Equilibrium Persistent Currents in "Normal" Metal Rings, a Simple solution to the Puzzle?

Yoseph Imry, WIS.

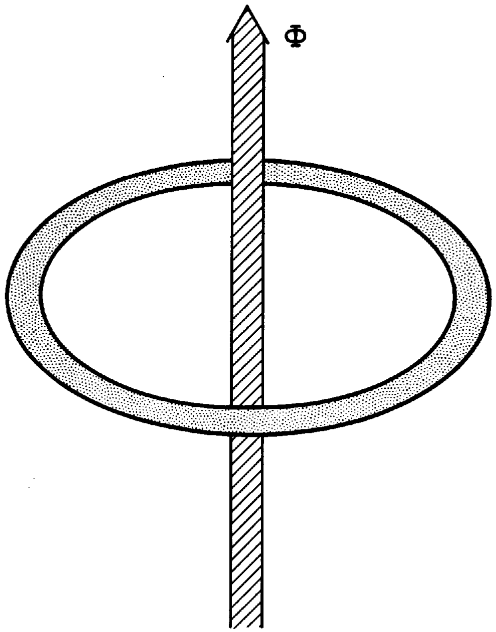
Work with:

L. Gunther, 1968-9;

M. Buttiker, R. Landauer, 1983;

H. Bary-Soroker and O. Entin-Wohlman, 2008.

See also: G. Schwiete and Y.Oreg, 2008, on a related problem.

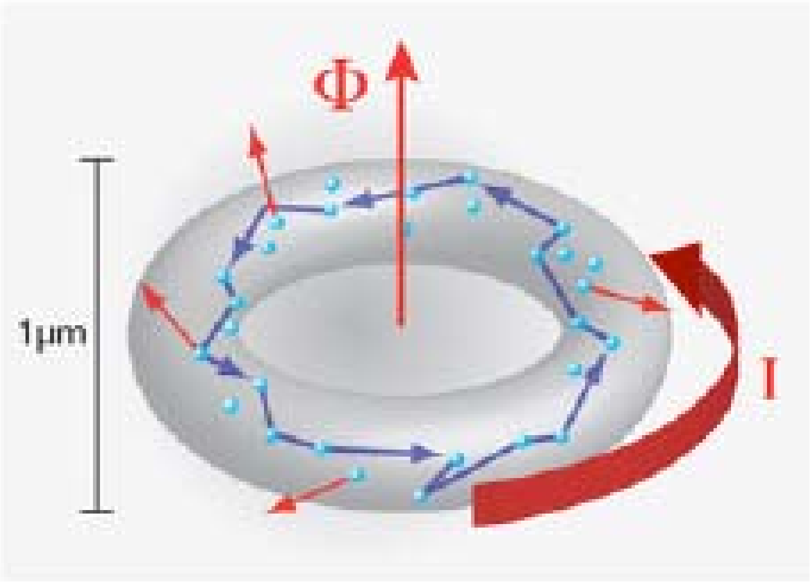


Purely quantum effect!

A-B Flux in an isolated ring

- A-B flux equivalent to boundary condition.
- Physics periodic in flux, period h/e (*Byers-Yang*).

- "Persistent currents" exist due to flux dependence of free energy.
- They do **<u>not</u>** decay by impurity scattering (*BIL*).

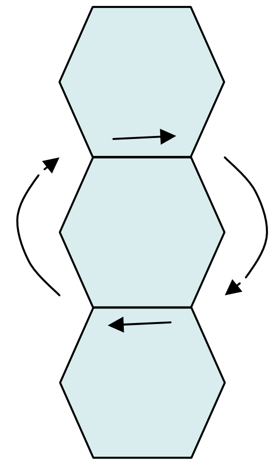


$$H = \frac{1}{2m}(-i\hbar\vec{\nabla} - e\vec{A})^2 + \quad , \qquad \vec{A} = \frac{\Phi}{L}\hat{\varphi}$$

Prehistory of microscopic persistent currents

L. Pauling: "The **diamagnetic Anisotropy** of Aromatic molecules", J. Chem. Phys. **4**, 673 (**1936**);

F. London: "Theorie Quantique des **Courants Interatomiques** dans les Combinaisons aromatiques", J. Phys. Radium **8**, 397 (**1937**);



Induced currents in anthracene

Even earlier work by F. Hund.

But: molecular scale only, no scattering!

Questions

- No orbital response in classical Physics - macroscopic scales, hence a Quantum effect! (coherence needed).

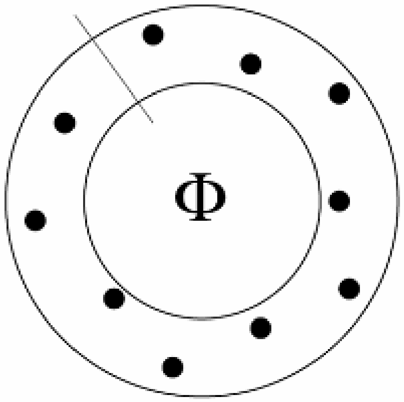
$$H = \frac{1}{2m}(-i\hbar\vec{\nabla} - e\vec{A})^2 \qquad , \qquad \vec{A} = \frac{\Phi}{L}\hat{\varphi}$$

- Anything interesting on intermediate scales?

- What about resistive rings? [L. Gunther and YI *Flux quantization without off-diagonal-long-range-order in a thin hollow cylinder.* **SSC 7**, 1391 (1969), Buttiker, Landauer, YI (1983)].

Flux = aperiodic boundary conditions

- Byers and Yang, Bloch:
  All physical properties are
  periodic in Φ with period $\Phi_0 = hc/e$



- Gauge transformation: $\psi' = e^{(ie/\hbar c)\int \vec{A} d\vec{x}'} \psi$
  Φ eliminated but ψ' not periodic
  phase changes by $\theta = 2\pi \Phi / \Phi_0$

Byers and Yang, Phys. Rev. Lett. 7, 46 (1961)

Elastic scattering - impurities and surface

Energy periodic in flux, period - Φ_0

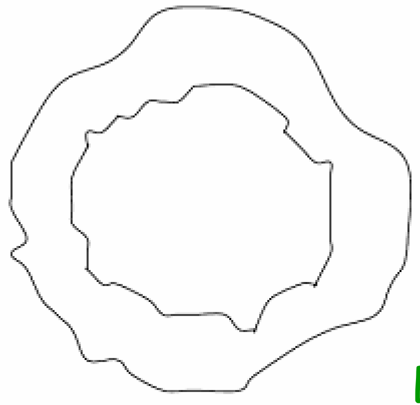
Equilibrium, non-dissipative !

