Virtual Photons in Imaginary Time: *Computing* Casimir Forces in New Geometries

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Outline

- Why?
- How?
- What?

Outline

- Why?
- How?
- What?

Hendrik Casimir, 1948

H. B. G. Casimir, *Proc. K. Ned. Akad. Wet.* **51** (1948) 793

Mathematics. — *On the attraction between two perfectly conducting plates.* By H. B. G. CASIMIR.

We are thus led to the following conclusions. There exists an attractive force between two metal plates which is independent of the material of the plates as long as the distance is so large that for wave lengths comparable with that distance the penetration depth is small compared with the distance. This force may be interpreted as a zero point pressure of electromagnetic waves.

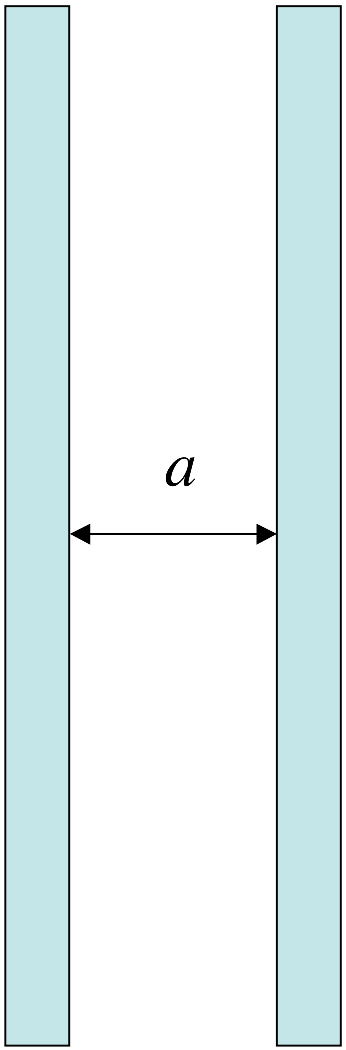
Although the effect is small, an experimental confirmation seems not unfeasable and might be of a certain interest.

Natuurkundig Laboratorium der N.V. Philips' Gloeilampenfabrieken, Eindhoven.)

[slide borrowed from F. Capasso]

The Casimir Force

[H. Casimir, 1948]



Parallel, neutral, perfect-metal plates, separation a

Quantum: attractive force $\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4}$

Ground-state energy: $$U = \sum_{\omega} \frac{\hbar \omega}{2}$$

depends on a!

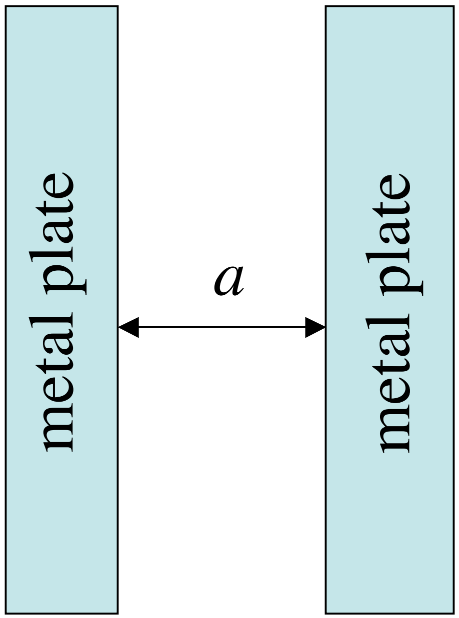
1d: $\omega = c \frac{\pi}{a} \cdot \#$

→ F = $-dU/da$

* U is infinite, so some care required

The Casimir Force

[H. Casimir, 1948]



$$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4}$$

attractive force,
monotonic decreasing

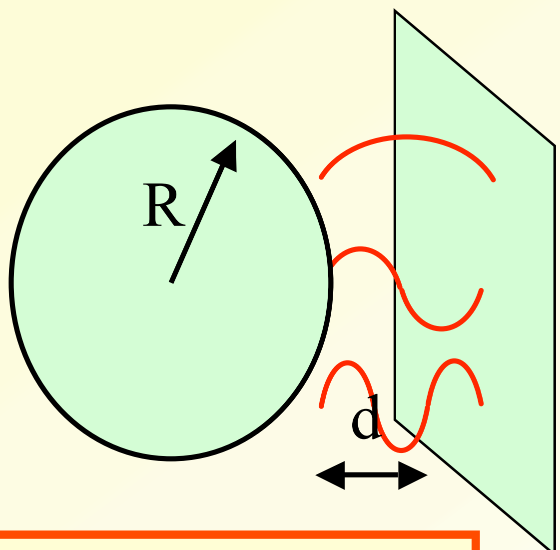
(10^{-7} N for a=1μm, A=1cm^2)

Numerous measurements in last decade
… mainly for sphere-plate
geometry

Sphere-plate measurements

slides courtesy
F. Capasso

Reduction of metallic reflectivity near plasma wavelength becomes important at comparable separation: lowering of the force



$$F_{Casimir} \approx -\frac{\pi^3 R}{360}\frac{\hbar c}{d^3}$$

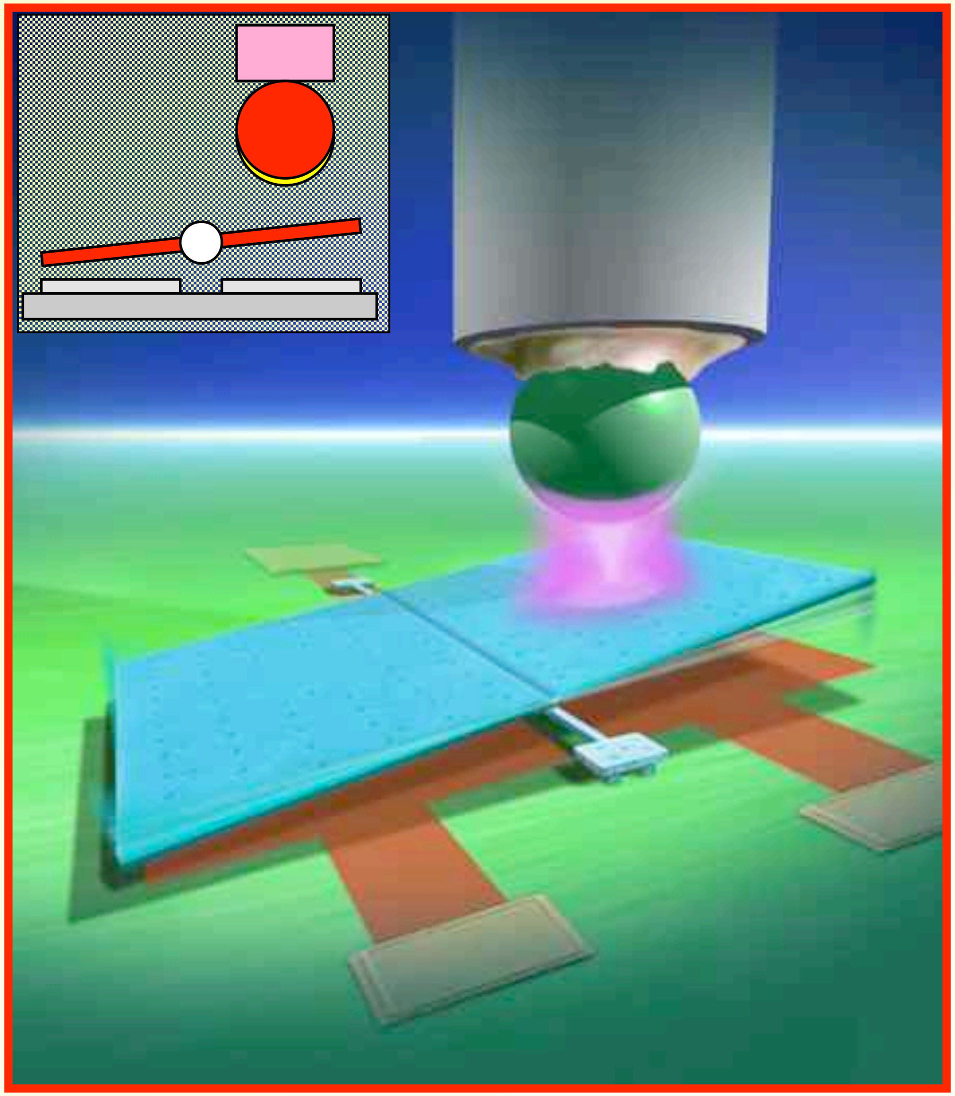
- **Van Blockland & Overbeek 1978**
  sphere-plate: first clear observation
- **Lamoreaux 1997**
  Torsional Pendulum
  first high precision experiment
- **Mohideen & Roy 1998 — AFM**
- **Chan, et al. 2001 — MEMS**
- **Decca et al 2003–2004 — MEMS**

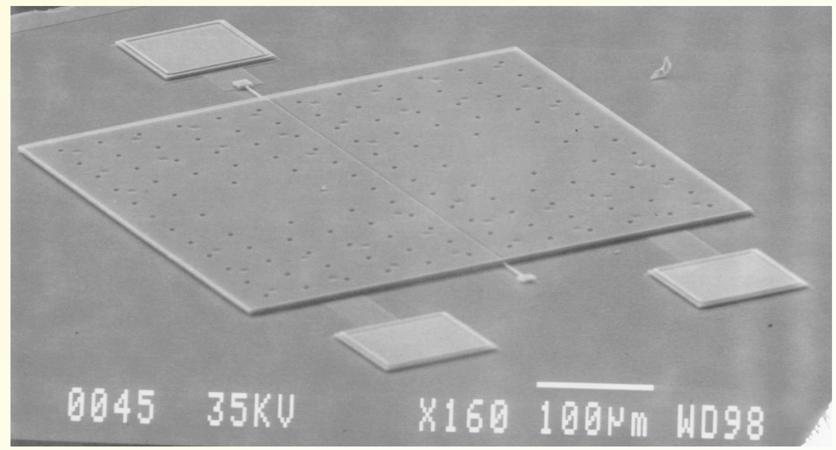
Experimental precision no better than 5%
and agreement with theory cannot be claimed to better that 10%

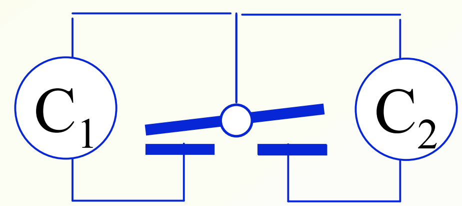
Measurement via MEMS

MicroElectroMechanicalSystem

slides courtesy
F. Capasso







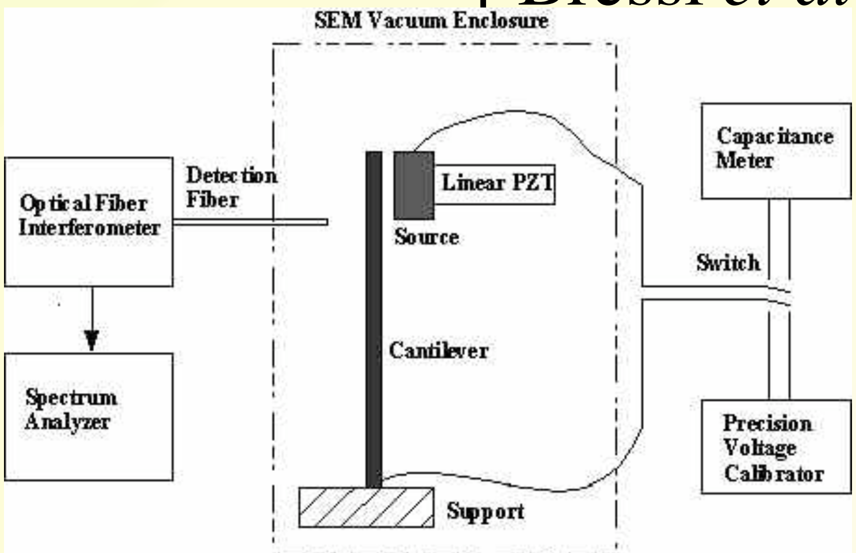
$$F_{min} \approx 10\ \text{pN}$$

H. B. Chan, V. A. Aksyuk, R. N. Kleinman, D. J. Bishop, and F. Capasso *Science* **291** (2001), p. 1941

Parallel Plate Measurement

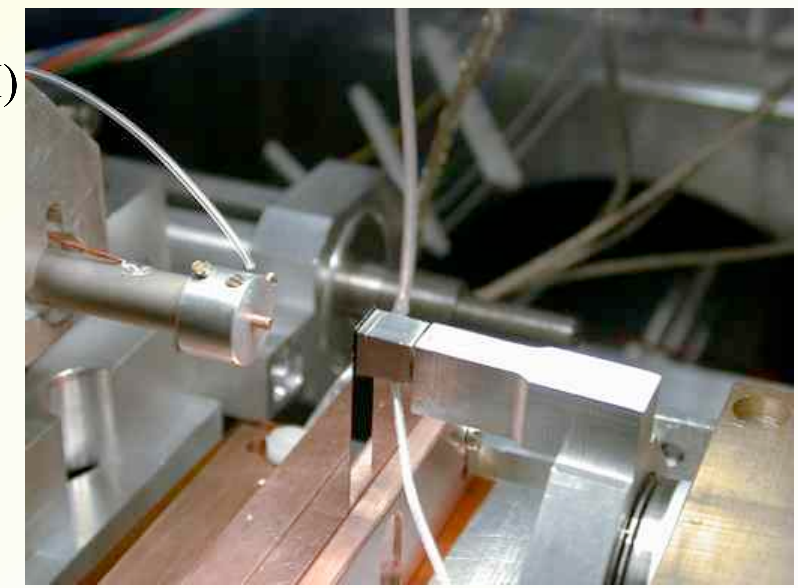
[Bressi *et al*., 2002]

slides courtesy
F. Capasso



- **Plane parallel** geometry
- **Silicon plates** with a 50 nm **chromium** deposit

- Apparatus inside **S**canning **E**lectron **M**icroscope (**SEM**) (pressure ~ 10^{-5} mbar)
- Mechanical decoupling between resonator and source
- SEM sitting on antivibration table
- System of actuators for parallelization
- Fiber-optic interferometer transducer
- SEM for final cleaning and parallelism monitoring
- Mechanical feedthroughs allow correct positioning of the apparatus in the electron beam
- Source approach to the cantilever using a linear PZT



Padova LNL 2002

Parallel Plate Measurement

[Bressi *et al*., 2002]

slides courtesy
F. Capasso

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Measurement of the Casimir Force between Parallel Metallic Surfaces

