Casimir energy between a plane and a sphere in electromagnetic vacuum

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Fluctuation-Induced Interactions - KITP-UCSB

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Collaborators

✓ Plane-sphere geometry

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✓ Different applications of the scattering approach

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Outline

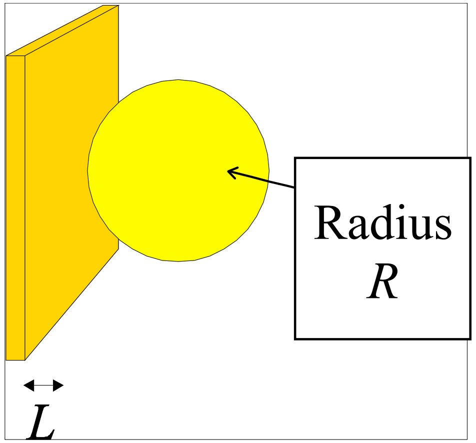
- Geometry and the Casimir effect
- Scattering approach
- Plane-sphere geometry

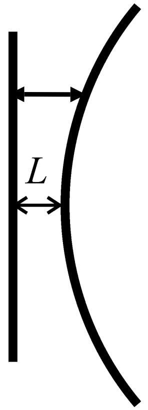
Geometry and the Casimir effect

Motivation:

- Many modern experiments use the plane-sphere geometry:

Lamoreaux, Mohideen, Capasso, Decca, ...





$R \gg L$ ⇒

Proximity Force (Derjaguin) Approximation (PFA): take the plane-plane result for the **local** distances

How accurate is the PFA for a given L/R ?

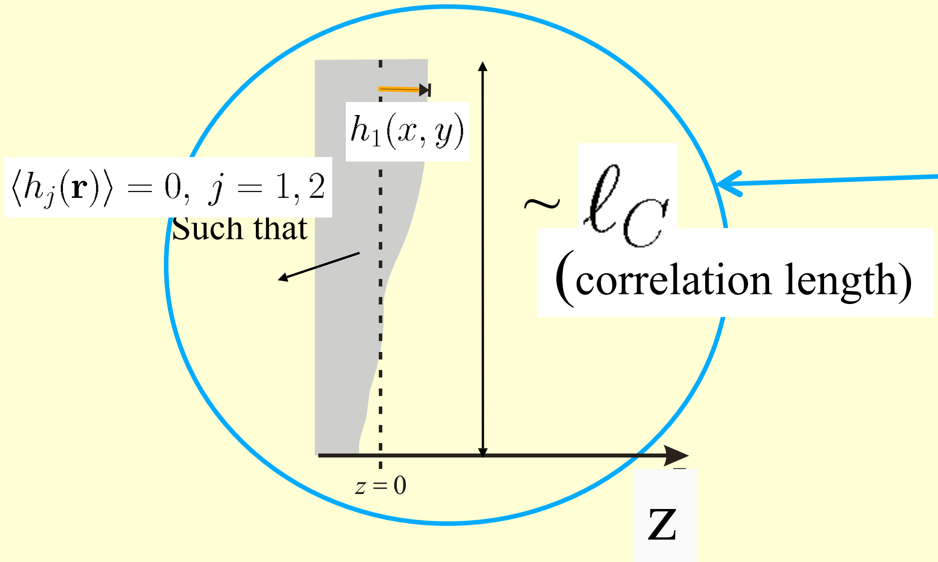
- See Krause+Decca+Lopez+Fischbach for experimental approach (2007)

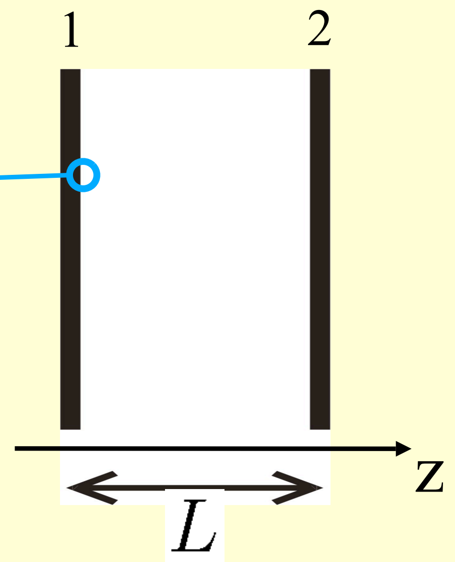
- Proposed experiment with plane and cylinder (Onofrio et al)

Geometry and the Casimir effect (motivation)

Another relevant issue for mastering the quality of theory/experiment comparisons…

❖ Roughness correction to the Casimir attraction





$$\sqrt{\langle h^2\rangle} \sim \mathrm{nm}$$

More important at short distances

Geometry and the Casimir effect

Theory/experiment comparison: we need to consider...

- the electromagnetic field
- real metals with finite conductivity

Geometry and the Casimir effect - theory

Scalar vs electromagnetic: not a simple factor 2 from polarization !

Example with **plane symmetry**: dissipative force on a single moving mirror (velocity $v(t)$, area A)

3+1 (nonrelativistic limit: $v/c \ll 1$)

Scalar vacuum field Ford+Vilenkin (1982)	Electromagnetic vacuum field PAMN (1994)
$F(t) = -\frac{1}{360\pi^2}\frac{\hbar A}{c^4}\frac{d^4v(t)}{dt^4}$	$F(t) = -\frac{1}{30\pi^2}\frac{\hbar A}{c^4}\frac{d^4v(t)}{dt^4}$

12 x larger !!

Geometry and the Casimir effect - theory

Why electromagnetic models ?

...transition to electromagnetic (EM) case is often simpler when geometry contains a direction of translational symmetry ..

