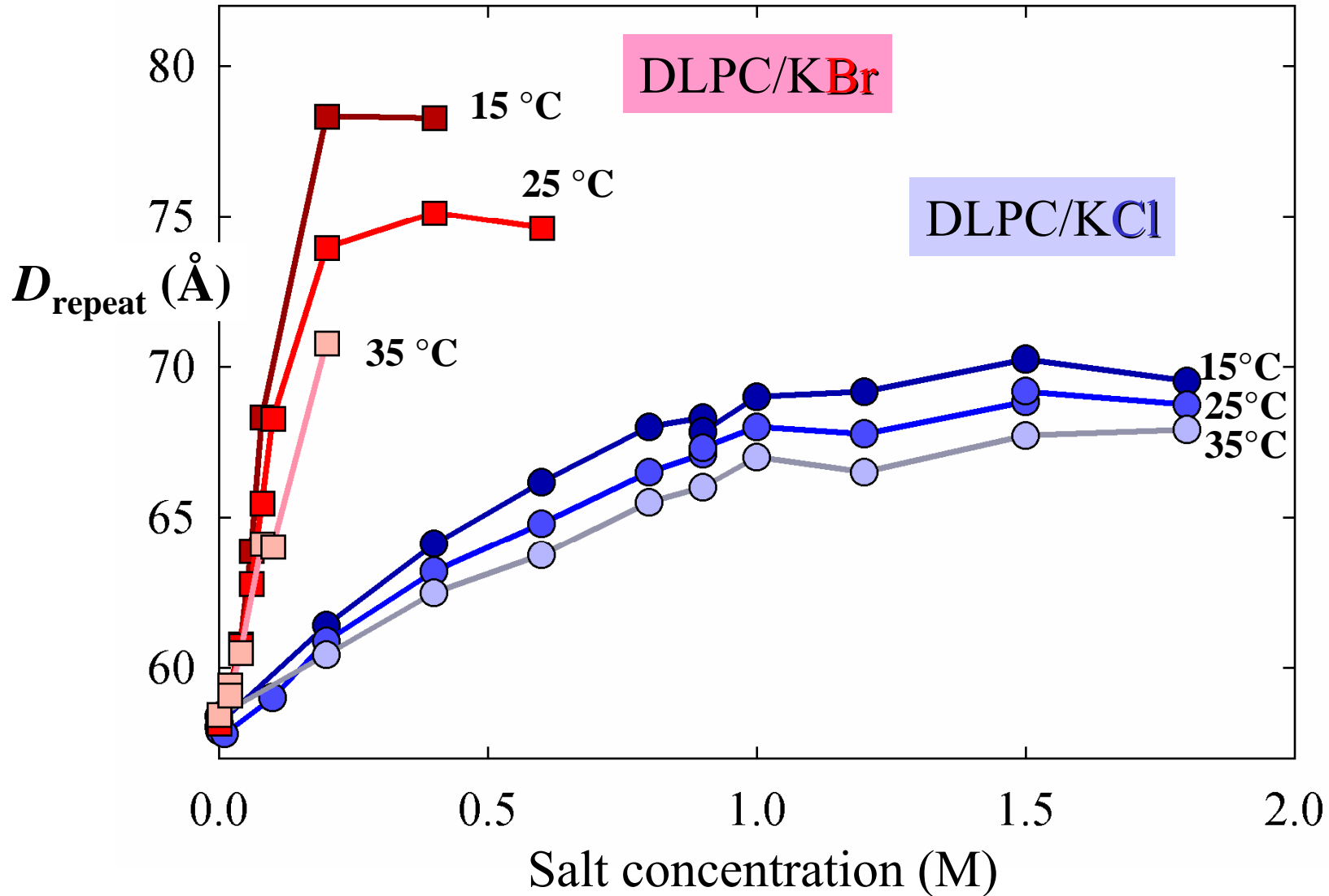
$$D = 2\pi/q \sim 60 \text{ \AA}$$





In excess solution, neutral lipids swell with added salt.

