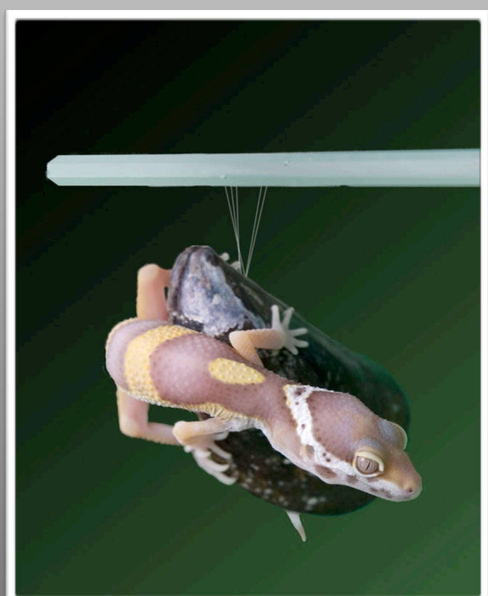
Anatomy of the Casimir Effect

Francesco Intravaia
Universitaet Potsdam, Institut fuer Physik, Am Neuen Palais 10, 14469 Potsdam, Germany & KITP

Kavli Institute for Theoretical Physics, November 19 2008

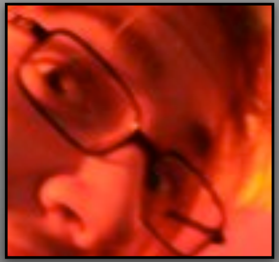
A. K. Geim et al., Nature online pubblication



Acknowledgments

- Kavli Institute for theoretical Physics and the Organizers
- Alexander von Humboldt Foundation
- Universität Potsdam
- Quantum Optics Group in Potsdam

Carsten Henkel



Harald Haackh

Contents

- Acknowledgments
- A very short overview of experiments
- Sum-over-the-mode approach:
  - ➡ Surface Plasmons and the Casimir effect
  - ➡ Metamaterials (see also Felipe's talk)
  - ➡ The role of dissipation
- Thermal and Entropy problem:
  - ➡ Description of the problem
  - ➡ Some new aspects
- Conclusions

Recent experiments

Alexander von Humboldt
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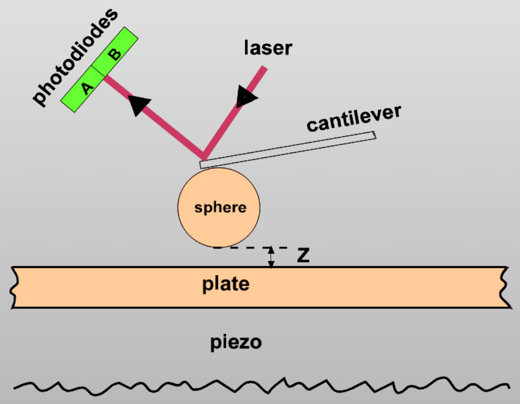
Atomic force microscope



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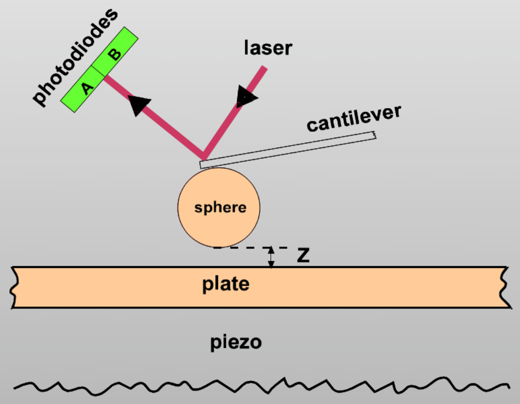

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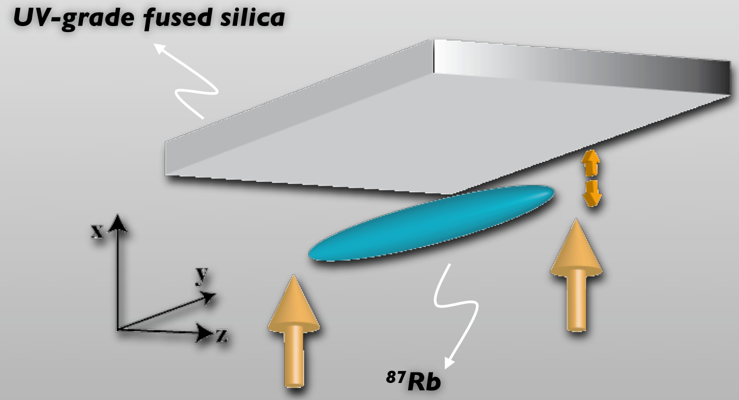
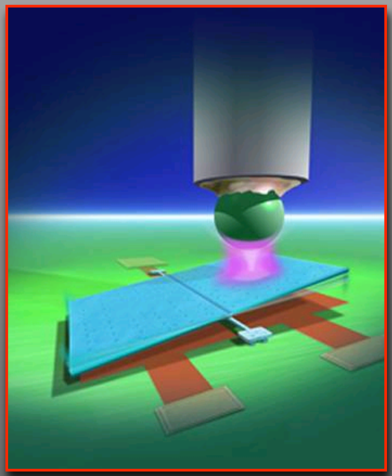


- D. M. Harber et al. *Phys. Rev. A* **72**, 033610 (2005)

MEMS



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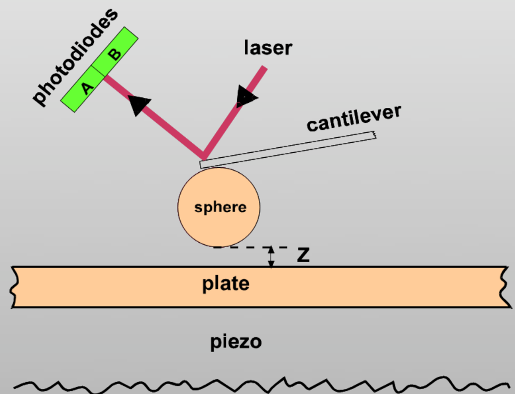

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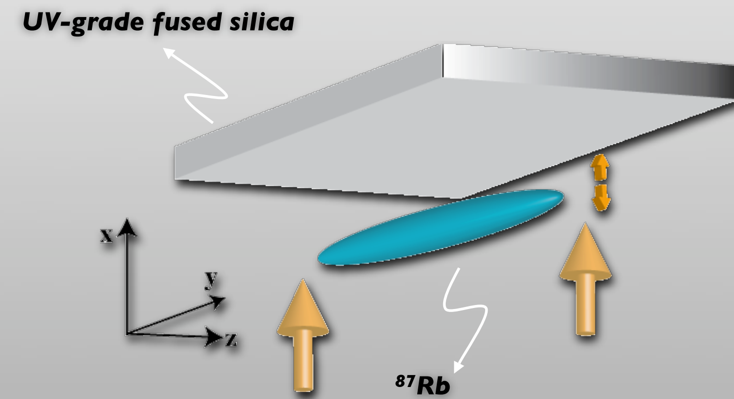

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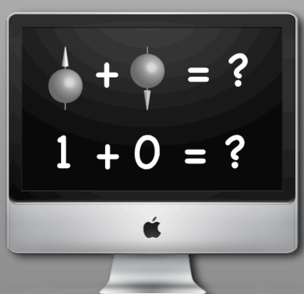
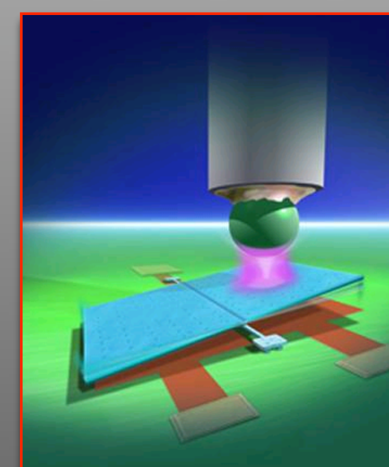


Quantum information
Atom-chips

Nanotechnologies
(Airbags, telescopes ...)