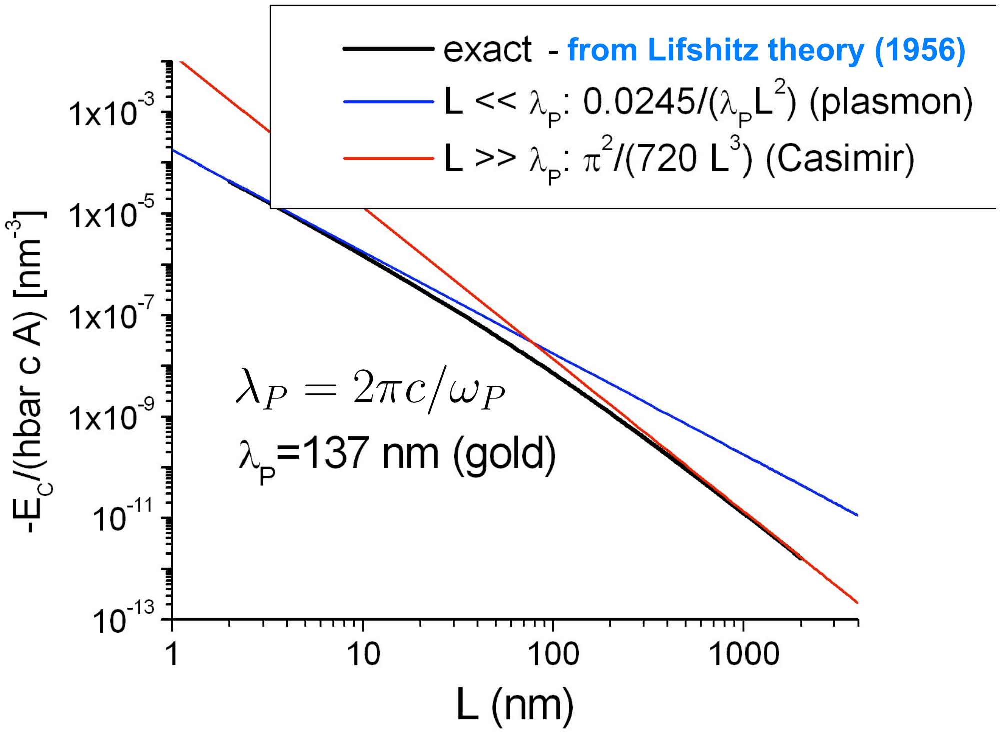
.....not trivial for spherical geometries...

Different physics involved: no s-wave scattering in the EM case !

Geometry and the Casimir effect - theory

Why real materials ?

Casimir energy (as a function of separation distance) with plasma model for metallic media



$c/L \gg \omega_P$ → plasma frequency

Surface plasmons (Van Kampen et al 1968)

$$E_{\rm PP}(L) = -0.0245\ \omega_P\ \frac{\hbar c A}{2\pi L^2}$$

$c/L \ll \omega_P$

Perfect reflectors - Casimir 1948

$$E_{\rm PP}(L) = -\frac{\pi^2}{720}\frac{\hbar c A}{L^3}$$

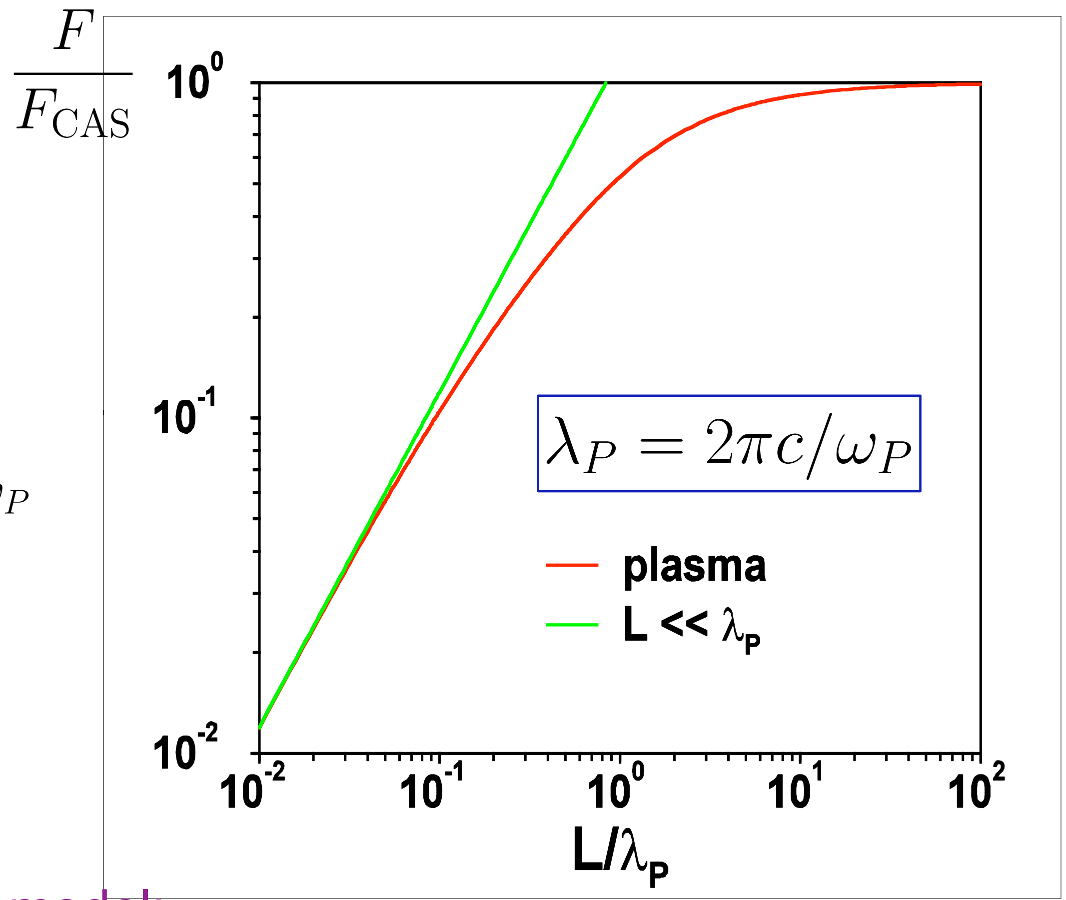
Power law modification as in Van der Waals – Casimir-Polder interatomic potential

Geometry and the Casimir effect - theory

Why real materials ?