At T=0 : $I = -\frac{e}{h}\frac{\partial E}{\partial \phi}$ $\phi = \Phi / \Phi_0$

At finite T : $I = -\frac{e}{h}\frac{\partial \Omega}{\partial \phi}$

Thouless scale: $E_c = \hbar D / L^2$

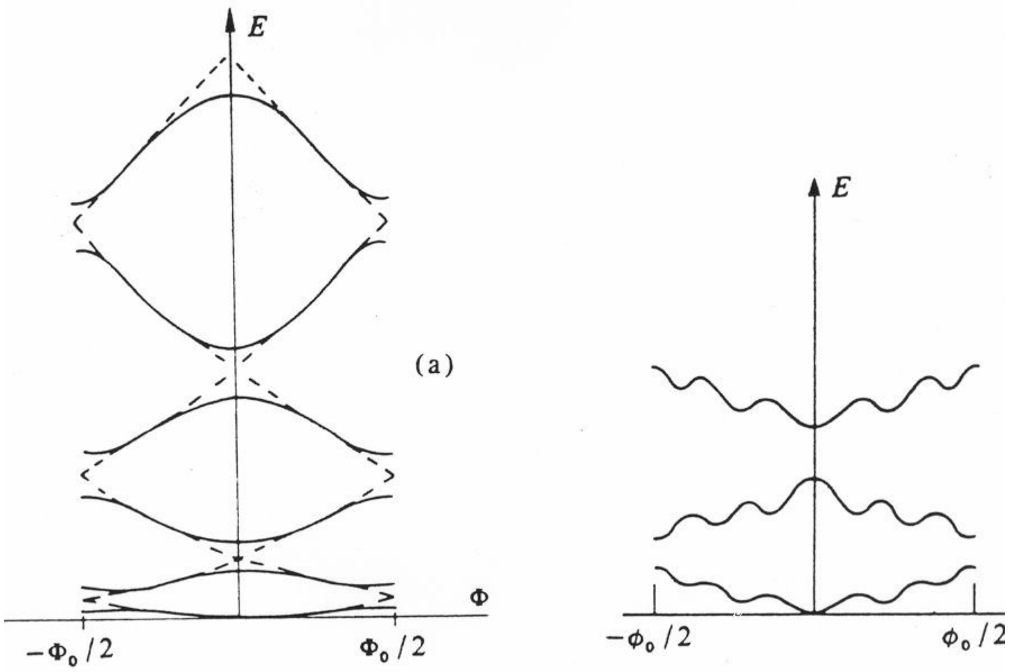
$D = v_F l / 3$, $l =$ elastic m.f.p



$l < L$ also in ballistic systems, BUT

Impurities suppress only as l / L

Buttiker, Imry and Landauer,Phys. Lett. A 96, 365 (1983)



Details of system (defect arrangement…) matter, hence ensemble average vs. "sample specific" issue.

Levy, Dolan, Dunsmuir and Bouchiat, 1990.
~10^7 Cu rings

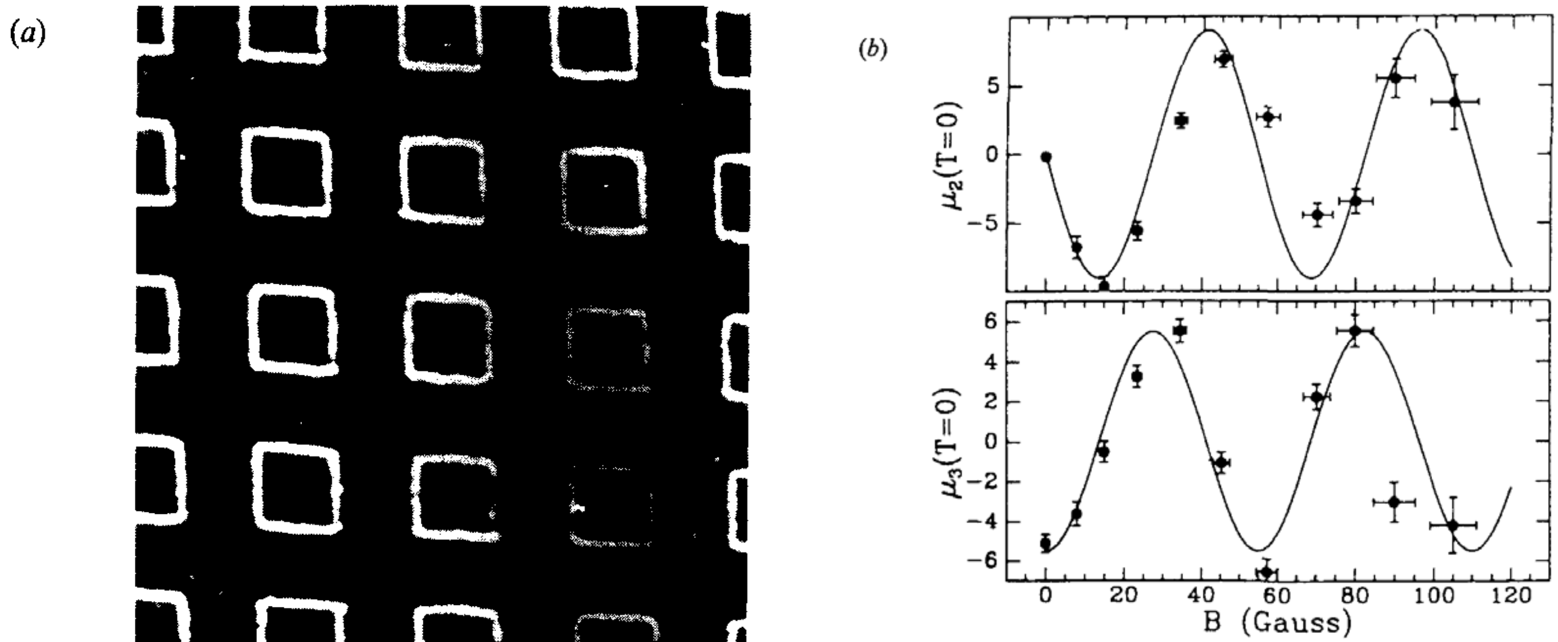


Figure 4.3 From Levy et al. (1990) who measured the nonlinear response of the ensemble of 10^7 copper rings a small part of which is shown is shown in (*a*), as function of flux (a value of *B* of 130 G corresponds to h/e). An a.c. signal at a low (~ 1 Hz) frequency and an amplitude of 15 G was employed and the second (μ_2) and third (μ_3) harmonics (double and triple the a.c. frequency) measured and shown in the figure. The nonlinearity arises from the periodic flux dependence and these results imply a persistent current with a period $h/2e$.