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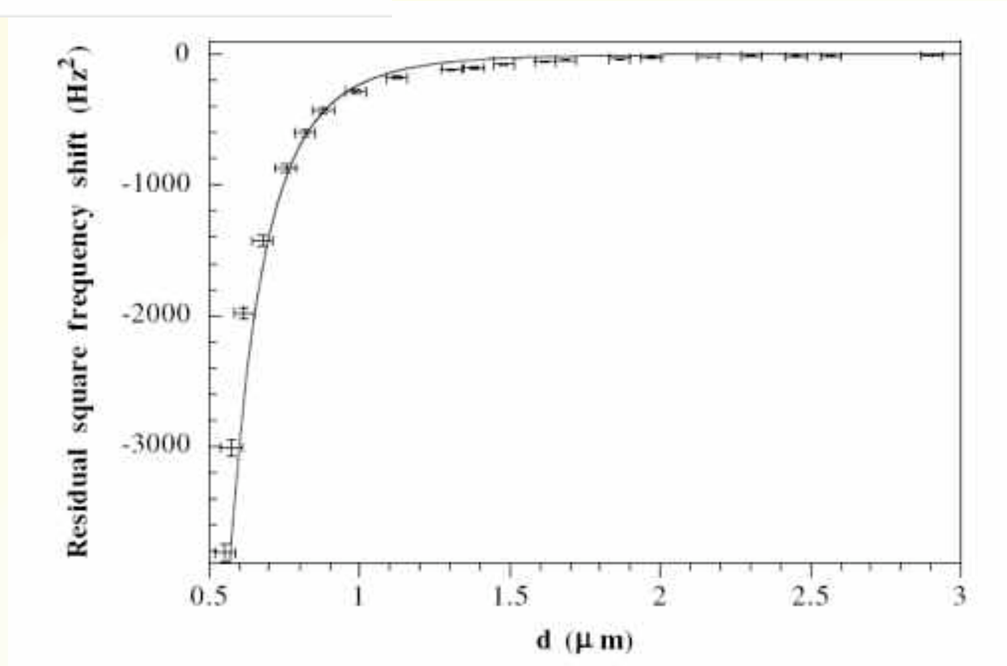
[3]Dipartimento di Fisica 'G. Galilei', Università di Padova, Via Marzolo 8, Padova, Italy 35131
(Received 10 October 2001; published 15 January 2002)

We report on the measurement of the Casimir force between conducting surfaces in a parallel configuration. The force is exerted between a silicon cantilever coated with chromium and a similar rigid surface and is detected by looking at the shifts induced in the cantilever frequency when the latter is approached. The scaling of the force with the distance between the surfaces was tested in the 0.5–3.0 μm range, and the related force coefficient was determined at the 15% precision level.

$$F = \frac{K_C}{d^4} S$$

$$K_C = (1.22 \pm 0.18) \cdot 10^{-27} \text{ N m}^2$$

$$K_C^{th} = \frac{\pi^2 \hbar c}{240} = 1.3 \cdot 10^{-27} \text{ N m}^2$$



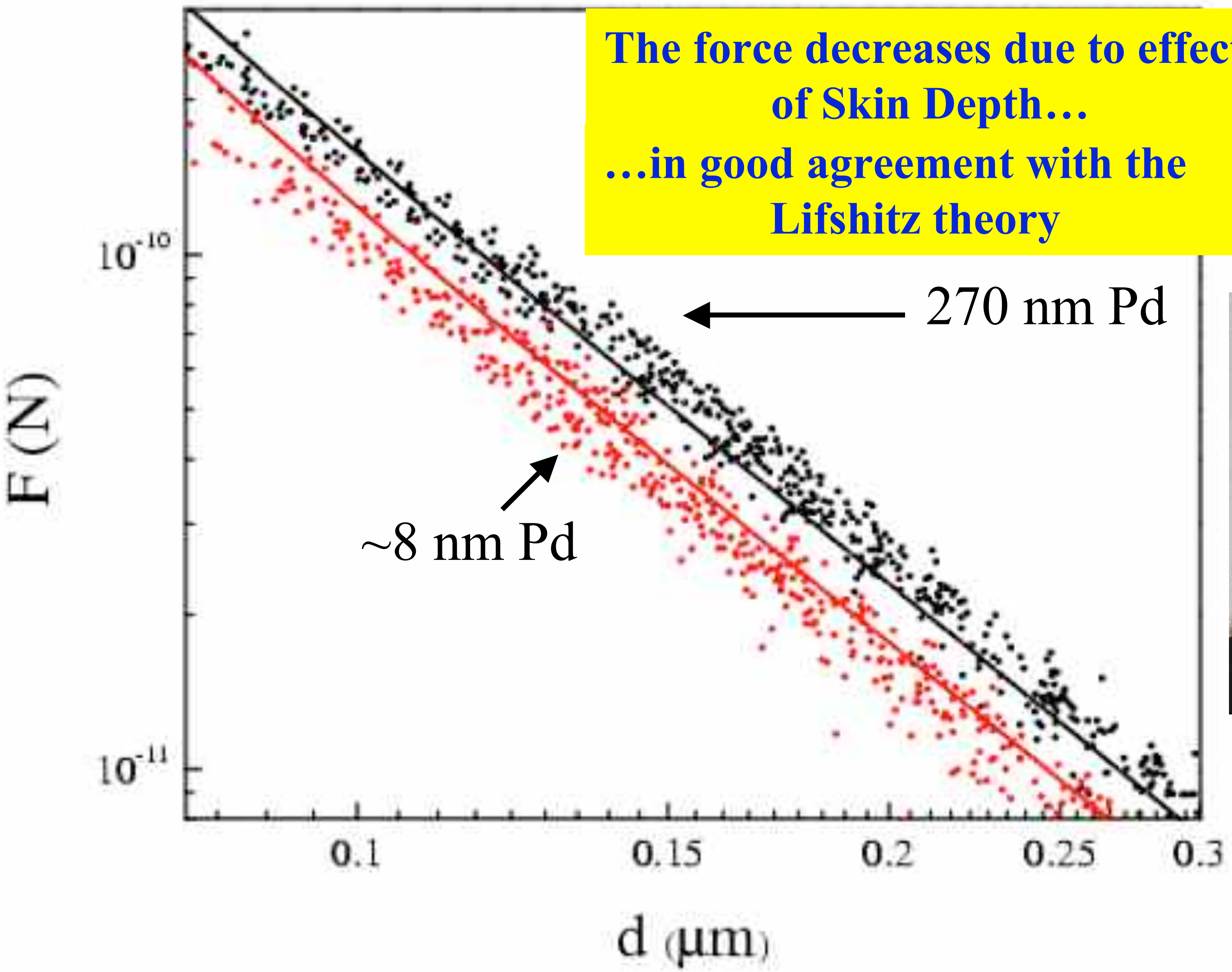
Padova LNL 2002

Finite skin-depth effect

[Lisanti et al, 2005]

slides courtesy
F. Capasso

The force decreases due to effect of Skin Depth…
…in good agreement with the Lifshitz theory

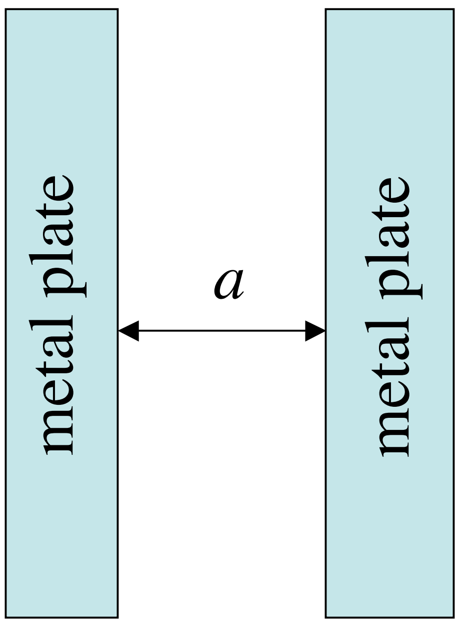


Mariangela Lisanti

M. Lisanti, D. Iannnuzzi, F. Capasso, *Proc. Nat. Acad. Sci.* 102, 11989 (2005)

Altering the Casimir Force

[H. Casimir, 1948]

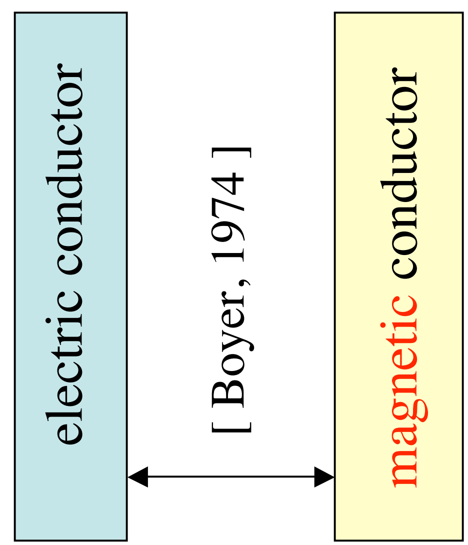


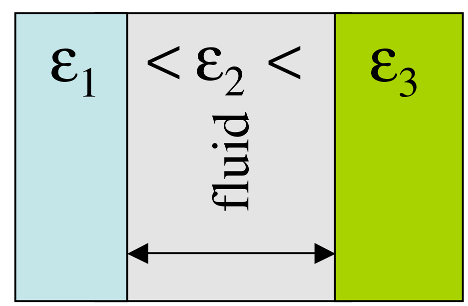
$$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4}$$

attractive force,
monotonic decreasing

(10^{-7} N for a=1μm, A=1cm^2)

obtaining *qualitatively different* behavior:
use "exotic" materials for repulsive force
(still monotonic)





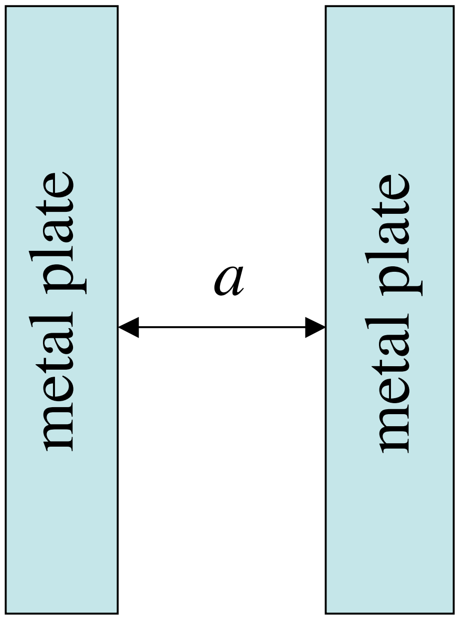
[Dzyaloshinskii, 1961; Munday & Capasso, 2007]

…or metamaterials
[Leonhardt, 2007; Dalvit, 2008]

…or excited atoms
[Sherkunov, 2005]

Altering the Casimir Force

[H. Casimir, 1948]

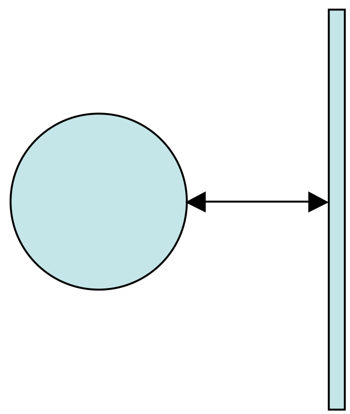


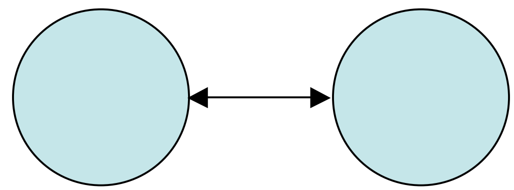
$$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4}$$

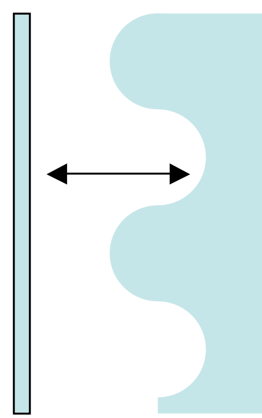
attractive force,
monotonic decreasing

(10^{-7} N for a=1μm, A=1cm^2)

obtaining *qualitatively different* behavior:
ordinary materials in complex *geometries*?







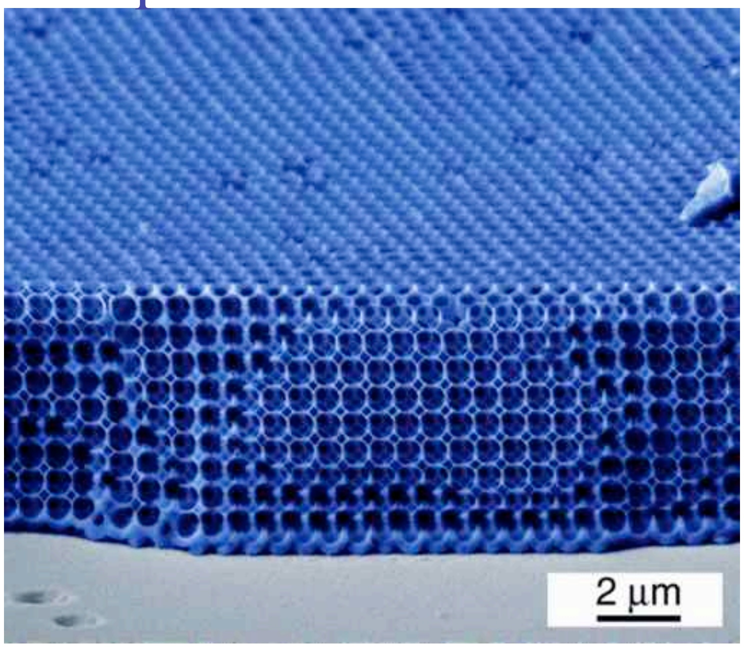
simplest strong-curvature
structures still give
monotonic attractive
forces 

(classical)

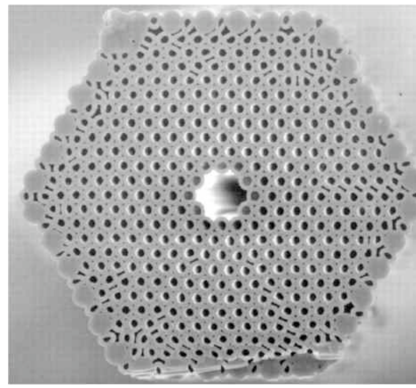
Nanophotonics:

classical electromagnetic effects can be greatly altered by λ-scale structures especially with *many* interacting scatterers

optical "insulators"



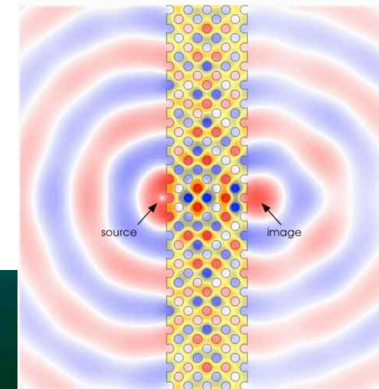
[D. Norris, UMN (2001)]



trapping/guiding light in vacuum

[R. F. Cregan (1999)]

flat "superlenses"

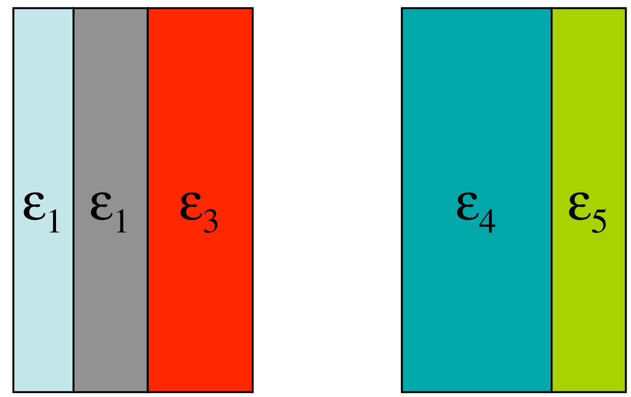


[Luo (2003)]

easy to study numerically, theory practically exact, well-developed scalable 3d methods for arbitrary materials

Casimir Nanophotonics?