Casimir energy
proportional to ω_P for
small ω_P ...

and satures at value
independent of ω_P for large ω_P



beyond the plasma model:
A. Lambrecht and S. Reynaud, Eur. Phys. J. D8 309 (2000)

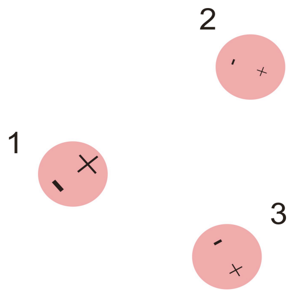
Geometry and the Casimir effect - theory

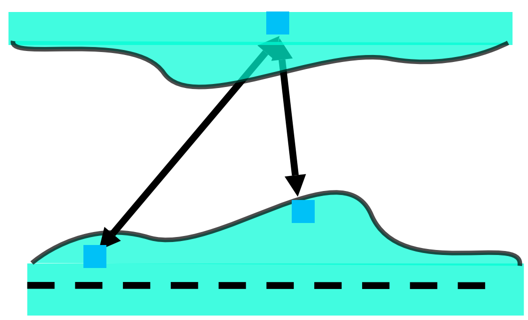
Approximation methods

- **Proximity Force Approximation (Derjagin) - take local distances**
- **Pairwise summation of vdWaals/Casimir-Polder interatomic potentials (Hamaker approach)**

Problem if medium is not rarefied: **van der Waals interaction is not additive !!**

(Dipole moments are not prescribed: they are induced by the other particles' dipole moments)





Plane-plane case must be corrected by comparison with exact result ...

... and then the *same* correction factor is employed for different geometries (ok for nearly plane surfaces – PFA limit)

Geometry and the Casimir effect - theory

Some theoretical tools

Numerical approaches

World-line Monte-Carlo – Gies, Langfeld (2001) - no EM implementation so far

Finite-difference numerical evaluation of Green function – Rodriguez et al (2007) – EM, real materials.

Scattering aproach and non-trivial geometries

Balian+Duplantier - Multiple scattering

Lambrecht+PAMN+Reynaud - Lifshitz formula generalized for non-planar scatterers

Kenneth+Klich - `TGTG' formula, Bordag, Bulgac+Magierski+Wirzba, Milton +Wagner, Emig+Graham+Jaffe+Kardar - displacement+T

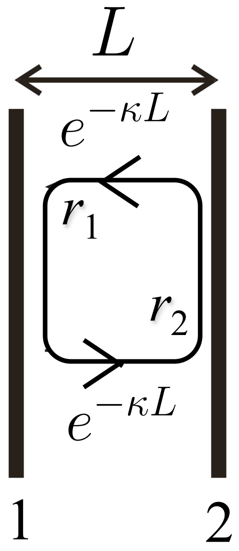
Scattering approach

Lifshitz formula

Sum over polarizations

$$E_{PP} = \hbar A \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2k}{(2\pi)^2} \sum_p \ln\left(1 - r_{1;p}(k) r_{2;p}(k)\, e^{-2\kappa L}\right)$$

$$\kappa = \sqrt{(\xi/c)^2 + k^2}$$



Closed loops

Lifshitz (1956) in a particular case (3 media with two plane interfaces)

Kats (1977): expression in terms of reflection coefficients

normal modes: $r_1 r_2 e^{-2\kappa L} E = E$

More general derivations along the time...

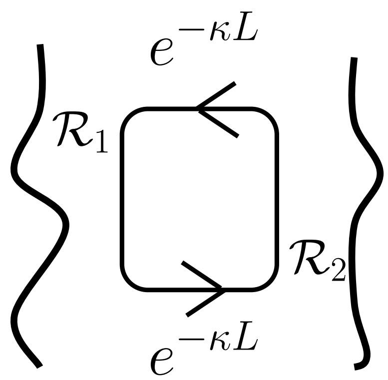
Lossy plates: Genet+Lambrecht+Reynaud Phys Rev A (2003)

Also applies for magnetic media, generalizations for anisotropic materials
(see Felipe da Rosa talk for applications to metamaterials)

Scattering approach

$$E_{\rm PP} = \hbar A \int_0^\infty \frac{\mathrm{d}\xi}{2\pi} \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \sum_p \ln\left(1 - r_{1;p}(k) r_{2;p}(k)\, e^{-2\kappa L}\right)$$

Closed loops with nonplanar surfaces



Generalizing for non-planar surfaces

$$\mathcal{E} = \hbar \int_0^\infty \frac{\mathrm{d}\xi}{2\pi} \operatorname{Tr} \ln\left(1 - \mathcal{R}_1(i\xi) e^{-\mathcal{K}L} \mathcal{R}_2(i\xi) e^{-\mathcal{K}L}\right)$$

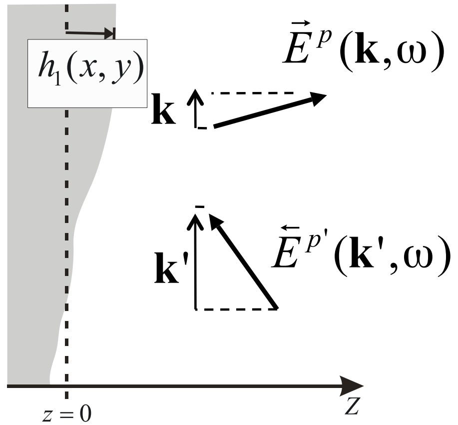
$$\operatorname{Tr}(...) \equiv \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \sum_p \langle \mathbf{k}, p | (...) | \mathbf{k}, p \rangle$$

Non-specular reflection operators

$\mathcal{R}_1$ and $\mathcal{R}_2$: change **k** and polarization *p*

(Diego's talk)

$$\overrightarrow{\mathbf{E}}_p\,(\mathbf{k},\omega) = \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \sum_{p'} \langle \mathbf{k}, p | \mathcal{R}(\omega) | \mathbf{k}', p' \rangle\ \overleftarrow{\mathbf{E}}_{p'}\,(\mathbf{k}',\omega)$$

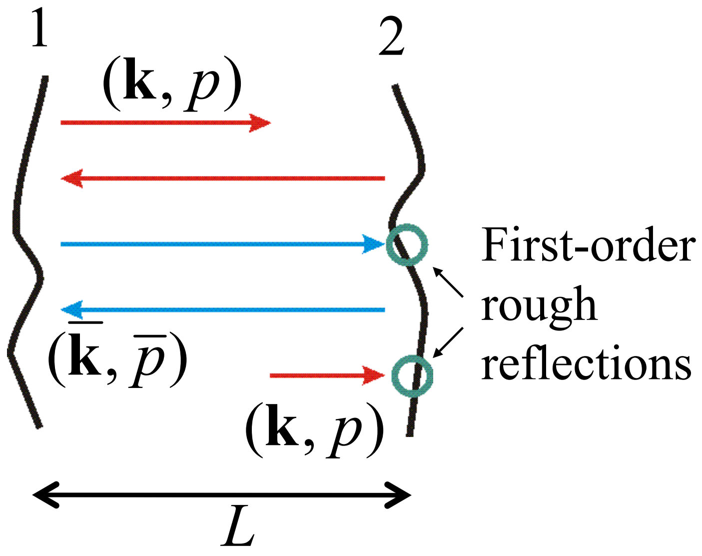


Lifshitz formula as a limiting case: $\mathcal{R}_1$ and $\mathcal{R}_2$ diagonal (specular reflection)

