Modulation of the Casimir Force with Light: Experiment and Theory

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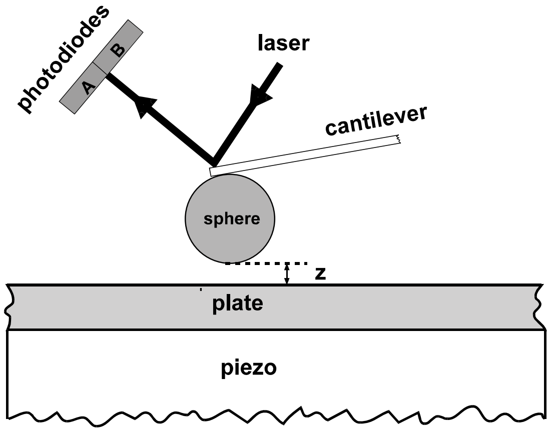
Background

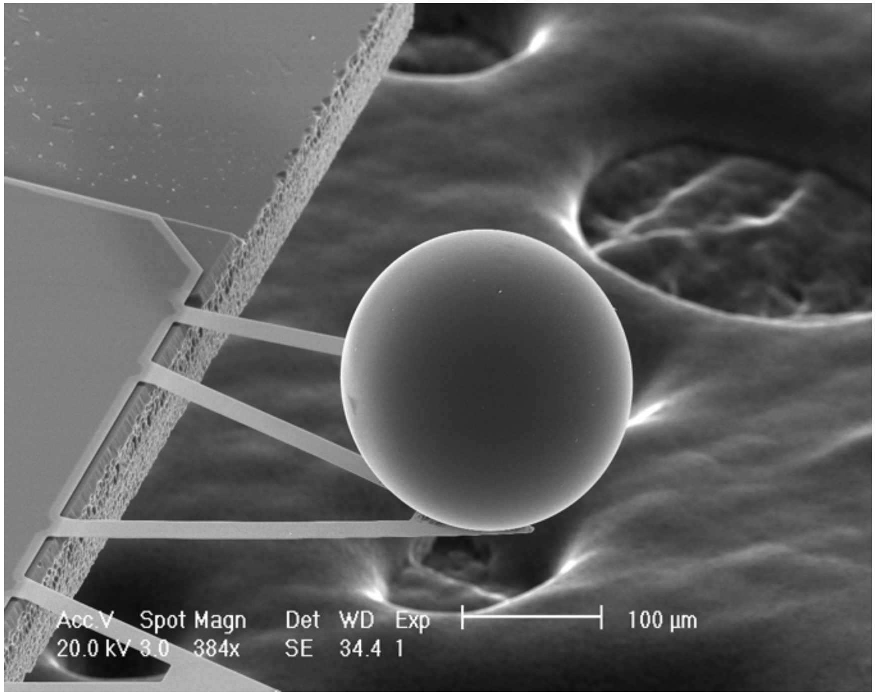
Use Senstivity of Atomic Force Microscope to make precision measurements of the Casimir force



Force Sensitivity 10^{-17} N possible

We achieve 10^{-13}N

200 micron Polystyrene Sphere on AFM Cantilever



Gold coating = 100 nm
Diameter = 200 µm

Using AFM

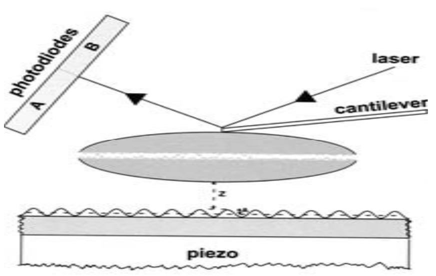
Normal Casimir Force From Metallic Surfaces

1. **U. M** and A. Roy, PRL, **81**, (1998).
2. A. Roy, C.Y. Lin, and **U. M**, PR D, **60**, (1999).
3. B.W. Harris, F. Chen and **U. M**, PR A, **62**, (2000). At 95% confidence level based on random + systematic error get 1.75% precision.

Theory Errors separate. Compare the force which is a nonlinear function with distance.

Casimir Force From Single Corrugated Surfaces

A. Roy and **U. M**, PRL, 82 (1999)



Lateral Casimir Force From Two Corrugated Surfaces

F. Chen, **U. M**, G.L.Klimchitskaya & V.M.Mostepanenko, PRL, **88** (2002), PRA (2003)

Force

Golestanian & Kardar PRL (1997), PRA (1998
Emig, Hanke, Golestanian and Kardar PRL (2001)

Material Dependences (Semiconductors) of the Casimir Force:

F. Chen, G.L.Klimchitskya, V.M. Mostepanenko, & **U. M**, PRA (2005), PRL, (2006)

Optical Modulation of the Casimir Force:

F. Chen, G.L.Klimchitskya, V.M. Mostepanenko, & U. M, Opt. Exp (2007), PRB (2007)

Motivation

- Demonstrate optical modulation of the Casimir force
- Probe puzzling applications of the Lifshitz theory for dielectric materials by taking a material from the dielectric phase to a metallic phase

Theory of the Casimir Force

1. H.B.G. Casimir , Proc. Kon. Nederl. Akad. Wet. 51 (1948) p. 793. → Ideal Metal plates

2. E.M. Lifshitz, Sov. Phys., JETP (USA)2 (1956)p. 73.
→ Real Materials

3. I.E. Dzyaloshinskii, E.M. Lifshitz, L.P. Pitaevskii, Sov. Phys. --USP (USA)4 (1961)p. 153

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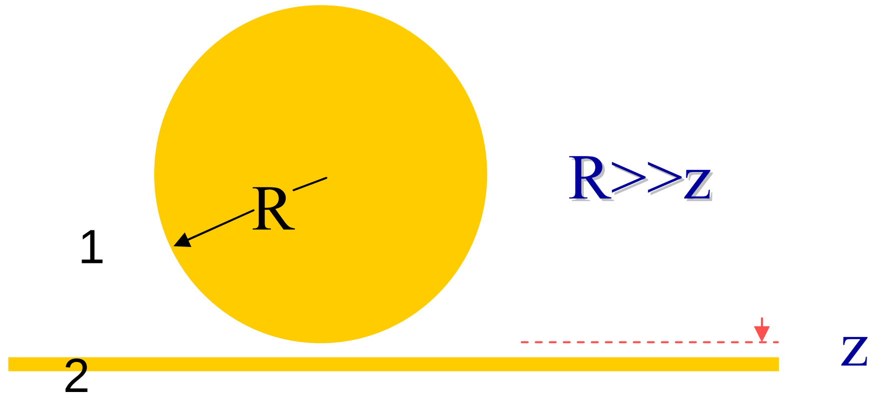
….

…

Lifshitz approach is general to non-ideal boundaries through use of ε and includes role of thermal photons

Lifshitz Formula



$$F_c(z)=K_BTR\sum_{l=0}^{\infty}(1-\tfrac{1}{2}\delta_{l0})\int_0^{\infty}K_\perp dK_\perp\{ln[1-r_{TM}^{(1)}(\xi_l,K_\perp)r_{TM}^{(2)}(\xi_l,K_\perp)e^{-2q_lz}]+ln[1-r_{TE}^{(1)}(\xi_l,K_\perp)r_{TE}^{(2)}(\xi_l,K_\perp)e^{-2q_lz}]$$

Reflection Coeffs: $$r_{TM}^{(k)}(\xi_i,K_\perp)=\frac{\varepsilon_l^{(k)}q_l-K_l^{(k)}}{\varepsilon_l^{(k)}q_l+K_l^{(k)}},\ r_{TE}^{(k)}(\xi_i,K_\perp)=\frac{K_l^{(k)}-q_l}{K_l^{(k)}+q_l}$$

Matsubara Freqs. $$\xi_l=\frac{2\pi k_BTl}{\hbar}$$ **At $l=0,\ \xi=0$**

$$q_l=(\frac{\xi_l^2}{c^2}+K_\perp^2)^{1/2},\ K_l^{(k)}=[\varepsilon^{(k)}(i\xi_l)\frac{\xi_l^2}{c^2}+K_\perp^2]^{1/2}$$

Puzzles in Application of Lifshitz Formula

$$r_{TM}^{(k)}(\xi_i, K_\perp) = \frac{\varepsilon_l^{(k)} q_l - K_l^{(k)}}{\varepsilon_l^{(k)} q_l + K_l^{(k)}}, r_{TE}^{(k)}(\xi_i, K_\perp) = \frac{K_l^{(k)} - q_l}{K_l^{(k)} + q_l}$$

For two metals and for large *separation distances* (or high T), *ξ=0 term dominates*

For ideal metals put $\varepsilon \rightarrow \infty$ first and l,ξ=0 next

Milton, DeRaad and Schwinger, Ann. Phys. (1978)

$$r_{TM}^{(k)}(0, K_\perp) = r_{TE}^{(k)}(0, K_\perp) = 1$$

Recover ideal metal Casimir Result

Puzzles in Applying Lifshitz Formula

For **Real Metals** if use Drude $\varepsilon(i\xi)=1+\frac{\omega_p^2}{\xi[\xi+\gamma^{(e)}]}$

$\omega_p=(\frac{ne^2}{m^*})^{1/2}$ and γ is the relaxation parameter

☞For ξ=0, $r_{TM}^{(k)}(0,K_\perp)=1, r_{TE}^{(k)}(0,K_\perp)=0$, only half the contribution even at z≈100 mm, where it should approach ideal behavior

☞Get large thermal correction for short separation distances z~100 nm

☞Entropy *S≠0 as T→0* (Third Law violation) for perfect lattice where γ(T=0)=0

If there are impurities γ(T=0) ≠0 , Entropy *S=0 as T→0*

Bostrom & Serenelius, PRL (2000); Physica (2004)
Genet, Lambrecht & Reynaud, PRA (2000).
Geyer, Klimchitskaya & Mostepanenko, PR A (2003)
Hoye, Brevik, Aarseth & Milton PRE (2003); (2005)
Svetovoy & Lokhanin , IJMP (2003)

Two Dielectric Surfaces

If the dielectric losses due to DC conductivity is included in the permittivity ε *(even a negligible amount)*

$$\varepsilon(i\xi) = \varepsilon_0(i\xi) + \frac{\omega_p^2}{\xi[\xi + \gamma\ ]}$$

ω_p represents the carrier concentration and γ is the relaxation parameter

Same Puzzling questions as with metals

True for negligible conductivity!!!