1956–1968:
planar multilayers
(Lifshitz formula, *etc*.)

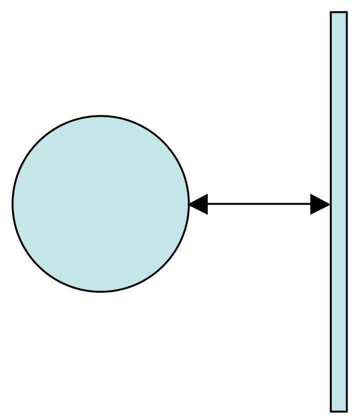
…


…

⋮

(various perturbative/asymptotic expansions)

2006:
cylinder/plate force
(numerical)
[Emig, Phys Rev. Lett. **96**, 080403]



perfect metals,
infinite cylinder/plate

(excluding *ad-hoc*, uncontrolled approximations)

e.g. pairwise "parallel-plate" interactions (PFA),
renormalized pairwise Casimir-Polder [Sedmik, 2006]
ray optics [Jaffe, 2004]

here: only "exact" numerical methods
= arbitrary accuracy given enough computing power

Outline

- Why?
- How?
- What?

How can this be problem be so hard?

non-interacting bosons — linear Maxwell-like PDEs,
continuum material models
polynomial complexity

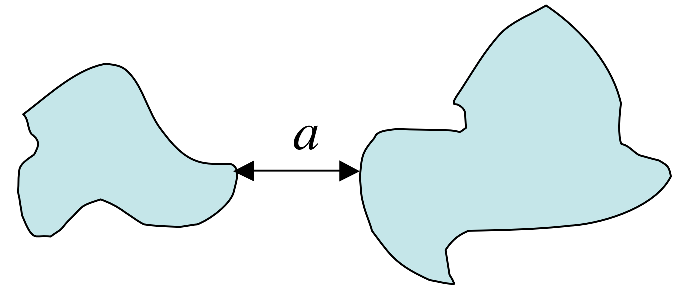
- Every current approach involves solving PDE's at least 1000's of times (usually much more!)

- Which PDE you solve makes a huge difference

— many equivalent formulations of Casimir force
… which is best-suited for numerics?

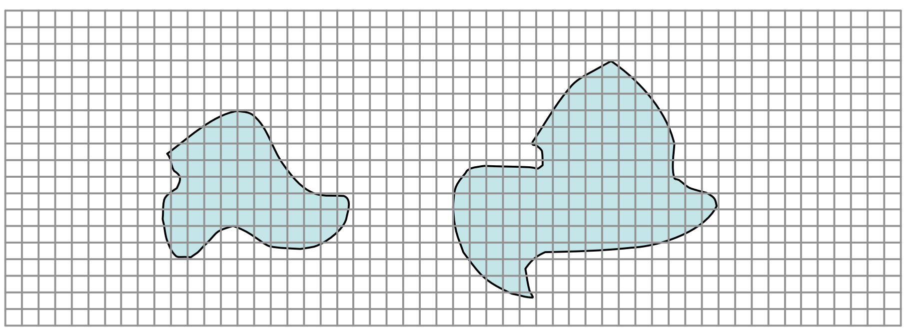
reasonably general, tractable numerical approaches demonstrated only recently
[Emig, 2001; Gies, 2003; Rodriguez, 2007; Emig, 2008]

A Simplistic Approach



zero-point energy (lossless media): $U = \sum_{\omega} \frac{\hbar\omega}{2} \quad \Rightarrow \quad F = -\frac{\partial U}{\partial a}$

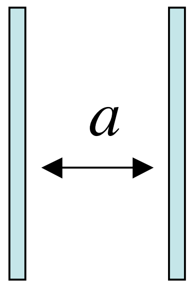
1) Compute (classical) eigenfrequencies numerically $\Rightarrow U$



any discretization
= regularization
= finite U

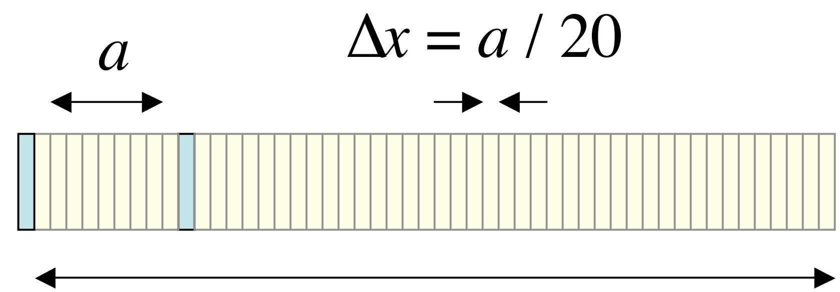
2) Numerical derivative $\Rightarrow F$

A Test for the Simplistic Approach

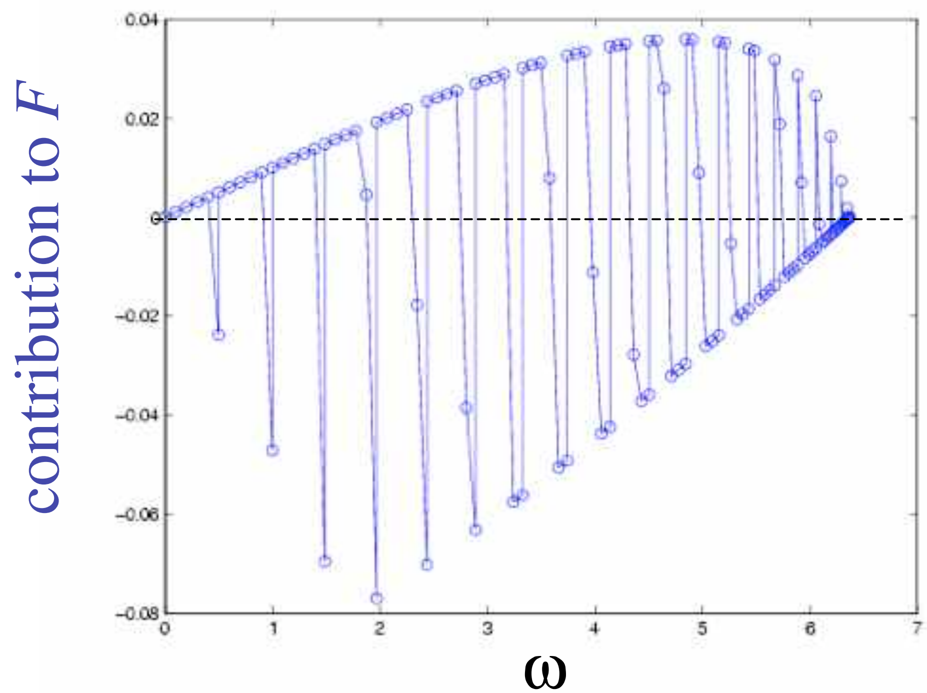


1d parallel plates

Discretized, finite:



$5a$, periodic boundaries



- wildly oscillating summand
- contributions up to Nyquist ω

$\Rightarrow$ need *all* eigenfrequencies
$O(N^3)$ work for N grid points
$O(N^2)$ storage
& with high accuracy

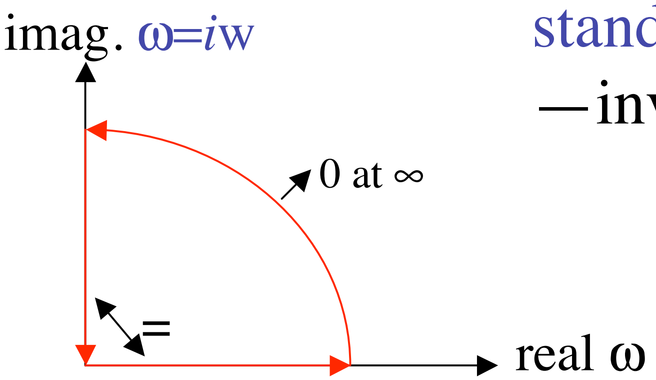
Real Benefits from Imaginary Time

- A reformulation: $$\sum_{\omega} \frac{\hbar\omega}{2} = \int_0^{\infty} \frac{\hbar\omega}{2} D(\omega) d\omega$$

$D(\omega)$ = density of states = trace of Green's function

= trace of inverse operator $$\frac{1}{\nabla\times\nabla\times - \omega^2\varepsilon(\omega)}$$

- Wick rotation (contour integration): real ω → imaginary ω = *iw*

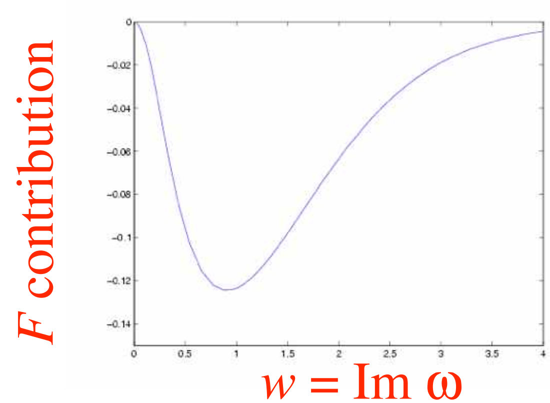


$$\frac{e^{i\omega r/c}}{r} \rightarrow \frac{e^{-wr/c}}{r}$$

standard numerical problem:

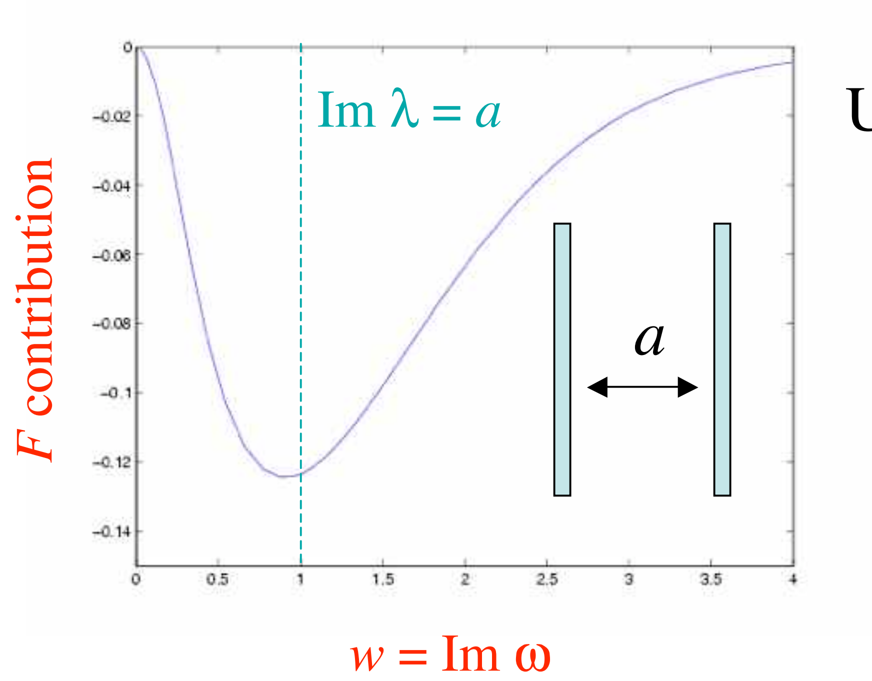
— inverting real-symmetric positive-definite

$$\nabla\times\nabla\times + w^2\varepsilon(iw)$$



non-oscillatory integrand, exponential cutoff

Better complexity, but not good enough



U = trace of Green's function

= integral of mean energy density

by fluctuation-dissipation theorem

[e.g. Tomas, *PRA* (2002)]

$$\sim \int_0^\infty dw \iiint_{\text{volume}} \mathrm{d}^3\mathbf{x} \, \frac{d(w^2\varepsilon)}{dw} \left\langle \mathbf{E}(\mathbf{x})^2 \right\rangle$$

= Green's function

= **E** at **x** from current at **x**

= solving one linear system

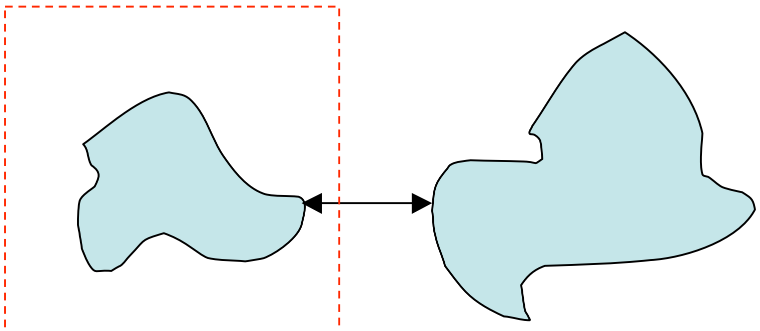
N degrees of freedom,
solving Green's = O(*N*) time
need at *every* **x** (*N* points)

= O(N^2) time

Better living through stress

We only *really* want the force, not U,

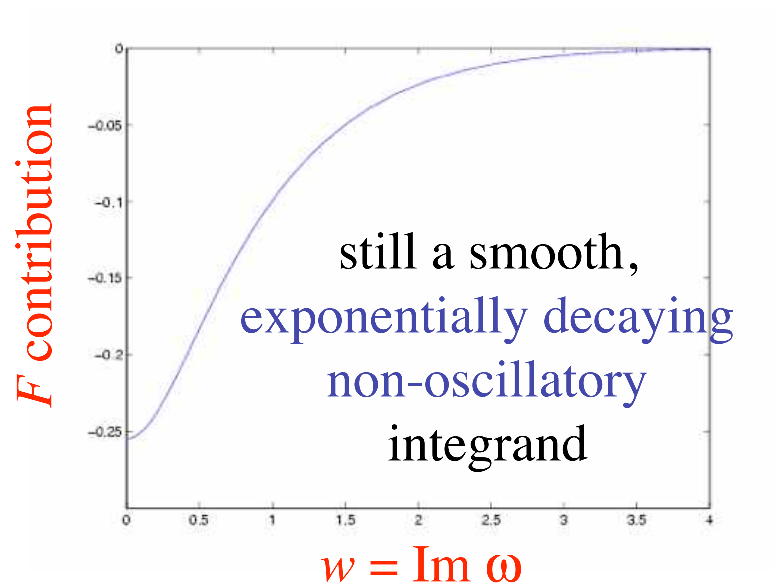
… so get force *directly* from *stress tensor*:



$$F = \int_0^\infty dw \oiint_{\text{surface}} \langle \mathbf{T} \rangle \cdot d\mathbf{A}$$

stress tensor $\sim \langle \mathbf{E}^2 \rangle + \langle \mathbf{H}^2 \rangle$ terms