Scattering approach

Second-order roughness correction –
PAMN, Lambrecht, Reynaud 2005

Second-order contribution of first-order reflection coefficients (at the same mirror) to closed loops



$$\delta g_p^{(2)}(\mathbf{k},\omega) = \sum_{\bar{p}} \int \frac{d^2\bar{k}}{(2\pi)^2} \Big(e^{ik_z L} r_{1;p}(\mathbf{k}) e^{ik_z L} \mathcal{R}^{(1)}_{2;p\bar{p}}(\mathbf{k},\bar{\mathbf{k}};\omega)\, e^{i\bar{k}_z L} r_{1;\bar{p}}(\bar{\mathbf{k}}) e^{i\bar{k}_z L} \mathcal{R}^{(1)}_{2;\bar{p}p}(\bar{\mathbf{k}},\mathbf{k};\omega) + \ldots$$
$$\ldots + \text{second order coeff.} \Big) \left| H_2(\bar{\mathbf{k}}-\mathbf{k}) \right|^2 + (\ldots) \left| H_1(\bar{\mathbf{k}}-\mathbf{k}) \right|^2$$

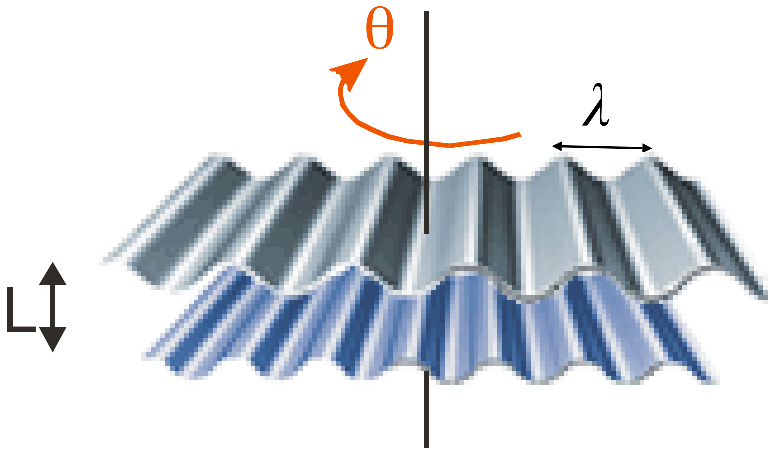
Results from scattering formula coincide with those from more ‘standard’ approaches !

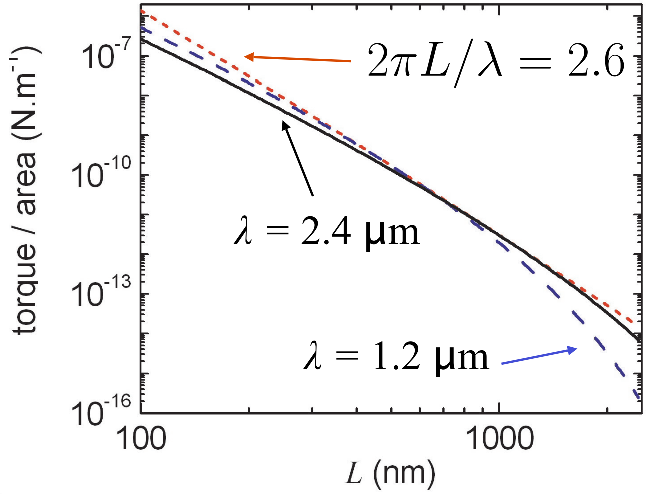
$$\mathcal{E} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \operatorname{Tr} \ln \left(1 - \mathcal{R}_1(i\xi) e^{-\mathcal{K}L} \mathcal{R}_2(i\xi) e^{-\mathcal{K}L}\right)$$

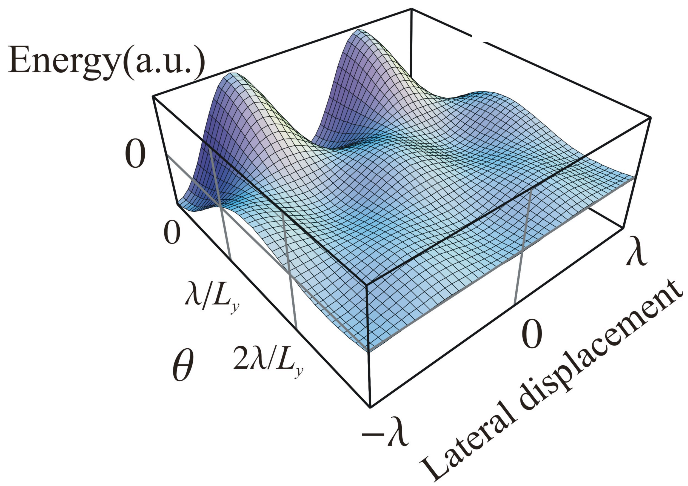
Scattering approach

another application:

Casimir Torque between corrugated surfaces: Rodrigues+PAMN+Lambrecht +Reynaud (2006)

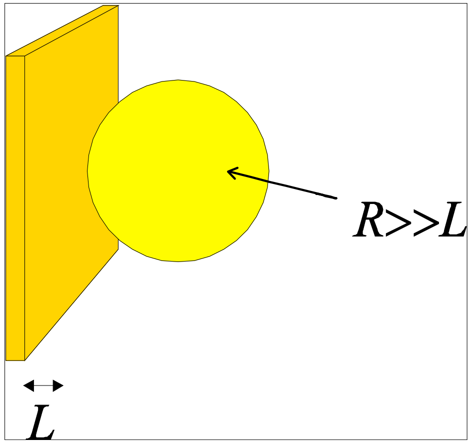






Scattering approach

Plane-sphere Casimir energy within the scattering approach



Bordag, Bulgac+Magierski+Wirzba: scalar field models, analytical results for first-order correction to PFA

Emig 2007: formula for a body in front of a perfectly reflecting plane, based on the method of images.

