Casimir Effects in Light of Quantum Field Theory

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Oct 30, 2008
KITP

Outline

I. Introduction: Casimir effects as a form of effective field theory

II. Six equivalent formulations and lessons

- $\int$ density of states
- $\int$ Trace Log S-matrix
- $\int$ Trace of Green's function
- Feynman diagrams
- Effective action for time independent backgrounds
- Functional Integral

III. Example: Quantum solitons in renormalizable QFT's

IV. Divergences (?) $\Rightarrow$ Breakdown of Casimir idealization

V. Dessert: Casimir effect $\leftrightarrow$ Dark Energy

Recent work on scattering approach to Casimir effects by our group: See upcoming talk by Jamal Rahi (and Thorsten Emig)
Emig, Graham, Kardar, Rahi, RLJ

Collaborators

T. Emig, E. Farhi, N. Graham†, P. Haagensen, M. Hertzberg†, S. Johnson, M. Kardar, V. Khemani†, M. Quandt, J. Rahi† , A. Rodriguez†, M. Scandurra, A. Scardicchio†, O. Schroeder, H. Weigel, S. Zaheer†

†present or former students

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Casimir physics — an extreme form of effective field theory

I. Introduction

All physics is quantum, but sometimes...

- ⋆ Separation of scales $\boxed{A}$ and $\boxed{B}$ with $\Lambda_A \ll \Lambda_B$

  Integrate out $\boxed{B}$ leaving $E_{\text{eff}}[A]$

- ⋆ Casimir $\equiv$ further idealization

  $\boxed{A}$ is time-independent

  classical

  macroscopic

  idealized by boundary condtions

  All of which may be relaxed

- ⋆ Quantum fluctuations of $\boxed{B}$ are evaluated with $\boxed{A}$ fixed, rigid, and without back reaction. a la Born-Oppenheimer

- ⋆ Casimir idealization fails if scales cannot be separated:

  Signature of failure: Sum over quantum fluctuations of $\boxed{B}$ diverges

Fundamental starting point is renormalized quantum field theory

$$\mathcal{L}_{\text{renormalized}}[A, B]$$

- Eg. QED, Scalar field in scalar background, Fermi-Dirac field in scalar background, ...
- Note: Renormalized
  - ⋆ Counterterms have been introduced and adjusted to cancel loop divergences order by order in perturbation theory.
  - ⋆ There are no counterterms available to cancel further divergences

    This is not a QFT defined with *ab initio* surfaces and surface counterterms (a la Symanzik)
  - ⋆ Must not encounter divergences
- $$\int \mathcal{D}[B] \exp\left(\frac{i}{\hbar} S_{\text{fundamental}}[A, B]\right) = \exp\left(-\frac{i}{\hbar} E_{\text{Casimir}}[A]\right)$$

Classic examples and extensions

- ⋆ **Casimir:** $E = -\frac{\hbar c \pi^2}{720 d^3}$
- ⋆ **Lifshitz:** **Boundary $\Rightarrow$ frequency dependent, but still rigid response, $\epsilon(\omega)$**

$$\int d\omega \int_{\mathcal{D}} d^3x(E^2 - B^2) \to \int d\omega \int d^3x(\epsilon(x,\omega)E^2 - \frac{1}{\mu(x,\omega)}B^2)$$

Scalar analogues

Potential $\int d\omega \int_{\mathcal{D}} d^3x\, ((\partial\phi)^2) \to \int d\omega \int d^3x\, ((\partial\phi)^2 - \sigma(x)\phi^2)$

Index of refraction $\int d\omega \int_{\mathcal{D}} d^3x\, ((\partial\phi)^2) \to \int d^3x \left(n^2(x,\omega)\dot{\phi}^2 - |\nabla\phi|^2 - \sigma(x)\phi^2\right)$

- ⋆ **Casimir Polder:** $E_{\text{Casimir}}[A] \Rightarrow V_{\text{Casimir}}[A] \Rightarrow H_{\text{Casimir}}[\dot{A}, A]$

Casimir energy becomes effective potential for adiabatic quantization of "slow" degrees of freedom $\boxed{A}$

Three scale problem $\Lambda_B \gg \Lambda_A \gg 1/T$, where T is scale of adiabatic motion.