MEMS



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Casimir's calculation:
Perfect mirrors (Sum over modes)

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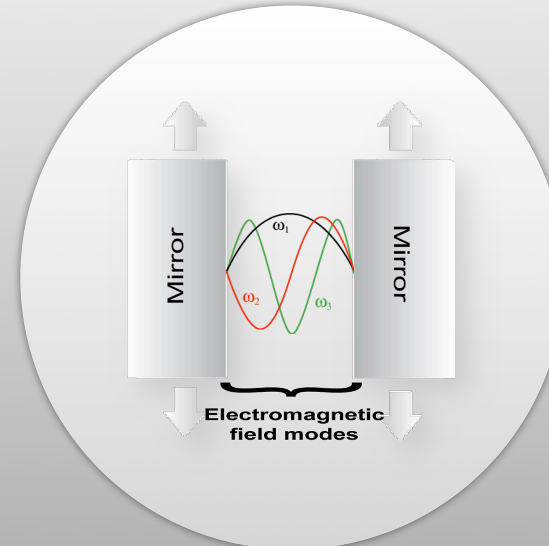
H. Casimir

Sum over the zero point energies

$$E = \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2}\left[\sum_n \omega_n^p\right]_L}_{\text{Infinite zero point energy}} \; \underbrace{-\sum_{p,\mathbf{k}} \frac{\hbar}{2}\left[\sum_n \omega_n^p\right]_{L\to\infty}}_{\text{Setting the zero}}$$



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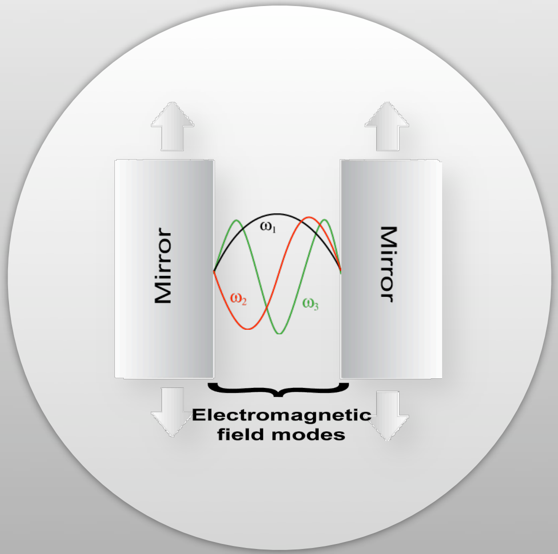

Lifshitz's calculation:
Real mirrors (Langevin-Maxwell theory)

- E.Lifshitz, Sov. Phys.-JETP (USA) 2, 73 (1956)

$$E = \hbar \,\mathrm{Im} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{\mathrm{p},\mathbf{k}} \ln\left(1 - \mathrm{r}_{\mathbf{k}}^{\mathrm{p}}[\omega]^2 \mathrm{e}^{2\mathrm{i}\mathrm{k}_z \mathrm{L}}\right)$$

E.Lifshitz

$r_{\mathbf{k}}^p[\omega]$ reflection coefficients

Fresnel (local) formulation: the medium properties are described through $\epsilon[\omega], \quad \mu[\omega]$ the dielectric function and the magnetic permittivity

Playing with surface plasmons

- F. I. and A. Lambrecht, *Phys. Rev. Lett.* **94**, 110404 (2005).
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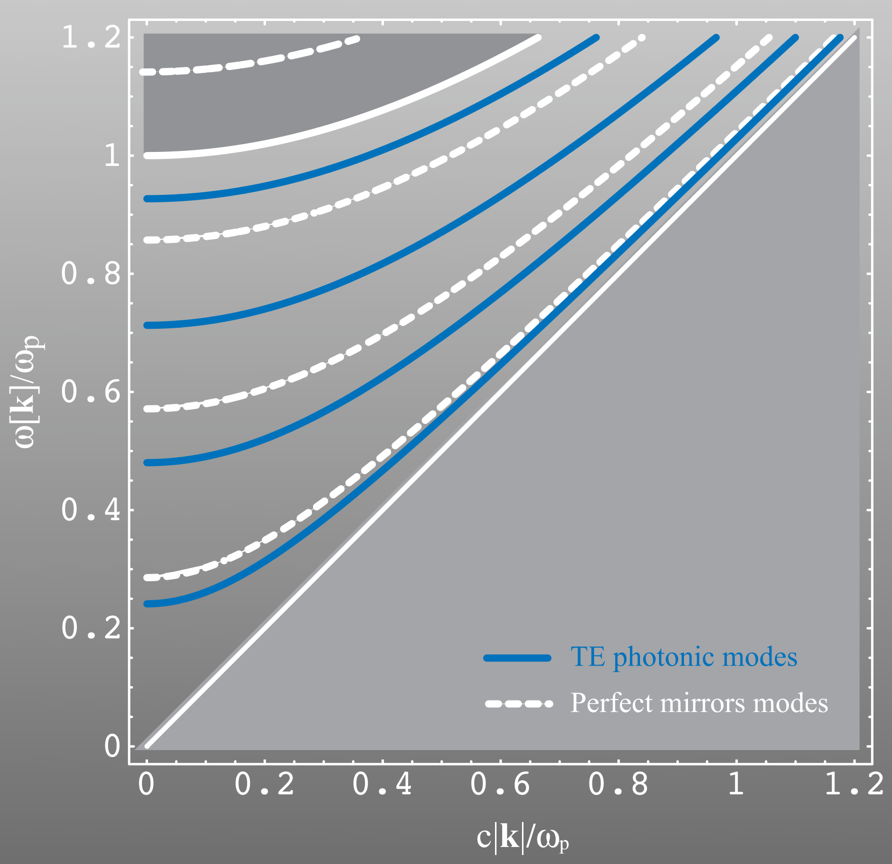

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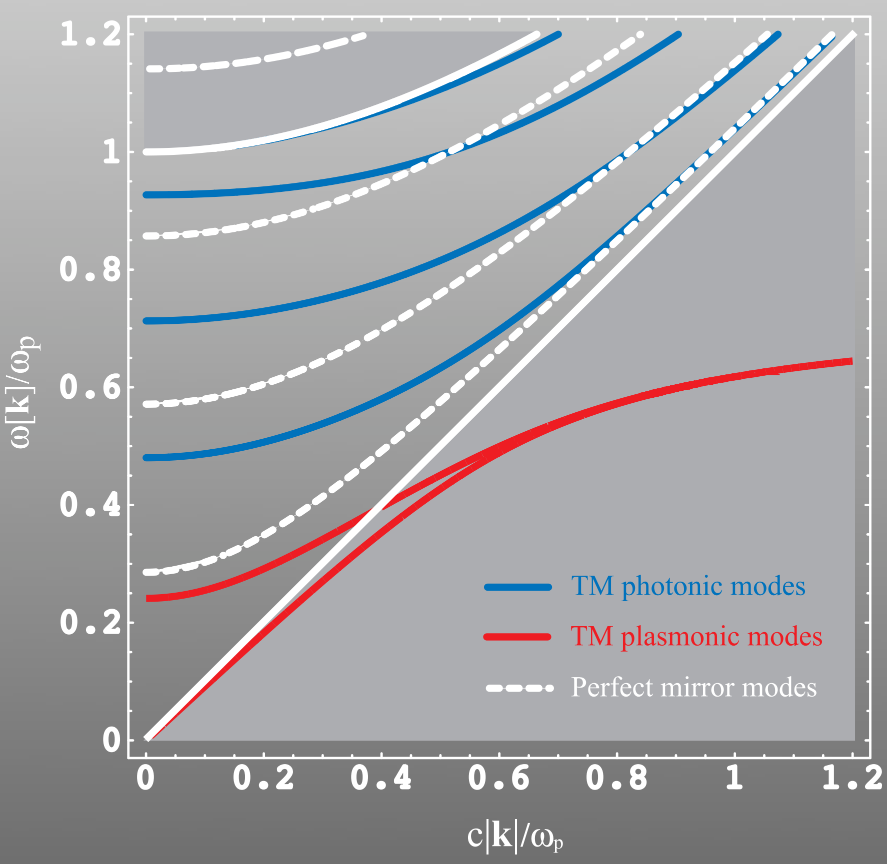

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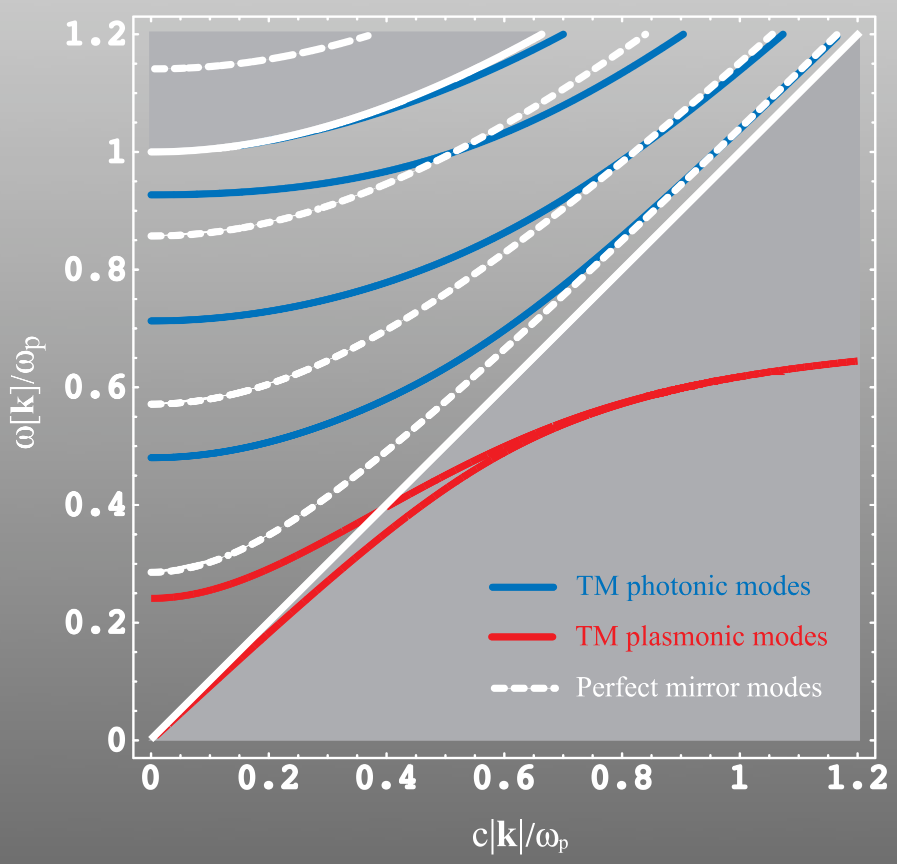

