Compact dimensions and the Casimir effect: the Proca connection

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Casimir Proca problem in the early 80's

Photon has a small mass *m:* Proca equation

Photon has 3 polarizations. As *m->0,* one still maintains 3 polarizations.

Does one recover Casmir's parallel plate result as m->0?

1981: paper obtains (3/2) times Casimir's result as *m->0.*

Work of G. Barton and N. Dombey (Nature,1984)

Three polarizations in presence of conducting parallel plates=
2 discrete (non-penetrating) modes + 1 continuum(penetrating) mode

2 discrete (non-penetrating) modes ↓

yields Casimir's result as m->0

m-dependent corrections

1 continuum(penetrating) mode ↓

thickness dependent correction

vanishes in the infinitely thin limit

Same issue 25 years later in a different context

$\Rightarrow$ extra dimension compactified on a circle of radius R

$M^4 \times S^1$ spacetime: photon has 3 polarizations

Photon maintains 3 polarizations as R->0.

Does one recover Casimir's parallel plate result as R->0?

Recent literature: three discrete modes were used to calculate Casimir force. (3/2) times Casimir's result as R->0.

Simple illustration: 2+1 dimensions with one dimension compactified to a circle

Kaluza Klein decomposition of 5D Maxwell into 4D massless and massive sector

$$S = \int d^4x \int dx_4 \left(-\frac{1}{4} F_{AB} F^{AB} \right)$$
$$= \int d^4x \int dx_4 \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F_{\mu 4} F^{\mu 4} \right)$$

A,B= 5D indices (0,1,2,3,4) μ, ν = 4D indices (0,1,2,3)

periodic boundary conditions along $x_4 = R\phi$: $A_\mu(x^\mu, \phi) = A_\mu^{(0)}(x) + \sum_{n=1}^{\infty} (A_\mu^{(n)}(x) e^{in\phi} + \text{h.c.}).$

one cannot go to axial gauge $A_4 = 0$ -> “almost” axial gauge where $A_4 = A_4^{(0)}(x)$

$$S = 2\pi R \int d^4x \left\{ -\frac{1}{4} F^{(0)}_{\mu\nu} F^{\mu\nu(0)} + \frac{1}{2} (\partial_\mu A_4^{(0)})^2 \right.$$

→ 4D Maxwell + extra 4D scalar field

$$\left. + 2 \sum_{n=1}^{\infty} \left[-\frac{1}{4} |\partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)}|^2 + \frac{1}{2} \frac{n^2}{R^2} |A_\mu^{(n)}|^2 \right] \right\}$$

→ 4D Proca fields, masses $m_n^2 = n^2/R^2$

note that both sectors have three “polarizations” or degrees of freedom

Mode decomposition in the presence of conducting plates

4D massless sector: radiation gauge

$$\left.\begin{aligned} A_0 &= 0 \\ A_1 &= c_1 \sin(k_z\, z)\, e^{i(\mathbf{k}.\mathbf{x}-\omega t)} \\ A_2 &= c_2 \sin(k_z\, z)\, e^{i(\mathbf{k}.\mathbf{x}-\omega t)} \\ A_3 &= c_3 \cos(k_z\, z)\, e^{i(\mathbf{k}.\mathbf{x}-\omega t)} \\ A_4 &= c_4\, e^{i(\mathbf{k}.\mathbf{x}+p_z\, z-\omega' t)} \end{aligned}\right\} \quad \begin{aligned} & k_z = \frac{n\,\pi}{a} \quad n = 1,2,3... \\ & \mathrm{k} = (k_x, k_y) \\ & \mathrm{x} = (x, y) \\ & \omega^2 = \mathrm{k}^2 + k_z^2 = \mathrm{k}^2 + \frac{n^2\,\pi^2}{a^2} \end{aligned}$$

2 independent discrete modes
(yields Casimir's result
$P_0 = -\pi^2/(240\, a^4)$)

1 continuum mode: scalar field
(no contribution)

4D Proca sector: discrete modes similar to Barton and Dombey ("m^2" replaced by $m_n^2 = n^2/R^2$)

$$\left.\begin{aligned} A_0 &= c_0 \sin(k_z\, z)\, e^{i(\mathbf{k}.\mathbf{x}-\omega t)} \\ A_1 &= c_1 \sin(k_z\, z)\, e^{i(\mathbf{k}.\mathbf{x}-\omega t)} \\ A_2 &= c_2 \sin(k_z\, z)\, e^{i(\mathbf{k}.\mathbf{x}-\omega t)} \\ A_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} & k_z = \frac{\ell\,\pi}{a} \quad (\ell = 1,2,..) \\ & \mathrm{k} = (k_x, k_y)\ ;\ \mathrm{x} = (x, y) \\ & \omega^2 = \mathrm{k}^2 + k_z^2 + m_n^2 \end{aligned}$$

2 independent discrete modes
(yields R-dependent correction)

there is also a continuum mode
(not shown here)

R-dependent corrections from Proca discrete modes

$$\omega = \sqrt{\mathbf{k}^2 + \frac{\ell^2\,\pi^2}{a^2} + n^2/R^2}$$

effect of compact dimension

$E_{proca} = 4\ E_D$

Derive manifestly negative formula for E_D in 4+1 dimensions including exterior

$$E_D = -\frac{1}{2\pi}\sum_{kmn=1}^{\infty}\sum_{\ell=1}^{\infty}\frac{\lambda_{kmn}}{\ell}\,K_1(2\,\ell\,\lambda_{kmn}\,a)\ \text{with}\ \lambda_{kmn} = \sqrt{(\pi k/b)^2 + (\pi m/c)^2 + (n/R)^2}$$