= Green's function
evaluated only on the surface
<< N times
<< $O(N^2)$ work
$O(N^{2-1/d})$ or even $O(N \log N)$

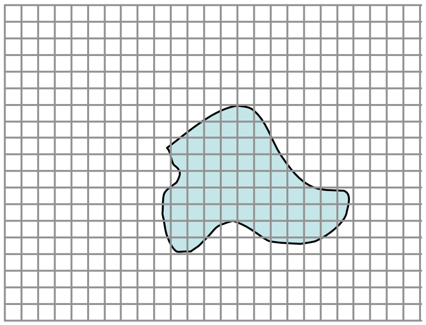


[analytical: Dzyaloshinskii, 1961] [numerical: Rodriguez, 2007]

Independent choices in numerical methods

- What PDE (or integral equation) are you solving?
  - — linear solver for imaginary-frequency Green's function
  - — stress tensor (may have inherent advantages over U)

- What discretization (what N degrees of freedom)?
  - — many *standard*, *well-developed* methods
  - finite elements & boundary elements (nonuniform mesh)
  - spectral (Fourier) methods
    - —exponentially fast convergence, somewhat geometry-specific
  - simple, dumb finite differences
    - —uniform grid, mediocre accuracy
    - —easy to implement proof-of-concept



- How to solve the linear equation?
  - — many fast iterative solvers ~ $O(N)$ time and storage

Outline

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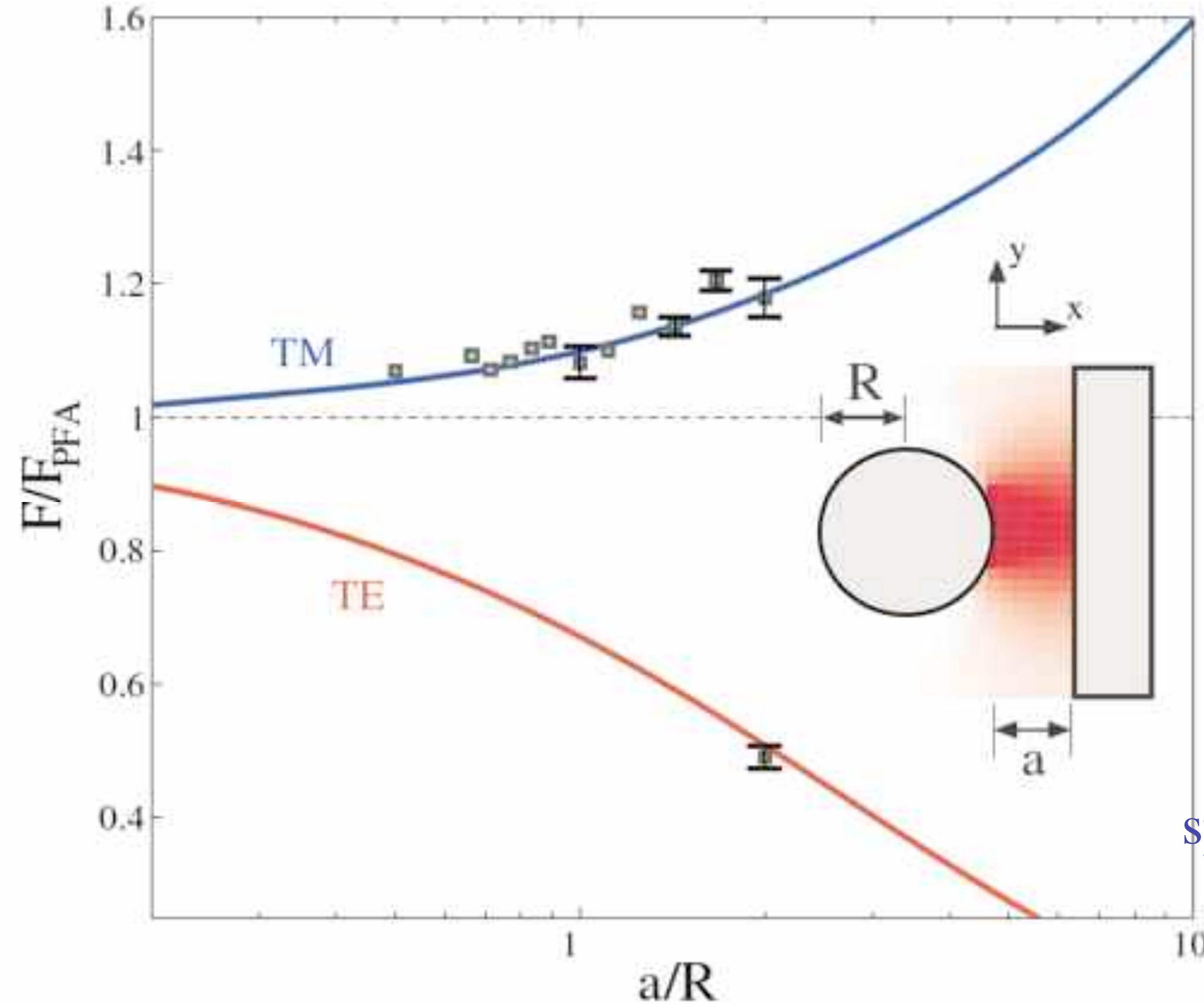
A more interesting test case



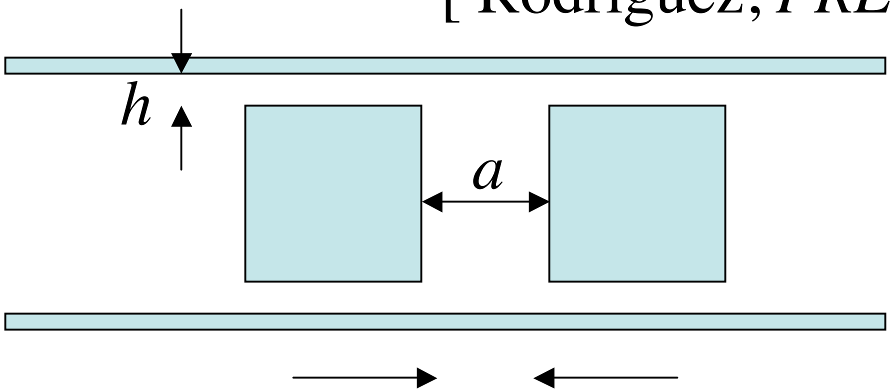
perfect-metal cylinder/plate force

dots = finite-difference stress tensor

lines = [Emig et al., *PRL* **96**, 080403 (2006)]
specially formulated for this geometry

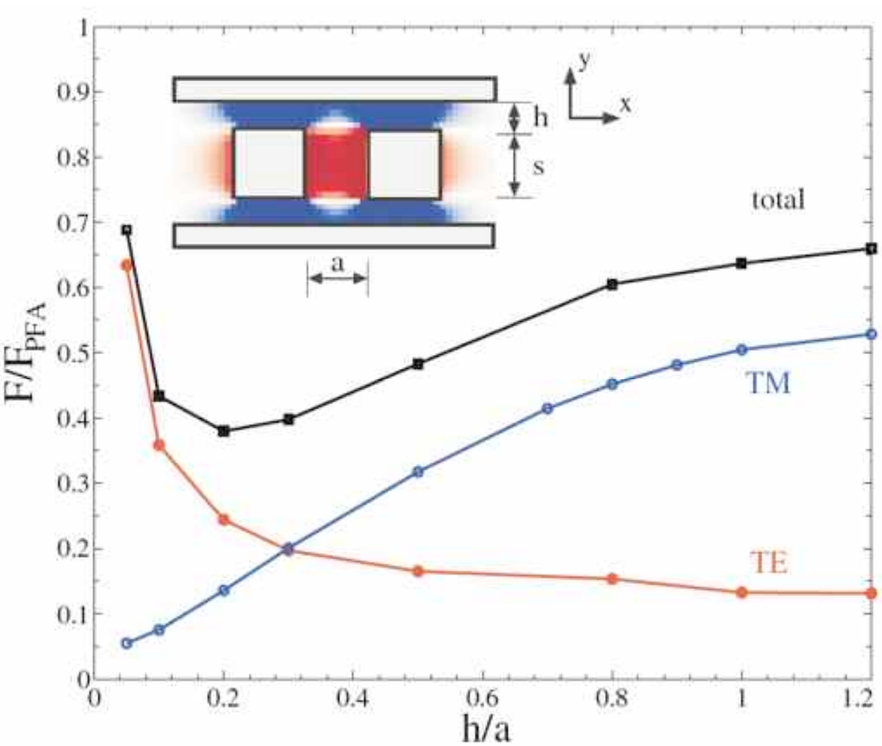
A multi-body interaction in 2d

[Rodriguez, *PRL* **99**, 080401 (2007)]

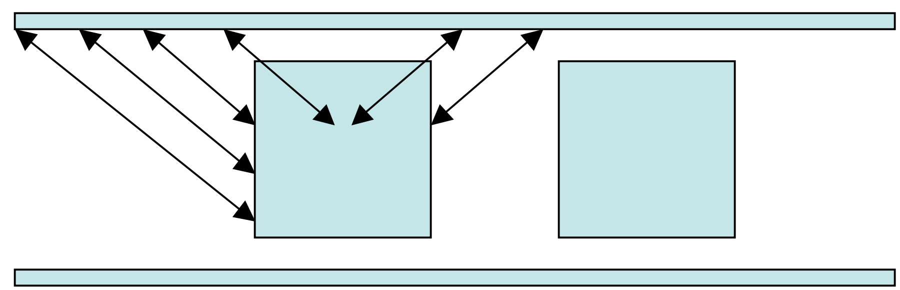


attractive force

Attractive force is a *non-monotonic* function of the sidewall separation *h*



ad-hoc pairwise interaction would predict force decreasing monotonically with *h* (if anything)

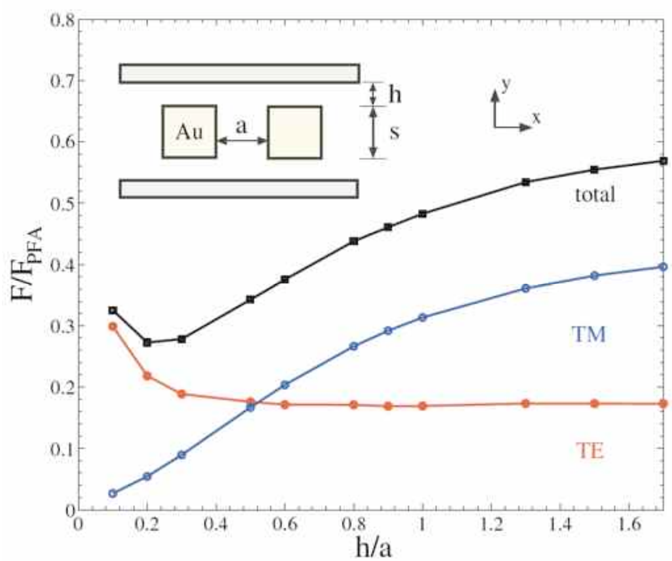


… although ray optics gives qualitatively correct behavior

[Zaheer, *PRA* **76**, 063816 (2007)]

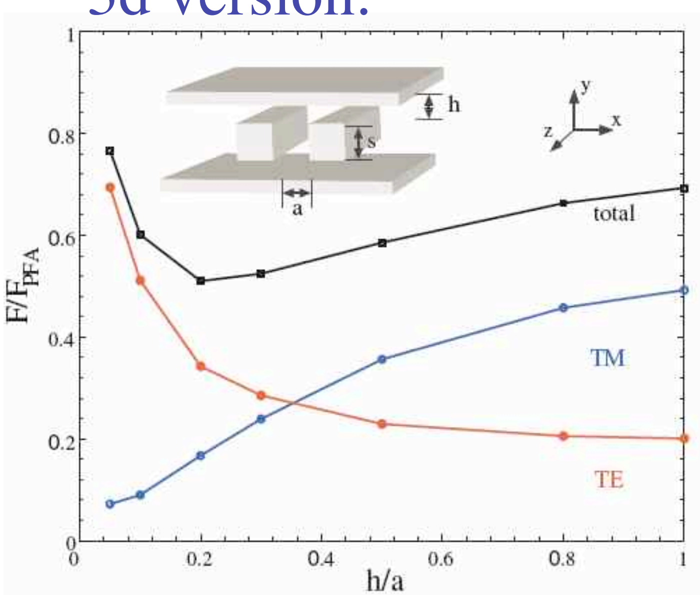
Other realizations

[Rodriguez, *PRL* **99**, 080401 (2007)]



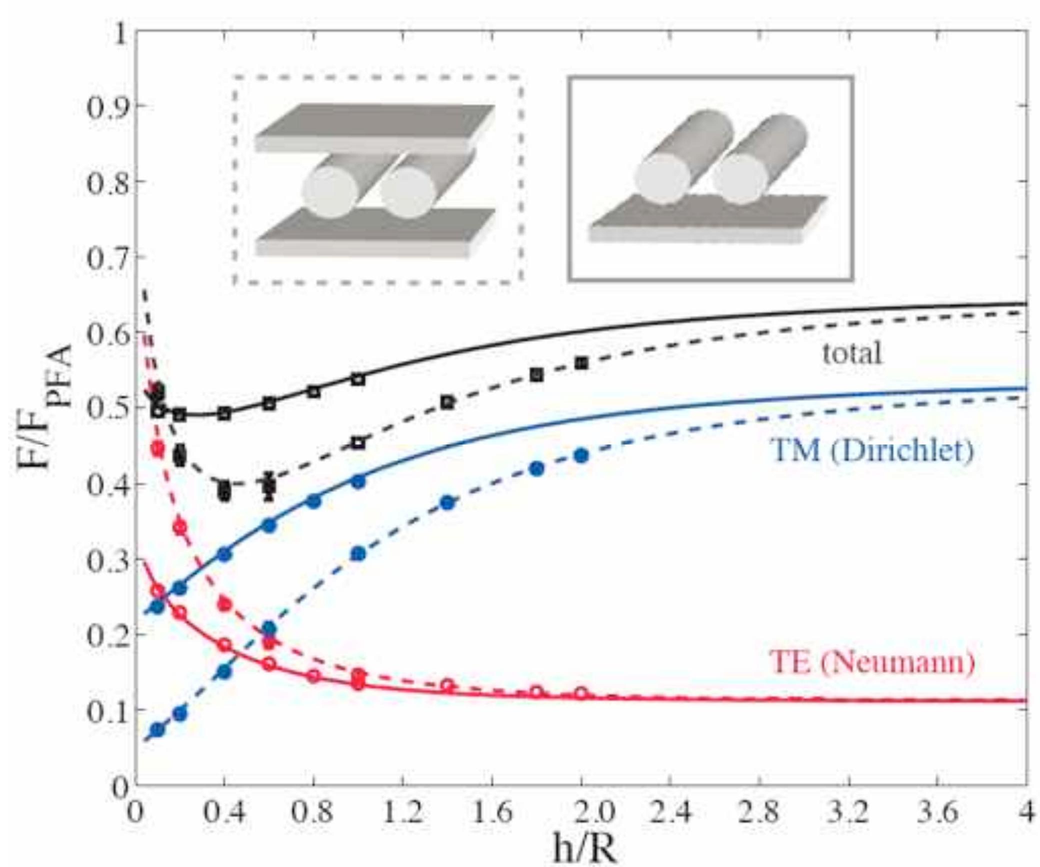
Force between gold squares (Drude model), $a = 1\mu$m