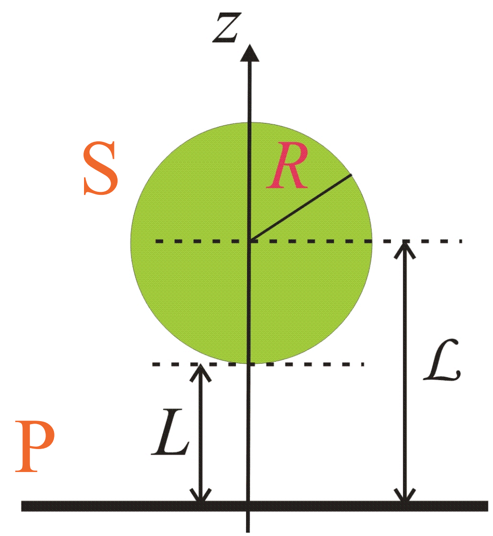
PAMN+Lambrecht+Reynaud (march 2008): formalism for real materials (ex: metals with finite conductivity)

Plane-sphere geometry

Casimir energy

$$\mathcal{E}_{\mathrm{PS}} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\mathcal{D}$$

$$\mathcal{D} = 1 - \mathcal{R}_{\mathrm{S}}\, e^{-\mathcal{K}\mathcal{L}}\, \mathcal{R}_{\mathrm{P}}\, e^{-\mathcal{K}\mathcal{L}}$$



Plane wave basis vs Multipole basis

better adapted to...

Scattering by plane (P) and free propagation	Scattering by sphere (S) and determinant evaluation

Plane-sphere geometry

Plane waves for a given
frequency ω

$$\{|\mathbf{k},\phi,p\rangle,\mathbf{k}\in\mathbb{R}^2,\phi=\pm1,p=\mathrm{TE},\mathrm{TM}\}$$

$$k_z=\phi\sqrt{\omega^2/c^2-k^2}$$

Multipoles for a given
frequency ω

$$\{|\ell,m,P\rangle,\ell=1,2,3...,m=-\ell,...,\ell,P=\mathrm{E},\mathrm{M}\}$$

E = electric multipoles, M = magnetic multipoles

Plane-sphere geometry

$$\mathcal{E}_{\mathrm{PS}} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\mathcal{D}$$

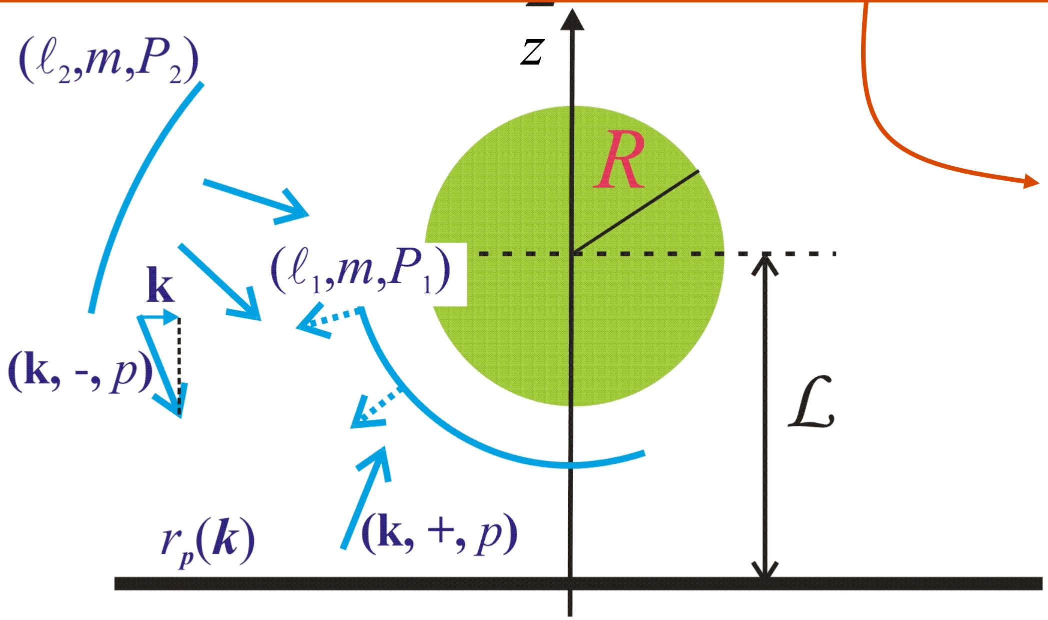
$$\mathcal{D} = 1 - \mathcal{R}_{\mathrm{S}}\, e^{-\mathcal{K}\mathcal{L}}\, \mathcal{R}_{\mathrm{P}}\, e^{-\mathcal{K}\mathcal{L}}$$

Axial symmetry:

$$\langle \ell_1, m_1, P_1 | \mathcal{D} | \ell_2, m_2, P_2 \rangle \propto \delta_{m_1,m_2}$$

$$\langle \ell_1, m, P_1 | \mathcal{D} | \ell_2, m, P_2 \rangle \equiv \mathcal{D}^{(m)}_{1,2}$$

$$\mathcal{D}^{(m)}_{1,2} = \delta_{1,2} - \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sum_{p=\mathrm{TE,TM}} \langle \ell_1 m P_1 | \mathcal{R}_S | \mathbf{k}, +, p \rangle r_p(\mathbf{k}) e^{-2\kappa\mathcal{L}} \langle \mathbf{k}, -, p | \ell_2 m P_2 \rangle$$



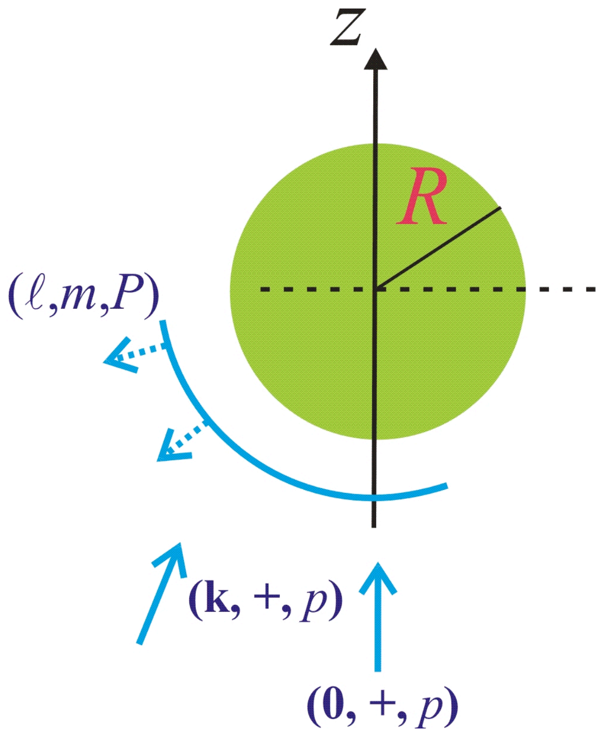
Usual Fresnel reflection coefficients (as in Lifshitz formula!)