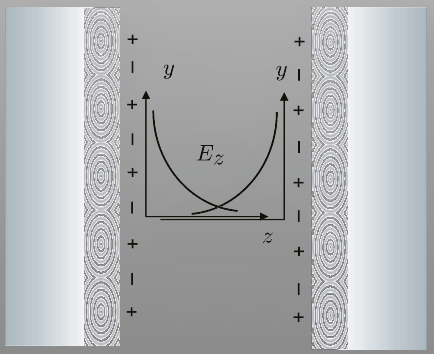

Surface Plasmons are evanescent modes of electromagnetic field associated with the electronic density oscillations at the vacuum/metal interface

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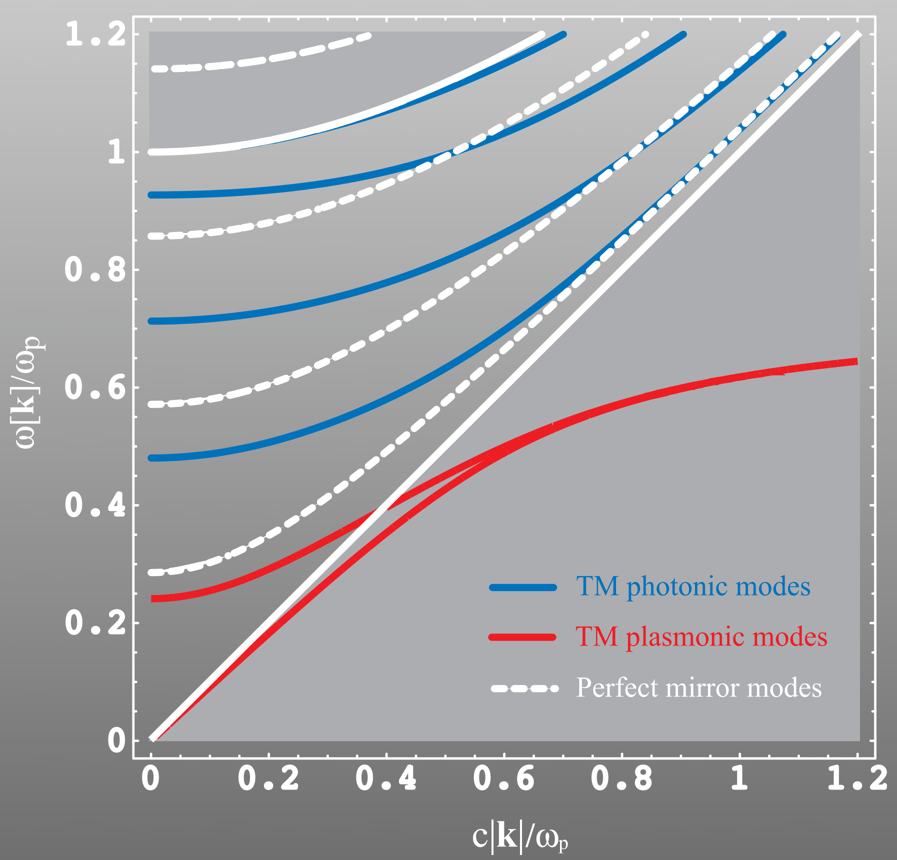

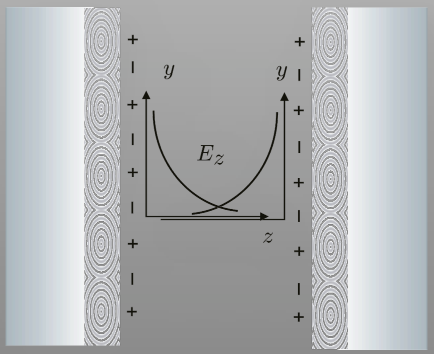

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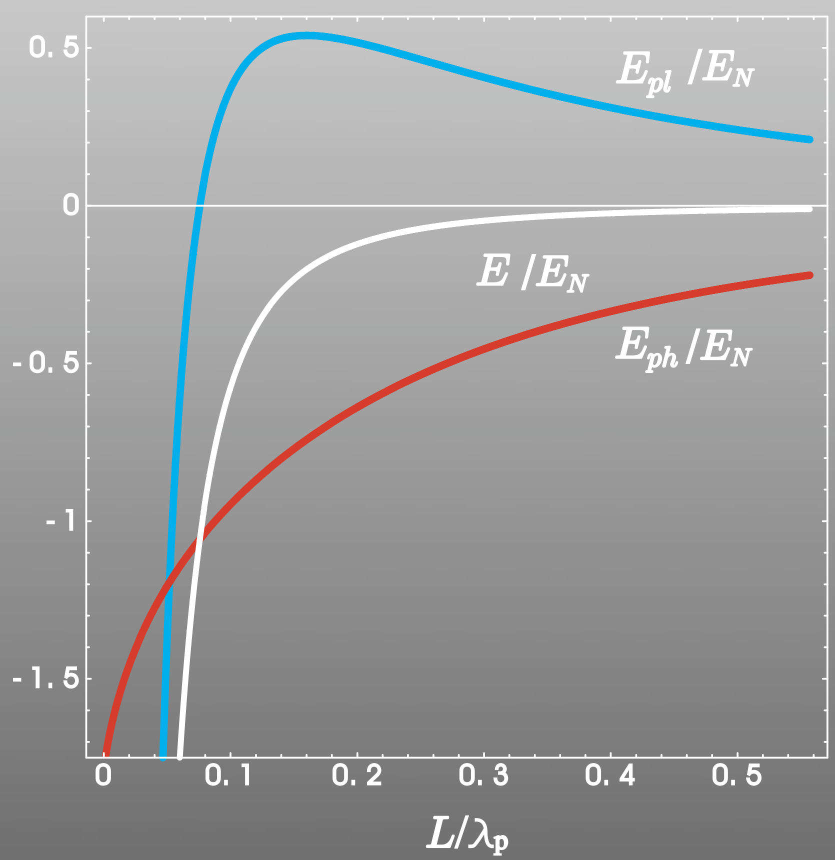
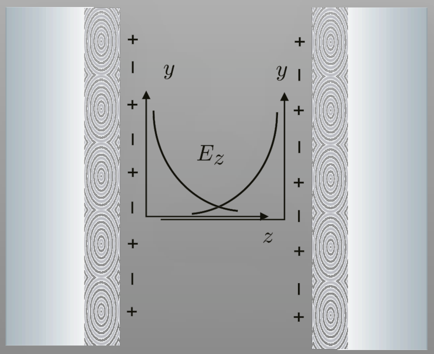


Short distances: For $L \ll \lambda_p$ the plasmonic contribution dominates

At long distances: The two contributions balance



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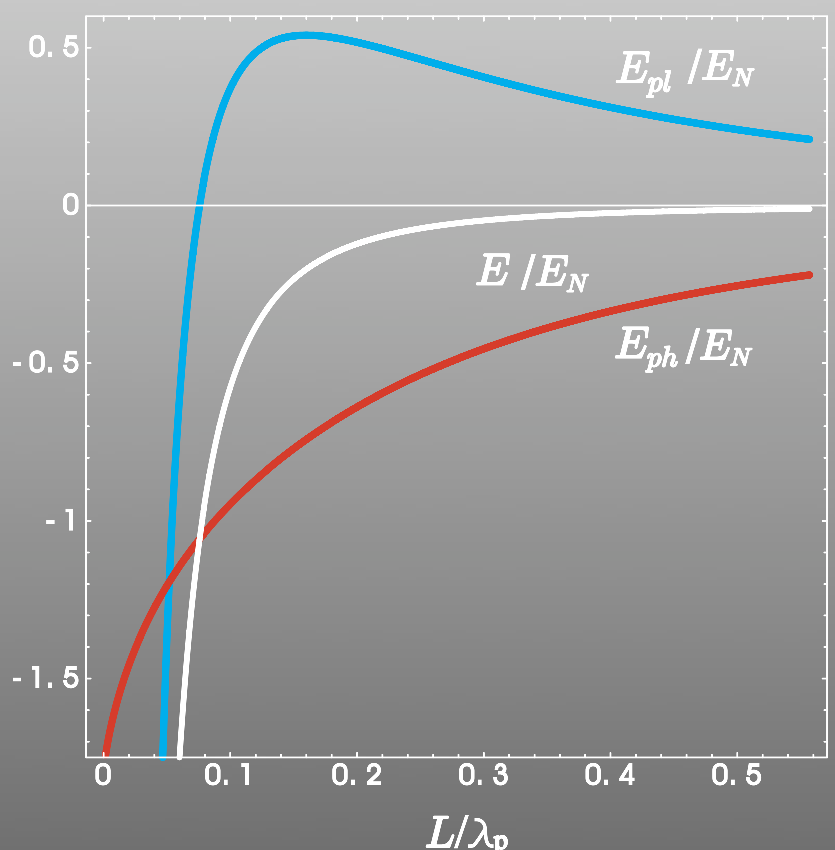

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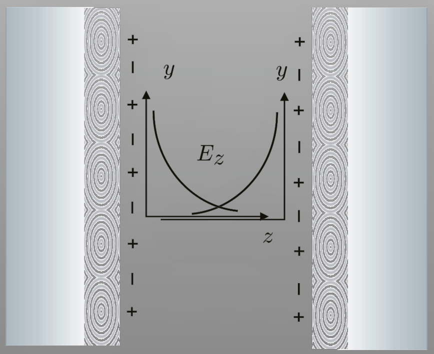


Surface Plasmons are evanescent modes of electromagnetic field associated with the electronic density oscillations at the vacuum/metal interface

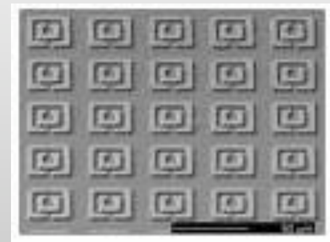
Can you get repulsion changing the balance of the two contributions?