3d version:



two (3d) cylinders with one or two sidewalls

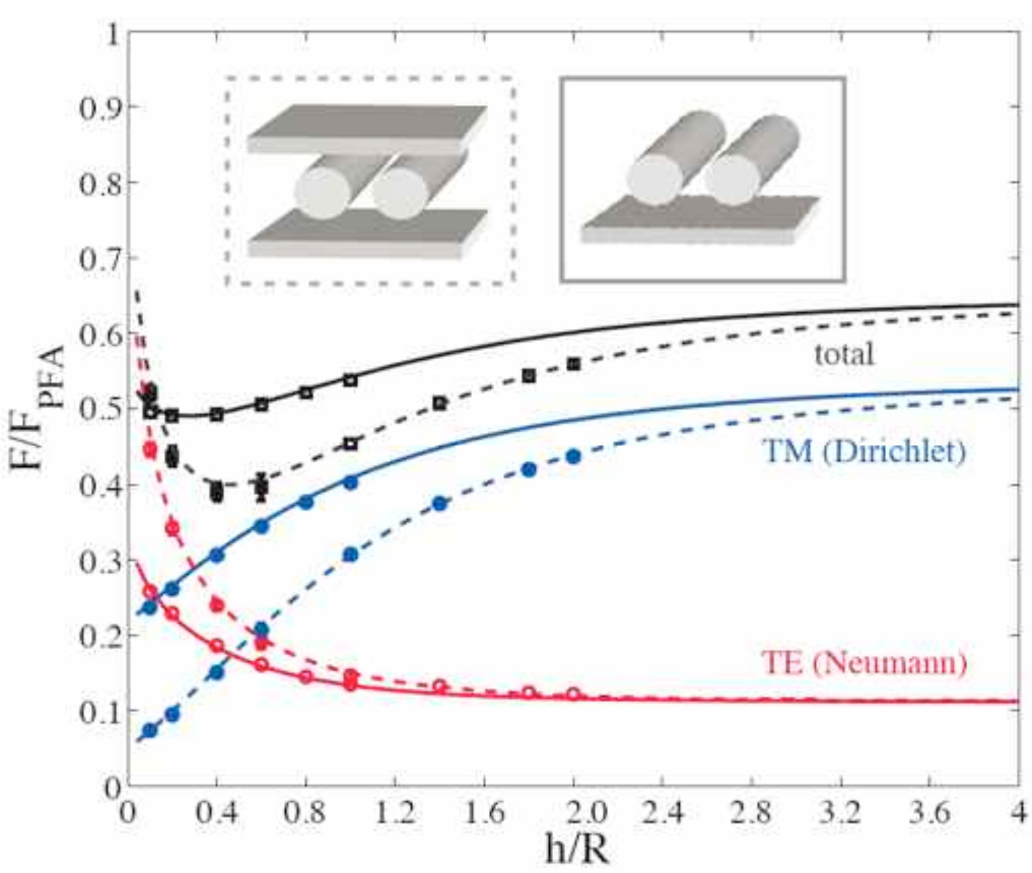
[Rahi, *PRA* **77**, 030101 (2007)]



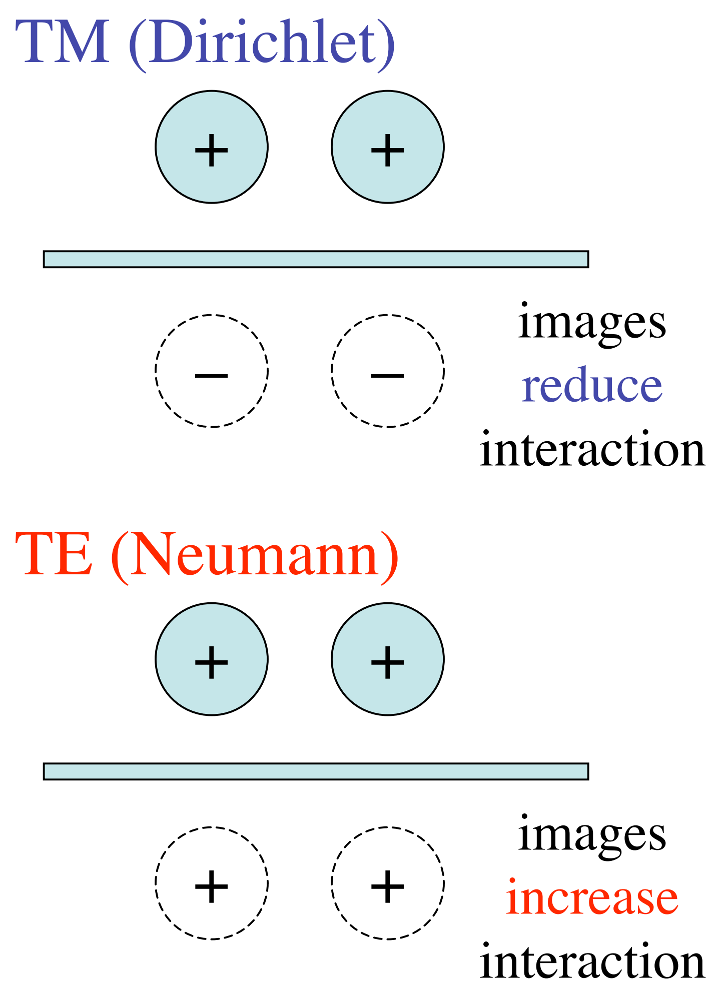
A simple explanation

[Rahi, *PRA* **77**, 030101 (2007)]

non-monotonic from **competition:**
TM forces *decrease* with *h*
and TE forces *increase*

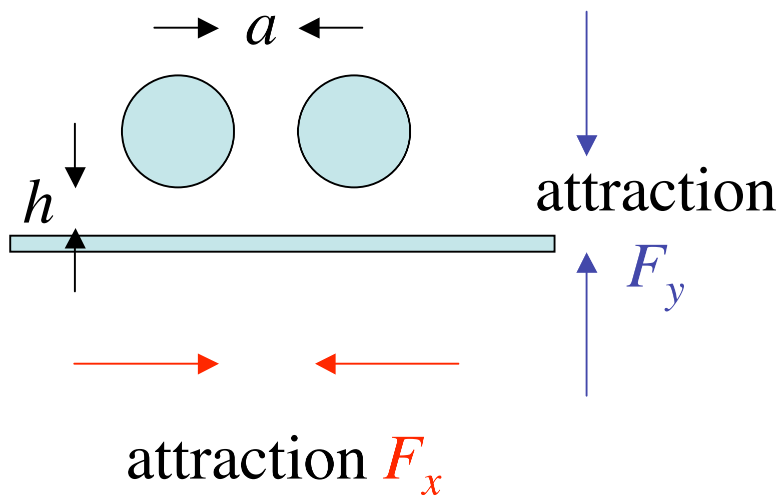


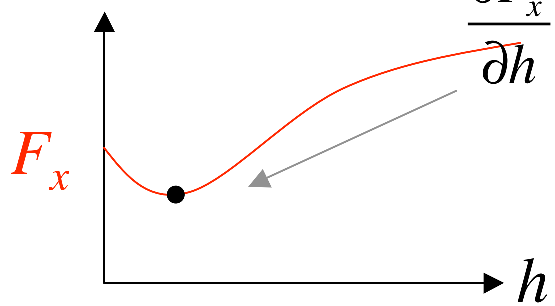
method of images



A corollary effect

[Rahi, *PRA* **77**, 030101 (2007)]

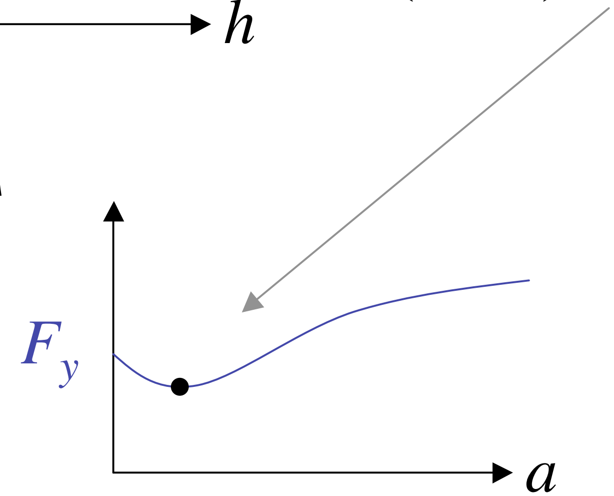




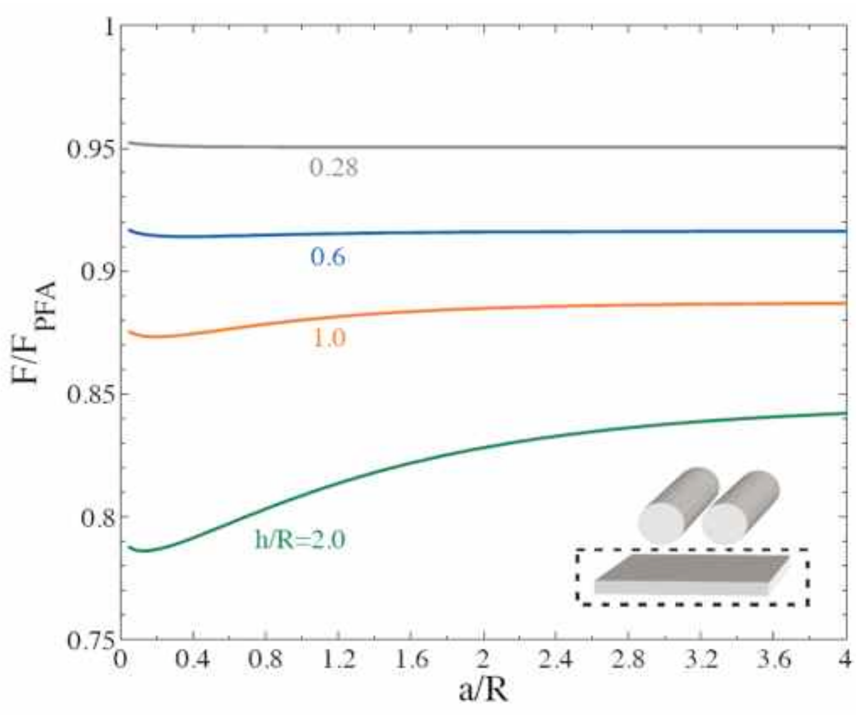
$$\frac{\partial F_x}{\partial h} = 0 = \frac{\partial}{\partial h}\left(-\frac{\partial U}{\partial a}\right)$$

$$= \frac{\partial}{\partial a}\left(-\frac{\partial U}{\partial h}\right) = \frac{\partial F_y}{\partial a}$$

$\Rightarrow$



cylinder-plate force depends non-monotonically on *a*

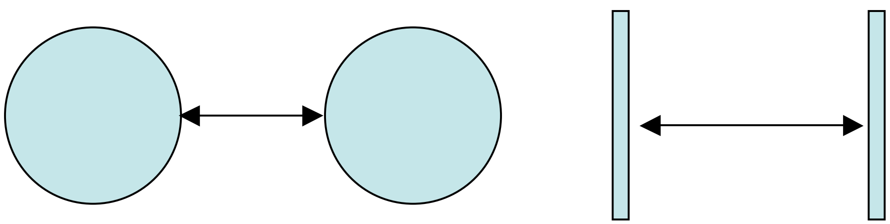


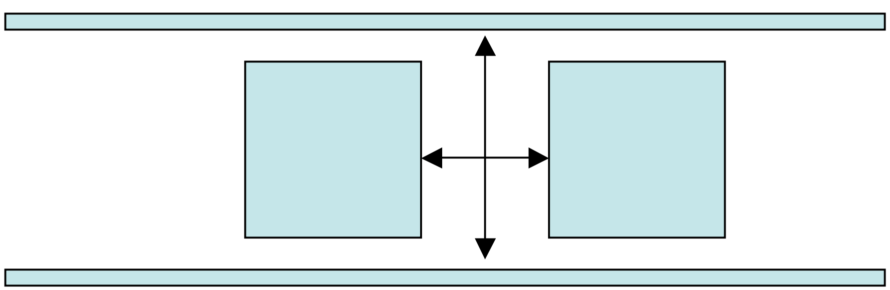
← confirmed by exact calculation

What about repulsive forces, and stable (1d) equilibria?

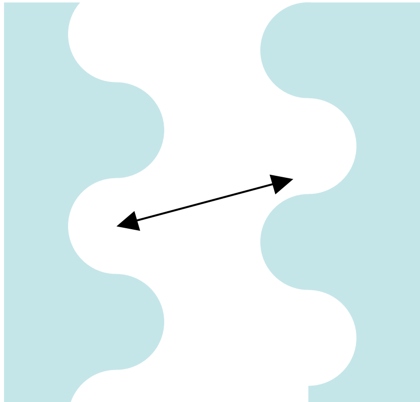
Theorem:
[Kenneth, 2006]

in a mirror-symmetric metal/dielectric [ε(iw) ≥ 1] structure, the Casimir force is always attractive





… but what about asymmetric structures?