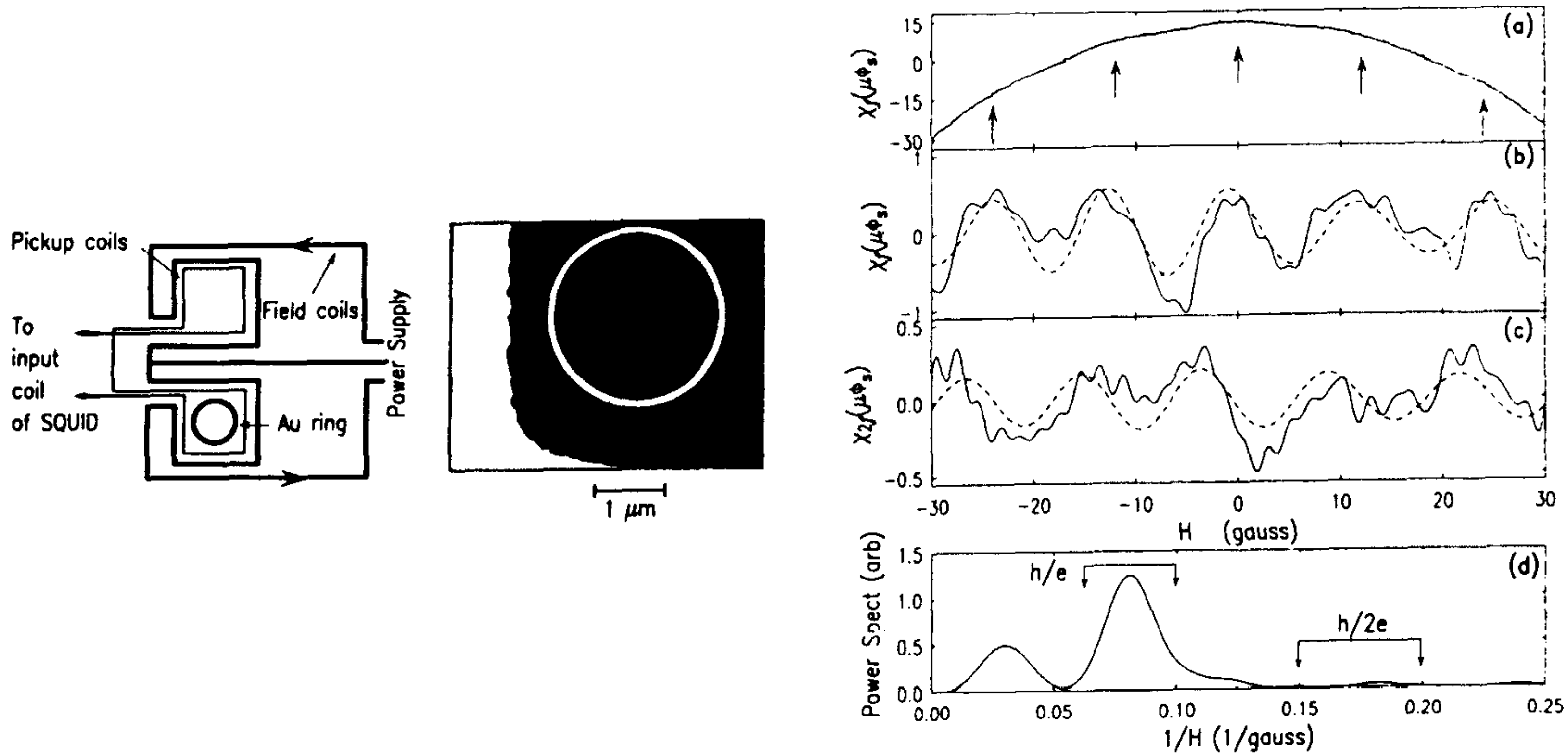
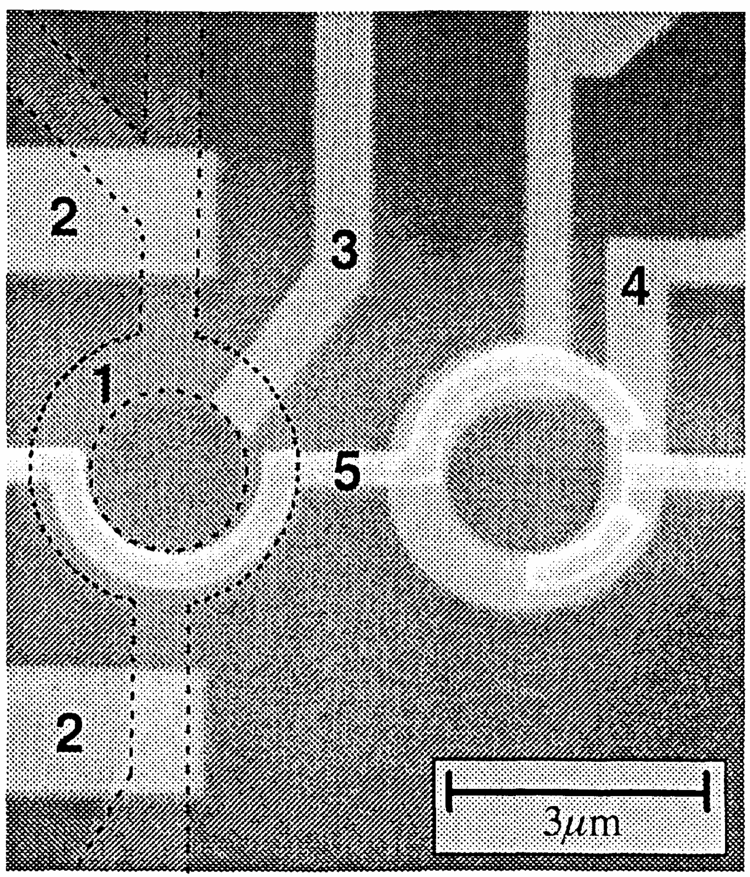


Figure 4.4 From Chandrasekhar et al. (1991). The experimental arrangement for obtaining the response of the single ring (shown) minus that of an empty substrate is depicted on the left. The results are shown on the right: the magnetic-field dependence of the amplitude of the f and $2f$ signals at 7.6 mK for the 1.4 μm × 2.6 μm gold loop. (a) f response with no signal processing. The arrows point to the maxima of the h/e periodic signal. (b) Data of (a), with the quadratic background subtracted. (c) $2f$ response, after subtraction of a linear background signal. The amplitude of the 4 Hz a.c. drive field was 4.12 G. (d) Power spectrum for the data displayed in (b). The h/e arrows show the region, centered about the expected frequency for h/e oscillations, over which the data in (b) and (c) were bandpassed to produce the dashed curves. The region where an $h/2e$ signal is expected to appear based upon the inside and outside area of the sample is also shown. The data in (b) and (c) has been digitally filtered to eliminate high-frequency contributions above 0.50 G^{-1}.