**Geyer,Klimchitskaya&
Mostepanenko, PRD (2005)**

One metal and one dielectric surface (Present Experiment):

If the dielectric losses due to DC conductivity included in the permittivity ε, then

$$r_{TM}^{(k)}(0,K_\perp)=1, r_{TE}^{(k)}(0,K_\perp)=0$$

Again same puzzles as with metals

True for negligible conductivity of dielectric !!!

Geyer, Klimchitskaya, Mostepanenko, IJMP A (2006)

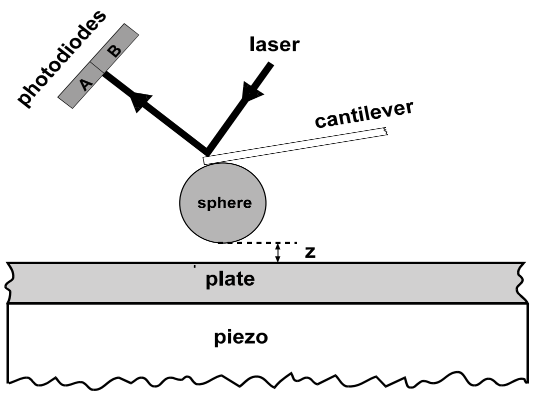
OPTICAL MODULATION OF THE CASIMIR FORCE

Experimental History on Optical Modulation

W. Arnold, S. Hunklinger, and K. Dransfeld, Phys. Rev. B, **19**, p. 6049 (1979)

1. Uncontrolled electrostatic systematic errors due to glass surface used.
2. Anamalous force decrease at short distances.

INSTRUMENT USED: ATOMIC FORCE MICROSCOPE



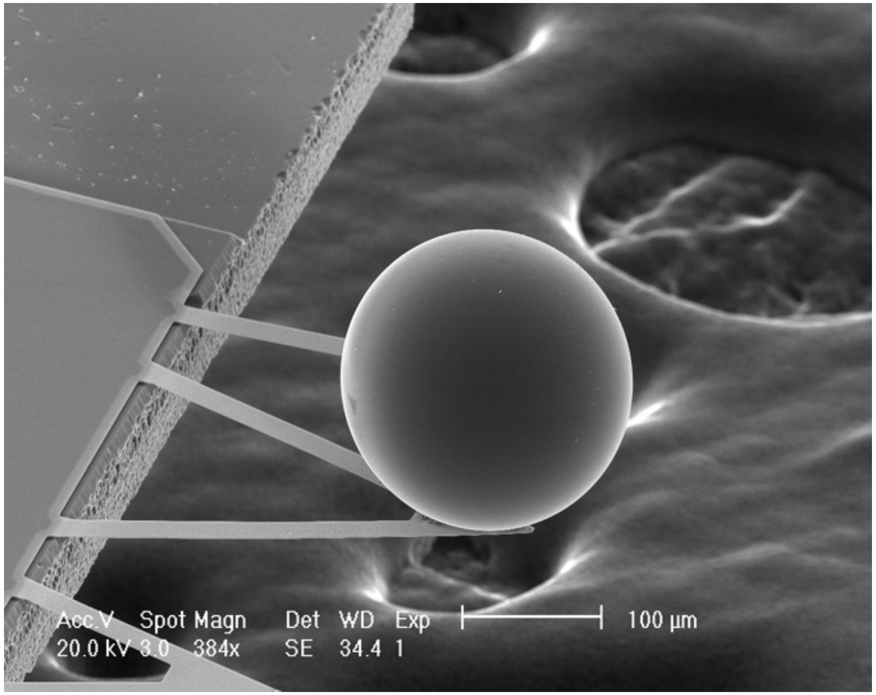
Force Sensitivity 10^{-17} N possible

We achieve 10^{-13}N

Room Temperature

10^{-7}-10^{-8} Torr vacuum

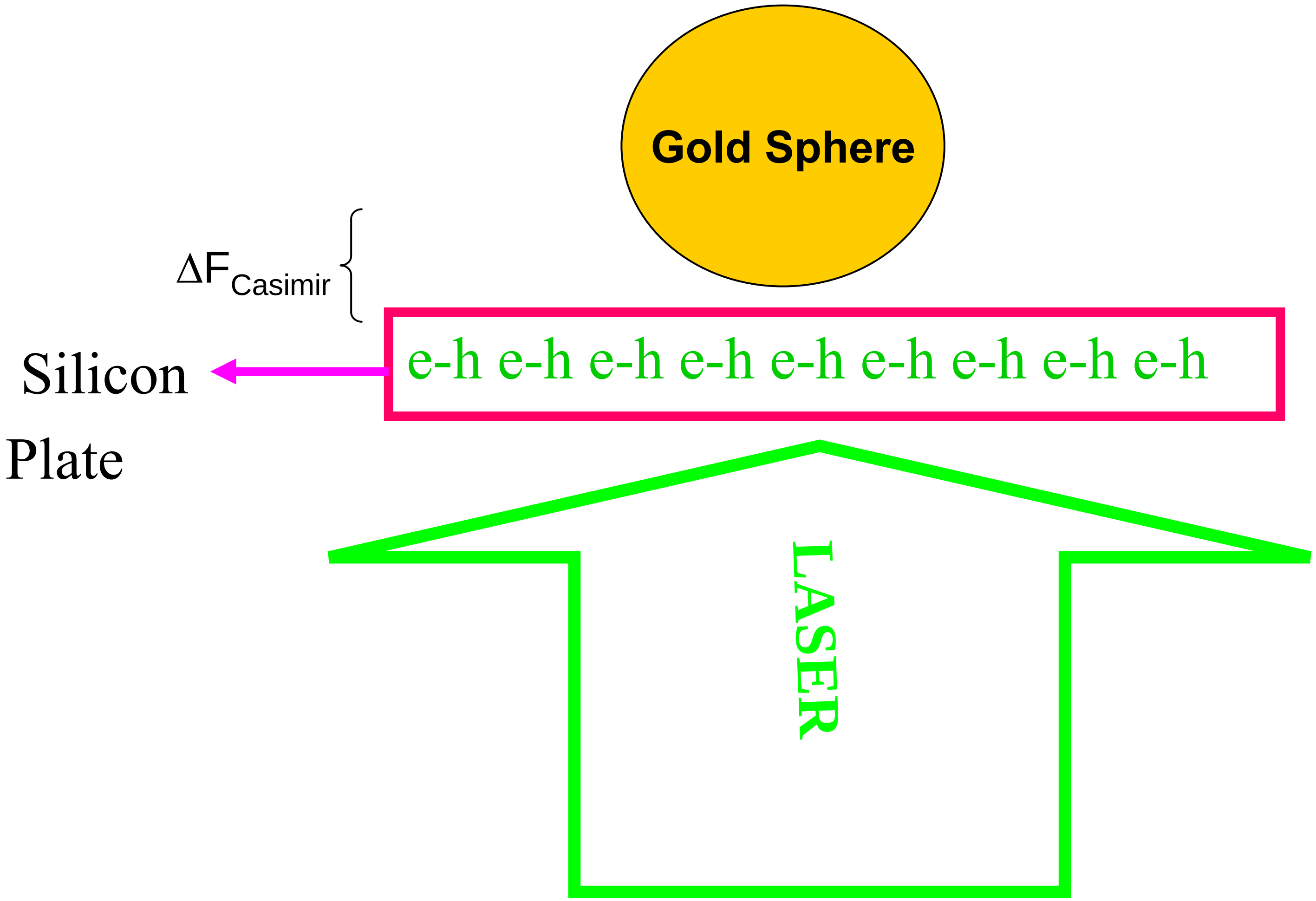
200 micron Polystyrene Sphere on AFM Cantilever



Gold coating = 82±2nm
Diameter = 197.8±0.3 µm

Optical Modulation of the Casimir Force In Semiconductors

Modulate Dielectric Constant of Boundary with Light



SAMPLE REQUIREMENTS

1. Need to increase Carrier Density from 10^{14} (impure dielectric) to 10^{19} /cc (metal)

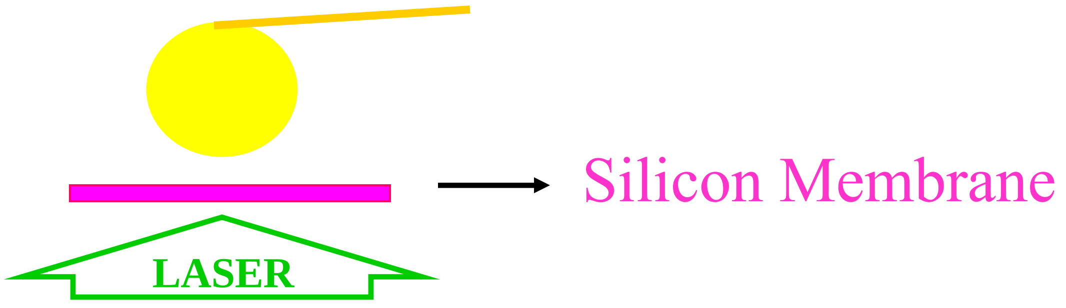
   ----- Long Lifetimes + Thin Membranes