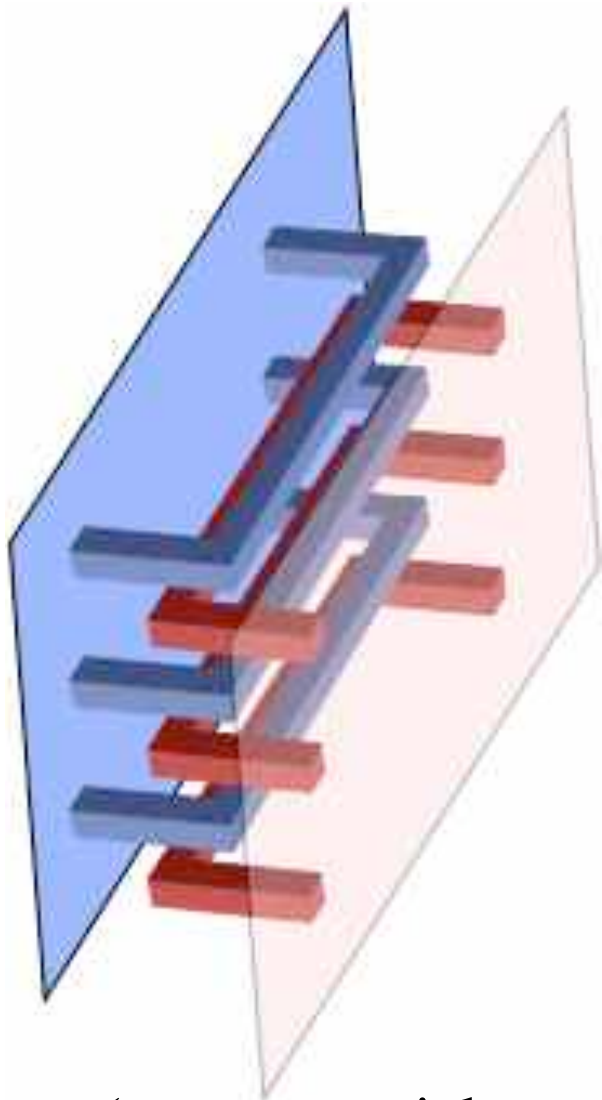


lots of interesting structures, e.g. with lateral forces, even Casimir "ratchets"

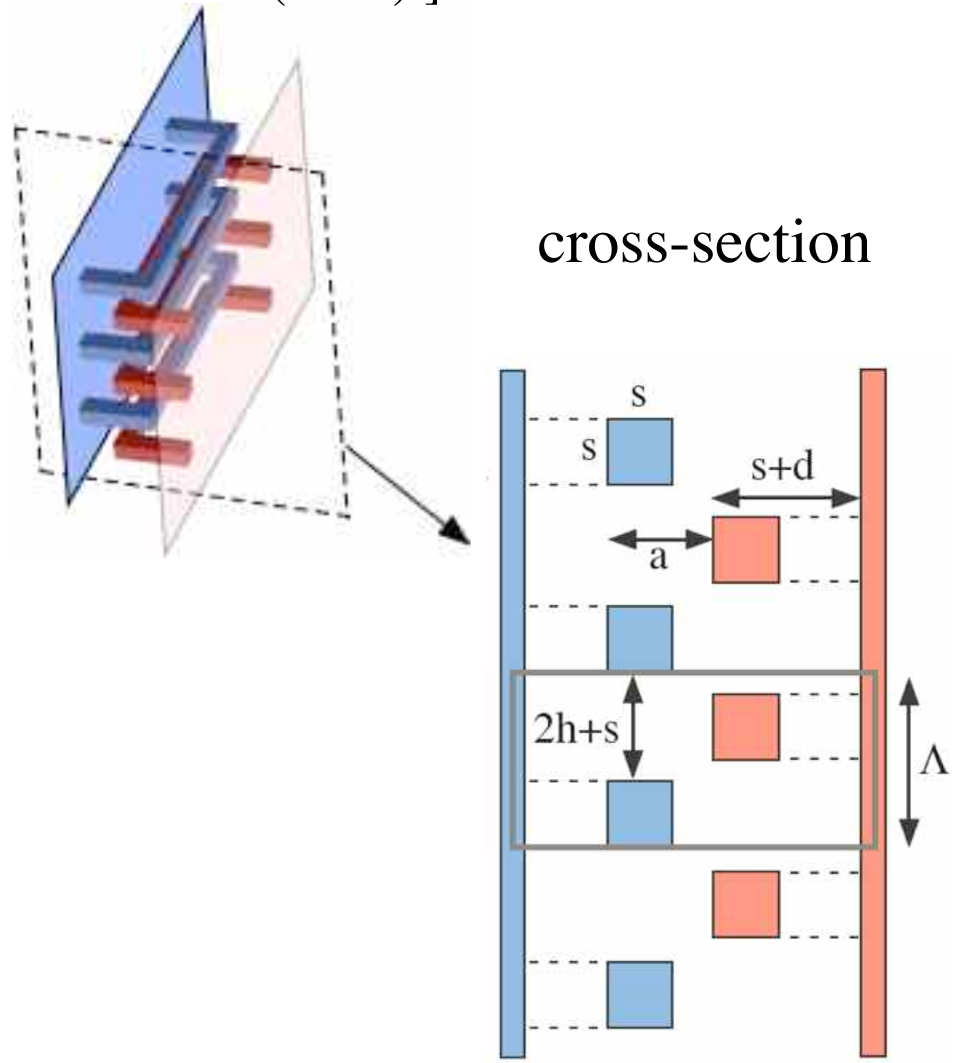
[Emig, arXiv cond-mat/0701641 (2007)]

A Casimir “zipper”

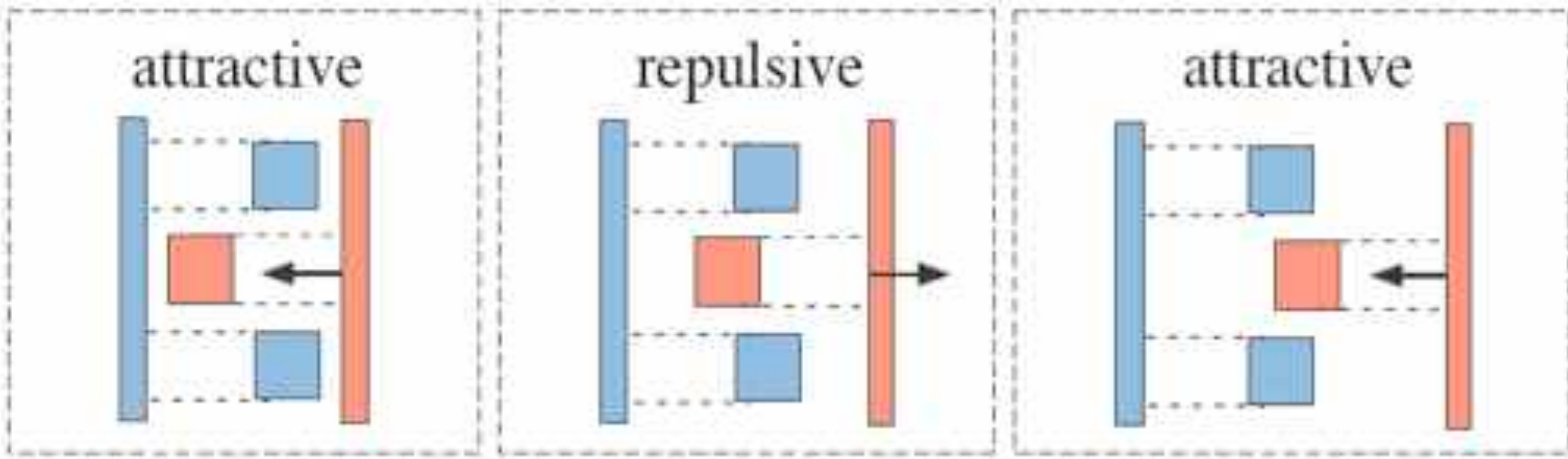
[Rodriguez, Joannopoulos, & Johnson, *arXiv* 0802.1494 (2008)]



(same materials, color for illustration only)



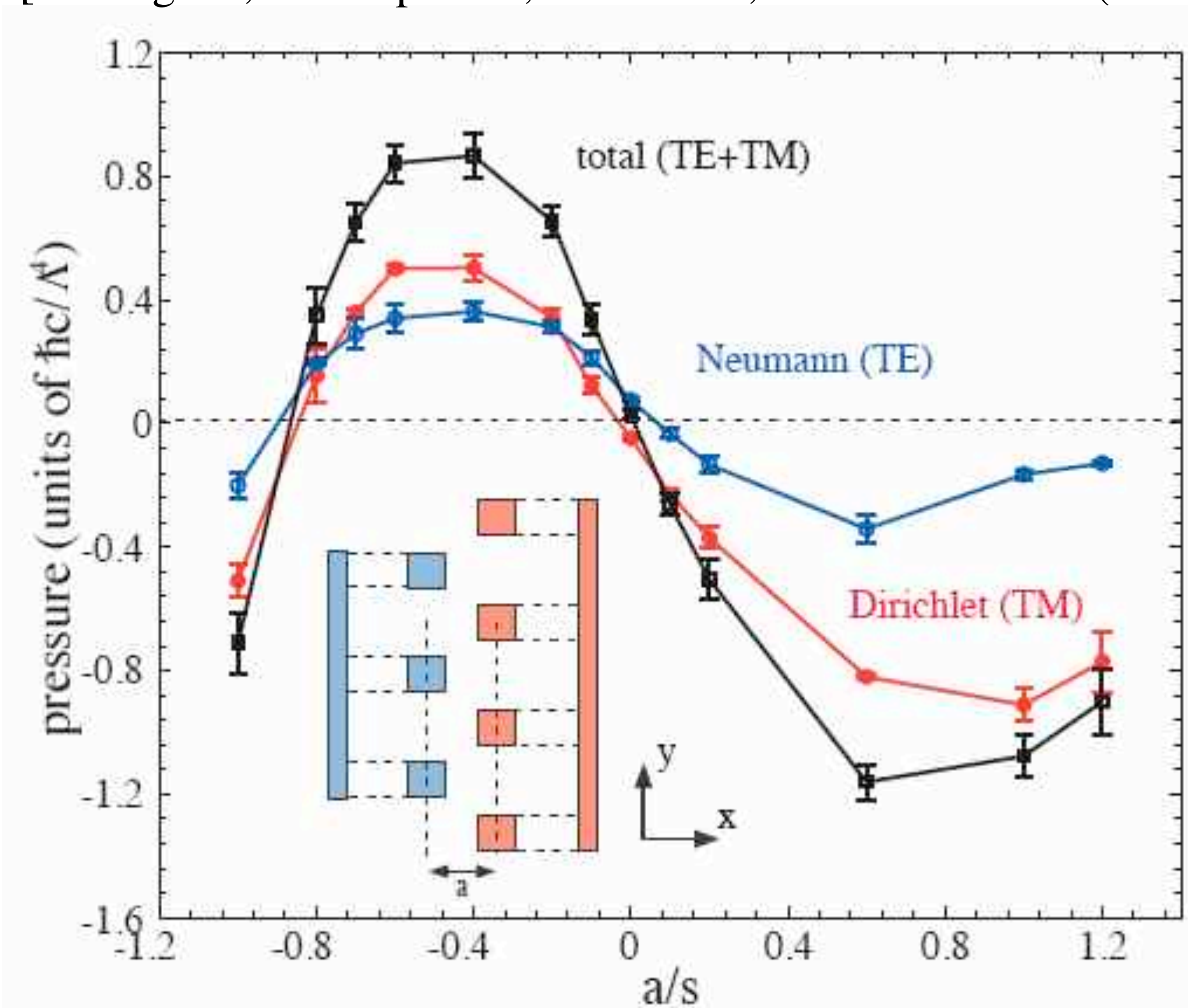
"Intuitive" pairwise-force picture



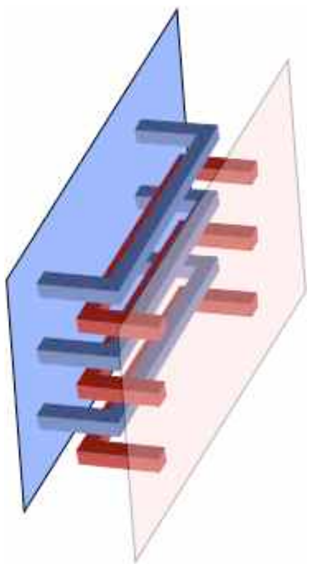
… but what if pairwise picture is totally wrong?

Exact numerical calculation

[Rodriguez, Joannopoulos, & Johnson, *arXiv* 0802.1494 (2008)]

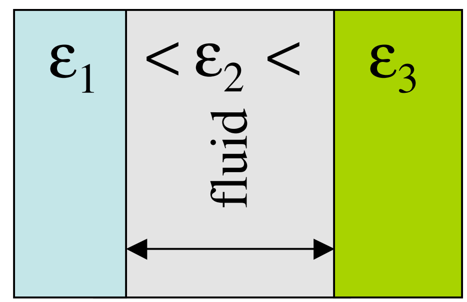


A "True" Repulsive Force?



repulsive,
but "made from"
attractive forces
… and laterally unstable

repulsive force between
fluid-separated plates
with ascending ε

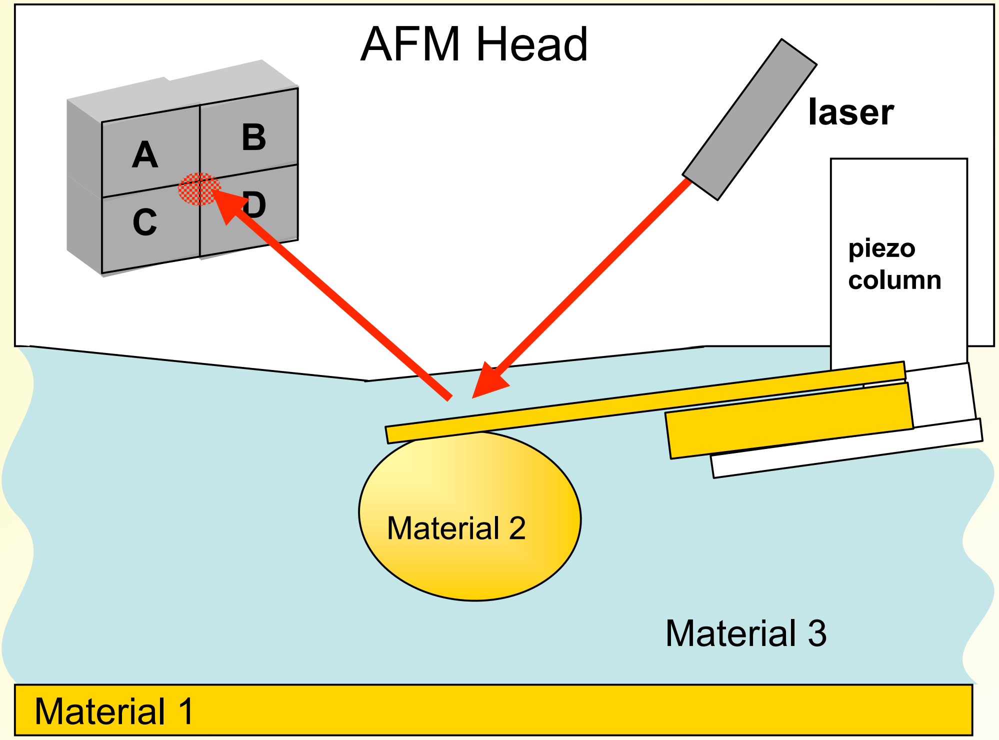


[Dzyaloshinskii, 1961;
Munday & Capasso,
2007]