Mailly, Chapelier and Benoit, 1993



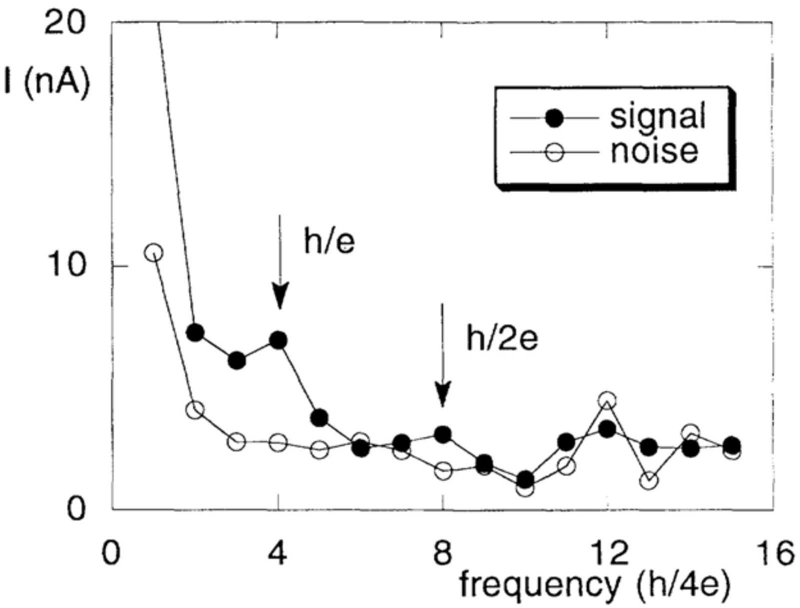
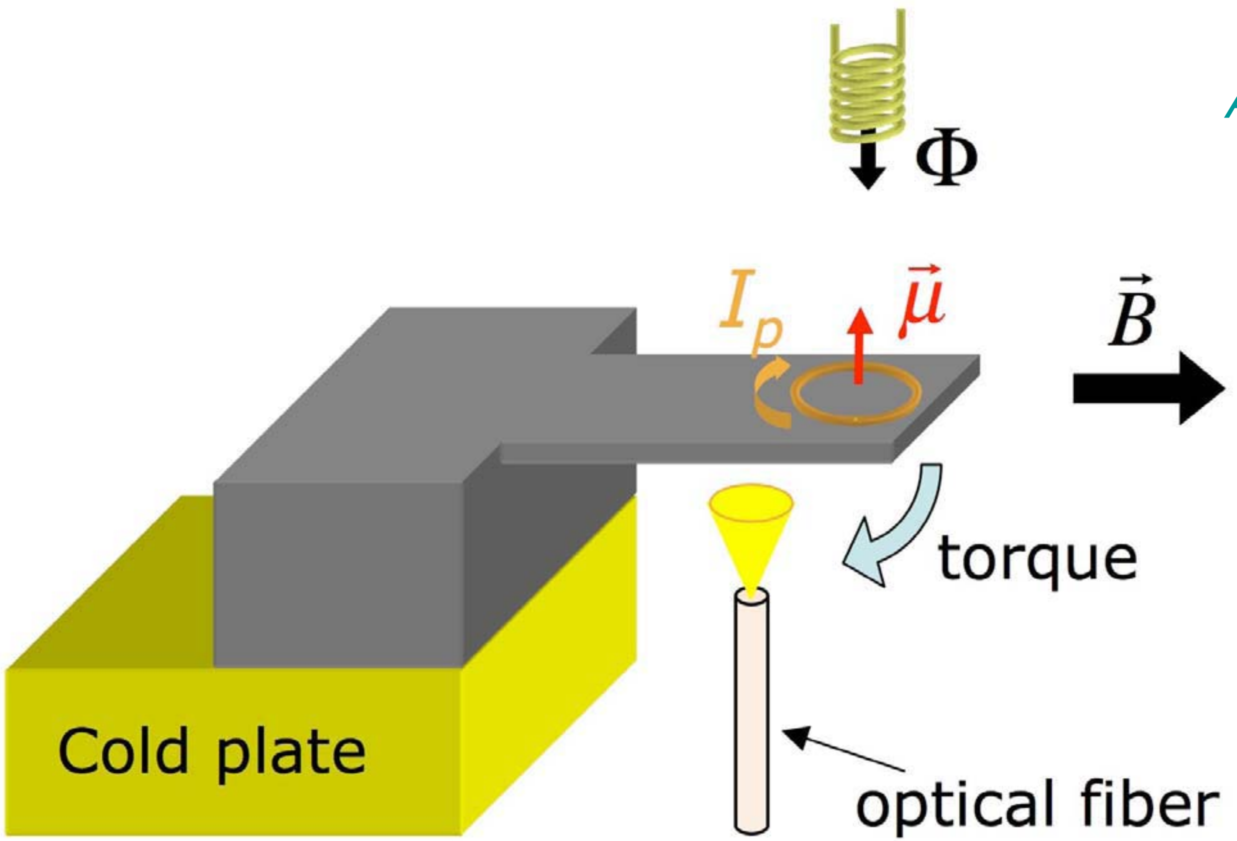
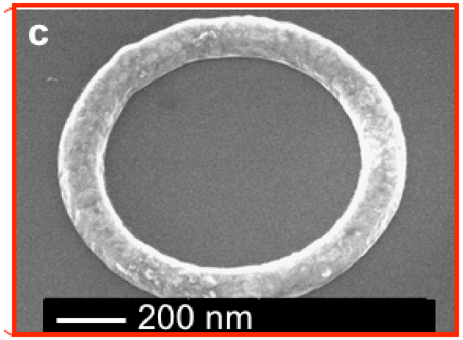


FIG. 3. Square root of spectrum power of magnetization of the ring. The values are converted into the equivalent current in the ring using the calibration coil. Open circles correspond to experimental noise, i.e., differences between measurements with ring open. Solid circles correspond to experimental signal, i.e., differences between measurements with ring closed and ring open. The two arrows indicate the position of period h/e and $h/2e$.

New measurement method—torque with AFM cantilever, laser interferometry, Harris, Yale



A.C. Bleszynski-Jayich, et al. 2007



Magnitude of persistent currents in diffusive rings

Noninteracting results:

- One ring - $I \sim E_c / \Phi_0$ (CGR89)
  Typically E_c / Δ levels are correlated.

- Ensemble averaged
  - Grand canonical ensemble: zero (EG89)
  - Canonical ensemble Δ / Φ_0 (AGI91)

Cheung, Gefen and Riedel,Phys. Rev. Lett. 52, 587 (1989)
Entin-Wohlman and Gefen,Europhys. Lett. 8, 477 (1989)
Altshuler, Gefen and Imry,Phys. Rev. Lett. 66, 88 (1991)

Experimental results

- Single rings (Chandrasekhar et al., 1991)
- Ensemble average (Levi et al., 1990)
  - Periodicity $\Phi_0 / 2$
  - Magnitude much larger than non-interacting value
    E_c / Φ_0 versus Δ / Φ_0 - ratio ≥100!
  - Diamagnetic response!