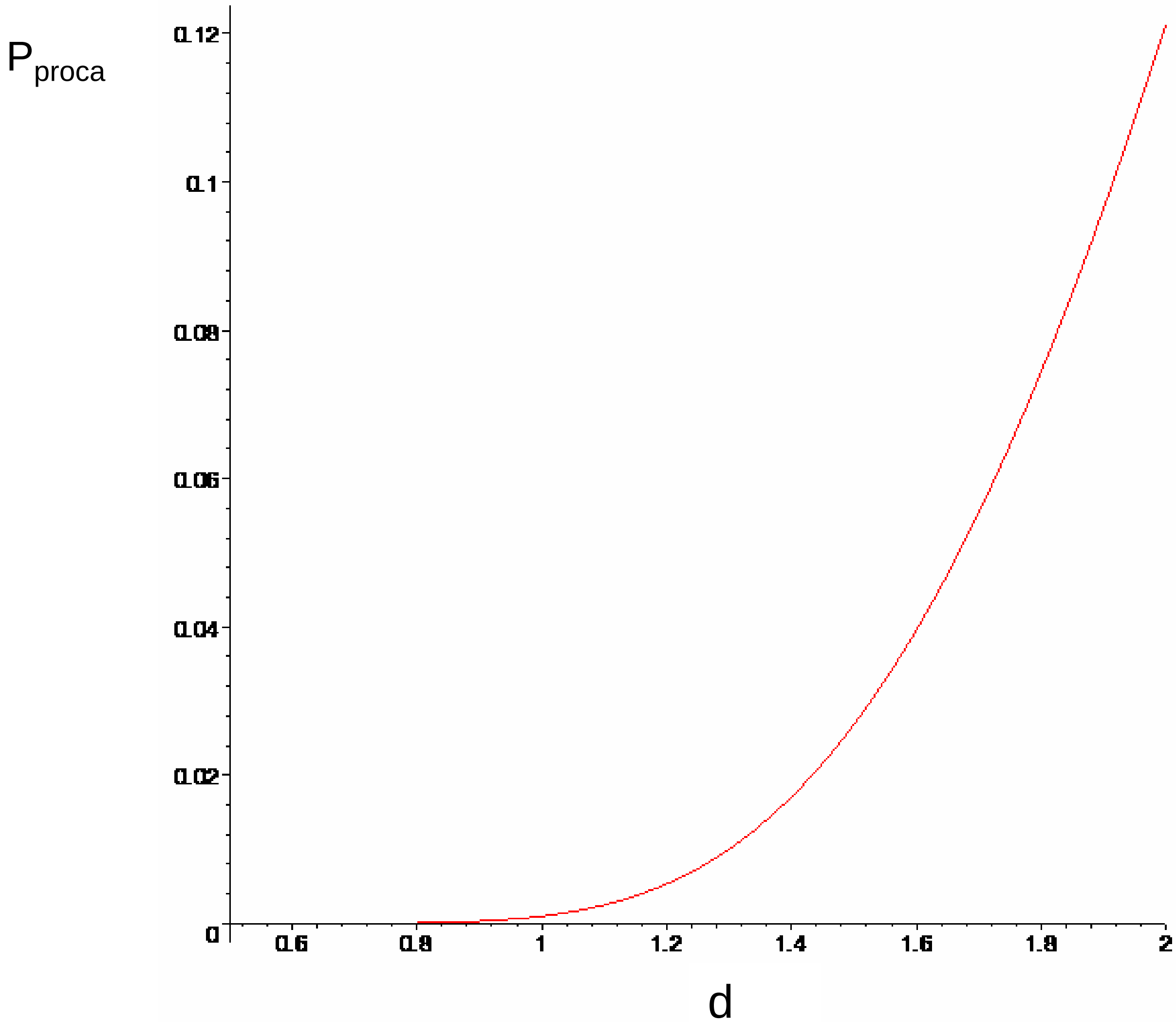
$$P_{proca} = \lim_{b,c\to\infty}\frac{F_{proca}}{b\,c} = -\sum_{n=1}^{\infty}\sum_{\ell=1}^{\infty}\left[\,\frac{3}{2}\frac{n^2}{\pi^2\,R^2\,a^2\,\ell^2}\,K_2(2\,\ell\,n\,a/R) + \frac{n^3}{\ell\,\pi^2\,R^3\,a}\,K_1(2\,\ell\,n\,a/R)\right]$$

P_{proca} is manifestly negative and increases the Casimir pressure.

Correction tends to zero exponentially as R->0. We recover Casimir's result.

Plot of correction P_{proca} as a function of circumference $d=2\pi R$

P_{proca} in units of Casimir's original result ; *d* in units of plate separation *a*



Role of exterior region when compact dimensions are present

When no compact dimensions are present, the exterior region can be neglected in the parallel plate case (as it makes no contribution).

When compact dimensions are present, the exterior plays an important role.

Example: for a=1 and d=2, $P_{proca} = -0.00499002227$

The exterior and interior contribution to this result is:

$P_{exterior} = -0.00984963256$

$P_{interior} = 0.00485961028$

The exterior has a larger magnitude than the interior.

Though $P_{interior}$ is positive, the total P_{proca} is negative.