predicted to play a role
in superfluid film wetting,
surface melting of metals…

Casimir forces across a fluid

slides courtesy
F. Capasso

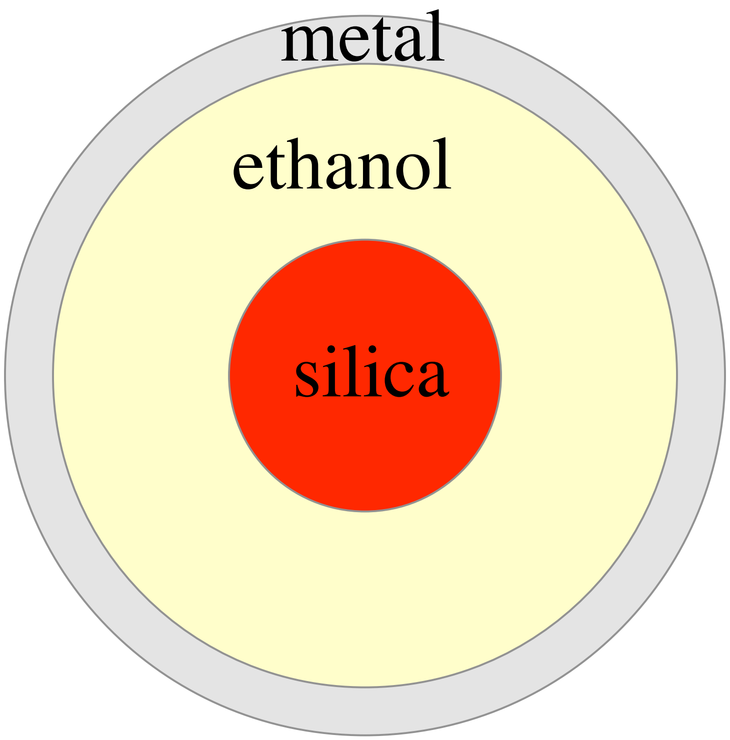


Jeremy Munday

Jeremy N. Munday and Federico Capasso
Physical Review A Rapid Comm. **75**, 60102 (2007)

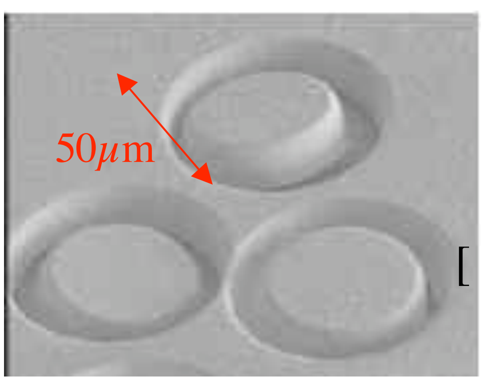
Stable Casimir Equilibria?

[Rodriguez, arXiv:0807.4166]

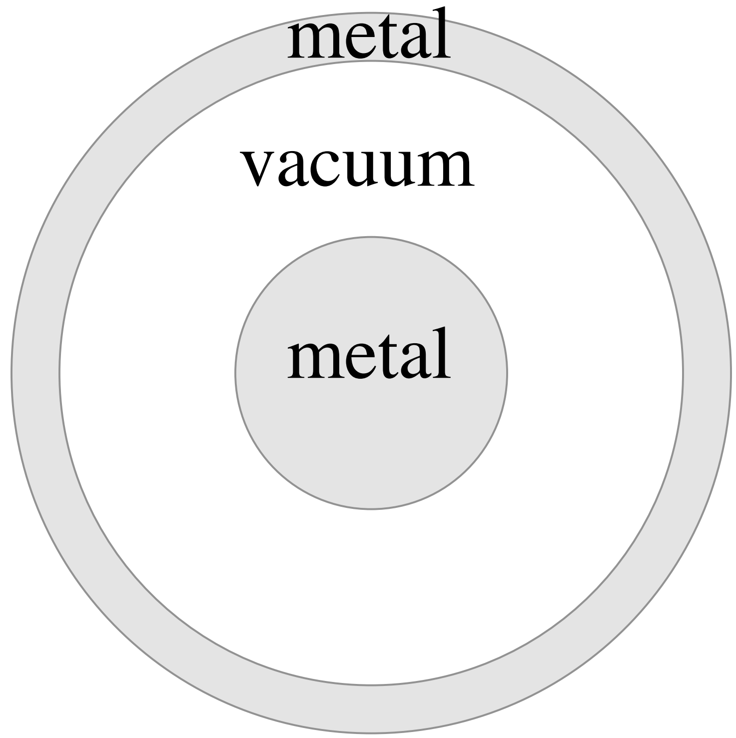


impossible with electrostatics

stable?

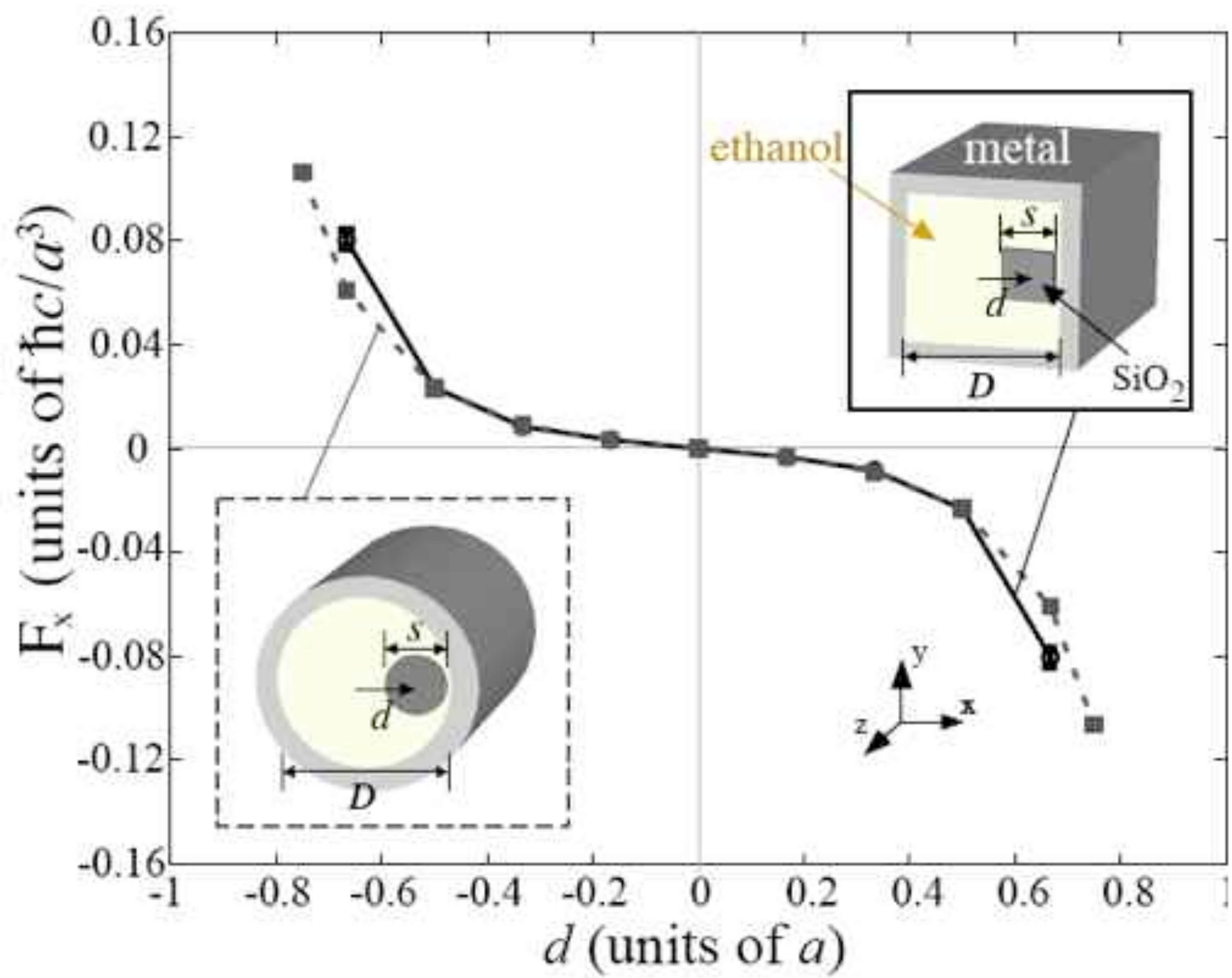


[Capasso, 2007]



unstable since force is $o(1/d)$

Stable Equilibria



$s/D = 0.25$

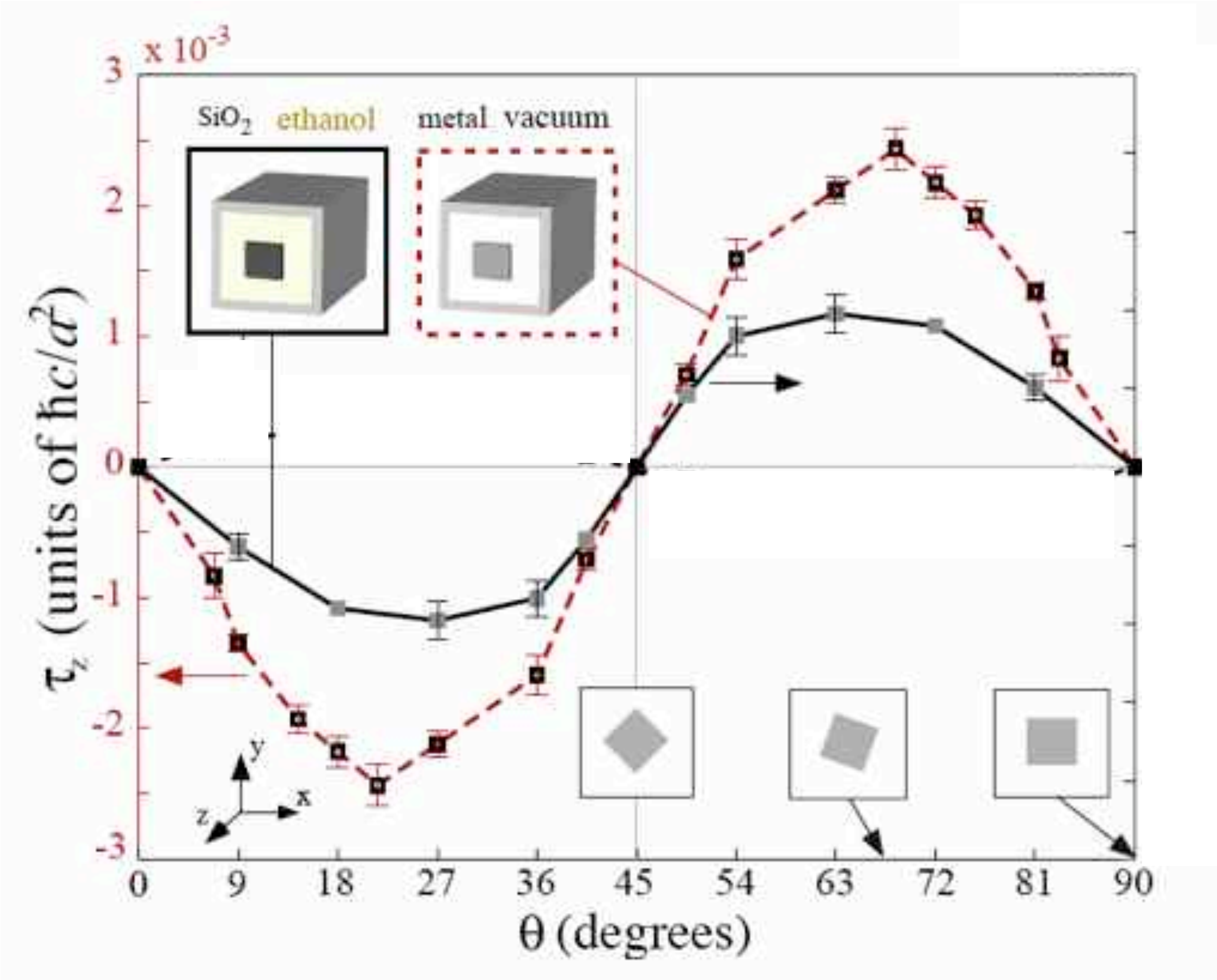
$a = (D{-}s)/2$

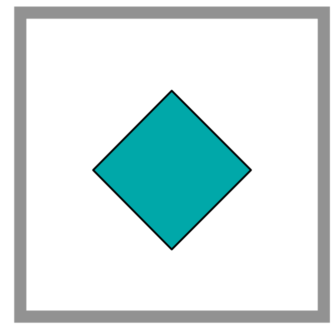
$= 0.096\mu\text{m}$

perfect metal,
but realistic
silica & ethanol

Casimir Torques

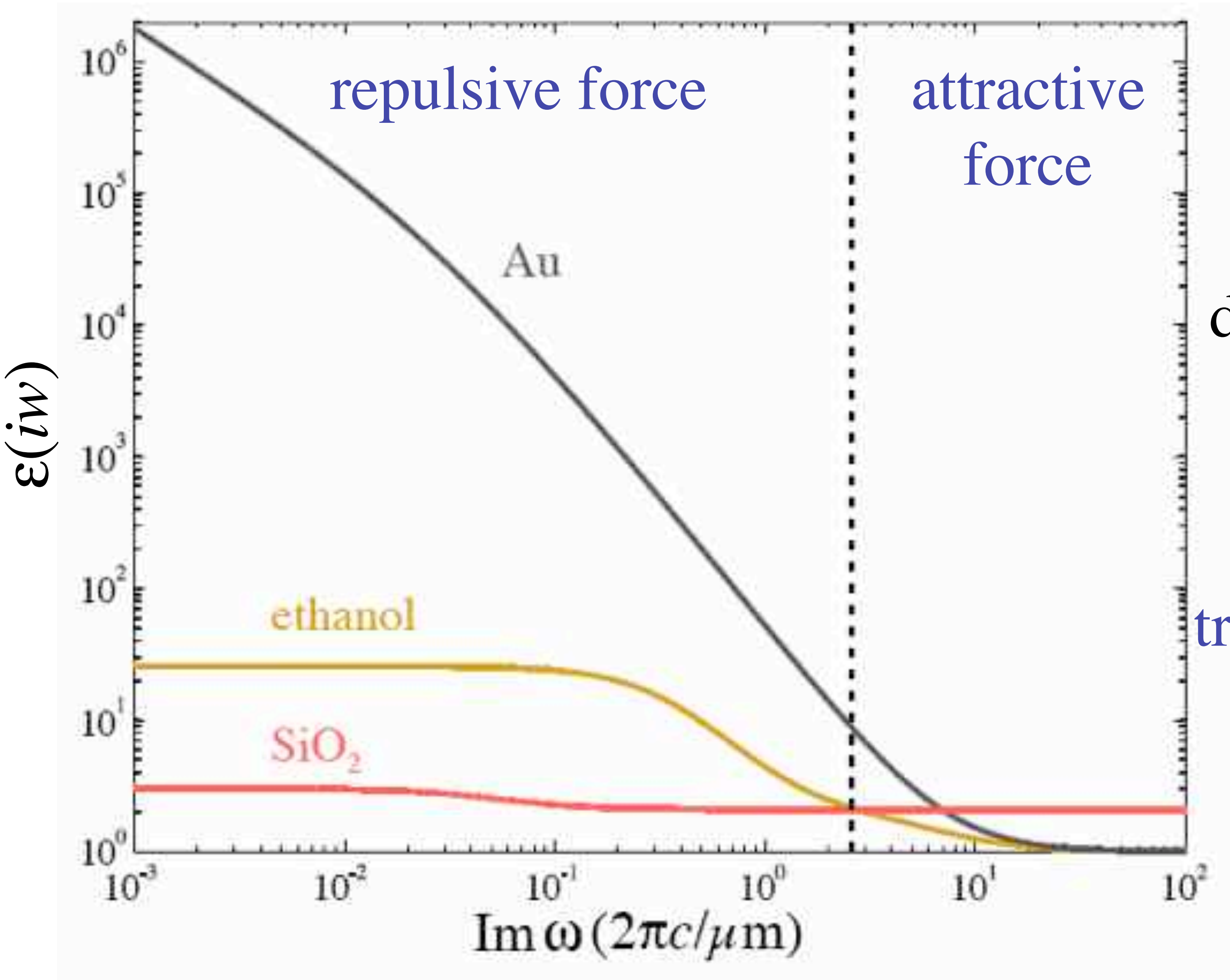
via $\int \mathbf{r} \times (\mathbf{T}\, d\mathbf{A})$





same orientation stable for *both* attractive and repulsive forces??