Some metamaterial structures

Some metamaterial structures

TARGET:

Strong magnetic response at the visible-light frequencies, including a band with negative μ.



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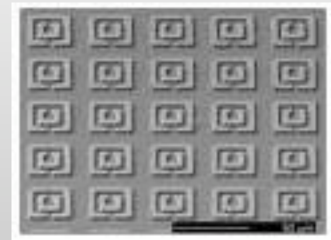

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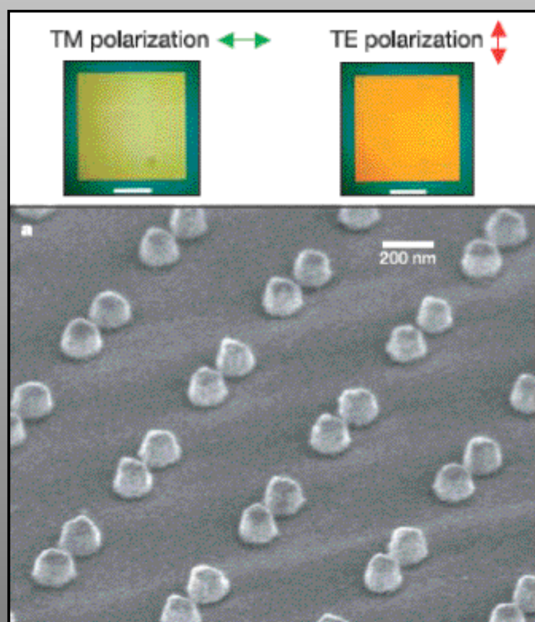
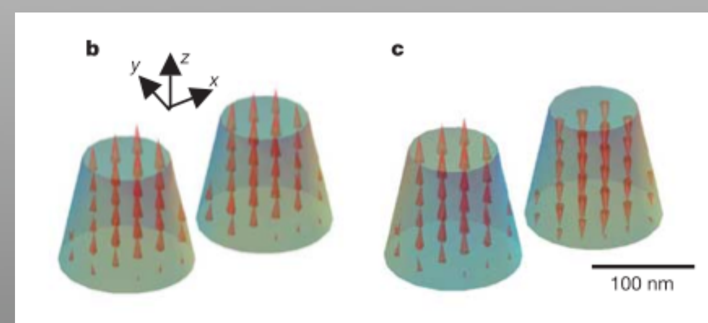


Playing with the Plasmons

The magnetism arises owing to the excitation of the antisymmetric plasmon resonance.



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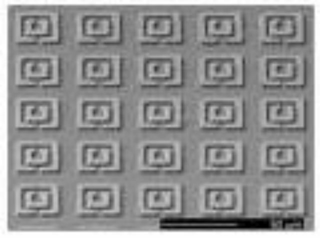

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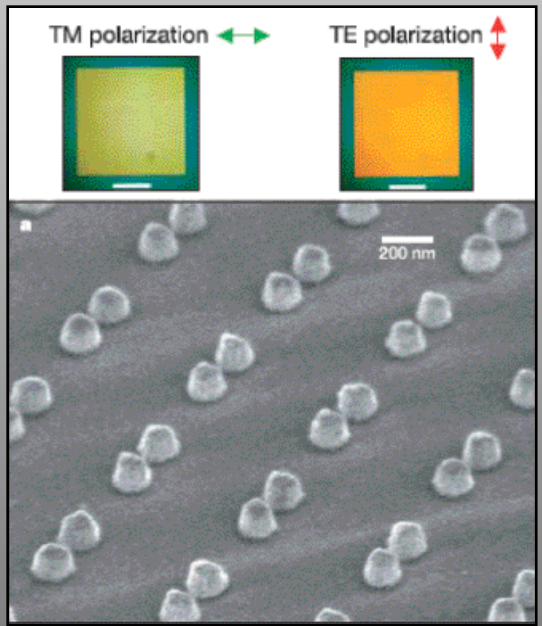

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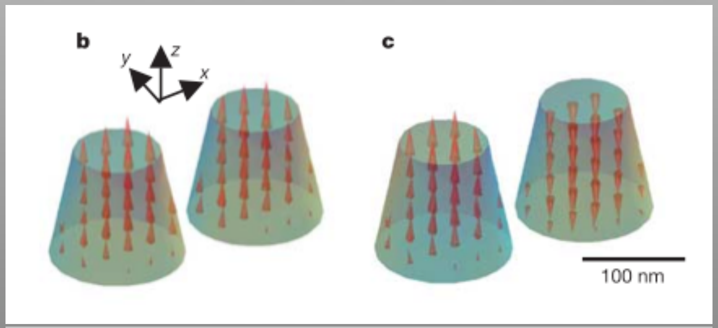
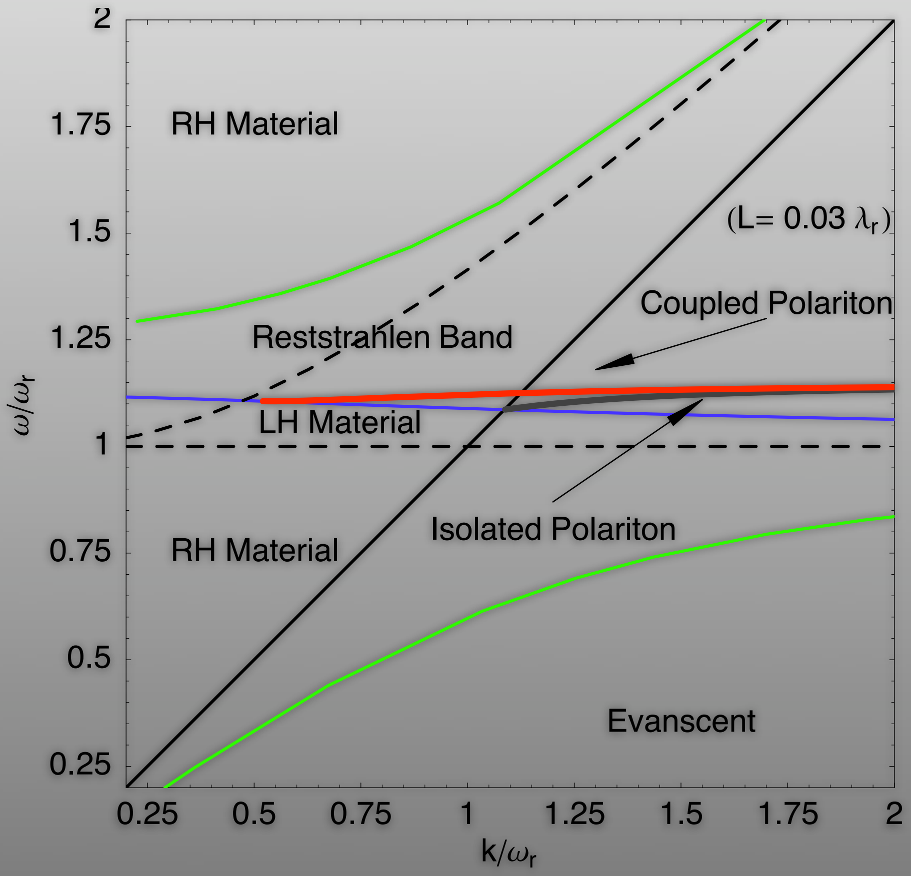


For more detail see Felipe's talk!!!