New directions

- ⋆ Variational optimization of $\boxed{A}$. Quantum solitons.

  Farhi, Graham, Haagensen, Khemani, Quandt, Scandurra, Weigel, RLJ.

  Classical background field ("soliton"): $\sigma[\{\alpha_j\}](x) \to E_{\text{Casimir}}[\{\alpha_j\}]$

$$\frac{\partial E_{\text{Casimir}}[\{\alpha_j\}]}{\partial \alpha_j} = 0$$

More details below

- Orientation dependent Casimir interaction energy

  Emig, Graham, Kardar, RLJ

  $\Rightarrow$ spin dependent Casimir effective potential for atoms or molecules with spin

Not discussed further here!

Limitations of Casimir idealization

⋆ Fails for total energy of $\boxed{A}$

⋆ Fails for any deformation energy

⋆ Examples of erroneous statements (including some urban legends of the field)

- "Casimir energy of a sphere is..."
- "Casimir pressure on a sphere is..."
- "Casimir 'force' on a cube is repulsive..."

⋆ And (less controversially) When back reaction on $\boxed{A}$ or equivalently the internal quantum structure of $\boxed{A}$ become significant

II. Six equivalent formulations and what they can teach us

$$E[A] = \frac{1}{2}\hbar \sum_B \left(\omega_B[A] - \omega_B^0\right)$$

$\boxed{1}$ **Change in density of states**

$$E[A] = \frac{1}{2}\hbar c \int_0^\infty dk\, k\, \Delta\rho_B(A,k)$$

$\boxed{2}$ **Scattering matrix ("Krein formula")**

$$E[A] = \frac{\hbar c}{2\pi i}\int_0^\infty dk\, k\, \frac{d}{dk}\left(\mathrm{Tr}\log S_B(k,A)\right)$$

In both cases the k integration contour can be rotated to the imaginary axis

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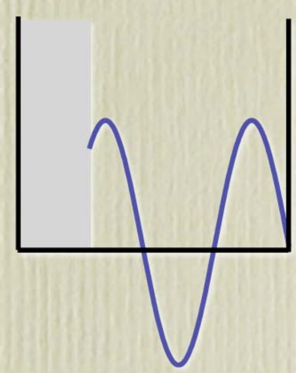
A little derivation (one dimension)

- $\psi(x) \propto \sin\left(kx + \delta(k)\right)$
- **Count states in (arbitrary) box:**



$$\begin{aligned}
kL + \delta(k) &= n\pi \\
L + \frac{d\delta}{dk} &= \pi\frac{dn}{dk} \\
\rho(k) = \frac{dn}{dk} &= \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk} \\
\Delta\rho(k) &= \frac{1}{\pi}\frac{d\delta}{dk} \quad \text{and} \\
S &= e^{2i\delta(k)}
\end{aligned}$$

$\boxed{3}$ Trace of Green's function

$$\mathrm{G}_{[\mathrm{A}]}(\mathrm{x},\mathrm{x}',\mathrm{k}) = \sum_{\mathrm{n}} \frac{\phi_{\mathrm{n}}(\mathrm{x})\phi_{\mathrm{n}}^{\dagger}(\mathrm{x}')}{\mathrm{k}-\mathrm{k}_{\mathrm{n}}-\mathrm{i}\epsilon}$$

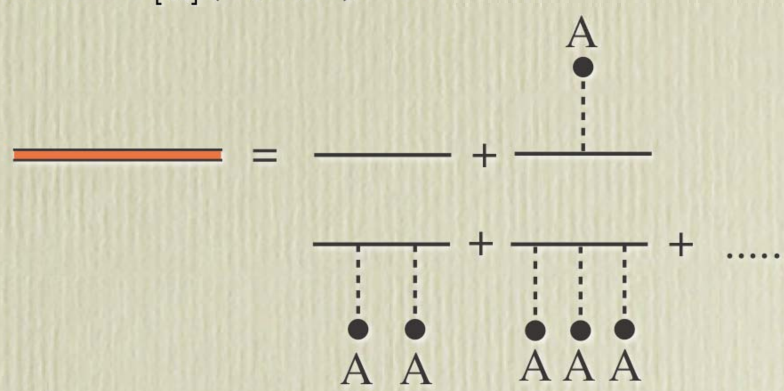
$$\mathrm{Im\,Tr}\int dx\, G_{[A]}(x,x,k) = \pi\sum_{n}\delta(k-k_n) = \pi\rho(k)$$