Electron - electron interactions !

V. Chandrasekhar et al., Phys. Rev.Lett. 67, 3578 (1991)
L. P. Levy et al., Phys. Rev.Lett. 64, 2074 (1990)

Interaction result
(1st order, renormalized coupling λ)

$\lambda = n(0)V$

$$I \sim \frac{E_c}{\Phi_0} \frac{\lambda}{1 + \lambda \ln(E_F / T)}$$

(Tolmachov; Anderson and Morel)

$\lambda > 0$ → higher order suppression

$\lambda < 0$ → $|\lambda| < 0.1$ (weak superconductor)

Result is too small by a factor of at least 5 !

Ambegaokar and Eckern, Phys. Rev. Lett. 65,381 (1990)
Ambegaokar and Eckern, Europhys. Lett. 13, 733 (1990)

summary: experiment versus theory

independent electrons: $I_{\rm exp}/I_{\rm theo} > 10^2$

interacting electrons: $I_{\rm exp}/I_{\rm theo} > 5$

- → attraction--diamagnetic
- → repulsion--paramagnetic

puzzle:

copper does not superconduct even at 10 micro-Kelwin. How could the attractive interaction in copper be so weak that it does not cause superconductivity, yet permits a large persistent current in the normal state?

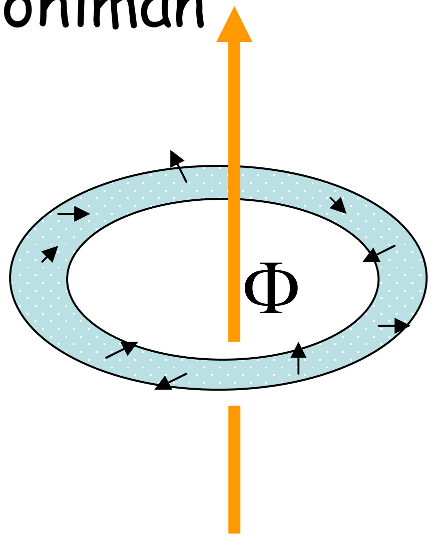
New results (2008):

Effect of pair-breaking on mesoscopic persistent currents well above T_c

with

Hamutal Bary-Soroker and Ora Entin-Wohlman

Capitalizing on recent exp results
on unwanted minute amounts of
magnetic imp's in the noble metals
(cf. Kravtsov and Altshuler, PRL 2000).

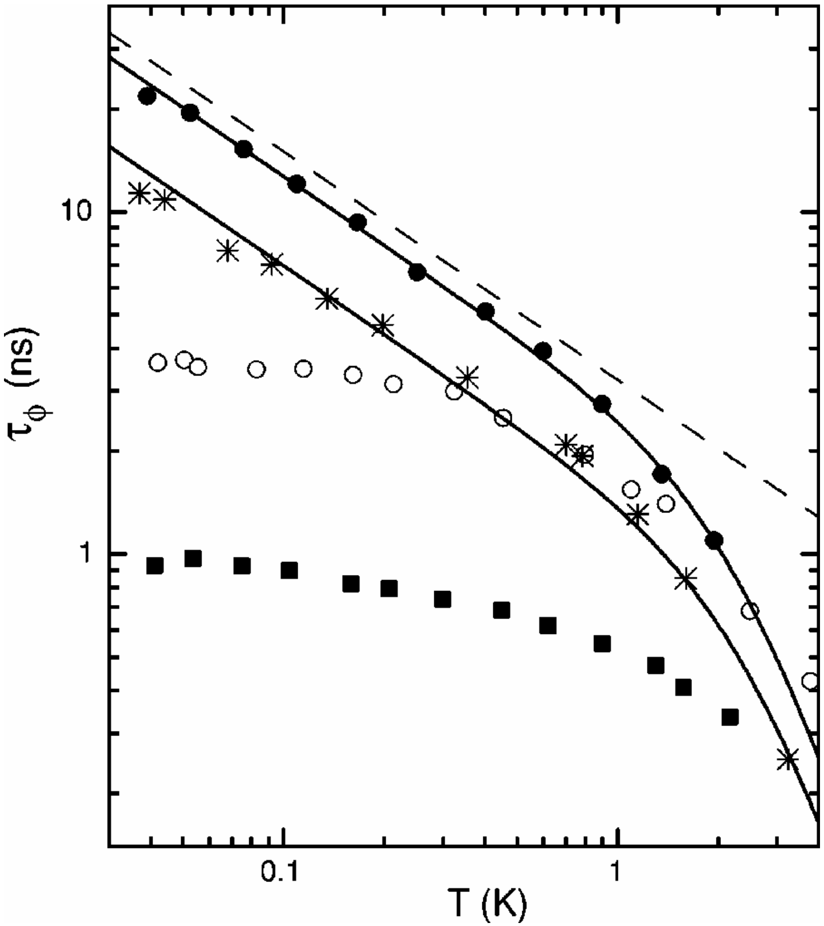


We propose a possible answer to an 18 years old puzzle

Dephasing of electrons in mesoscopic metal wires

F. Pierre,[1,2,3,*] A. B. Gougam,[1,†] A. Anthore,[2] H. Pothier,[2] D. Esteve,[2] and Norman O. Birge[1]

Apparent "saturation" of dephasing at low T



due to ~10^{-6} amount of magnetic imp's

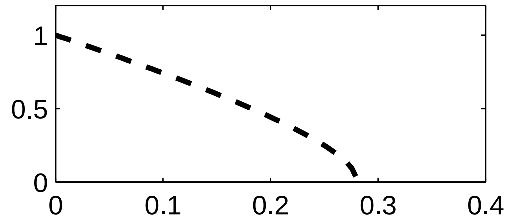
FIG. 3. Phase coherence time τ_ϕ versus temperature in wires made of copper Cu(6N)b (■), gold Au(6N) (*), and silver Ag(6N)c (●) and Ag(5N)b (○). The phase coherence time increases continuously with decreasing temperature in wires fabricated using our purest (6N) silver and gold sources as illustrated respectively with samples Ag(6N)c and Au(6N). Continuous lines are fits of the measured phase coherence time including inelastic collisions with electrons and phonons [Eq. (4)]. The dashed line is the prediction of electron–electron interactions only [Eq. (3)] for sample Ag(6N)c. In contrast, the phase coherence time increases much more slowly in samples made of copper (independently of the source material purity) and in samples made of silver using our source of lower (5N) nominal purity.

1ns corresponds to ~5mK,!