“Plasmons” and Metamaterial

- F.I. and C. Henkel, in preparation

A metallic mirror model in front of a Metamaterial mirror



Model

- The metallic mirror is described by the plasma model
- The metamaterial is described by the Drude-Lorentz model

One coupled Surface-Polariton in the TE Polarization

Including dissipation

Including dissipation

"Matter point of view"

Dissipation is described by the reflection coefficients

$$E = \hbar \, \mathrm{Im} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{\mathrm{p},\mathbf{k}} \ln \left(1 - \mathrm{r}^{\mathrm{p}}_{\mathbf{k}}[\omega]^2 \mathrm{e}^{2\mathrm{i}\mathrm{k_z}\mathrm{L}}\right)$$

E.Lifshitz

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E.Lifshitz

"Field point of view"

The modes are characterized by complex frequencies

$$E = \sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m\right]_{L\to\infty}^{L} \qquad ?$$

H. Casimir

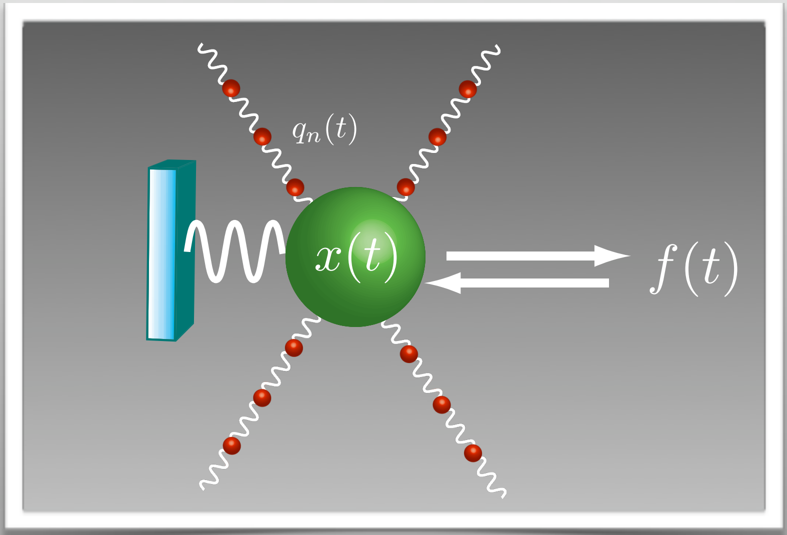
Including dissipation :

The continuum of oscillators - The origin of complex frequencies

As an example: Dissipative Oscillator

$$x[\omega] = G[\omega] f[\omega]$$

$$G[\omega] = \left[\omega_0^2 - \omega^2 - \omega^2 \sum_n \frac{\omega_n^2}{\omega_n^2 - \omega^2}\right]^{-1}$$



$$H = \frac{1}{2}\left(p^2 + \omega_0^2 x\right) + \sum_n \frac{1}{2}\left(p_n^2 + \omega_n^2 [q_n - x]^2\right) - x f(t)$$

All poles on the real axis!

Alexander von Humboldt Stiftung/Foundation

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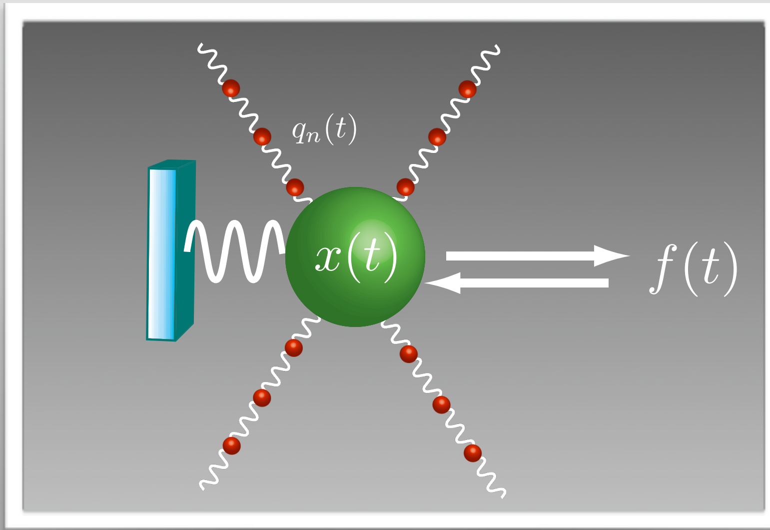

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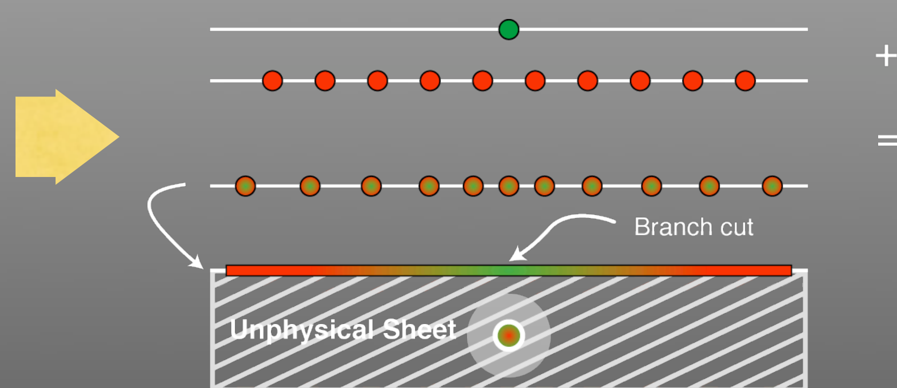
Continuum Limit:



Displacement in the unphysical sheet

$$G[\omega] = \left[\omega_0^2 - \omega^2 - \imath\omega\Gamma(\omega)\right]^{-1}, \quad \Gamma(\omega) \propto \frac{\pi\Omega^2}{2}\left(\frac{\omega}{\omega + \imath\Omega}\right)$$

Dissipative case

The frequencies (ω_m) are complex.

$$E = \sum_{p,\mathbf{k}} \frac{\hbar}{2} \, \mathrm{Re} \left[\sum_m \omega_m - \frac{2\mathrm{i}\omega_\mathrm{m}}{\pi} \ln \frac{\omega_m}{\Lambda} \right]_{L\to\infty}^{L}$$

Sum Rule!

$$\left[\sum_m \mathrm{Im} \; \omega_m^p \right]_{L\to\infty}^{L} = 0$$

- F.I. and C. Henkel, J. Phys. A: Math. Gen. 41, 164018 (2008)

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Analogy with the quantum dissipative oscillator :

- K. E. Nagaev and M. Buttiker., *Europhys. Lett.* **58**, 475 (2002) - F. I. et al. *Phys. Rev. A* **67**, 042108 (2003)

- Mean energy of the ground state

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Short distances with Drude model

- C. Henkel et al., *Phys. Rev. A* **69**, 023808 (2004)

- Plasmons still dominate in the limit $L \ll \lambda_p$
- Leading term correction ($O[\gamma]$)

$$\begin{cases} \mu[\omega] = 1 \\ \epsilon[\omega] = 1 - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\gamma)} \end{cases}$$

$$\downarrow$$

$$\eta \approx \frac{3}{2}\alpha \frac{L}{\lambda_p} - \gamma L \frac{45}{2\pi^4}\zeta(3)$$

Thermal and Entropy problem

$$E = E_0 + \boxed{\Delta E(T)}$$

Hot discussion (more then 25 articles, comments and replies!!)

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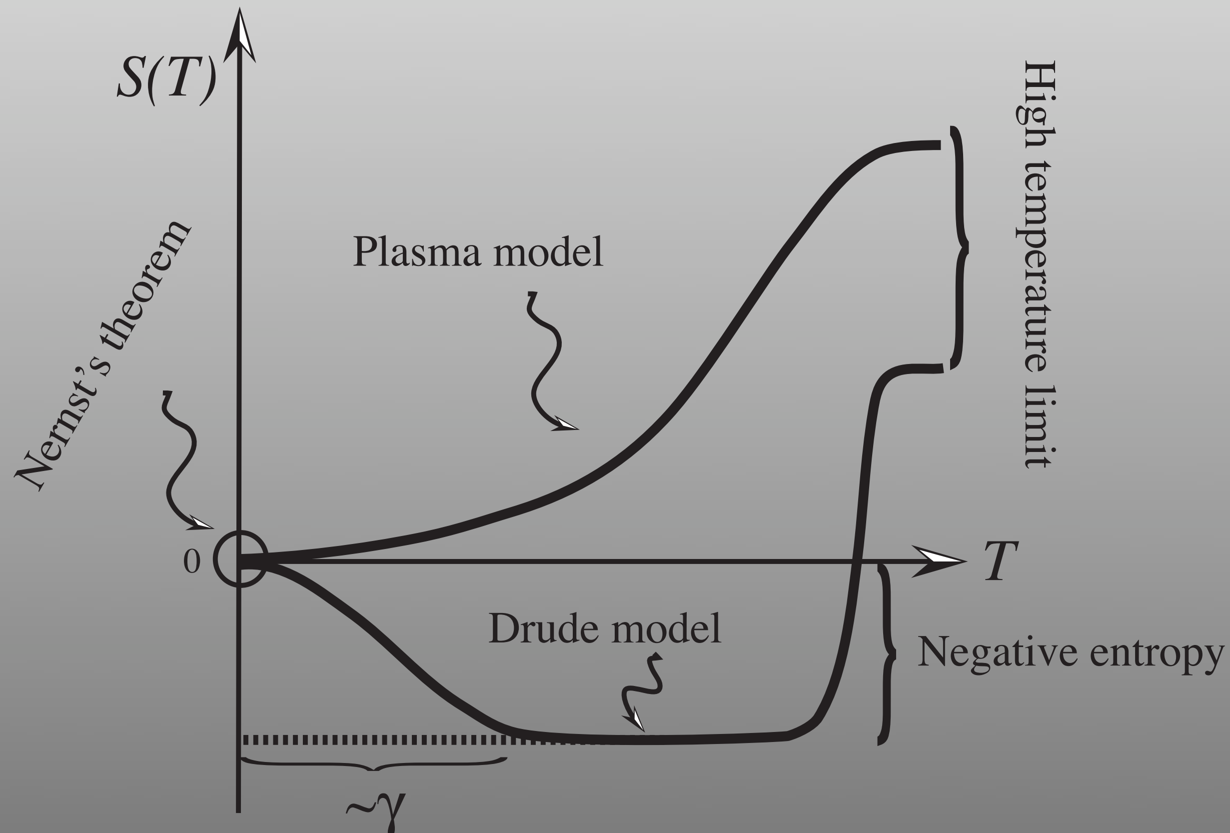