So

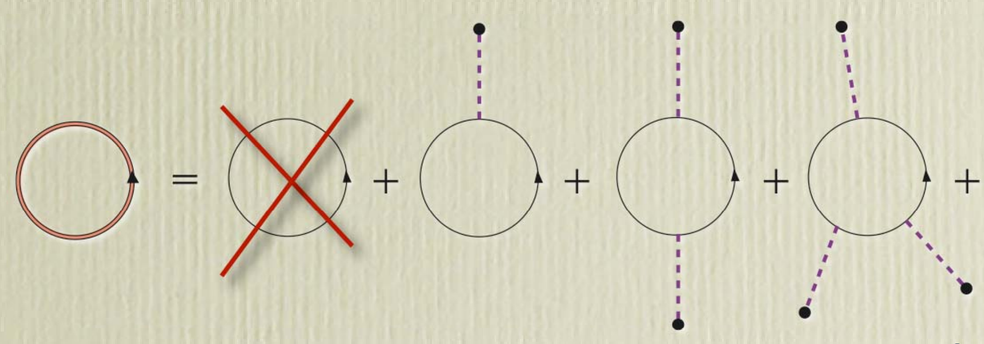
$$E[A] = \frac{\hbar c}{2\pi}\,\mathrm{Im\,Tr}\int_0^\infty dk\,k\int dx\,\Delta G_{[A]}(x,x,k)$$

$\boxed{4}$ Feynman diagrams

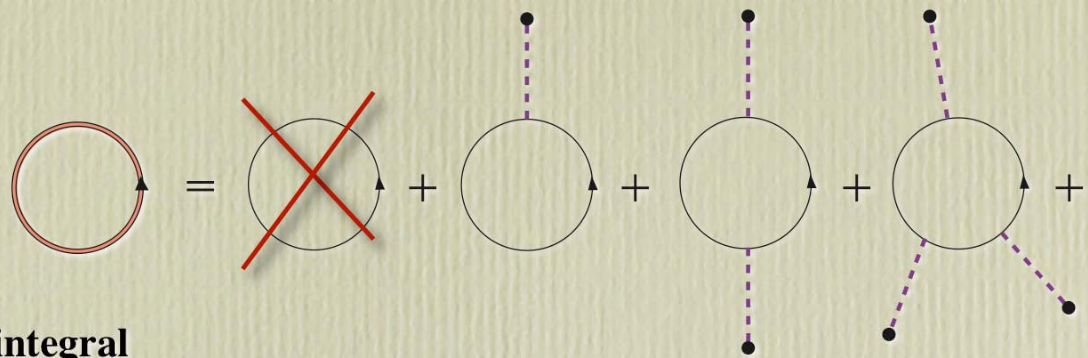
$\mathrm{G}_{[\mathrm{A}]}(\mathrm{x},\mathrm{x}',\mathrm{k})$



$\mathrm{Tr}\ \Delta\mathrm{G}_{[\mathrm{A}]}(\mathrm{x},\mathrm{x}',\mathrm{k})$



$\boxed{5}$ Effective action (for time independent background)



$\boxed{6}$ Functional integral

$$\frac{\int \mathcal{D}[B] \exp \frac{i}{\hbar} S_{\text{renormalized}}[A, B]}{\int \mathcal{D}[B] \exp \frac{i}{\hbar} S_{\text{renormalized}}[B]} = e^{-\frac{i}{\hbar} E[A]}$$

For example: a massless scalar field coupled to a smooth background potential, $A(x)$

$$S_{\text{renormalized}}[\phi, A] = \int d^4x \left((\partial_\mu \phi)^2 - gA(x)\phi^2(x) + c_1(\epsilon)A(x) + c_2(\epsilon)A^2(x) \right)$$

Renormalization counter terms

ϵ is a cutoff (eg. fractional dimension $d = 4 - \epsilon$)

Counter terms cancel loop divergences subject to renormalization conditions

Example of application to quantum solitons in the Standard Model:

> **Standard solitons are classical ($\hbar = 0$)**
>
> **Perhaps a classically unstable background field provides a better place for quantum fields to fluctuate.**
>
> **So Casimir energy may stabilize a soliton.**
>
> **In the Standard Model a very heavy fermion may have this character**

The Casimir Effect in Particle Physics

What happens to very heavy fermions in the Standard Model?