Horia Petrache et al (2006)



# Salt screening/weakening of vdW forces: three new ideas

H. I. Petrache, S. Tristram-Nagle, D. Harries, N. Kucerka, J. F. Nagle, V. A. Parsegian, Swelling of phospholipids by monovalent salt J. Lipid Res. 47, 302-309 (2006)

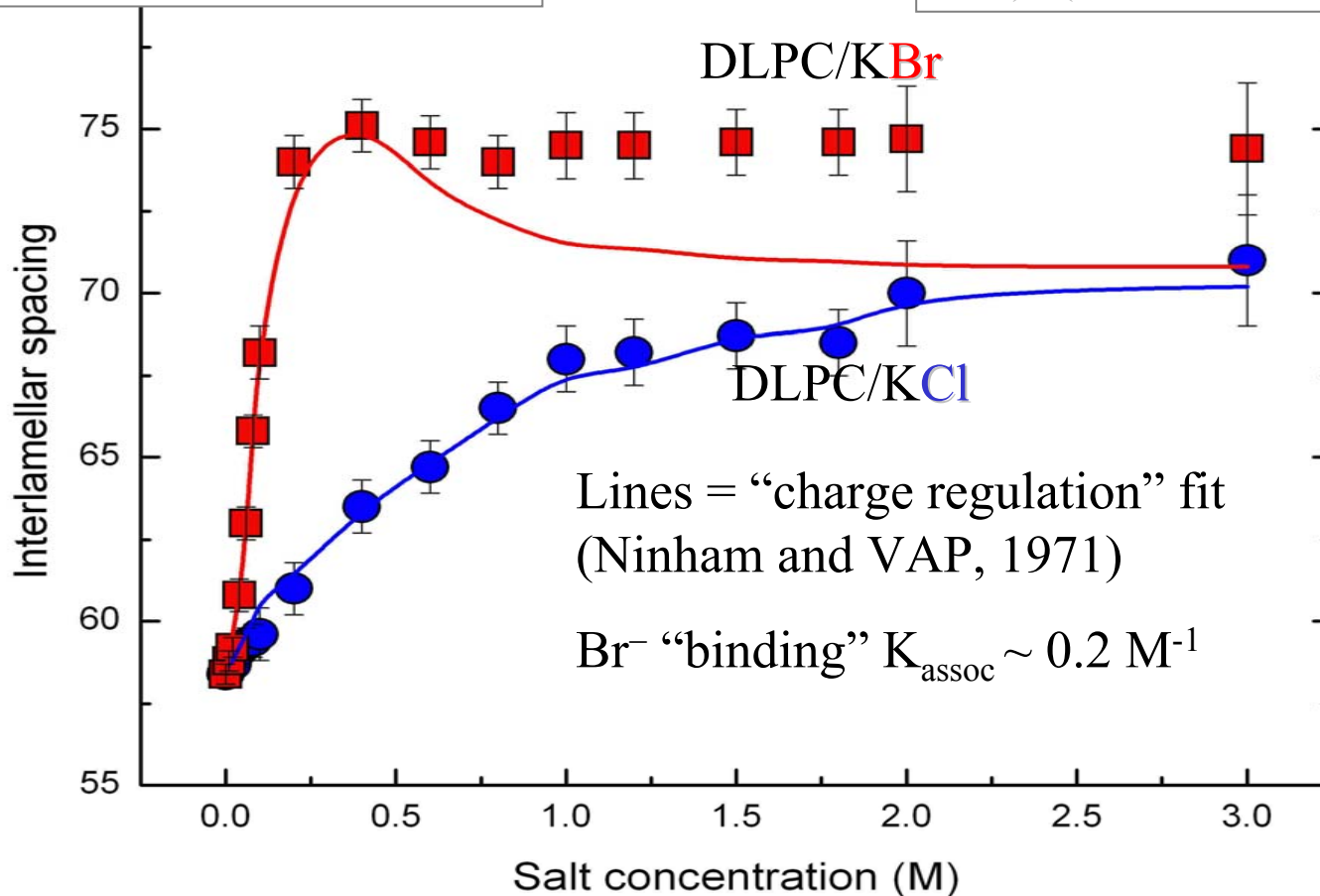
## Low salt:

\*screening of zero frequency vdW attraction (Ninham & VAP)

\*electrostatic repulsion from Br binding via vdW forces (Ninham)

## High salt:

\* vdW weakening at optical frequencies (refractive index of salt solutions increases with salt). (Rand & VAP)



Kevin Cahill:

How about first-order interactions?"

distance; cf. §02, PROBLEM 1, and §01, PROBLEM 1.

‡ This, of course, does not imply that the mean value of the interaction energy of the atoms is precisely zero. It diminishes exponentially with distance, i.e. more rapidly than every finite power of  $1/r$ , and hence each term of the expansion vanishes. This occurs because the expansion of the interaction operator in terms of the multipole moments involves the assumption that the charges of the two atoms are at a large distance  $r$  apart, whereas in quantum mechanics the electron density distribution has finite (though exponentially small) values even at large distances.

Landau & Lifshitz, Quantum Mechanics, footnote page 341

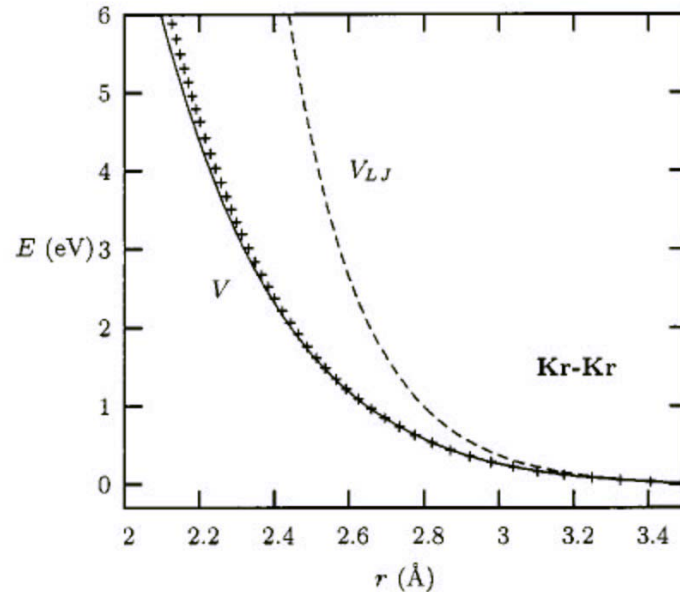
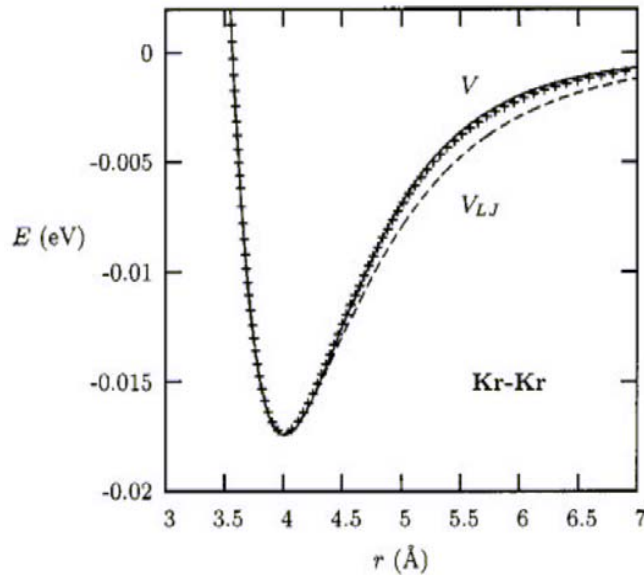
# First-order van der Waals forces atom-atom attraction