2. Flat bands at surface and no surface charge traps

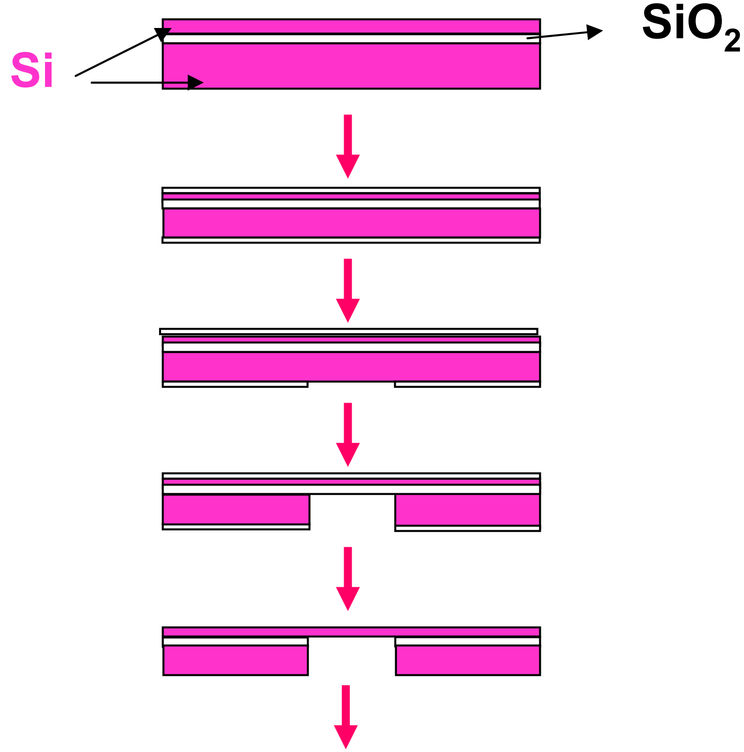
   → control electrostatic forces

3. Allow excitation from bottom to reduce photon pressure systematics

4. Need 2-3 micron thick samples to reduce transmitted photon force (Optical absorption depth of Silicon= 1 micron)



Membrane Fabrication

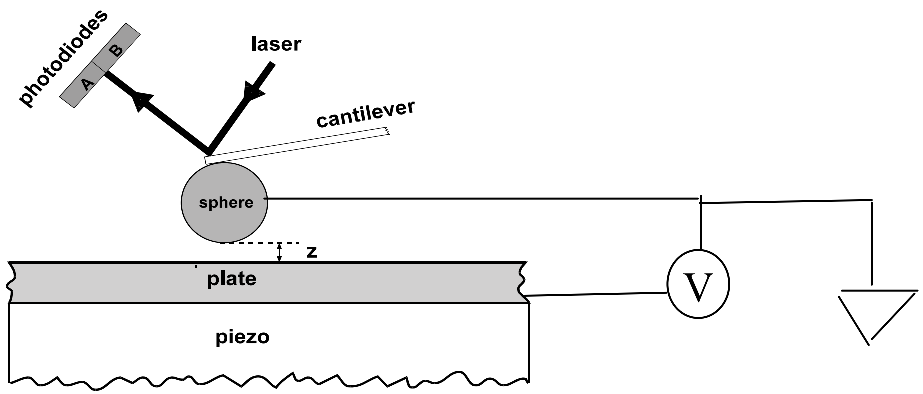


Hydrogen Passivation

5 μm Si <100> top layer
400nm SiO_2
600 μm Si substrate
n~5x 10^{14} cm^{-3} p-type

Membrane Thickness=4±0.3μm

ELECTROSTATIC CALIBRATION OF CANTILEVER



$$F(z) = 2\pi\varepsilon_o (V - V_0)^2 \sum_{n=1}^{\infty} \operatorname{csch} n\alpha \left(\coth\alpha - n\coth n\alpha\right)$$

where $\alpha = \cosh^{-1}(1 + z/R)$

$$F(z) = -2\pi\varepsilon_0 (V - V_0)^2 \sum_{i=-1}^{\infty} c_i \left(\frac{z}{R}\right)^i \equiv -X(z)(V - V_0)^2$$

$$C_{-1} = -0.5, C_0 = 1.18260, C_1 = -22.2375, C_2 = 571.366,$$

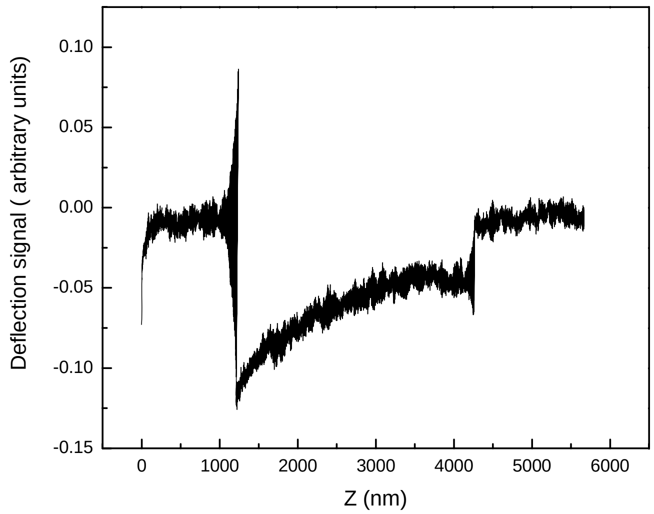
$$C_3 = -9592.45, C_4 = 90200.5, C_5 = -383084, C_6 = 300357,$$

V= voltage on plate

V_0= Residual voltage difference between uncharged sphere and plate

Electrostatic Calibration, Measurement of Contact Separation and Deflection Coefficient

DC Voltages between 0.65 to -0.91 V + square pulses between 1.2to -0.6V applied to plate



Apply to horizontal axis:

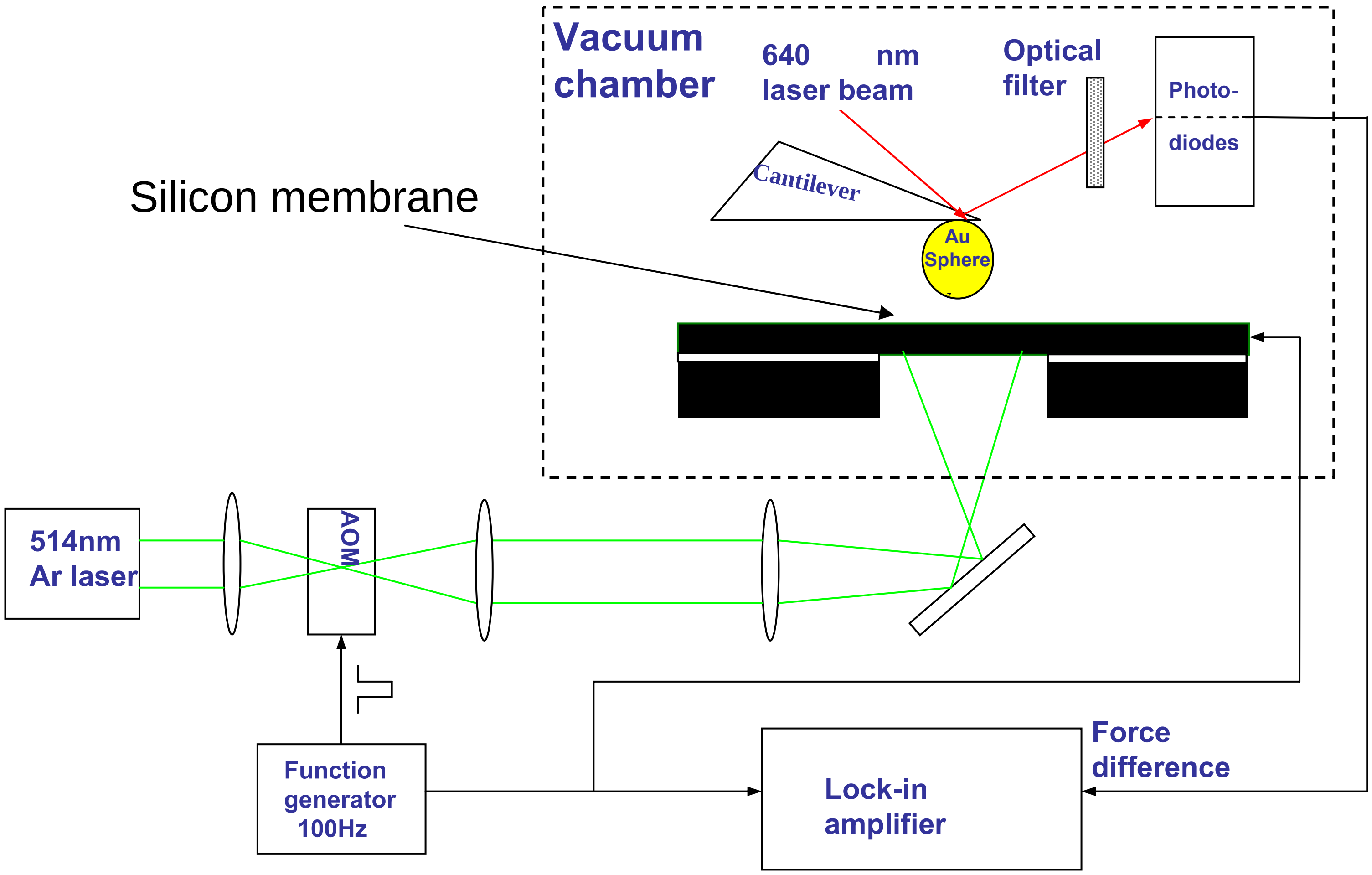
$z = z_{piezo} + z_0 -$ Deflection signal * m

1. z_{piezo}= Distance moved by piezo → **Calibrated interferometrically**

2. z_0 = Surface separation on contact = 97 nm → **From fitting electrostatic curve**

3. m= rate of change in distance due to cantilever tilt=137.2±0.6 nm per unit Def. → **Larger applied V lead to earlier contact**

Experimental Setup



Experimental Measurement of Modulation of Casimir Force due to Light

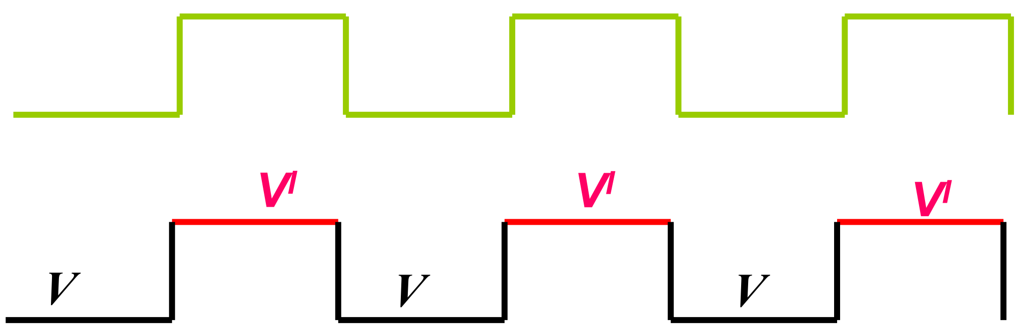
$$\Delta F_{tot}(z) = \boxed{-X(z)[(V^l - V_0^l)^2 - (V - V_0)^2]} + \Delta F_C(z)$$