Noah Graham, RLJ, Vishesh Khemani, Herbert Weigel, and Eddie Farhi

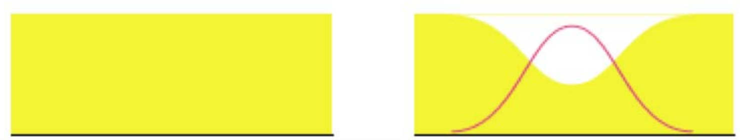
- ⋆ Quarks and leptons get their masses from their coupling to the Higgs condensate.

$$\mathcal{L} = \cdots + g\bar{\Psi}\phi\Psi$$

$$\langle\phi\rangle = v \quad \Rightarrow \quad \mathcal{L} = \cdots + [gv]\bar{\Psi}\Psi$$

$$m \sim gv$$

- ⋆ Would seem to favor "evacuation" of Higgs condensate near a fermion

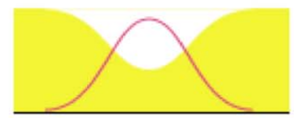


★ Energy balance:

$\boxed{-}$ Higgs departs from preferred constant value:

$V(\phi) > 0$

$\frac{1}{2}|\vec{\nabla}\phi|^2 > 0$



$\boxed{+}$ Fermion is strongly bound in deformed Higgs background:

$$\hbar\omega_0 \ll m$$

★ Inconsistent! All fermion eigenfrequencies are shifted in deformed background:

$$\sum_{j\neq 0} \hbar\omega_j \approx \hbar\omega_0$$

★ Generalization of Casimir problem from boundary condition to smooth background.

★ But how to calculate $\sum \hbar\omega$?

- ⋆ Combine Casimir Sum with Feyman diagram methods.
- ⋆ Combine Feyman diagrams with counterterms and renormalize.

$$E_{\text{RENORMALIZED}}[\phi, g, m]$$

- ⋆ Unambiguous treatment of renormalization. Feynman graph divergences are cancelled by divergent counterterms. Ambiguities are resolved by imposing perturbative renormalization conditions on low-order Green's functions.
- ⋆ Practical for numerical calculation. Subtracted Casimir "sums" are now regulator independent and convergent.
- ⋆ Suitable for variational approach:

$$\left.\frac{\delta E_{\text{RENORMALIZED}}}{\delta\phi}\right|_{g,m} = 0$$

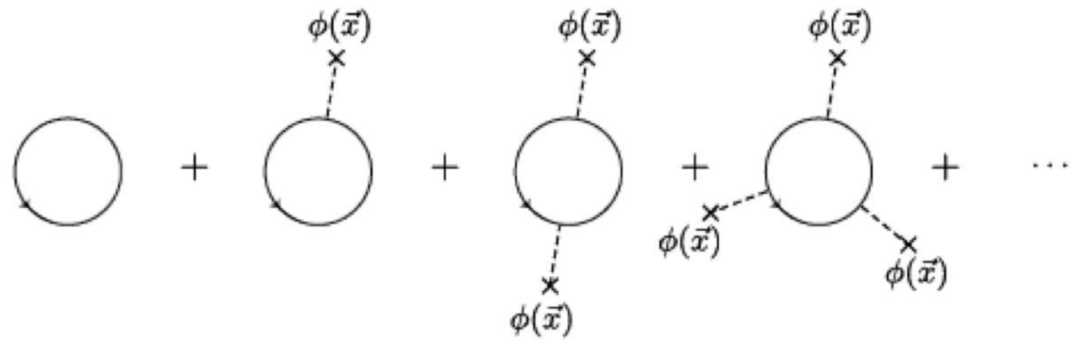
Effective action formalism. For time-independent fields,

$$S_{\text{EFF}}[\phi(\vec{x},T)] \to T\ E[\phi(\vec{x})]$$

To one-loop order,

$$E[\phi] = E_{\text{classical}} + E_{\text{1-loop}} + E_{\text{counterterm}}$$

$E(1-\text{loop})$:



$$\begin{aligned} E_{1-\text{loop}} - E_{\text{vacuum}} &= \pm\sum_k \frac{1}{2}\hbar(|\omega_k| - |\omega_k^0|) \\ &\equiv E_{\text{Casimir}}[\phi] \end{aligned}$$