A Rydberg-like potential  $V_{\text{Rydberg}}$ ,

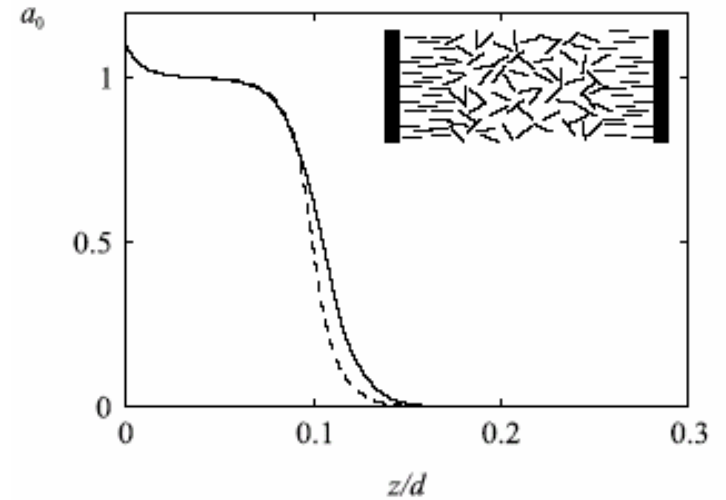
better than Lennard-Jones  $V_{\text{LJ}}$  6-12 potential generally used.

$$V_{\text{Rydberg}} = V(r) = ae^{-br} (1 - cr) - \frac{d}{r^6 + er^{-6}} \quad V_{\text{LJ}}(r) = |V(r_0)| \left[ \left( \frac{r_0}{r} \right)^{12} - 2 \left( \frac{r_0}{r} \right)^6 \right]$$

+ symbol “exact” numerical solution Meath and Aziz, Molec. Phys., 52, 225 (1984).



Pseudo Casimir effect for non-EM fields described with similar equations.



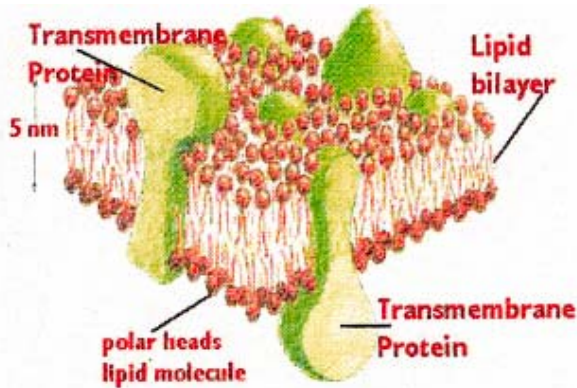
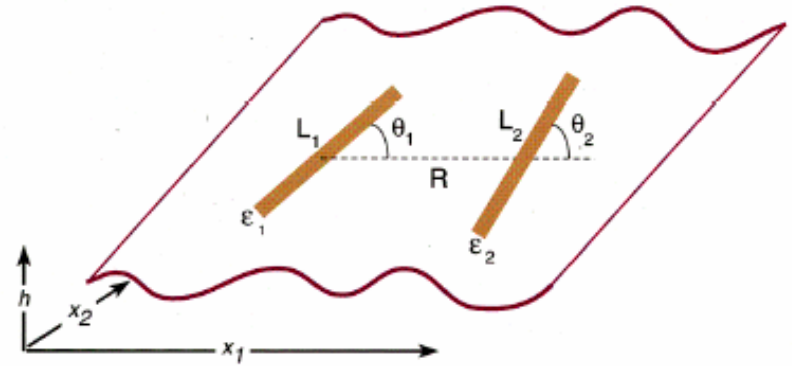
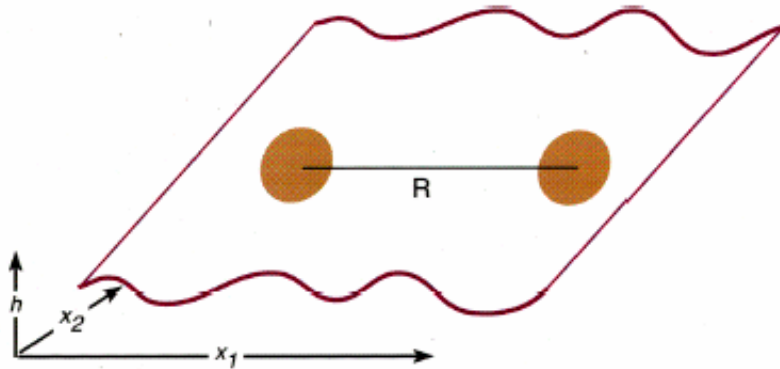
Nematic film with stiff boundaries  
(Ajdari, Duplantier, Hone, Peliti, Prost, 1982; Mikheev, 1989).