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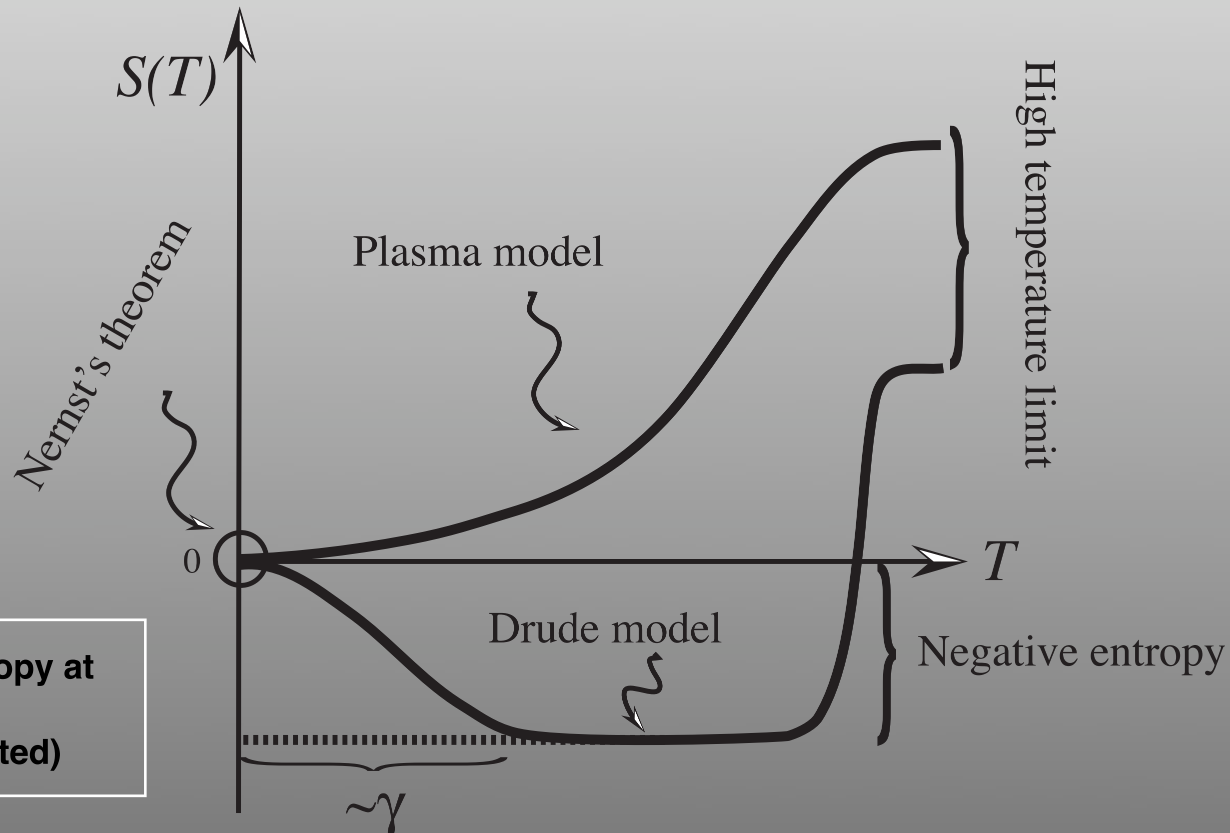
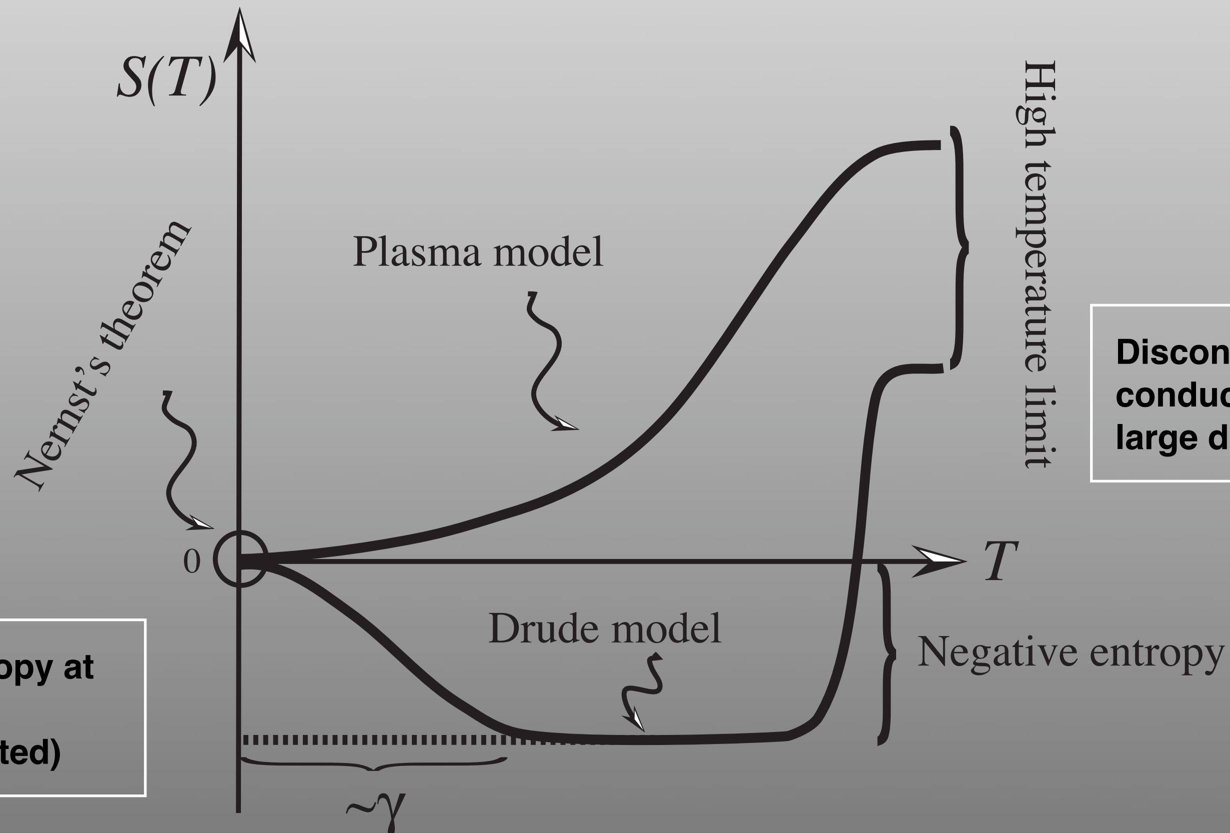


Negative entropy at T -> 0 (Nernst theorem violated)

Thermal and Entropy problem

$$E = E_0 + \boxed{\Delta E(T)}$$

Hot discussion (more then 25 articles, comments and replies!!)



Discontinuity real/perfect conductor (high "T" & large distances)

Negative entropy at T -> 0 (Nernst theorem violated)

Thermal and Entropy problem

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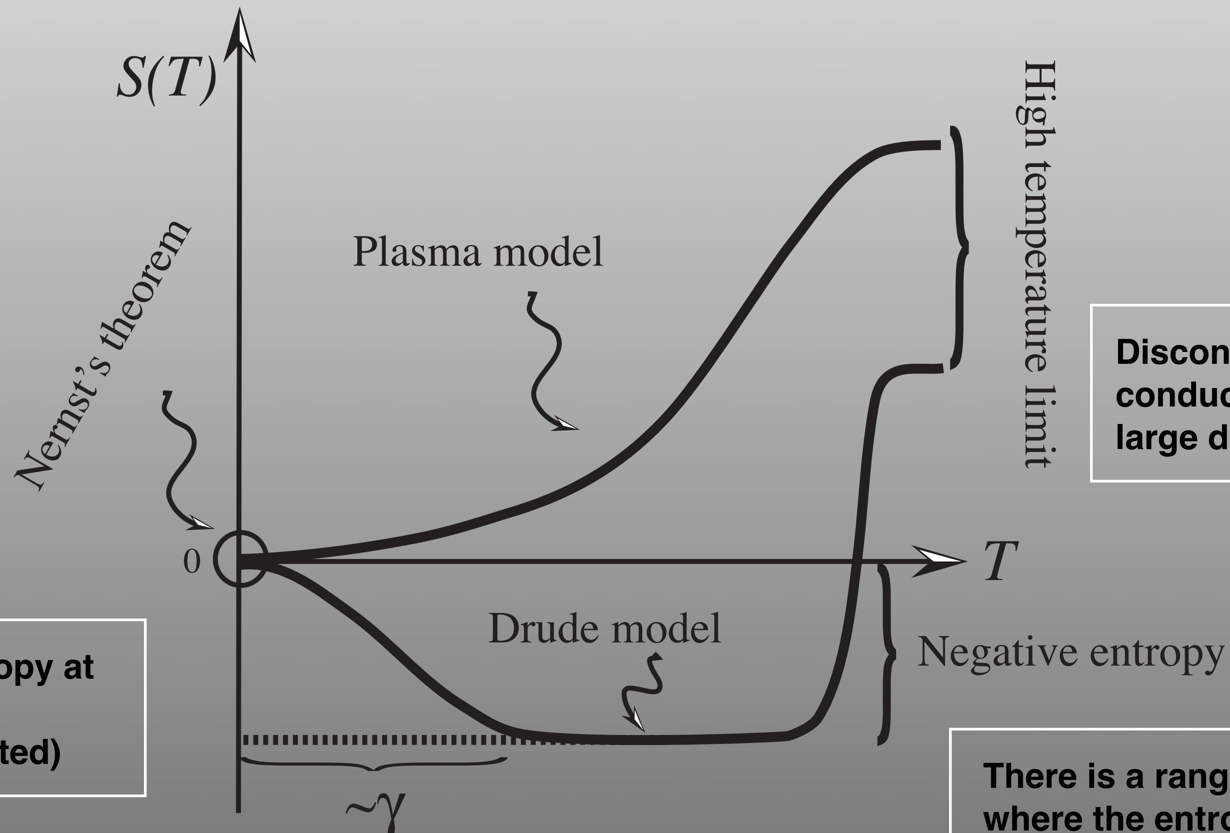