Plane-sphere geometry

Scattering by the sphere: in classical optics, one usually needs just the matrix elements (propagation along *z*-axis)

$\langle \ell m P | \mathcal{R}_S | \mathbf{0}, +, p \rangle$

scattering amplitudes: Mie coefficients	
Electric multipoles: P=E	Magnetic multipoles: P=M
$a_\ell(\mathrm{i}\xi)$	$b_\ell(\mathrm{i}\xi)$

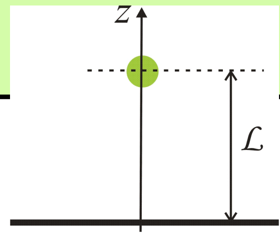


Rotating the incident plane wave with the help of finite rotation matrix elements

$$d^\ell_{m,m'}(\theta) \equiv \langle \ell m | e^{-iJ_y\theta/\hbar} | \ell m' \rangle$$

$$\langle \ell m \mathrm{E} | \mathcal{R}_\mathrm{S} | \mathbf{k}, +, \mathrm{TE} \rangle \sim a_\ell(i\xi) \left(d^\ell_{m,1}(\theta_k) + d^\ell_{m,-1}(\theta_k) \right)$$

Plane-sphere geometry



Mie scattering with very small spheres: **Rayleigh limit**

- If $R << \lambda_{vac}/n, \lambda_{vac},$ then a_1 (electric dipole) dominates over higher multipoles (including magnetic dipole b_1)

- Translating to Casimir theory: $\lambda_{vac} \sim \mathcal{L}$, then condition reads

$$R << \mathcal{L}, \mathcal{L}/n$$

- We find from the previous results (α is the sphere polarizability)

$$a_1(i\xi) \approx -\frac{2}{3}\left(\frac{\xi}{c}\right)^3 \frac{\alpha}{4\pi\epsilon_0} \qquad \boxed{\alpha = 4\pi\epsilon_0 \frac{\epsilon - 1}{\epsilon + 2}}$$

$$\mathcal{E}_C = \frac{\hbar}{2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\frac{d^2k}{(2\pi)^2}\, e^{-2\kappa\mathcal{L}}\,\frac{\hat{\xi}^2}{\kappa}\,\alpha(i\xi)\left[r_{\rm TE}(k,\xi) - (1 + 2k^2/\hat{\xi}^2)r_{\rm TM}(k,\xi)\right]$$

→ vdWaals/Casimir-Polder interaction !!

Diego's talk

Plane-sphere geometry

$$\mathcal{D}^{(m)}_{1,2} = \delta_{1,2} - \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sum_{p=\mathrm{TE,TM}} \langle \ell_1 m P_1 | \mathcal{R}_S | \mathbf{k}, +, p\rangle r_p(\mathbf{k}) e^{-2\kappa\mathcal{L}} \langle \mathbf{k}, -, p | \ell_2 m P_2 \rangle$$

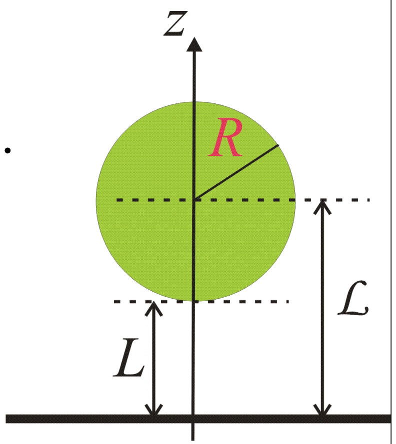
Simpler expressions in the case of perfect reflectors, taken as the limiting case of the plasma model for very short plasma wavelengths λ_P

- For the plane: $r_{TE} = -1$, $r_{TM} = 1$ ⟹ allows us to add analitically over p

- For the sphere, it is not sufficient to consider the limit $n \gg 1$ if the sphere is small...one also needs $R \gg \lambda_{vac}/n$ ...

..translating to Casimir theory: $\lambda_P \ll \mathcal{L}$ **and** $\lambda_P \ll R$

No intersection with Rayleigh limit !



Plane-sphere geometry

Perfectly-reflecting limit:

$$\mathcal{D}^{(m)}_{\ell_1\mathrm{E},\ell_2\mathrm{E}} = \delta_{\ell_1\ell_2} + \tfrac{1}{2}\sqrt{(2\ell_1+1)(2\ell_2+1)}a_{\ell_1}\mathcal{F}^{(+)}_{\ell_1,\ell_2,m}$$
$$\mathcal{D}^{(m)}_{\ell_1\mathrm{M},\ell_2\mathrm{M}} = \delta_{\ell_1\ell_2} - \tfrac{1}{2}\sqrt{(2\ell_1+1)(2\ell_2+1)}b_{\ell_1}\mathcal{F}^{(+)}_{\ell_1,\ell_2,m}$$
$$\mathcal{D}^{(m)}_{\ell_1\mathrm{E},\ell_2\mathrm{M}} = \tfrac{i}{2}\sqrt{(2\ell_1+1)(2\ell_2+1)}a_{\ell_1}\mathcal{F}^{(-)}_{\ell_1,\ell_2,m}$$
$$\mathcal{D}^{(m)}_{\ell_1\mathrm{M},\ell_2\mathrm{E}} = \tfrac{i}{2}\sqrt{(2\ell_1+1)(2\ell_2+1)}b_{\ell_1}\mathcal{F}^{(-)}_{\ell_1,\ell_2,m}$$