Nematic wetting (Ziherl, Podgornik and Zumer, 1998).

Smectic films (Li and Kardar, 1992).

# Membrane inclusions

(Goulian, Bruinsma, Pincus 1993, Golestanian, Goulian and Kardar, 1996)



Interaction between (lipid) membrane inclusions such as proteins.

Important in understanding aggregation of membrane proteins.

Thanks

# Lifshitz Theory and dielectric spectra of SWCNT

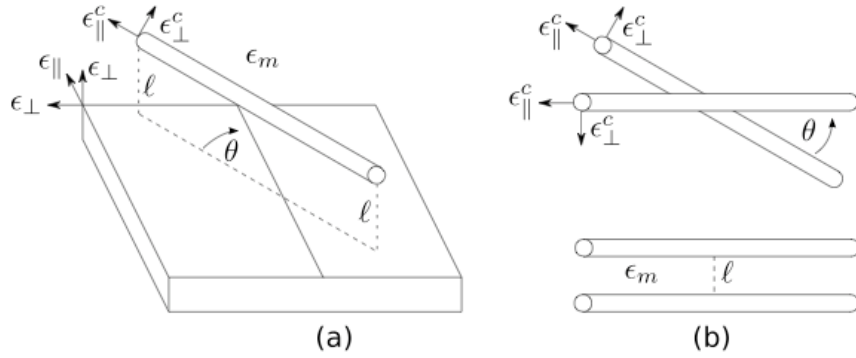


Figure 4: Schematic showing the geometry of the (a) anisotropic cylinder-anisotropic planar substrate and (b) anisotropic cylinder-anisotropic cylinder systems.

$$g(\ell, \theta) = -\frac{k_B T (\pi a^2)}{4\pi \ell^3} \left( \mathcal{H}^{(0)} + \mathcal{H}^{(2)} \cos^2 \theta \right).$$

$$\mathcal{H}^{(0)} + \mathcal{H}^{(2)} = \frac{1}{2\pi} \sum_{n=0}^{\infty} \int_0^{2\pi} d\phi \Delta_{\mathcal{L}m}(\phi) \left( \Delta_{\perp} + \frac{1}{4}(\Delta_{\parallel} - 2\Delta_{\perp}) \cos^2 \phi \right),$$