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$$E_{\text{Casimir}}[\phi] = \pm \sum_k \frac{1}{2}\hbar(|\omega_k| - |\omega_k^0|)$$

Work in the continuum: $\sum_k \to \sum_{\text{boundstates}} + \int dk$

$$\sum \frac{1}{2}(|\omega| - |\omega^0|) \rightarrow \sum_j \frac{1}{2}|\omega_j| + \int_0^\infty \frac{|\omega|}{2}(\rho(k) - \rho^0(k))dk$$

- ⋆ where ω_j are bound states, $|\omega| = \sqrt{k^2 + m^2}$ on the right hand side, and $\rho(k)$ is density of states.
- ⋆ Assume (generalized) spherical symmetry (spherical, grand spin, reduces to symmetric and antisymmetric as $n \to 1$).

$$\rho(k) - \rho^0(k) = \sum_\ell D_\ell \frac{1}{\pi}\frac{d\delta_\ell(k)}{dk}$$

$$\left[\text{General result:} \quad \frac{dn}{dk} = \frac{1}{2\pi i}\frac{d}{dE}\text{Tr}\ln S(E)\right]$$

- ⋆ $\delta_\ell(k)$ sums phase shifts for $\pm|\omega(k)|$.
- ⋆ n – space dimension – suppressed on degeneracy factor D_ℓ and δ_ℓ.
- ⋆ Identify potentially divergent terms and regularize through the Born Approximation.

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⋆ Born expansion (in n dim.) $\delta_\ell(k) = \sum_{i=1}^{\infty} \delta_\ell^{(i)}(k)$

⋆ One-to-one correspondence between Born contributions to density of states and Feynman diagrams

⋆ Subtract N Born approximants to regulate

$$\delta_\ell(k) \;\Rightarrow\; \bar{\delta}_\ell(k) \equiv \delta_\ell(k) - \sum_{i=1}^{N} \delta_\ell^{(i)}(k) \quad \text{So} \quad E_{\text{cas}} \Rightarrow \bar{E}_{\text{cas}}$$

Regulated $\bar{E}_{\text{Casimir}}$ is both finite and cutoff independent.

- In theory, because divergent diagrams have been subtracted.
- In practice, because leading large k & large ℓ have been subtracted.

⋆ Add back in Feynman diagrams

$$\Rightarrow \sum_{n=1}^{N} \Gamma^{(n)}[\phi, \Lambda]$$

Regulate in traditional fashion, combine with counterterms and renormalize.

$E[\phi(\vec{x}), \{g\}, \{m\}]$

For a case where Feynman 1- and 2-point functions are potentially divergent as $n \longrightarrow$ integer...

$$E[\phi(\vec{x}), \{g\}, \{m\}] = E_{\text{cl}}[\phi(\vec{x}), \{g\}, \{m\}] +$$

$$\left\{\Gamma^1[\phi,\epsilon] + \Gamma^2[\phi,\epsilon] - c_1(\epsilon)\phi - c_2(\epsilon)\phi^2 - c_3(\epsilon)|\vec{\nabla}\phi|^2\right\}$$

$$+\frac{1}{2}\sum_j (E_j - m) - \frac{1}{2\pi}\int_0^\infty dk\,(|\omega(k)| - m)\sum_{\ell=0}^{\infty} D_\ell \frac{d}{dk}\bar{\delta}_\ell(k)$$

- ⋆ Classical energy.
- ⋆ Potentially divergent Feynman diagrams plus counterterms.
- ⋆ Regulated "Casimir" energy. Finite and smooth as $n \longrightarrow$ integer.
- ⋆ Subtraction of mass protects against infrared divergences and is an identity following from Levinson's theorem.
- ⋆ Renormalization $\bar{\Gamma}^1[\phi] = 0$

$$\left.\frac{d\bar{\Gamma}^2}{dp^2}\right|_{p^2=0} = 1 \qquad \left.\bar{\Gamma}^2[\phi]\right|_{p^2=0} = -m^2$$

With standard scale and scheme dependence as expected.