Electric Force

V^l, V = Applied synchronous voltage pulse for light on and light off respectively

V_0^l, V_0 = Residual voltage for light on and light off respectively

$\Delta F_C(z) = F_C^l(z) - F_C(z)$ = Change in Casimir Force for light on and light off



→ 5 ms wide Light Pulse on Membrane

→ Voltage Pulse V^l with $V \sim V_0$ on Membrane

Generate Parabolas in V^l at every z. Maxima of Parabola lead to V_0^l =-.303±.002V

Repeat with different V and fixed V^l every z. V_0 =-.225±.002V

Measurement of Potential Differences between surfaces

$$\Delta F_{tot}(z) = \boxed{-X(z)[(V^{l} - V_0^{l})^2 - (V - V_0)^2]} + \Delta F_C(z)$$

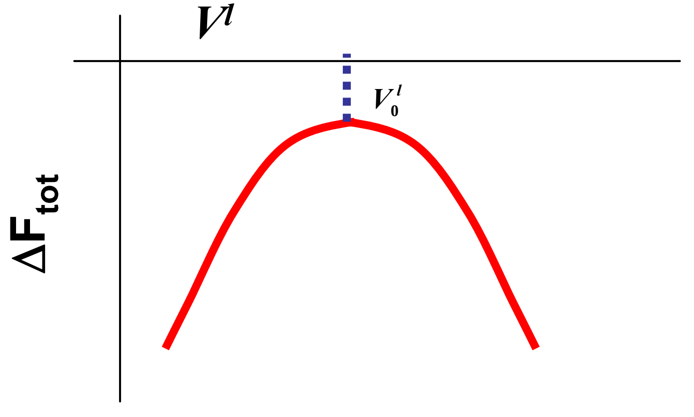
Electric Force

V^{l}, V = Applied synchronous voltage pulse for light on and light off respectively

V_0^{l}, V_0 = Residual voltage for light on and light off respectively

Blokland & Overbeek

TFCS,78



Generate Parabolas in V^l at every z . Maxima of Parabola lead to V_0^{l} =-.303±.002V

Repeat with different V and fixed V^l every z . V_0 =-.225±.002V

Use V_0^l =-.303±.002V

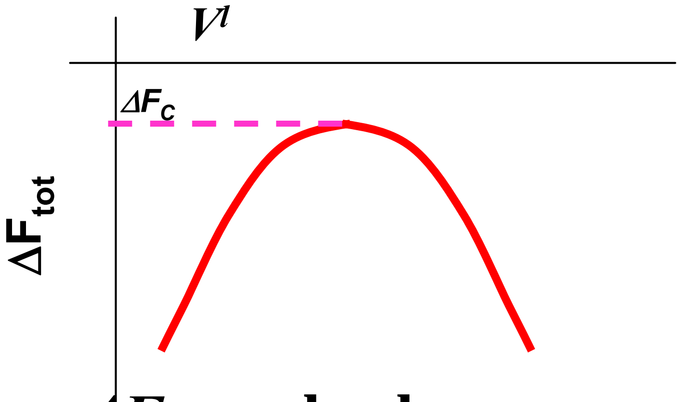
V_0 =-.225±.002V

in

$$\Delta F_{tot}(z) = \boxed{-X(z)[(V^l - V_0^l)^2 - (V - V_0)^2]} + \Delta F_C(z)$$

Electric Force

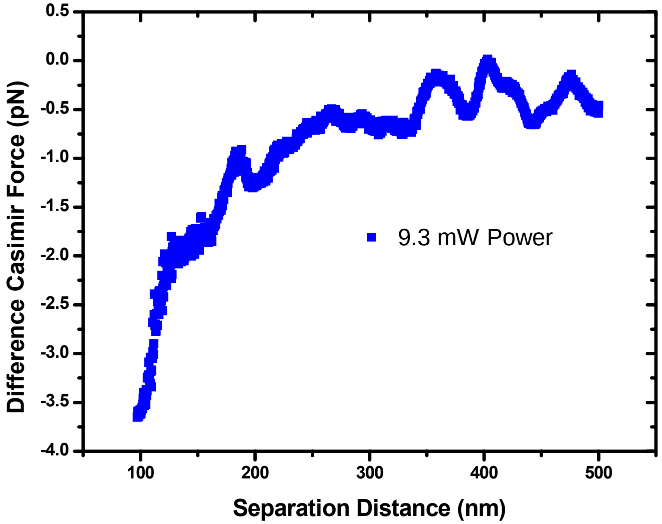
Apply small synchronous voltages $V^l \sim V_0^l$, $V \sim V_0$



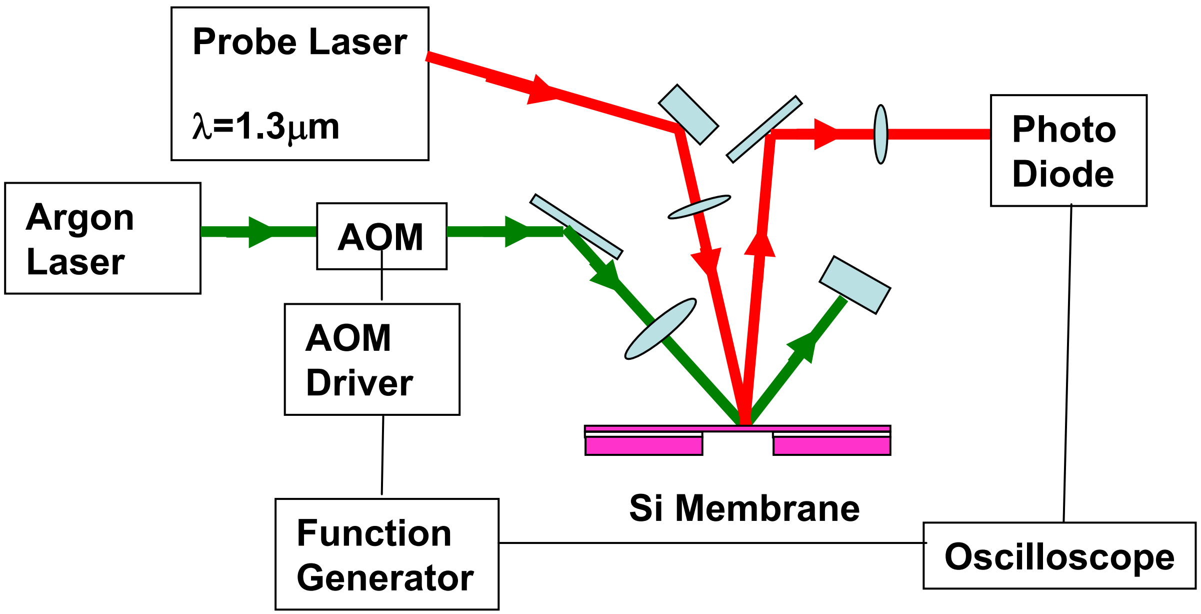
Measure ΔF_{tot} and subtract out the Electric force to get ΔF_C, the change in the Casimir Force due to absorption of light

Experimental Results

Average of 31 Repititions

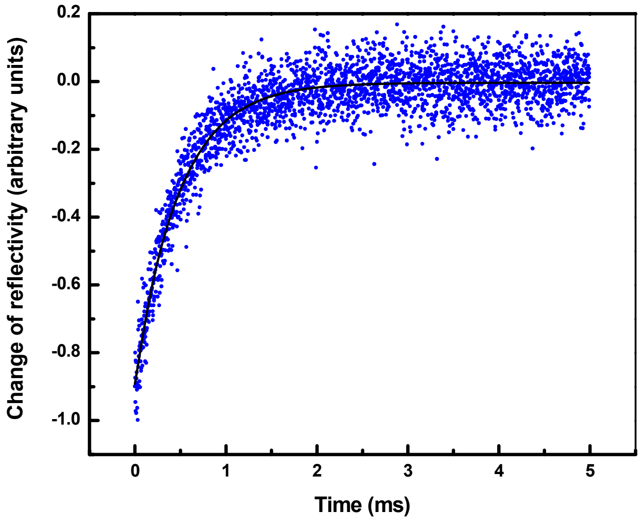


Excited Carrier Lifetime Measurement

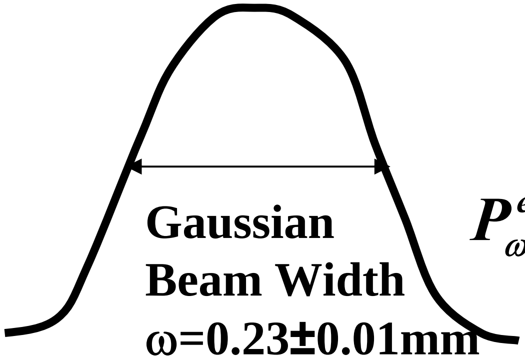


$\tau_{9.3mW} = \tau_{8.5\,mW} = 0.38 \pm 0.03$ ms

$\tau_{4.7mW} = 0.47 \pm 0.01$ ms



Excited Carrier Density



P_{ω}^{eff} =Power in ω is uniformly distributed= 0.393 P_{total}

Excited Carrier Density $$n = \frac{4P_{\omega}^{eff}\tau}{\hbar\omega d\pi\omega^{2}}$$

Recombination Rate τ measured earlier

9.3 mW $n=(2.1\pm0.4)\times 10^{19}$ /cc
8.5 mW $n=(2.0\pm0.4)\times 10^{19}$ /cc
4.7 mW $n=(1.4\pm0.3)\times 10^{19}$ /cc