and for  $\theta = \frac{\pi}{2}$ ,

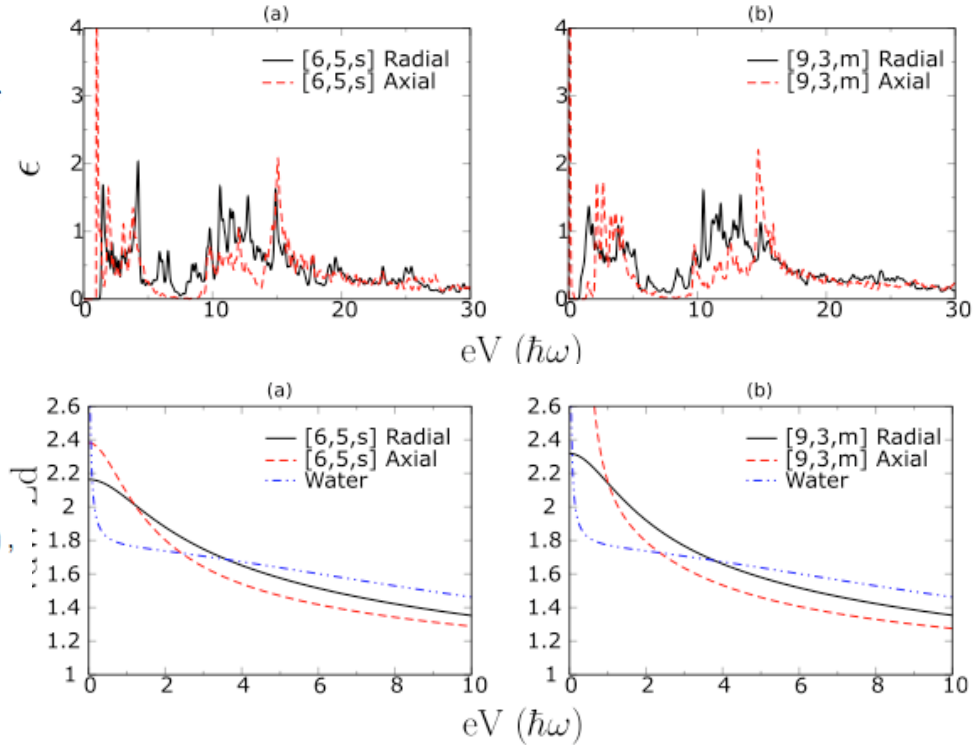
$$\mathcal{H}^{(0)} = \frac{1}{2\pi} \sum_{n=0}^{\infty} \int_0^{2\pi} d\phi \Delta_{\mathcal{L}m}(\phi) \left( \Delta_{\perp} + \frac{1}{4}(\Delta_{\parallel} - 2\Delta_{\perp}) \sin^2 \phi \right).$$

$$\Delta_{\perp} = \frac{\epsilon_{\perp}^c - \epsilon_m}{\epsilon_{\perp}^c + \epsilon_m}; \Delta_{\parallel} = \frac{\epsilon_{\parallel}^c - \epsilon_m}{2\epsilon_m}$$

van der Waals - London Dispersion Interactions for Optically Anisotropic Cylinders: Metallic and Semiconducting Single Wall Carbon Nanotubes

Rick Rajter††, Rudi Podgornik†\*, V. Adrian Parsegian†, Roger H. French §, W.Y. Ching§†

Phys Rev B Sub'td





$$G_{\text{AmB}}(l) = -\frac{A_{\text{Am/Bm}}(l)}{12\pi l^2} \quad A_{\text{Am/Bm}}(l) \approx \frac{3kT}{2} \sum_{n=0}^{\infty} \bar{\Delta}_{\text{Am}} \bar{\Delta}_{\text{Bm}} R_n(l)$$

$$\bar{\Delta}_{\text{Bm}} = \frac{\varepsilon_{\text{B}} - \varepsilon_{\text{m}}}{\varepsilon_{\text{B}} + \varepsilon_{\text{m}}} \quad \bar{\Delta}_{\text{Am}} = \frac{\varepsilon_{\text{A}} - \varepsilon_{\text{m}}}{\varepsilon_{\text{A}} + \varepsilon_{\text{m}}} \quad \varepsilon_i = \varepsilon_i(i\xi_n)$$

$$h\xi_n \equiv 2\pi kTn \quad \xi_n(T_{\text{room}}) = 2.41 \times 10^{14} n$$

## Finite temperature form

The first sampling frequency after  $\xi_0 = 0$ , corresponds to infra-red vibrations.

$$\lambda_1 = 2\pi c / \xi_1 = 7.82 \times 10^{-4} \text{ cm} = 7.82 \text{ } \mu\text{m} = 7.82 \times 10^4 \text{ Angstroms.}$$