Discontinuity real/perfect conductor (high "T" & large distances)

Negative entropy at T -> 0 (Nernst theorem violated)

There is a range of temperatures where the entropy is negative

Alexander von Humboldt
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Foucault/Eddy currents

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Alexander von Humboldt Stiftung/Foundation

Principle (Classical View)

- A changing magnetic field generates swirling current.
- These circulating eddies of current create electromagnets with magnetic fields that opposes the change of the magnetic field

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- Structural Testing: Eddy current techniques are used for condition monitoring of a large variety of metallic structures

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Quantization

- Damped modes: `system+bath' paradigm
- Heat-conduction-like equation for their vector potential:

$$\nabla^2 A + \frac{1}{D}\partial_t A = 0, \quad D = (\mu_0 \sigma)^{-1}$$

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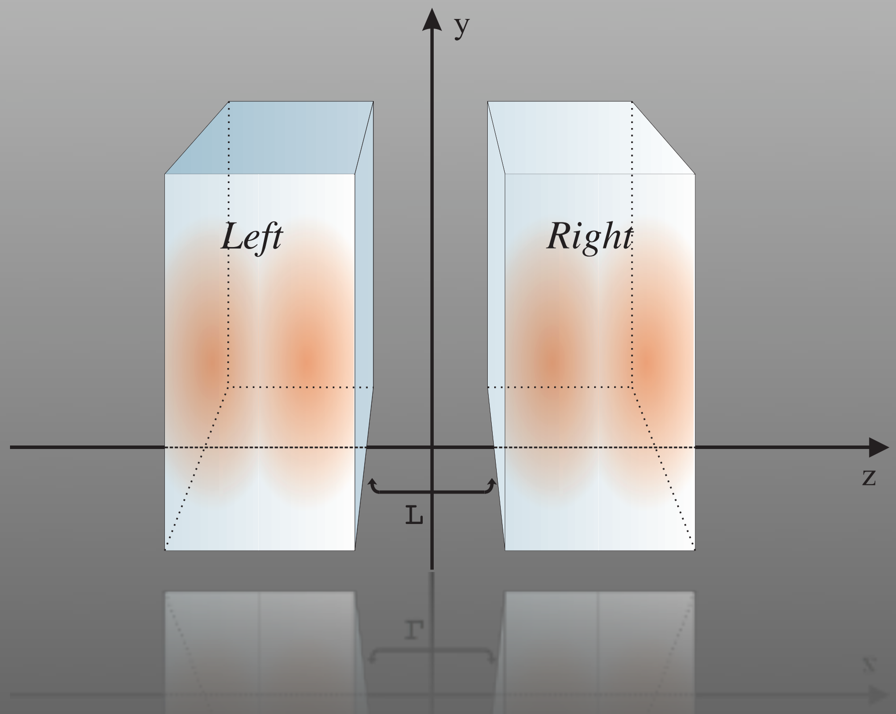
Two interacting bulks

- DOS depending on L
- Bulk modes
- Transversal evanescent modes
- Low frequency modes



Eddy currents and the Casimir Effect:

Exact Contribution

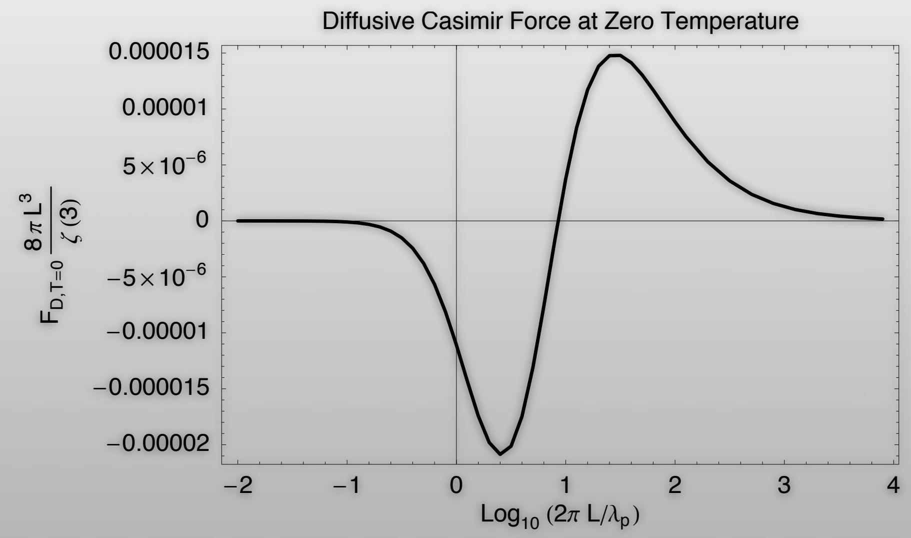
- F.I. and C. Henkel, to be submitted

Eddy currents and the Casimir Effect:

Exact Contribution

Zero temperature: TE polarization

- Change in sign with the distance (as for the plasmonic modes).
- Very small contribution



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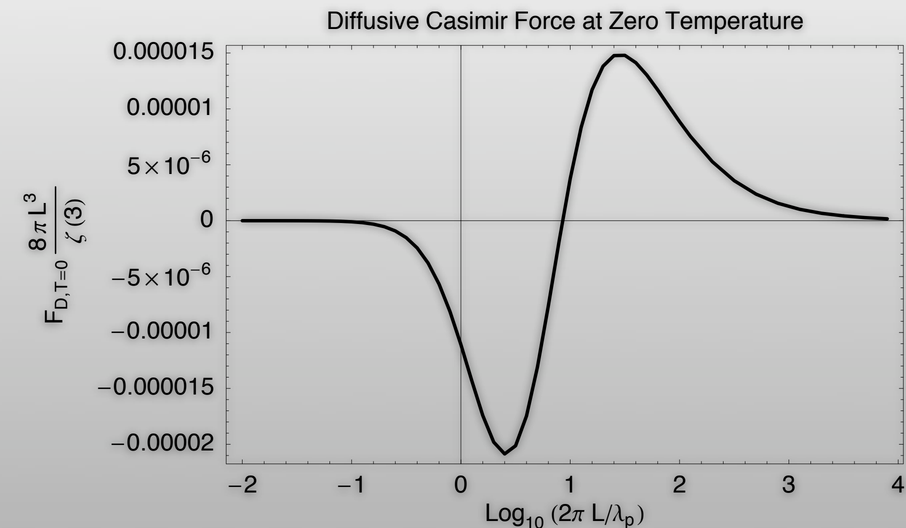
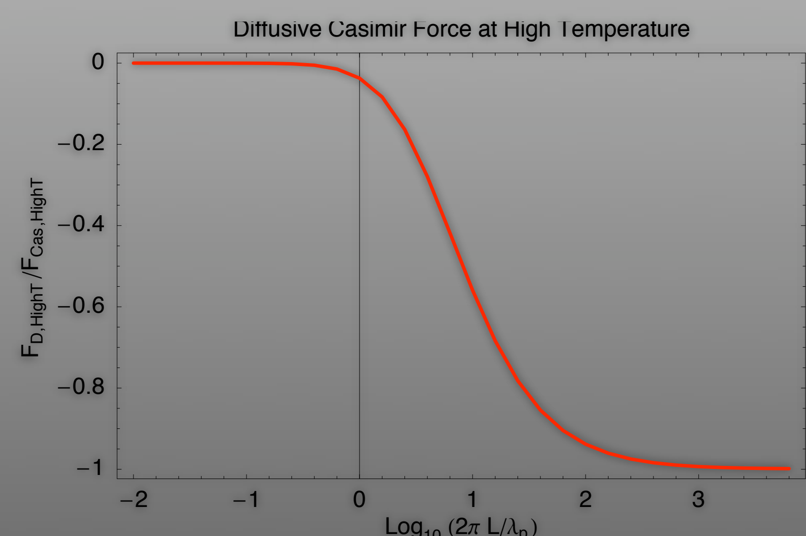


High temperature: TE polarization

- Their contribution is equal but opposite in sign to the value obtained with perfect mirrors.
- They cancel the contribution coming from the propagating waves.