- ⋆ Numerical calculations are convergent and quick.

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III. Finite Casimir Effects (or) What divergences tell us!

- ⋆ Casimir energies are one-loop effective energies in renormalizable quantum field theories. Therefore finite.
- ⋆ But $\sum \frac{1}{2}\hbar(\omega_B[A] - \omega_B^0)$ is horribly divergent!
- ⋆ And there has been a major industry of computing Casimir energies that are superficially divergent, then regulating, and "renormalizing" them ("Zeta-function regularization","Heat kernel expansion"...)
- ⋆ In brief:

  If a Casimir calculation diverges it is a sign that the idealization has failed and that the quantity in question is not protected from the high energy (frequency) scale, Λ_B.

- ⋆ The essence can be extracted from the Balian & Bloch multiple reflection expansion for the asymptotic density of states as a function of geometry

R. Balian & C. Bloch

Ann. Phys. (NY)
60, 401 (1970)

Geometrical expansion of the density of states

- Attempting to address M. Kac's problem: "Can you hear the shape of a drum"!
- Derived an integral equation (multiple reflection expansion) for cavity Green's fn.
- Leads to asymptotic expansion for density of states

$$\rho(k) = \frac{1}{2\pi^2}\Big(\overset{1}{k^2V} - \overset{2}{\frac{\pi}{4}kS} + \overset{3}{\frac{1}{3}\oint d^2s\frac{1}{2}\Big(\frac{1}{R_1}+\frac{1}{R_2}\Big)}$$
$$+ \underset{4}{C\ln k\oint d^2s\frac{1}{2}\Big(\frac{1}{R_1}-\frac{1}{R_2}\Big)} + \underset{5}{C'\frac{1}{k}\oint d^2s\frac{1}{R_1R_2}} + \ldots\Big)$$

- V volume
- S surface area
- R_1 R_2 principal radii of curvature

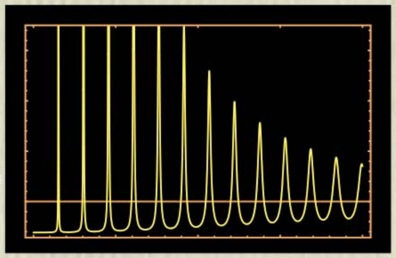
1 quartic divergence
2 cubic divergence
3 quadratic divergence

- What are all these divergences?
- How can one identify and compute the underlying (presumably finite) term that contributes to the force that is buried beneath?

$$\rho(k) = \frac{1}{2\pi^2}\left(k^2V - \frac{\pi}{4}kS + \frac{1}{3}\oint d^2s\frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right.$$
$$\left. + \; C\ln k\oint d^2s\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right) + C'\frac{1}{k}\oint d^2s\frac{1}{R_1R_2} + ...\right)$$

Observations:

- ⋆ B& B expansion is for smoothed density of states, $\lim_{\gamma\to 0}\rho(k+i\gamma)$, and fails to capture oscillatory terms at "finite" k.
- ⋆ All smooth terms in $\rho(k)$ yield divergences in the Casimir energy!
- ⋆ All divergences are local on the surfaces — manifest from multiple reflection expansion — and therefore cancel out of forces and torques where surfaces are displaced rigidly
- ⋆ The divergences have nothing to do with the short distance behavior of QFT. They are exotic: eg. cubic divergence in $3+1$ dimensions
- ⋆ Casimir forces come entirely from oscillatory terms in $\rho(k)$ and therefore are extremely difficult to obtain by direct computation of $\int k\rho(k)$



- ⋆ Total (Casimir) energy and deformation dependent quantities like the pressure seem to diverge and do
- ⋆ The same phenomenon is visible in the Casimir energy density, which is finite away from surfaces, but diverges like $1/\epsilon^3$ as surface is approached
- ⋆ Divergences arise from trying to constrain a fluctuating field (eg. to vanish on $\partial\mathcal{D}$) for all frequencies

Where the divergences are coming from:

- ⋆ No finite strength interaction can force all frequency components of a quantum field to obey a BC at a point. Trying to enforce a BC introduces arbitrarily high energy scales ⇒ divergences

- ⋆ Simple example in one dimension

  The "Dirichlet point"

  - One point: $\phi(0) = 0 \Rightarrow E_1$
  - Two points: $\phi(-a) = \phi(a) = 0 \Rightarrow E_2(a)$

"Standard" Approach (flawed)

- ⋆ One point: $\phi(0) = 0$

  Constrained modes: $\sin kx, \sin k|x|$ for $k > 0$.