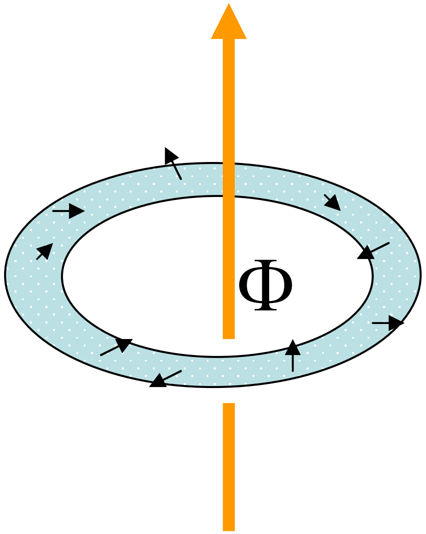
The effect of magnetic impurities

- Magnetic impurities are hard to avoid (Pierre et al. 2003)
- T_c vanishes when $1/\tau_s \geq T_c^{\,0}$ (Abrikosov and Gorkov 1961)

$T_c / T_c^{\,0}$



$$s = \frac{1}{\pi T_c^{\,0} \tau_s}$$



$T_c^{\,0}$ - the bare transition temperature

T_c - the real transition temperature

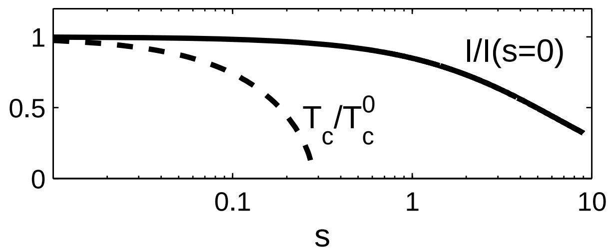
τ_s - the m.f.t due to scattering from the magnetic impurities

Hamiltonian: $H \to H + \int d\vec{r}\, u_2(\vec{r}) \psi_\alpha^\dagger(\vec{r}) \vec{S}\vec{\sigma}^{\alpha\gamma} \psi_\gamma(\vec{r})$

What is the effect of magnetic impurities on the current ?

Our new results, for $T_c^{\,0} \leq 1/\tau_s \leq E_c$

The current depends (mostly) on $T_c^{\,0}$ not T_c !



$E_c = 10T_c$

$$s = \frac{1}{\pi T_c^{\,0} \tau_s}$$

If the magnetic impurities have (nearly) no effect, what is new here ?

Ambegaokar and Eckern: $I(T_c)$ ⟹ No magnetic impurities $T_c = T_c^0$ Realistic values of Tc give a too small current.

Our result: $I(T_c^{\,0}, \tau_s)$, ⟹ Our theory can fit experiments **even though (measured) Tc is very small (or zero) in Cu.**

The Hamiltonian

$$H = \int d\vec{r} \left(\psi_\alpha^\dagger(\vec{r}) \left[\left(H_0 + u_1(\vec{r}) \right) \delta_{\alpha\gamma} + u_2(\vec{r}) \vec{S} \vec{\sigma}^{\alpha\gamma} \right] \psi_\gamma(\vec{r}) - \frac{g}{2} \psi_\alpha^\dagger(\vec{r}) \psi_\gamma^\dagger(\vec{r}) \psi_\gamma(\vec{r}) \psi_\alpha(\vec{r}) \right)$$

The formula for the persistent current, obtained by generalization of the AE derivation:

$$I = -eE_c \sum_{m=1}^{\infty} \frac{\sin(4\pi m\phi)}{m^2} \sum_{\nu} \int_0^{\infty} \frac{x \sin(2\pi x) \Psi' \left(\frac{1}{2} + \frac{|\nu| + 2/\tau_s}{4\pi T} + \frac{\pi E_c x^2}{m^2 T} \right) dx}{\ln\left(\frac{T}{T_c{}^0} \right) + \Psi\left(\frac{1}{2} + \frac{|\nu| + 2/\tau_s}{4\pi T} + \frac{\pi E_c x^2}{m^2 T} \right) - \Psi\left(\frac{1}{2} \right)}$$

The Thouless energy is $E_c = \hbar D / L^2$

$$I = (e/h)\, \partial T \ln \mathcal{Z} / \partial \phi.$$

$$\mathcal{Z} = \prod_{q,\nu} [\epsilon(q,\nu)]^{-1}, \qquad \epsilon(q,\nu) \equiv \ln\left(\frac{T}{T_c^0}\right) + \psi\left(\frac{1}{2} + \frac{Dq^2 + |\nu| + 2/\tau_s}{4\pi T}\right) - \psi\left(\frac{1}{2}\right)$$

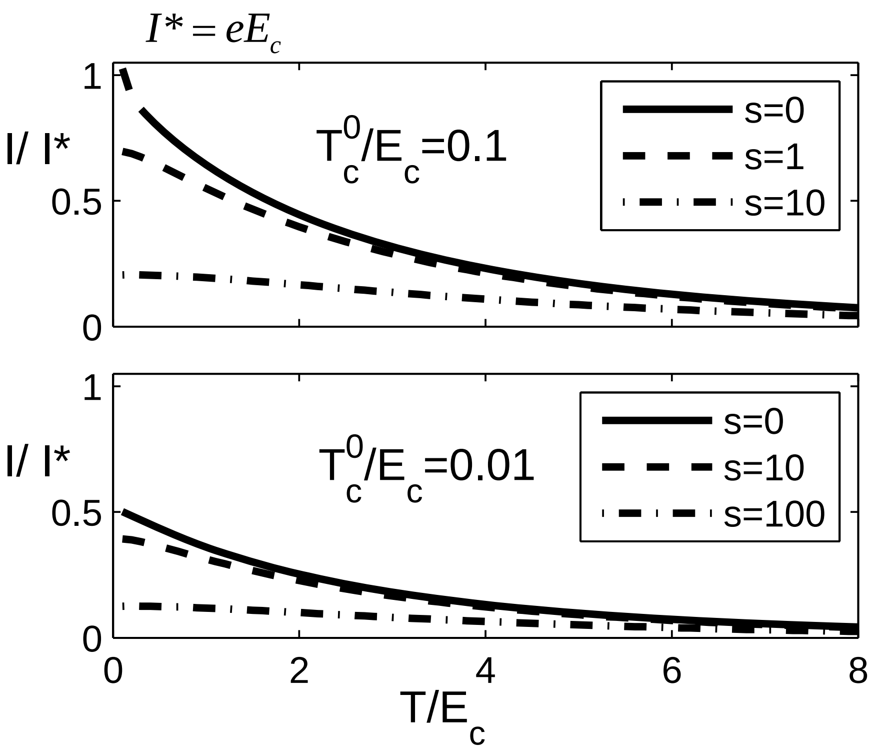
ψ = digamma function, $q = \frac{2\pi}{L}(n + 2\phi)$ $\quad \nu = 2\pi n T$

ν = 0, q =0 determine depression of Tc by magnetic imp's

(Abrikosov-Gor'kov 1961).