Material dispersion

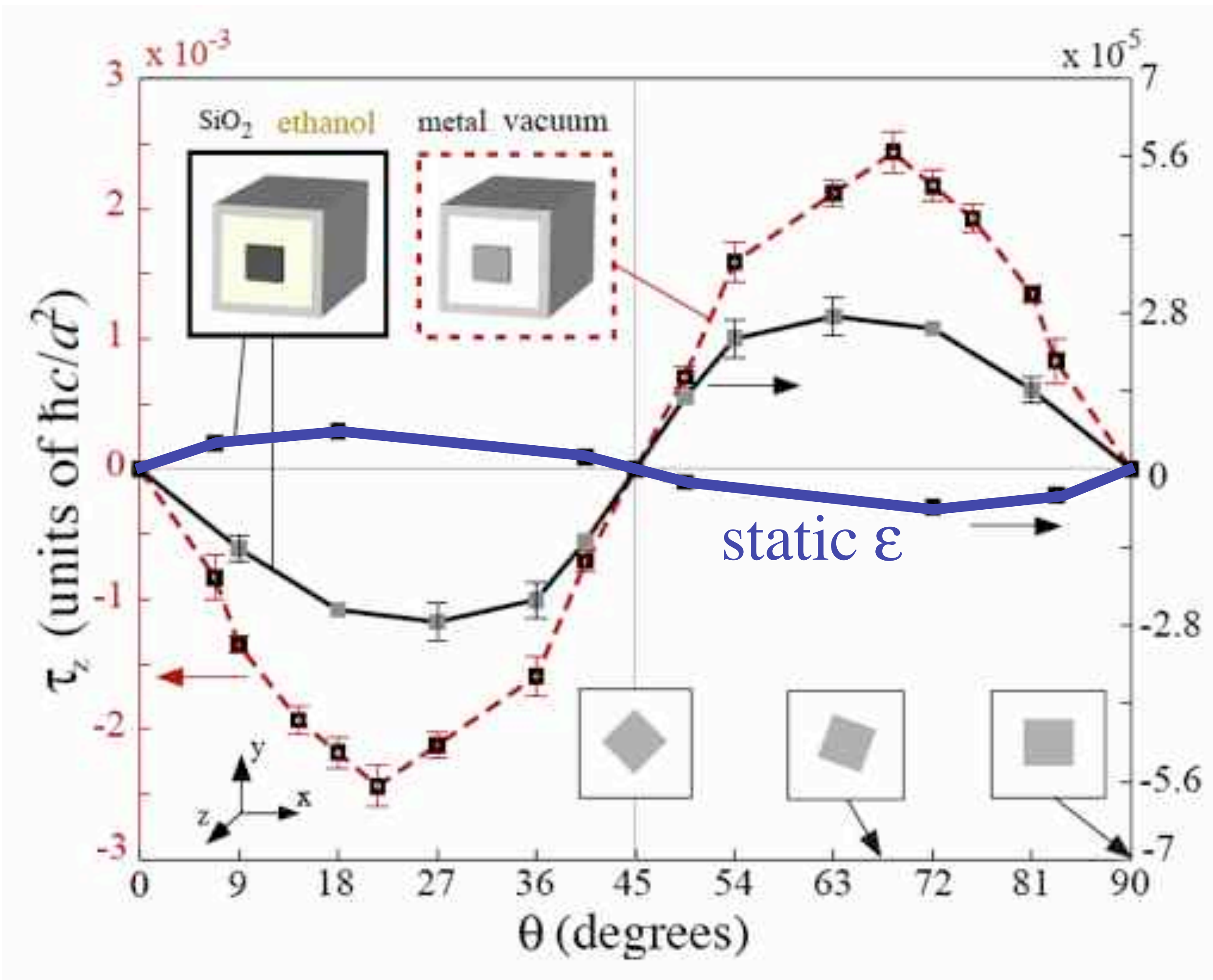


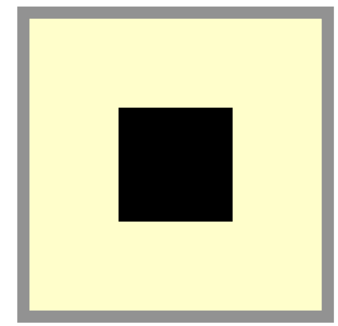
force at small/large separation dominated by large/small Im ω

transition from repulsive to attractive at ~ 0.02μm

Casimir Torques

via $\int \mathbf{r} \times (\mathbf{T}\, d\mathbf{A})$



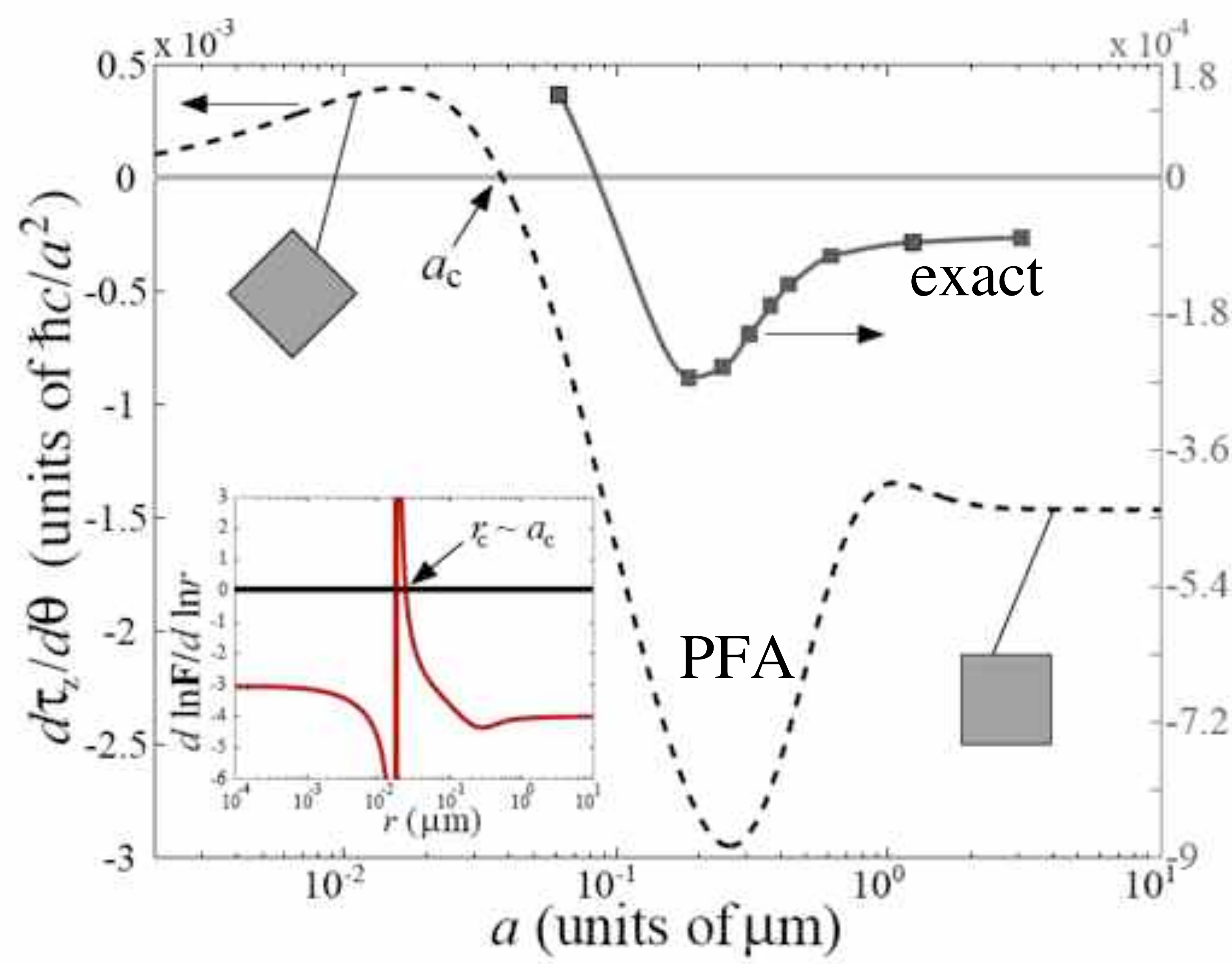


opposite stable orientations for non-dispersive ($a \to \infty$) materials

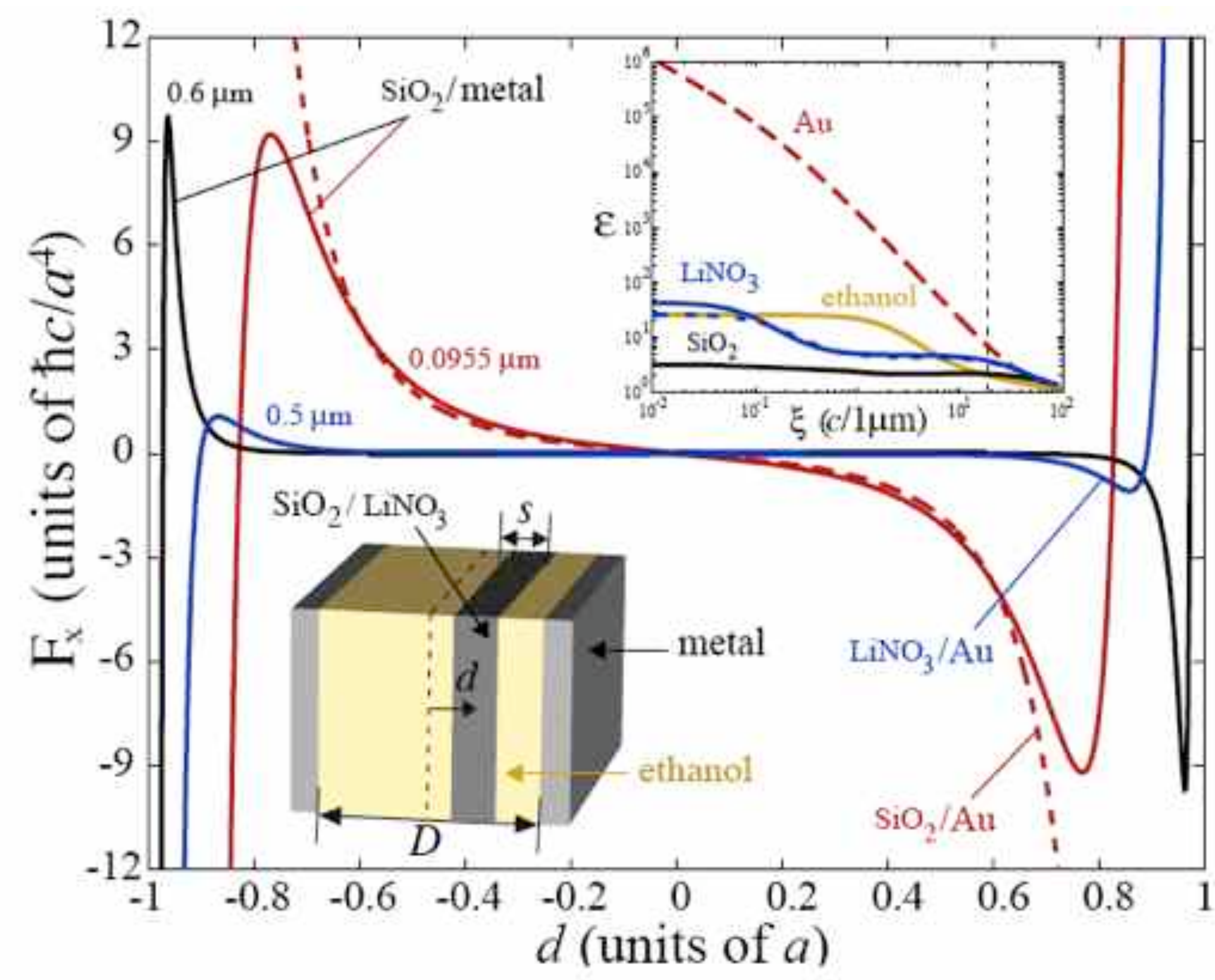
or

for $a > \sim 0.1\mu$m

Casimir Torque slope at θ=0



A 1d Equilibrium



repulsive/ attractive transitions for several material pairs

Summary

- Using very non-planar geometries to get unusual Casimir forces is almost unexplored — almost every geometry never tried

- "Exact" (no uncontrolled approximations) numerical techniques are finally becoming available to probe novel geometries by applying highly-developed, general, and scalable techniques from classical computational electromagnetism.

Thanks again: A. Rodriguez, M. Ibanescu, J. D. Joannopoulos ,
D. Iannuzzi, F. Capasso, S. Zaheer, S. J. Rahi, R. L. Jaffe,
M. Kardar, T. Emig, D. Dalvit

papers/preprints online: http://math.mit.edu/~stevenj