  Unconstrained modes: $\sin kx, \cos kx$ for $k > 0$.

  Spectra identical so $\tilde{E}_1 = 0$

- ⋆ Standard answer:

$$\tilde{E}_2(a) = -\frac{\hbar c \pi}{48a}$$

"Derivation"

- Modes inside $\sin k(x + a)$ with $k = n\pi/2a$
- Modes outside identical to unconstrained case (and therefore cancel)

$$\tilde{E}_2 = \frac{\hbar c}{2} \sum_{n=0}^{\infty} \frac{n\pi}{2a} = \frac{\hbar c \pi}{4a} \sum_{n=1}^{\infty} n$$

- Manifestly divergent!
- But, "Zeta-function" regulate: $\sum_{n=1}^{\infty} n^{-s} = \zeta(s)$
- And $\zeta(-1) = -1/12$ which gives

$$\tilde{E}_2 = -\frac{\hbar c \pi}{48a}$$

$\tilde{E}_1 = 0$ and $\tilde{E}_2 = -\frac{\hbar c \pi}{48 a}$?

Critique

- $a \to 0$ on physical grounds $E_2 \to E_1$, but $E_1 = 0$ and $\lim_{a\to 0} E_2(a) = \infty$
- Also, massless scalar QFT does not exist in 1-dimension.

  It has a $\ln m$ divergence as $m \to 0$!

Careful approach

- Couple ϕ to a scalar background, compute renormalized one loop effective energy, and then let background approach limit in which it constrains all modes of the field.

$$\mathcal{L}_{\text{int}} = g\sigma(x)\phi^2 + c_1(\epsilon)\sigma(x)$$

- Only divergence comes from tadpole Feynman graph, which is renormalized to zero (choice of scheme) by the counter term
- It suffices to take background to be a $\delta(x)$ (could choose smooth function and take δ-function limit)

$$E_1(g,m) = \frac{\hbar c}{2\pi}\int_m^\infty dt \frac{(t\ln(1+g/2t) - g/2)}{\sqrt{t^2-m^2}}$$

$$E_1(g,m) = \frac{\hbar c}{2\pi}\int_m^\infty dt \frac{(t\ln(1+g/2t) - g/2)}{\sqrt{t^2-m^2}}$$

- E_1 **converges for any** g **and** $m \neq 0$**, but diverges logarithmically as** $m \to 0$**.**

$$E_2(g,m,a) = \frac{\hbar c}{2\pi}\int_m^\infty dt \frac{1}{\sqrt{t^2-m^2}}(t\,\ln(1+\frac{g}{t}+\frac{g^2}{2t^2}(1-e^{-4at})) - g)$$

- **(Both** E_1 **and** $E_2(a)$ **were computed using** S**-matrix version with Feynman diagram subtraction and renormalization.)**

- **Check:**

  ⋆ $a \to \infty \qquad E_2(g,m,a) \to 2E_1$

  ⋆ $a \to 0 \qquad E_2(g,m,a) \to E_1(2g,m)$

  ⋆ $m \to 0 \qquad E \sim ln(m).$

- $\lim_{g\to\infty}$ **is the "Dirichlet limit".**

  ⋆ $\lim_{g\to\infty} E_2(g,m,a) \sim -g\ln g \to -\infty$

  So there is no finite Casimir energy in the boundary condition limit.