Spin scattering cuts-off superconducting correlations!

The effect of magnetic impurities on the current



A method to measure $T_c^{\,0}$!

Need $T_c^{\,0} \leq 1/\tau_s \leq E_c$

Levy's results correspond to

$$T_c^{\,0}{}_{(Cu)} \sim 1\ mK$$

Their E_C ~ 15mK

Need just .2 - .3 ppm mag imp's!

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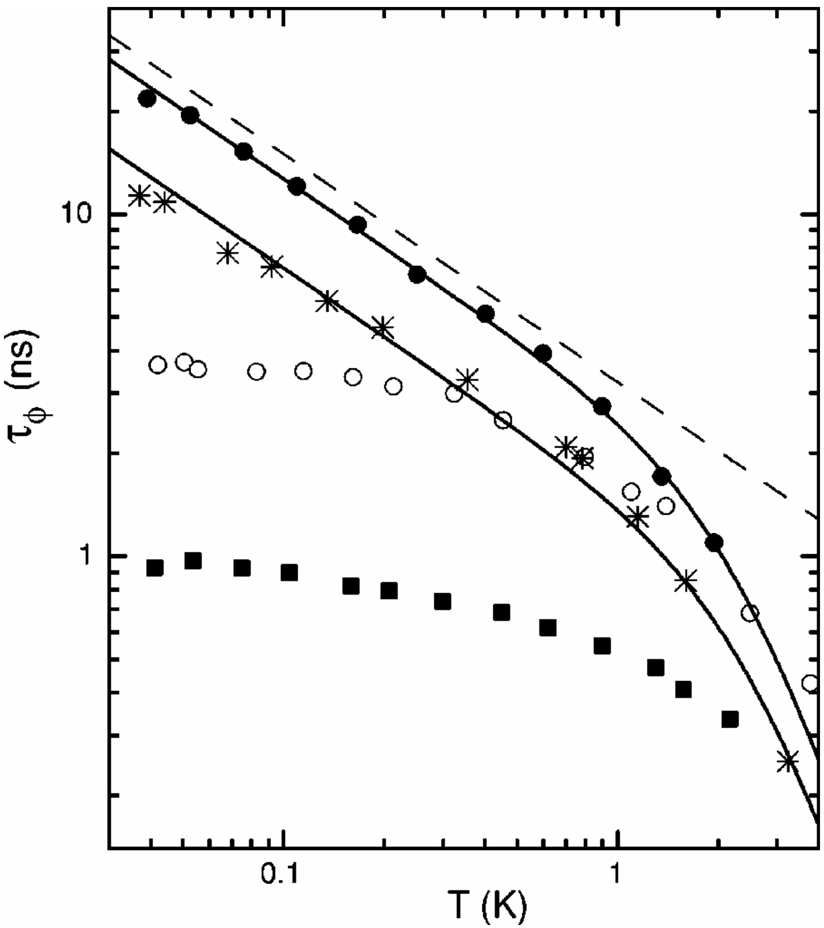

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1ns corresponds to ~5mK,!

details of the calculation

Hamiltonian:

$$\mathcal{H} = \int d\mathbf{r}\Big(\psi^\dagger_\alpha(\mathbf{r})\Big[(\mathcal{H}_0 + u_1(\mathbf{r})\delta_{\alpha\gamma} + u_2(\mathbf{r})\mathbf{S}\cdot\sigma^{\alpha\gamma}\Big]\psi_\gamma(\mathbf{r}) - \tfrac{g}{2}\psi^\dagger_\alpha(\mathbf{r})\psi^\dagger_\gamma(\mathbf{r})\psi_\gamma(\mathbf{r})\psi_\alpha(\mathbf{r})\Big)$$

partition function:

$$\mathcal{Z} = \int \mathcal{D}\Big(\Psi(\mathbf{r},\tau),\overline{\Psi}(\mathbf{r},\tau)\Big)\mathcal{D}\Big(\Delta(\mathbf{r},\tau),\Delta^*(\mathbf{r},\tau)\Big)e^{-\mathcal{S}}$$

action:

$$\mathcal{S} = \int d\mathbf{r}\int_0^\beta d\tau\Big(\frac{|\Delta(\mathbf{r},\tau)|^2}{g} - \frac{1}{2}\overline{\Psi}(\mathbf{r},\tau)G^{-1}_{\mathbf{r},\mathbf{r};\tau,\tau}\Psi(\mathbf{r},\tau)\Big)$$

inverse Green function:

$$G^{-1} = \begin{bmatrix} -\partial_\tau - h^\phi_\uparrow & -2u_2S_- & 0 & \Delta \\ -2u_2S_+ & -\partial_\tau - h^\phi_\downarrow & -\Delta & 0 \\ 0 & -\Delta^* & -\partial_\tau + h^{-\phi}_\uparrow & 2u_2S_+ \\ \Delta^* & 0 & 2u_2S_- & -\partial_\tau + h^{-\phi}_\downarrow \end{bmatrix}$$

details of the calculation (cont.)

partition function:

$$\mathcal{Z} = \prod_{\mathbf{q}\nu} \left(\frac{1}{\lambda} - \Pi(\mathbf{q}, \nu) \right)^{-1}$$

$$\lambda = \frac{V}{g\mathcal{N}(0)}$$

$$\Pi(\mathbf{q}, \nu) = \Psi\left(\frac{1}{2} + \frac{2\omega_{\mathrm{D}} + |\nu| + Dq^2}{4\pi k_{\mathrm{B}} T} \right) - \Psi\left(\frac{1}{2} + \frac{|\nu| + Dq^2 + 2/\tau_{\mathrm{S}}}{4\pi k_{\mathrm{B}} T} \right)$$

spin-flip time:

$$\frac{1}{\tau_{\mathrm{S}}} = 2\pi \mathcal{N} N_i S(S+1) u_2^2$$

bare transition temperature is found from the pole of the partition function at zero wave vector and zero frequency:

$$\frac{1}{\lambda} = \Psi\left(\frac{1}{2} + \frac{\omega_{\mathrm{D}}}{2\pi k_{\mathrm{B}} T_{c0}} \right) - \Psi\left(\frac{1}{2} \right)$$

details of the calculation (cont.)

the persistent current is found from the partition function:

$$I = \frac{e}{h}\frac{\partial k_B T \ln Z}{\partial \Phi}$$

$$q_{\parallel} = \frac{2\pi}{L}\left(n + 2\frac{\Phi}{\Phi_0}\right)$$

leading to the result:

$$I = -8\frac{eE_c}{h}\sum_{m=1}^{\infty}\frac{\sin(4\pi m\Phi/\Phi_0)}{m^2}$$

$$\times \sum_{\nu}\int_0^{\infty} dx \frac{x\sin(2\pi x)\Psi'(F(x,\nu))}{\ln(T/T_{c0})+\Psi(F(x,\nu))-\Psi(1/2)}$$

$$F(x,\nu) = \frac{1}{2} + \frac{|\nu|+2/\tau_S}{4\pi k_B T} + \frac{\pi E_c x^2}{m^2 k_B T}$$