Dielectric Permittivity of Silicon Membrane

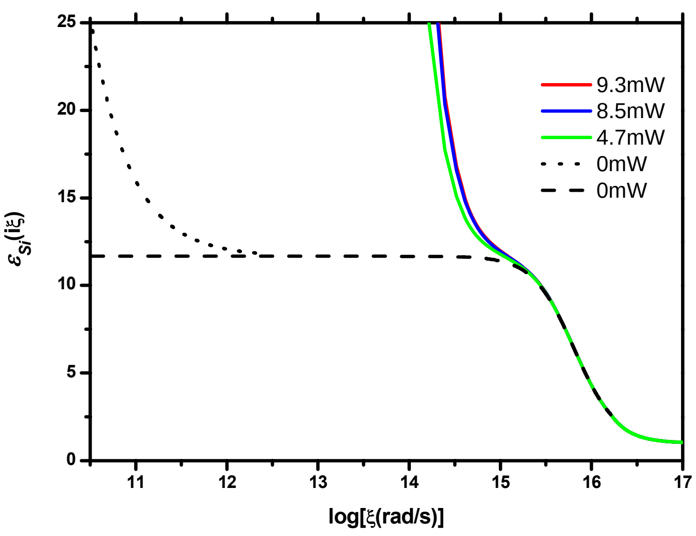
$$\varepsilon_{Si}(i\xi) = \varepsilon(i\xi) + \frac{{\omega_p^{(e)}}^2}{\xi[\xi + \gamma^{(e)}]} + \frac{{\omega_p^{(p)}}^2}{\xi[\xi + \gamma^{(p)}]},$$

$$\omega_p^{(e,p)} = \left(\frac{ne^2}{m^*_{e,p}}\right)^{1/2},$$

$$\omega_{P,a}^{(e)} = (5.1 \pm 0.5) \times 10^{14}\, rad/s, \qquad \omega_{P,a}^{(p)} = (5.7 \pm 0.6) \times 10^{14}\, rad/s,$$

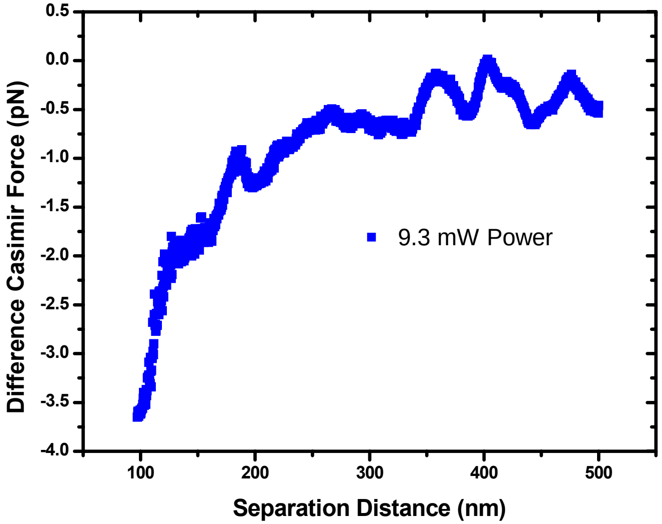
$$\omega_{P,b}^{(e)} = (5.0 \pm 0.5) \times 10^{14}\, rad/s, \qquad \omega_{P,b}^{(p)} = (5.6 \pm 0.5) \times 10^{14}\, rad/s,$$

$$\omega_{P,c}^{(e)} = (3.7 \pm 0.4) \times 10^{14}\, rad/s, \qquad \omega_{P,c}^{(p)} = (4.1 \pm 0.4) \times 10^{14}\, rad/s,$$

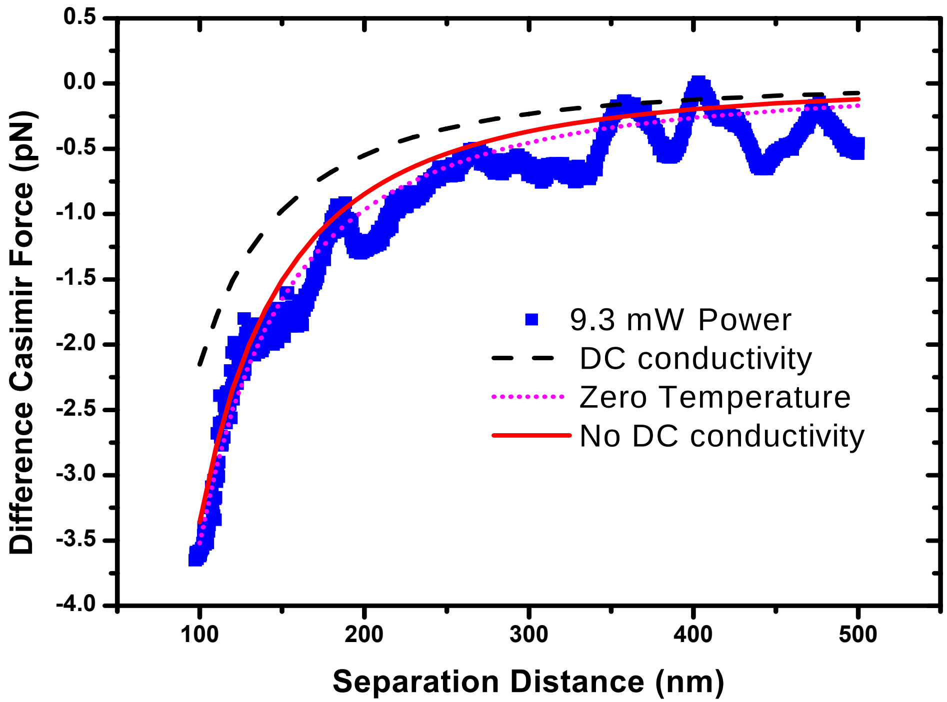


Experimental Results

Average of 31 Repititions

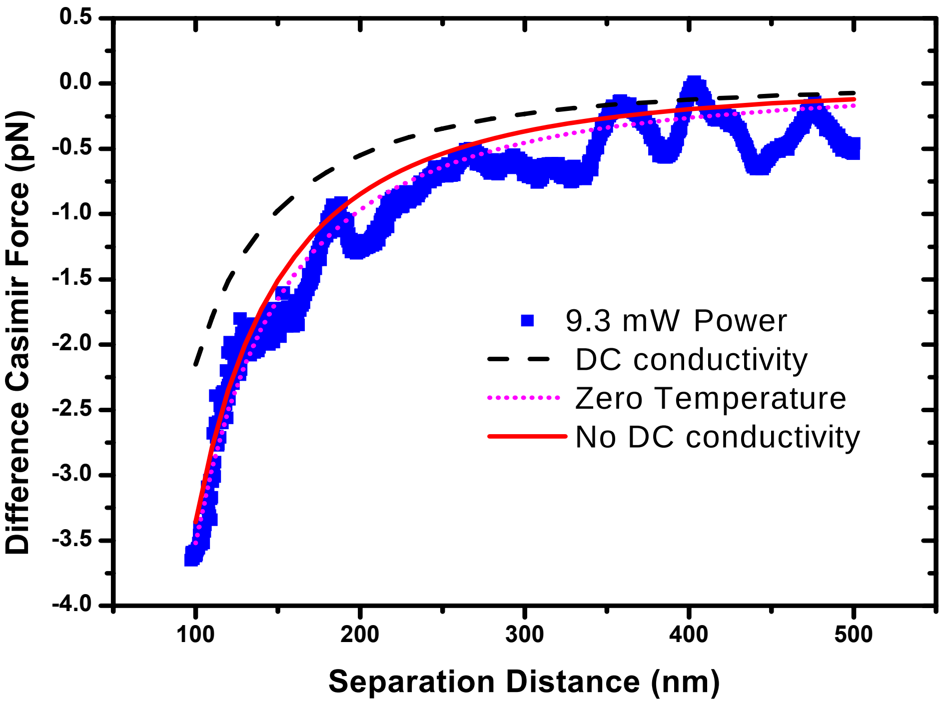


Comparison of Experiment with Theory



Inclusion of DC conductivity for high resistivity Si (dark phase) does not agree with results

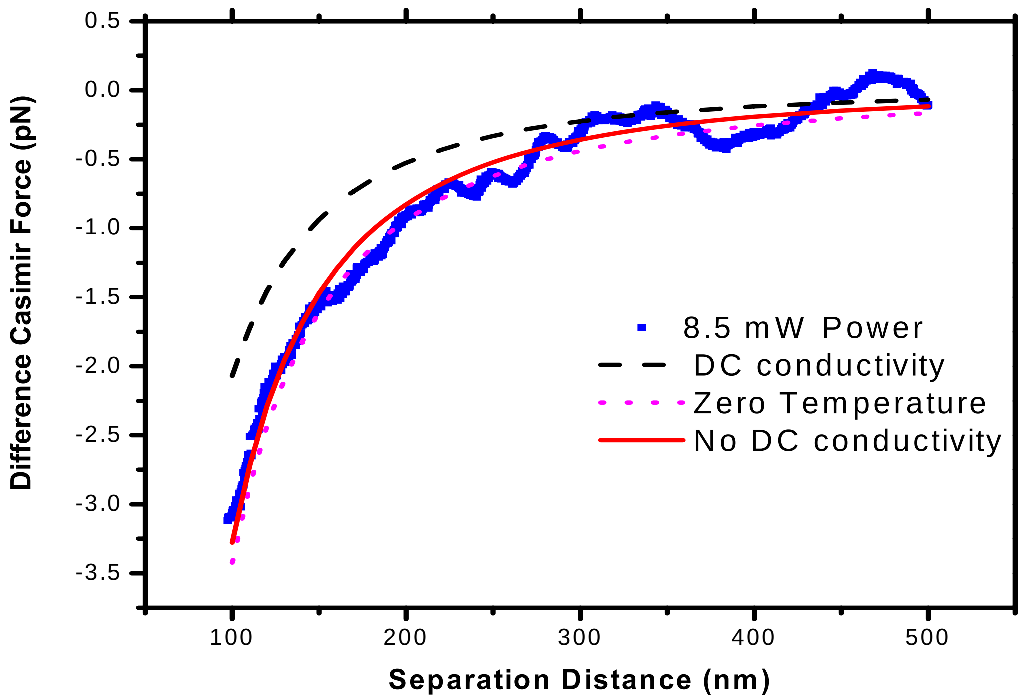
Comparison of Experiment with Theory



Inclusion of DC conductivity for high resistivity Si (dark phase) does not agree with results. Same puzzle as in theory.