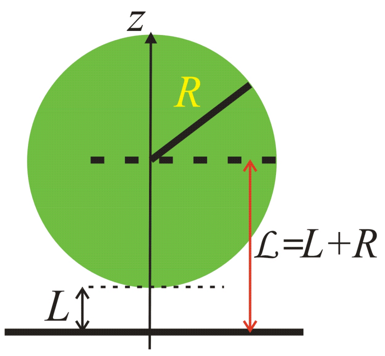
$$\mathcal{F}^{(\pm)}_{\ell_1,\ell_2,m} = (-)^{\ell_2+m}\int_1^\infty \mathrm{d}\cos\theta\; e^{-2\xi\mathcal{L}\cos\theta/c}\left[d^{\ell_1}_{m,1}(\theta)d^{\ell_2}_{m,1}(\theta) \pm (-)^{\ell_1-\ell_2}d^{\ell_1}_{m,1}(\pi-\theta)d^{\ell_2}_{m,1}(\pi-\theta)\right]$$

PFA limit:

When $L \ll R$, we have $\xi R/c \gg 1$ (for typical values of ξ) and then $a_\ell(i\xi)$, $b_\ell(i\xi) \sim \exp(2\xi R/c)$

$$\Rightarrow \quad a_\ell\,\mathcal{F}^{\pm},\, b_\ell\,\mathcal{F}^{\pm} \sim \exp\left(-\frac{2\xi L}{c}\right)$$

Edge showing up !

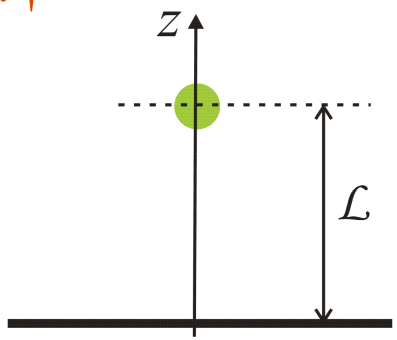


Plane-sphere geometry

Small, perfectly-reflecting sphere: $\lambda_P << R << \mathcal{L}$

Electric and magnetic dipoles are of the same order !

$$a_1(i\xi) \approx -2b_1(i\xi) \approx -\frac{2}{3}\left(\frac{\xi R}{c}\right)^3$$



Neglect higher multipoles ⟹

'Casimir-Polder' with magnetic dipole contribution

$$\mathcal{E}_{\rm PS} = -\frac{9\,\hbar c}{16\pi}\frac{R^3}{\mathcal{L}^4}$$

T. Emig (2008) from model of perfect reflectivity

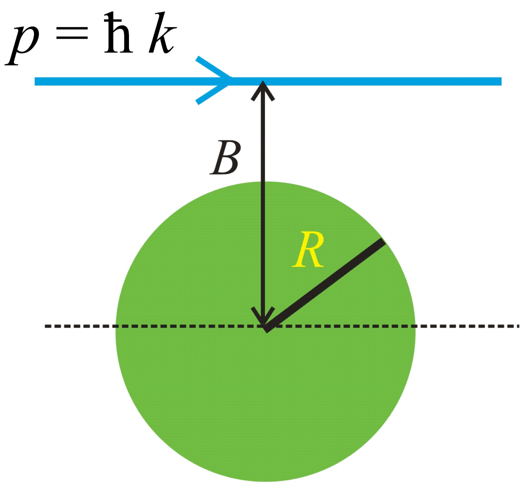
no intersection with Rayleigh limit
($R << \lambda_P << \mathcal{L}$)

$$\mathcal{E}_{\rm PS} = -\frac{3\hbar c}{8\pi}\frac{R^3}{\mathcal{L}^4}$$

Plane-sphere geometry

What are the typical values of angular momentum ℓ when we approach the PFA limit ?

Localization principle



When $\lambda = 2\pi/k \ll R$, ℓ corresponds to rays with impact parameter B given by

$$\ell = k\, B$$

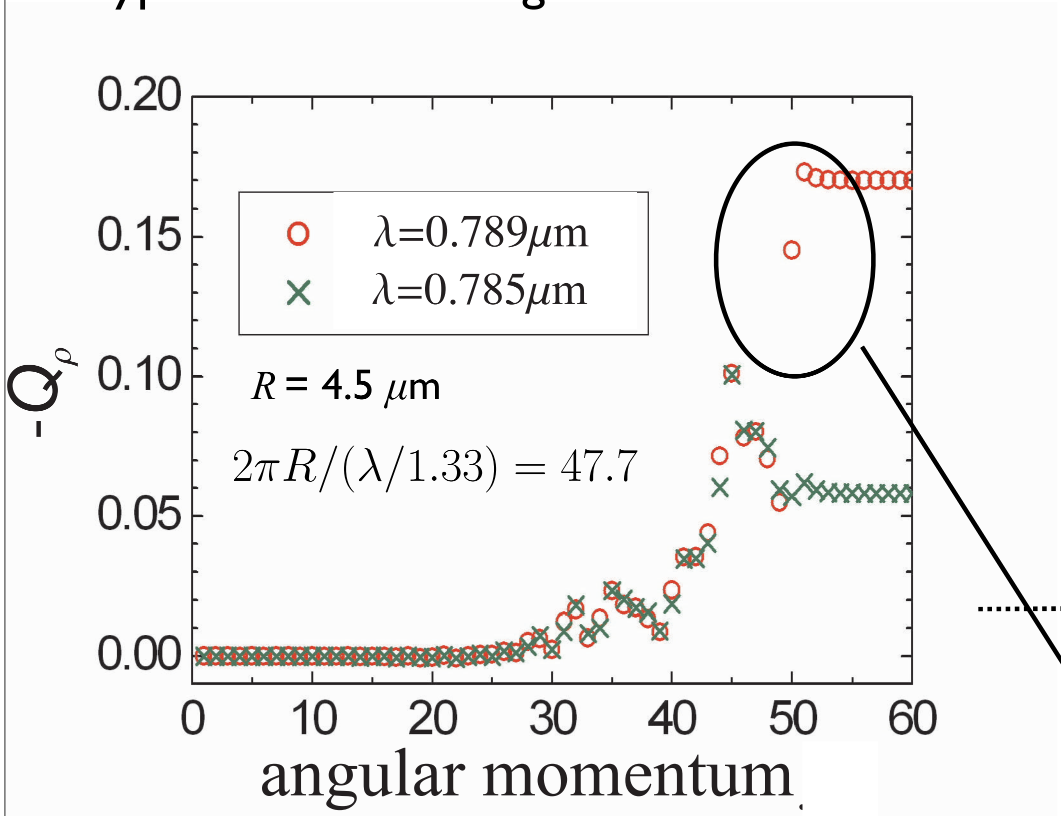
Rays with $B > R$ provide negligible contributions