Renormalization of Interaction

$$\lambda_{\text{eff}} = \frac{\lambda}{1 + \lambda \ln(E_> / E_<)}$$

Competition of reduced Coulomb and attractive Frohlich at the Debye scale determines if there the interaction is attractive. **If so, λ_{eff} diverges at Tc.**

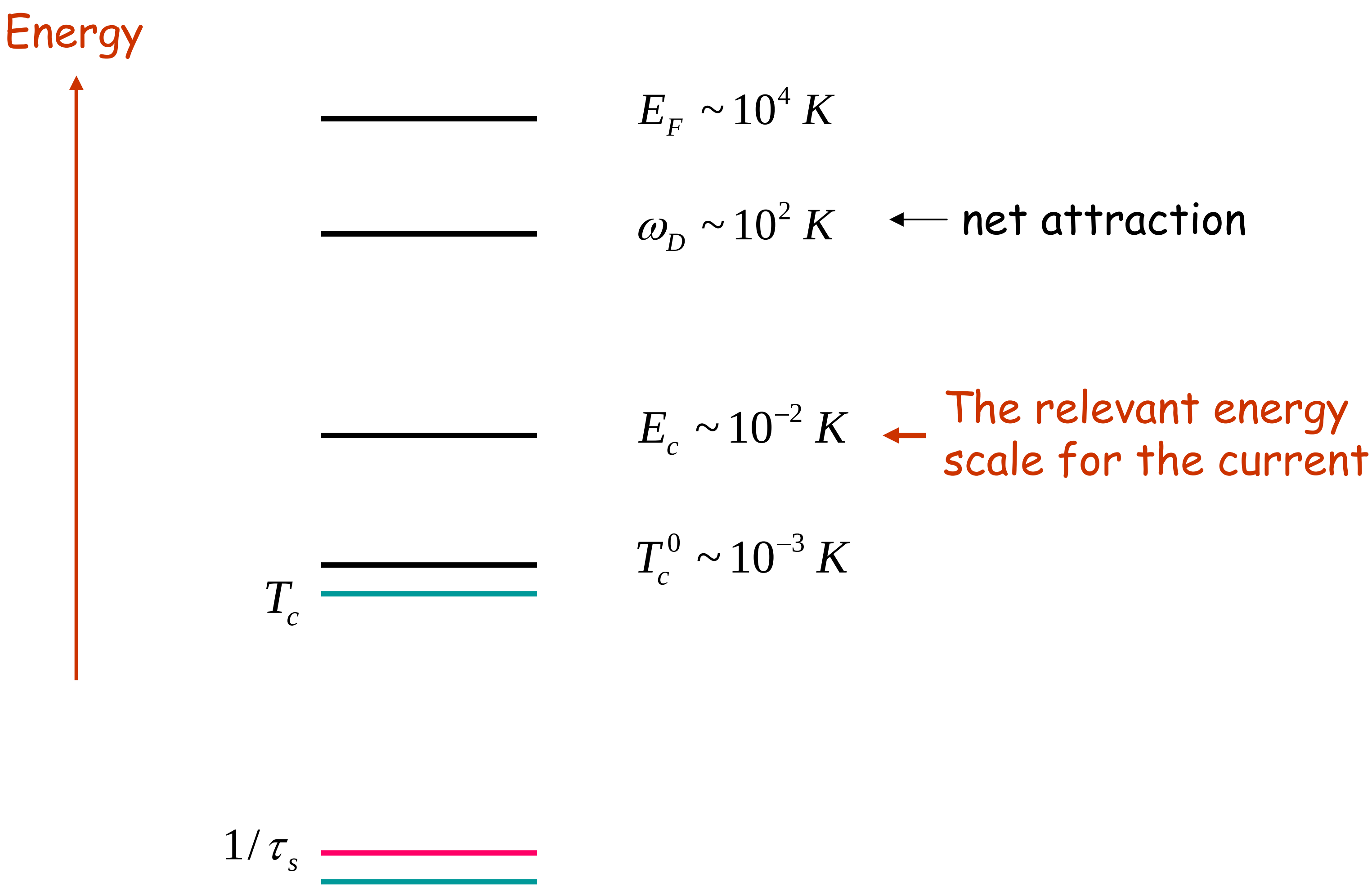
Persistent current **is determined on the Thouless scale**.

Pair breakers eliminate Tc, by stopping the renormalization above it, but persistent current is hardly affected, **if**

$$T_c^{\ 0} \le 1/\tau_s \le E_c$$

Why does Tc decay but the current doesn't ?

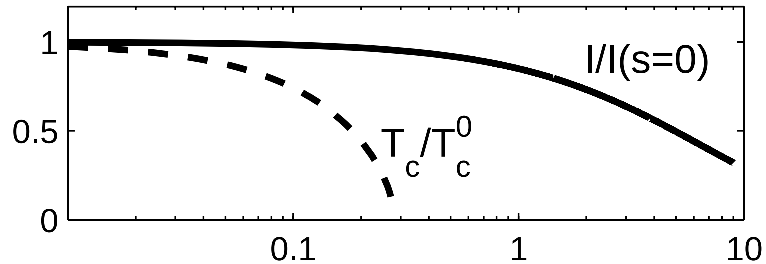
renormalization



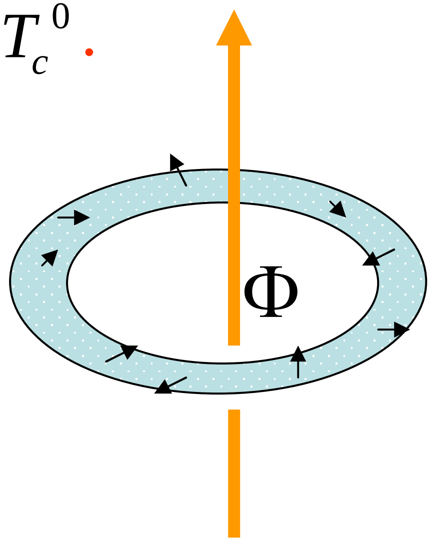
Conclusions

- We can explain the large current observed in experiments **assuming** $T_c^0 \sim 1mK$ for Cu. $T_c^0 \le 1/\tau_s \le E_c$

- Measuring the PC is a tool to determine T_c^0.



$$s = \frac{1}{\pi T_c^0 \tau_s}$$



Cond-mat: arXiv:0804.0702 April 2008, and PRL, **101**, 057001 (2008). Review (H. Bouchiat): Physics 1, 7 (2008) .

Thank you!