Comparison of Experiment with Theory

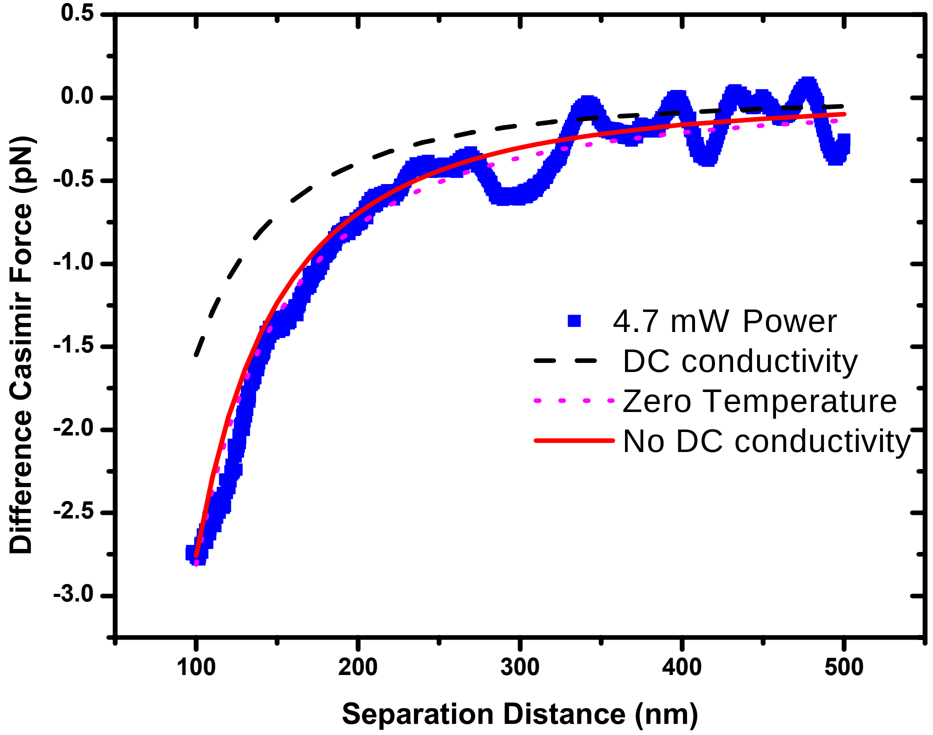
Experiment: Average of 41 Repititions



Inclusion of DC conductivity for high resistivity Si (dark phase) does not agree with results

Comparison of Experiment with Theory

Experiment: Average of 33 Repititions



Inclusion of DC conductivity for high resistivity Si (dark phase) does not agree with results

Experimental Errors

Systematic Errors	
Force Calibration	0.6%
Instrument Noise	0.08 pN
Instrument Resolution	0.02 pN
Total Systematic Error **@ 95% confidence**	0.092 pN to 0.095 pN
Experimental Random Error **@ 95% confidence**(8.5 mW)	0.32 pN @ 100nm 0.23 pN @ 250nm

Roughness Effects

Roughness Correction

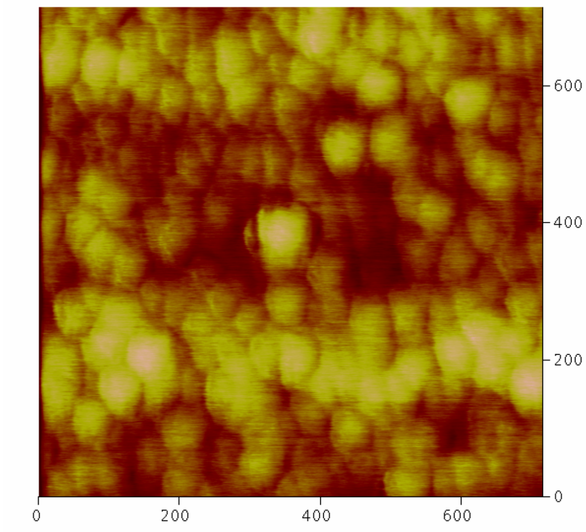
$$F_c = F_c \left[1 + 6\left(\frac{A}{z}\right)^2 + \ldots.. \right]$$

Roughness Amplitude A (measured with AFM)

~<1% effect

Maradudin & Mazur, PR B (1981)
Chen,Klimchitskaya,U.M, & Mostepanenko, PR A (2003)
Genet, Lambrecht Neto & Reynaud, Euro P.L (2003)
Emig, et al PRA (2003)

AFM Image of surface



$$\Delta F_C^{theor}(z_i) = \sum_{k=1}^{33}\sum_{l=1}^{17} v_k v_l \Delta F_C(z_i + H_0^{(1)} + H_0^{(2)} - h_k - h_l),$$

$$\sum_{k=1}^{33}(H_0^{(1)} - h_k)v_k = \sum_{l=1}^{17}(H_0^{(2)} - h_l)v_l = 0,$$

Roughness Contribution to Force

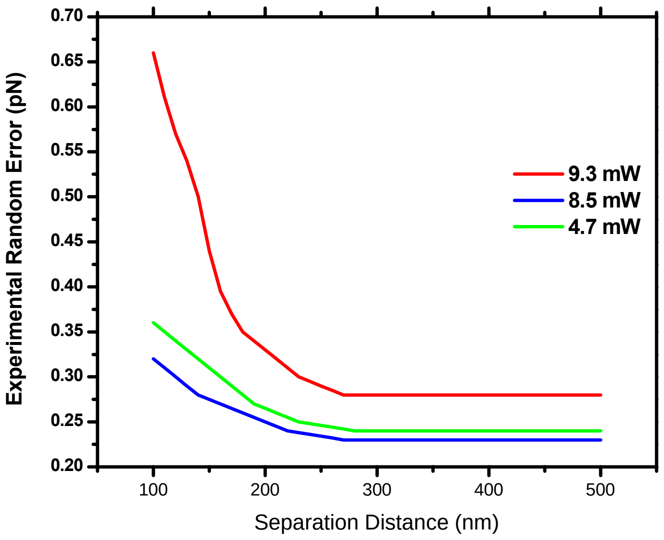
1.2% @ 100nm

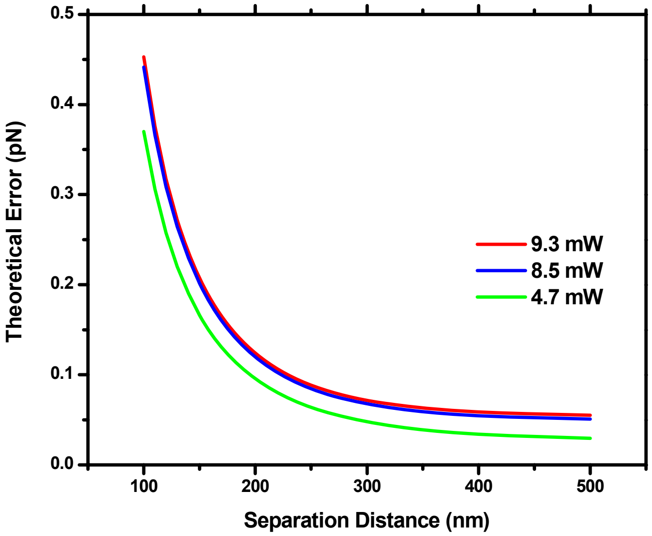
0.5% @ 150nm

Theoretical Errors

Uncertainty in Excited Charge Carrier Concentration	±20% (lead to 12% error in Force)
±1 nm Uncertainty in Separation Distance	< 3% error in Force @100 nm
Light Pressure Force	< 0.05-0.02 pN

Experimental and Theoretical Errors as a Function of Distance

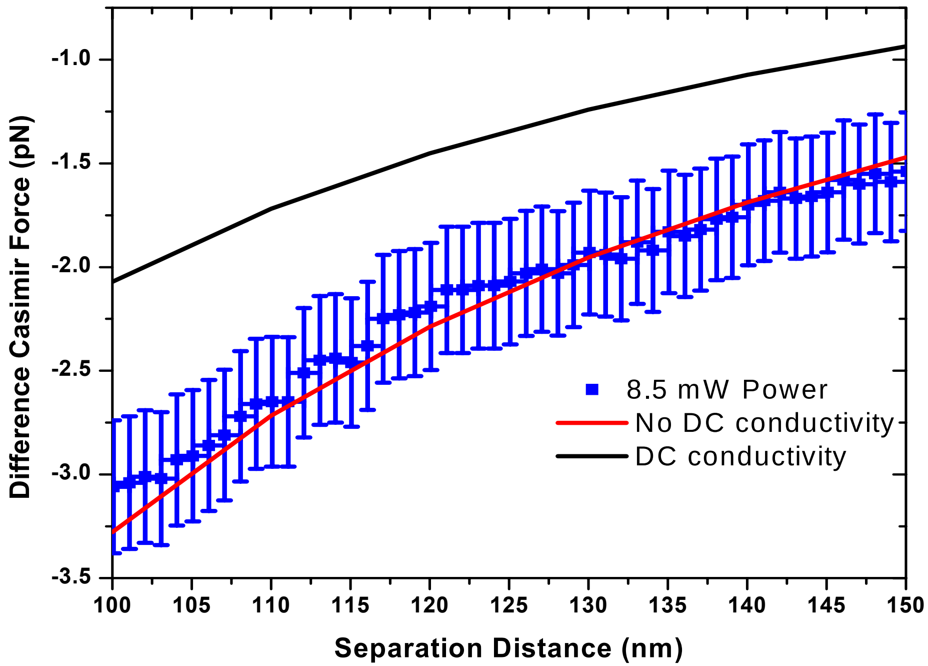




$$\Xi_{\beta} = \min\{\Delta^{tot}(\Delta F_C^{\mathrm{expt}}) + \Delta tot(\Delta F_C^{theor}), k_{\beta}^{(2)}\sqrt{[\Delta^{tot}(\Delta F_C^{\mathrm{expt}})]^2 + [\Delta^{tot}(\Delta F_C^{theor})]^2}\}$$