➡ Partial results:
- G. Bimonte. Johnson noise and the thermal casimir effect. New J. Phys., 9(8):281, 2007. (One dimension)
- V. B. Svetovoy. Evanescent character of the repulsive thermal casimir force. Phys. Rev. A, 76(6):062102, 2007. (Separating Evanescent and Propaganting wave)



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Eddy currents and the Thermal Problem

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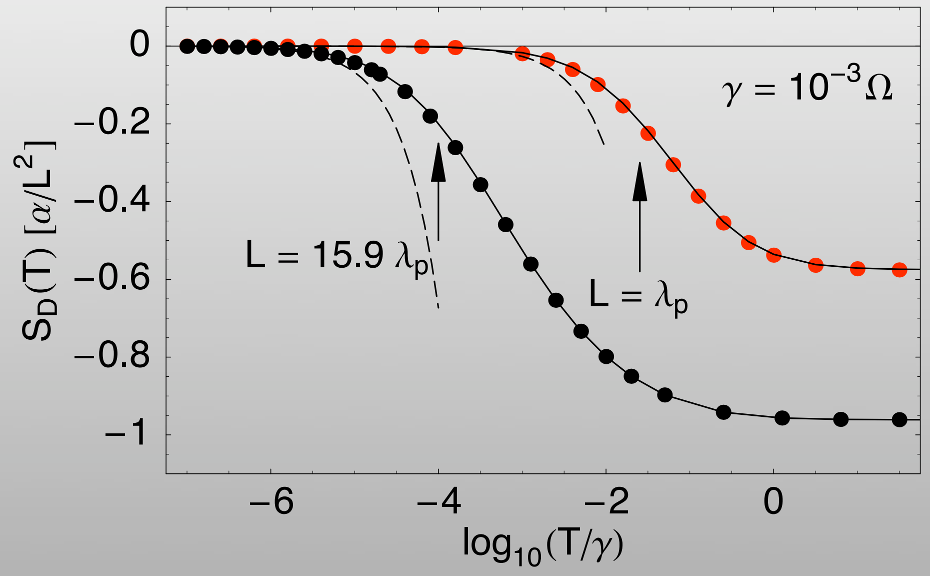
Eddy currents and the Thermal Problem

Nernst Theorem

- The diffusive modes then always stay in the high-temperature regime and contribute a constant value to the entropy.



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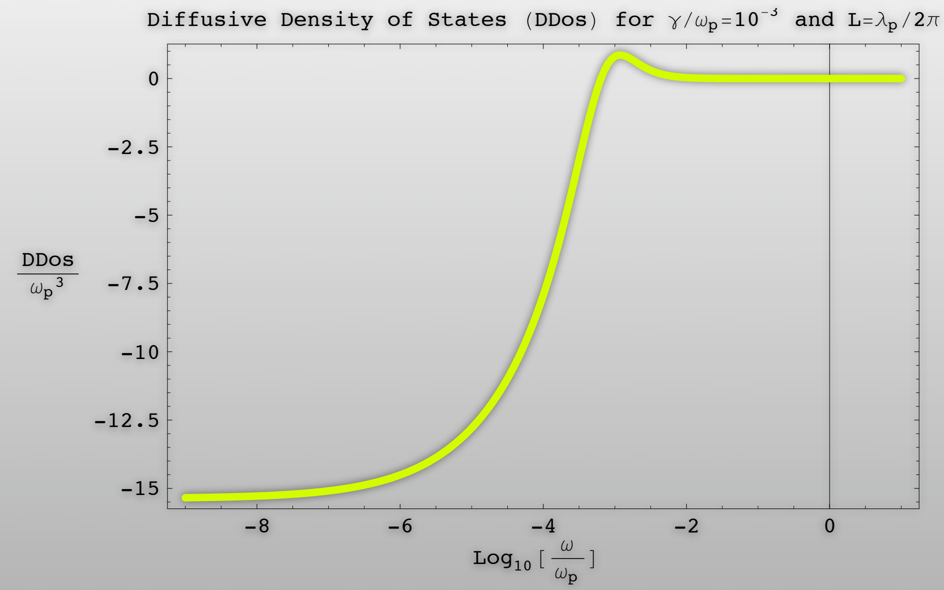

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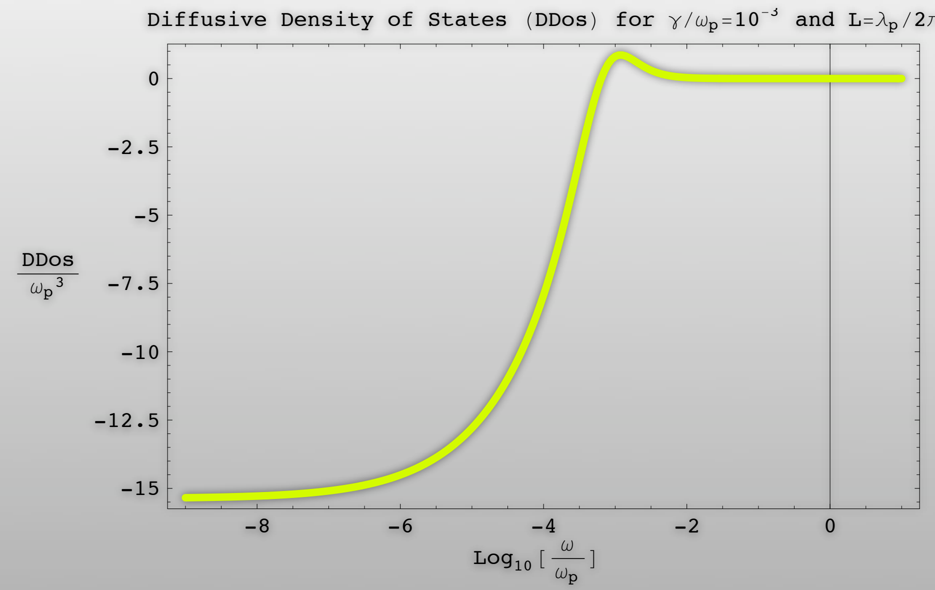
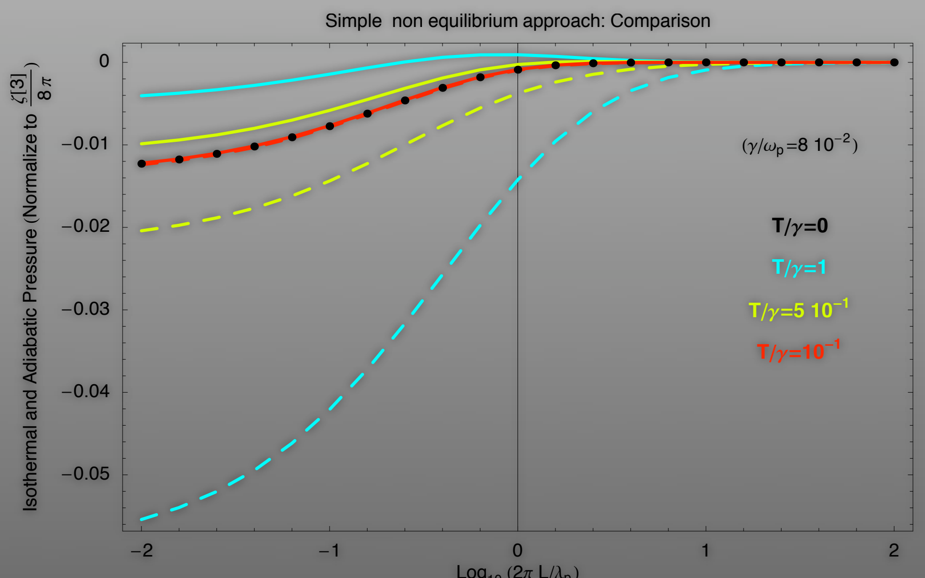


Get rid of them: adiabatic approach

- Extremely low frequencies/small decay constants: experiment with a finite characteristic duration, the diffusive modes are **not in thermal equilibrium**
- The simplest alternative to an isothermal change of state: an **adiabatic** change.

$$p_{ad} = -\left.\frac{\partial U}{\partial L}\right|_T + \left.\frac{\partial S}{\partial L}\right|_T \left(T - \frac{S}{\partial S/\partial T|_L}\right)$$



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Conclusions and Outlines

- Sum over modes analysis an very useful tool:
  - Different contributions to the Casimir Effect
  - Plasmonic and Photonic Contributions
  - Metamaterials influence
  - Role of dissipation
  - Thermal Problem

- Results:
  - Plasmons & Metamaterial could be a resource to taylor the Casimir Force
  - Generalization of Casimir Formula
  - Eddy Currents help to understand the thermal problem
  - Orthodox Nernst's theorem can be violeted: Highly degenerate sub space
  - High temperature behavior: non-equilibrium dynamic (adiabatic pressure)