$$\Rightarrow \quad \ell \lesssim kR \sim \frac{R}{L} \gg 1$$

H. M. Nussenzveig, *Diffraction Effects in Semiclassical Scattering,* Cambridge 1992

Plane-sphere geometry

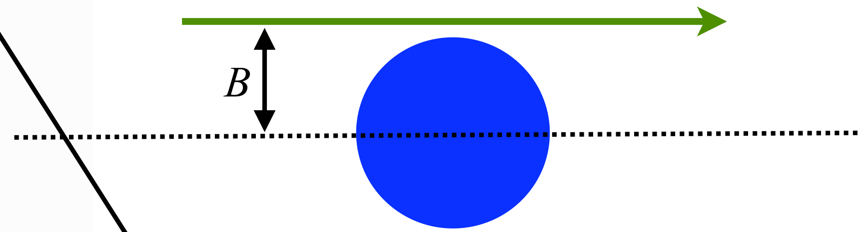
a typical Mie scattering numerical calculation



from **R S Dutra , N B Viana , PAMN and H M Nussenzveig,** J. Opt. A: Pure Appl. Opt. **9** (2007) S221

- converges at $\ell \sim \omega R/c$
- partial sum oscillates with ℓ
- strong variation with frequency

localization principle $\ell = \omega B/c$

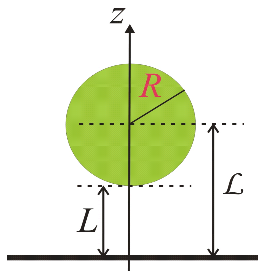


Mie resonances:

- ☑ optical whispering gallery modes
- ☑ contribution of (slightly) above-edge rays: $\ell > \omega R/c$
- ☑ lie close to the real axis ....ouufff...!

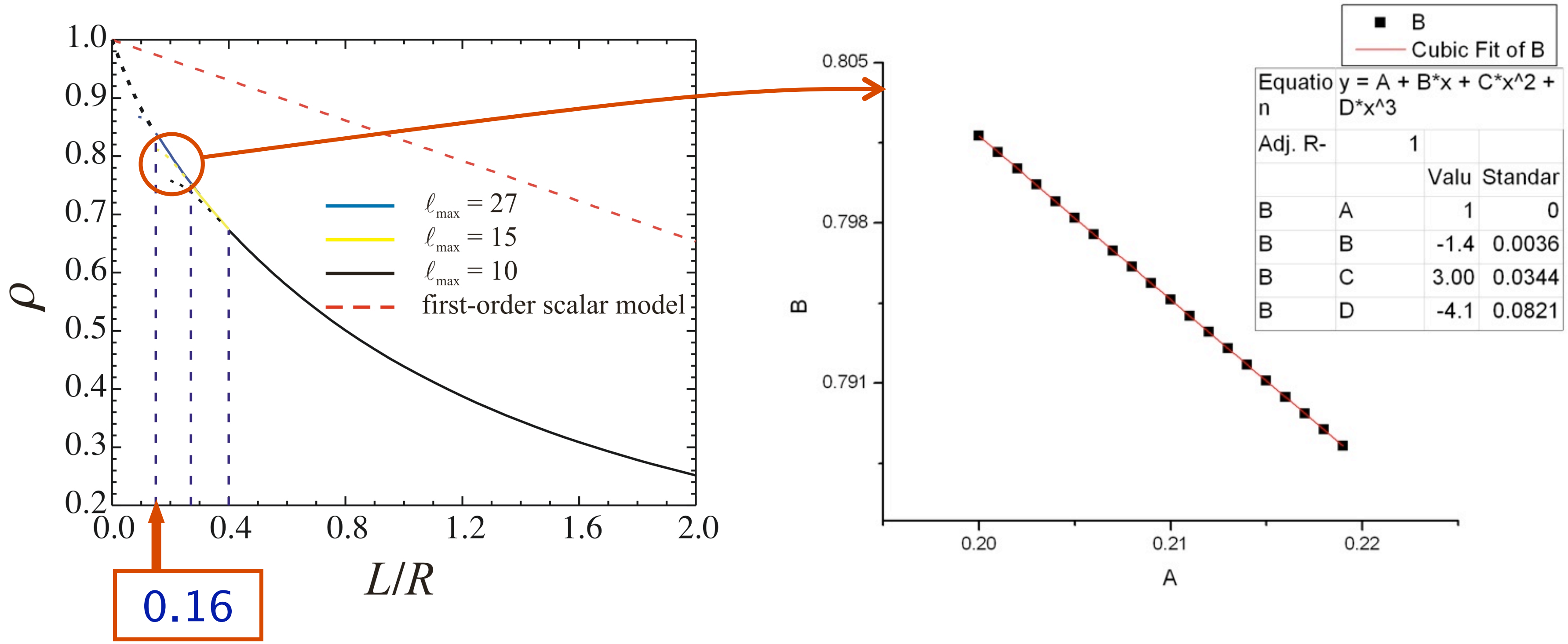
Plane-sphere geometry

Numerical calculation



Comparing with the Proximity Force Approx (PFA) result

$$\rho \equiv \frac{\mathcal{E}}{\mathcal{E}^{\rm PFA}}, \qquad \mathcal{E}^{\rm PFA} = -\frac{\hbar c \pi^3 R}{720 L^2}$$



Summary

- Scattering approach was employed to compute the Casimir interaction energy between a plane and a sphere
- Our approach allows for the computation in the case of real metals with finite conductivity
- Numerical calculation in the case of perfectly-reflecting surfaces. PFA is less accurate in the electromagnetic case (than in the scalar model) – correction is ~ 8 x larger in the EM case!
- Numerical calculation in the more general case will provide the correction to the Proximity Force Approx. result for a given L/R under realistic conditions

For details see PAMN, A Lambrecht and S Reynaud, Phys. Rev. A **78**, 012115 (2008)