Comparison between Experiment and Theories

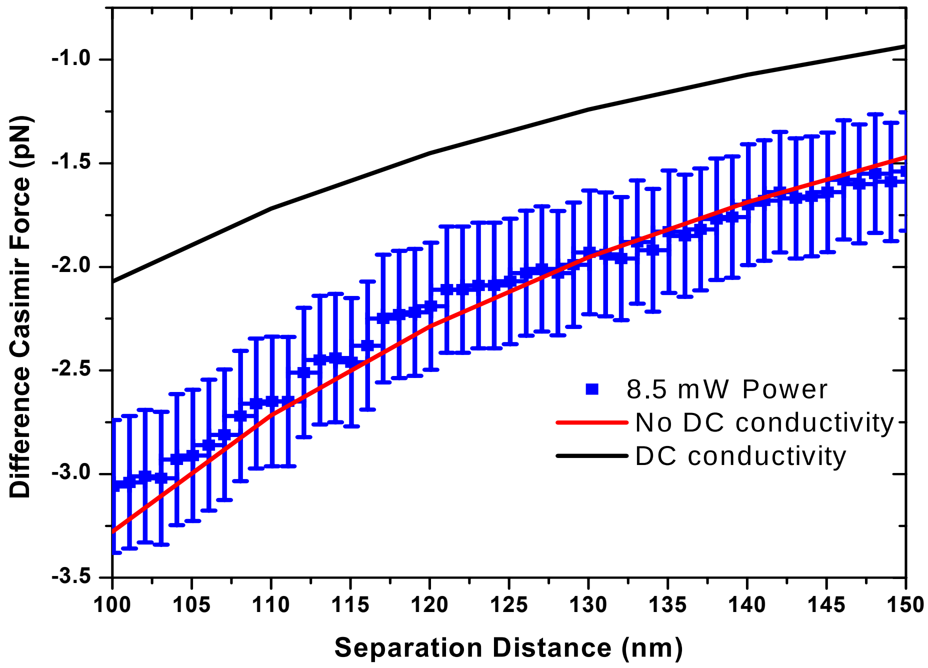
Error bars represent 95% confidence level



Chen, Klimchitskaya, Mostepanenko, Mohideen PRB (2007)

Comparison between Experiment and Theories

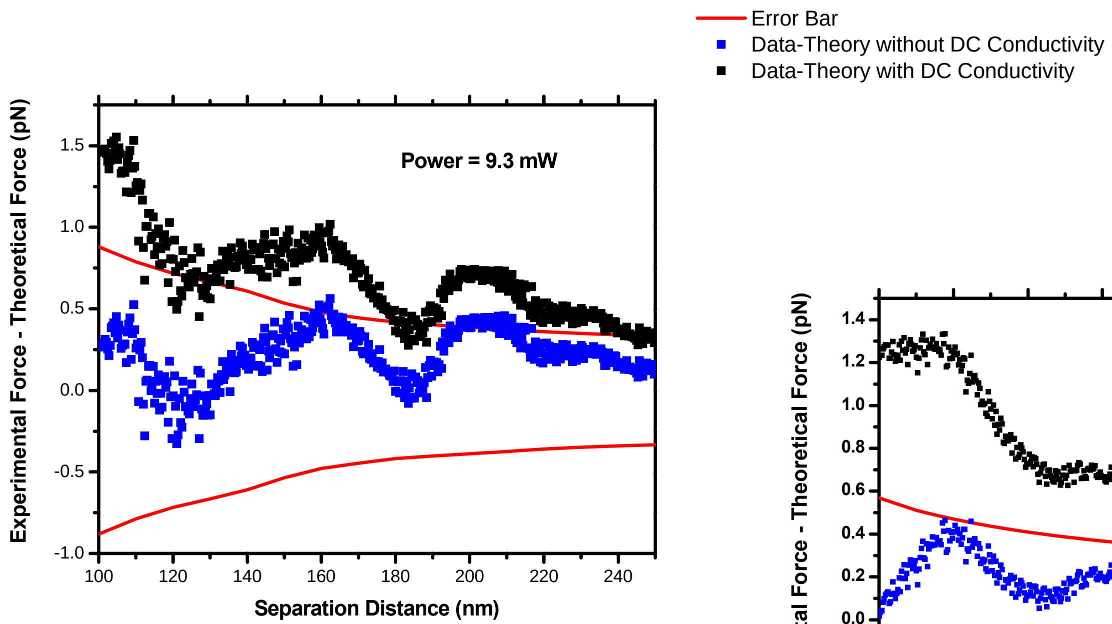
Error bars represent 95% confidence level

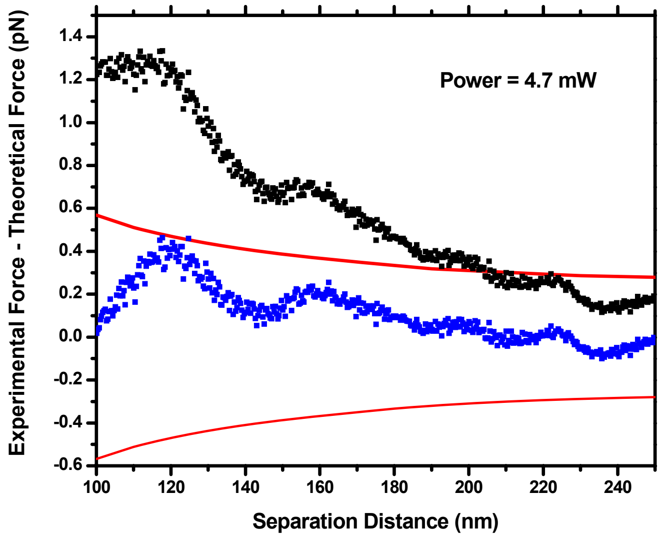


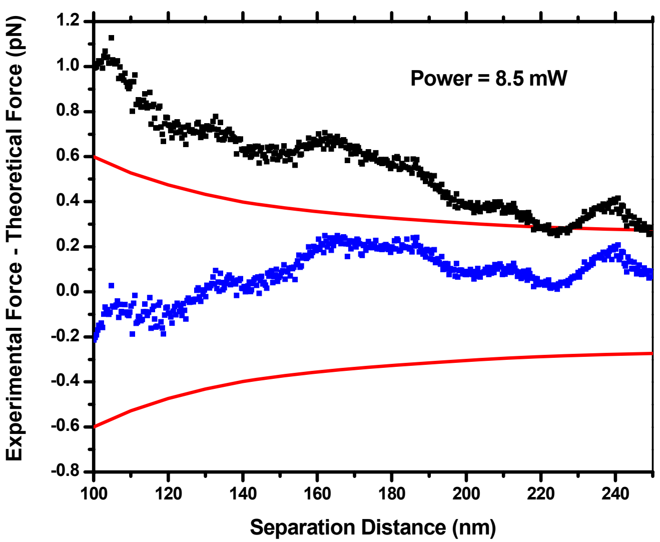
Inclusion of DC conductivity for high resistivity Si (dark phase) does not agree with results

Chen, Klimchitskaya, Mostepanenko, Mohideen PRB (2007)

Comparison of Difference Force (Theories -Expt.) with Total Errors at 95% Confidence Level







Inclusion of DC conductivity for high resistivity Si (dark phase) does not agree with results

Chen, Klimchitskaya, Mostepanenko, Mohideen PRB (2007)

Dielectric Permittivity of Silicon Membrane using Plasma Model

Drude Model

$$\varepsilon_{Si}(i\xi) = \varepsilon(i\xi) + \frac{\omega_p^{(e)^2}}{\xi[\xi + \gamma^{(e)}]} + \frac{\omega_p^{(p)^2}}{\xi[\xi + \gamma^{(p)}]},$$

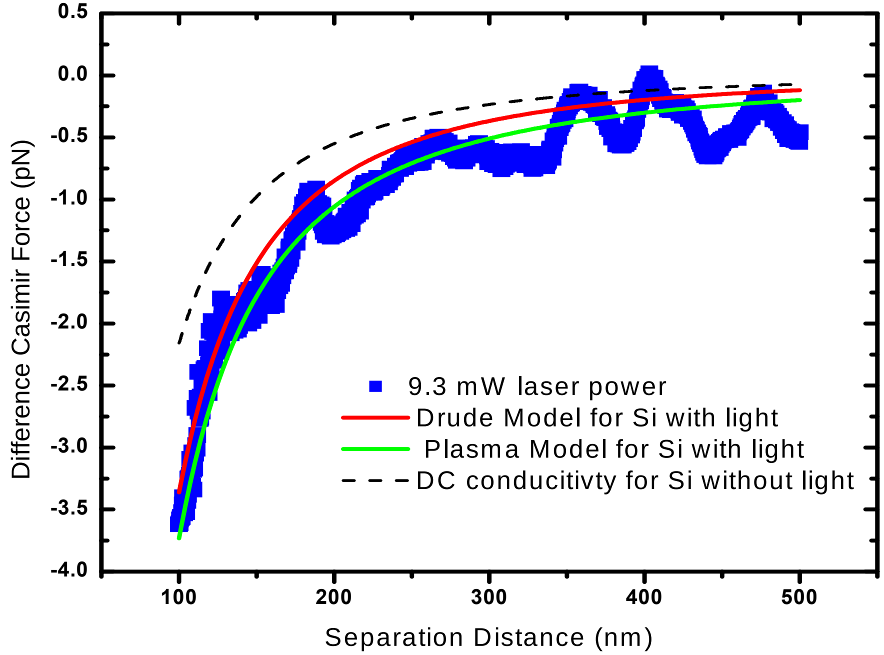
Plasma Model

$$\varepsilon_{Si}(i\xi) = \varepsilon(i\xi) + \frac{\omega_p^{(e)^2}}{\xi^2} + \frac{\omega_p^{(p)^2}}{\xi^2},$$