  ⋆ **But force is finite and agrees with "zeta-function" approach**

$$\lim_{g\to\infty}\left(-\frac{\partial E_2}{\partial a}\right) = -\frac{\hbar c\pi}{48a^2}$$

Problems get worse in higher dimensions

- Same interaction in $3+1$ dimensions. Look at contribution of renormalized two point function

$$E^{(2)}[\sigma] = \frac{g^2}{64\pi^2}\int d^3p\, \tilde{\sigma}(p)\tilde{\sigma}(-p)\int dx \ln\frac{m^2+x(1-x)p^2}{m^2+x(1-x)\mu^2}$$

- Renormalized (note μ^2) and finite for any $\sigma(p)$ that vanished fast enough with p.
- Divergence can only originate in bad large p behavior of $\sigma(p)$.
- In $3+1$ dimensions one cannot even take the local (ie. δ-function) limit.

Saves reading lots of papers

- Boyer – repulsive Casimir pressure on a sphere
- Ambjorn & Wolfram – repulsive Casimir forces on cuboid
- ...

The Casimir Energy and the quantum vacuum...

Is the Casimir effect definitive evidence that quantum fluctuations of the vacuum are "real"?

Why is this interesting?

Dark energy = cosmogical constant = vacuum energy ~ [cutoff]^4

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

Are Casimir forces a "property of the vacuum"?

No! They are quantum forces between material objects computed in a clever, but deceptive way.

$$\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$

Casimir's formula actually depends on the fine structure constant, and the force vanishes as $\alpha \rightarrow 0$!

Weinberg Rev. Mod. Phys. 61, 1 (1989)

"Perhaps surprisingly, it was along time before particle physicists began seriously to worry about quantum zero point fluctuation contributions to the cosmological constant despite the demonstration in the Casimir effect of the reality of zero-point energies."

Carroll Living Rev. Rel. 4, 1 (2001) [arXiv:astro-ph/0004075

" ... And the vacuum fluctuations themselves are very real, as evidenced by the Casimir effect."

Sahni and Starobinsky Int. J. Mod. Phys. D 9, 373 (2000) [arXiv:astro-ph/9904398]

"The existence of zero-point vacuum fluctuations has been spectacularly demonstrated by the Casimir effect."

RLJ, Phys. Rev. D72:021301 (2005)

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- **Let the fine structure constant go to zero: Conductivity vanishes -- Casimir force vanishes**
- **Physical cutoff -- indeed as anticipated by Casimir -- is the plasma frequency**

$$k_m^2 \propto \omega_p^2 = \frac{4\pi e^2 n}{m}$$

- **Dominant frequencies contributing to Casimir effect are ~ c/d**

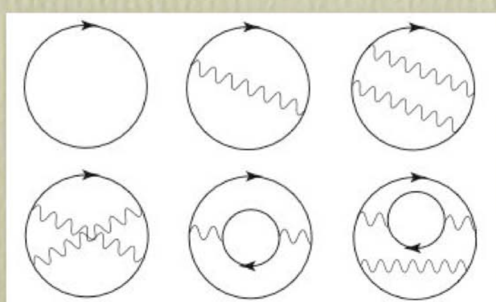
$$\omega_p \gg c/d \quad \rightarrow \quad \alpha \gg \frac{mc}{4\pi\hbar n d^2} \quad \rightarrow \quad \alpha \gtrsim 10^{-5}$$

- **The introduction of a fluctuating field is a calculational convenience that can be dispensed with (Schwinger)**

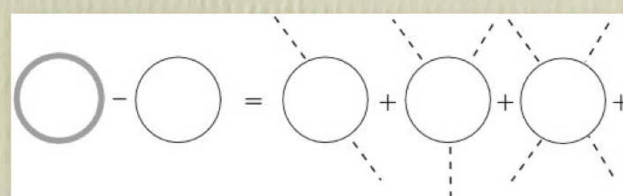
$$\mathcal{E} = \int d^3x d^3y \frac{\rho(\vec{x})\rho(\vec{y})}{|\vec{x}-\vec{y}|} = \frac{1}{8\pi}\int d^3x \; \vec{E}^2(\vec{x})$$

- **So the Casimir force is the infinite coupling limit of a function of the coupling.**

Diagrams contributing to the vacuum fluctuation energy density



Diagrams contributing to the Casimir force



Casimir force is one of many one-loop QFT effects that are computed in a formalism where the vacuum is embued with zero point energy. Nevertheless, none of them give any direct evidence for the "reality" of vacuum fluctuation energy density

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