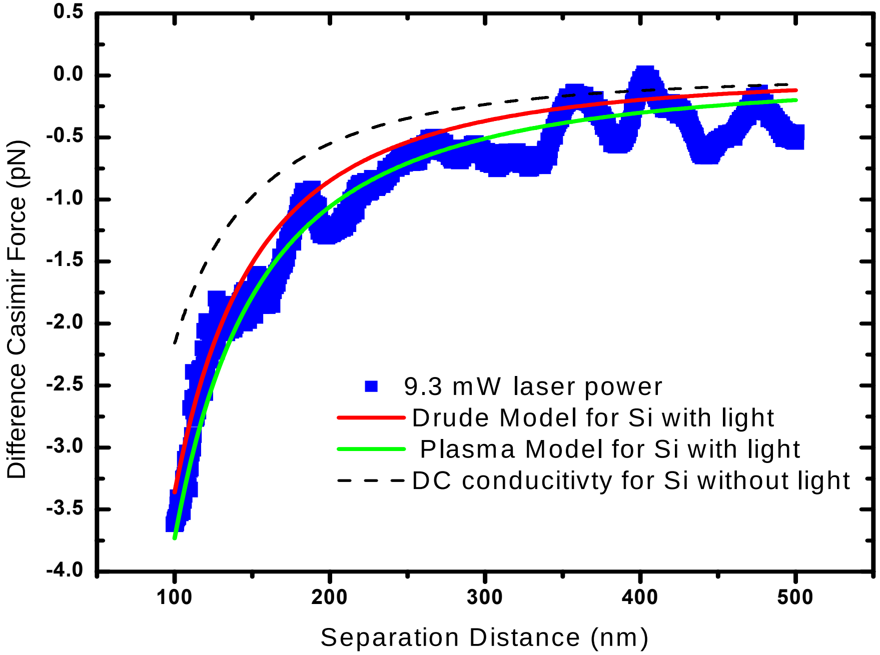
$$\omega_p^{(e,p)} = \left(\frac{ne^2}{m_{e,p}^*}\right)^{1/2}$$

For 9.3 mW laser power $\omega_{P,a}^{(e)} = (5.1 \pm 0.5) \times 10^{14}\, rad/s,$ $\omega_{P,a}^{(p)} = (5.7 \pm 0.6) \times 10^{14}\, rad/s$

Comparison of Experiment with Theory using Different Models of Conductivity



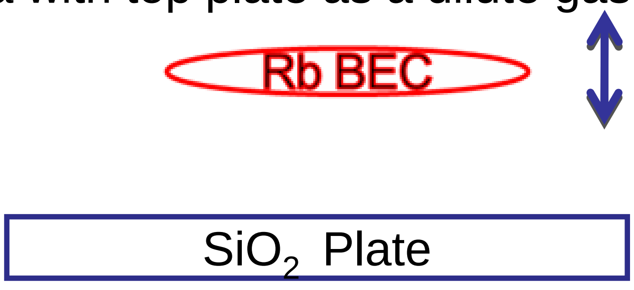
Comparison of Experiment with Theory using Different Models of Conductivity



Within Error Bars Cannot Discriminate between Drude and Plasma Model for High Conductivity Silicon

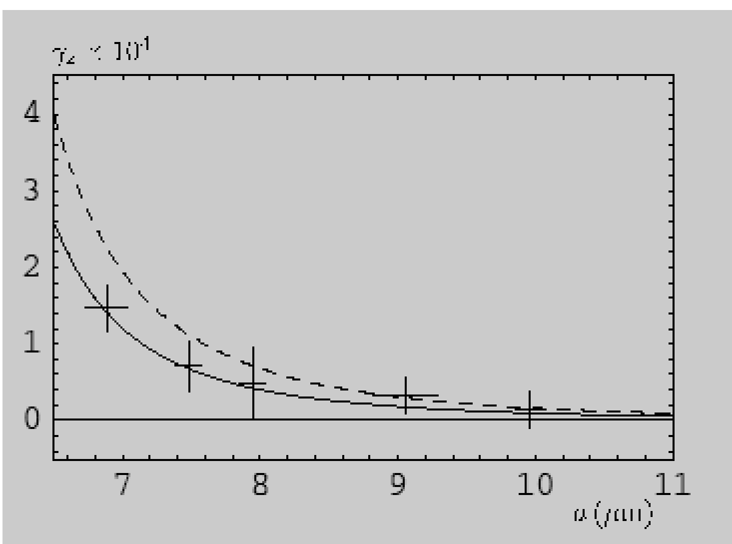
Experimental Measurement of Thermal Casimir-Polder Force

Theory from Lifshitz formula with top plate as a dilute gas



Oscillation freq. related to Casimir-Polder force

Obrecht, Wild,Antezza,Piteavskii Stringari &Cornell, PRL 2007



Crosses= Data from Obrecht et al 2007
Solid line= SiO_2 as ideal dielectric
Dashed line= DC conductivity of SiO_2 included

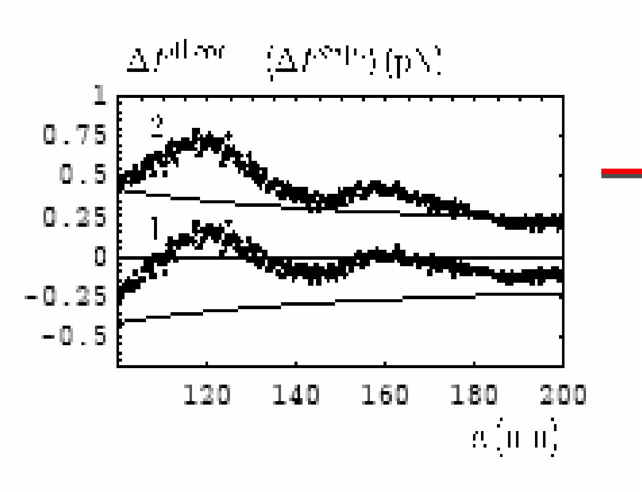
Klimchitskaya& Mostepanenko, 2008 JPA

Emergent Models-1

1. Piteavskii 2008 arXiv:0801.0656– Introduces Debye-Huckel screening of the l=0 (static field) mode

Parsegian 2005
Geyer, Klim.,Most. 2007 JPA

S≠0 as n≠0 as T→0



1= Data from Chen et al PRB (2007)- Lifshtiz theory with no DC conductivity in dark phase
2= Data- Emergent Theory

Solid line error bars at 95% confidence level

Emergent Models-2

1. Dalvit & Lamoreaux 2008 arXiv:0805.1676– Introduces Debye-Huckel screening of the l=0 (static field) mode by charges by including diffusion and drift currents with Boltzmann Eqn.

$$r_{TM}^{(mod)}(\xi_i, K_\perp) = \frac{\varepsilon_l^{(k)} q_l - K_l^{(k)} - \frac{K^2{}_\perp}{\eta(\xi)} \frac{\varepsilon(i\xi) - \varepsilon_c(i\xi)}{\varepsilon_c(i\xi)}}{\varepsilon_l^{(k)} q_l + K_l^{(k)} + \frac{K^2{}_\perp}{\eta(\xi)} \frac{\varepsilon(i\xi) - \varepsilon_c(i\xi)}{\varepsilon_c(i\xi)}}$$

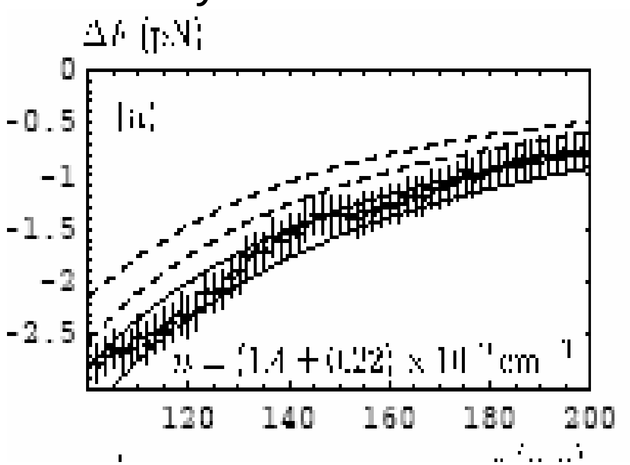
Parsegian 2005
Geyer, Klim.,Most. 2007 JPA

$$\varepsilon(i\xi) = \varepsilon_c(i\xi) + \frac{\omega_p^2}{\xi[\xi + \gamma^{(e)}]},$$

ε_c =Response of core electrons

Debye screening length $R_D = \sqrt{\frac{\varepsilon_c(0) k_B T}{4\pi e^e n}}$

S→0 at T→0 only for intrinsic semiconductors with n~ exp(-E/T)

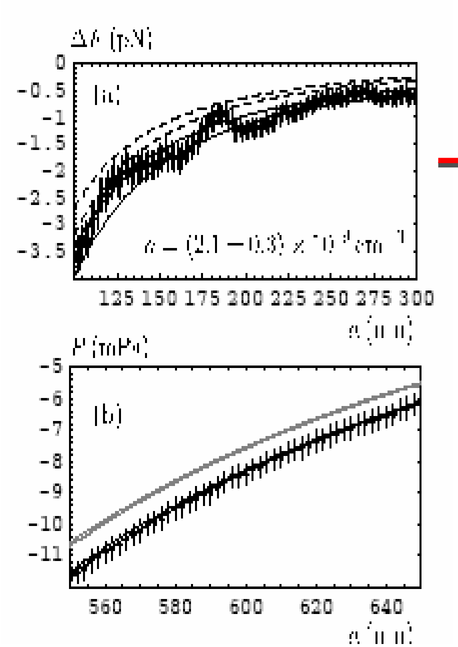


Crosses= Data
Dotted band= Proposed Model
Solid line band= No DC conductivity in dark phase

Emergent Models-3

1. Svetovoy 2008 arXiv:0809.3901– Introduces spatial dispersion into the dielectric permittivity calculated using Random Phase Approximation (specular reflection at the boundaries) . Nonlocal permittivity due to Thomas Fermi screening

S≠0 at T→0 for anything but intrinsic semiconductors



Data from Chen et al PRB (2007)
Dashed line and band – Emergent model
Solid line and band= No DC conductivity in dark phase

Data from Decca et al Eur P.J. (2007)
Dashed line and band – Emergent model
Solid line and band= No DC conductivity in dark phase

Conclusions

1. Optical Modulation of the Casimir Force was demonstrated.
2. The results have the correct distance dependence
3. Inclusion of DC conductivity in the Lifshitz formula for the high resistivity Si leads to disagreement with the experimental results. (Related problems in Casimir-Polder force for BEC- Klimchitskaya & Mostepanenko J. Phys A 2008)

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Experiment

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