Precise Measurements of the Casimir Force using Microelectromechanical Torsional Oscillators

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Collaborators

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Funding

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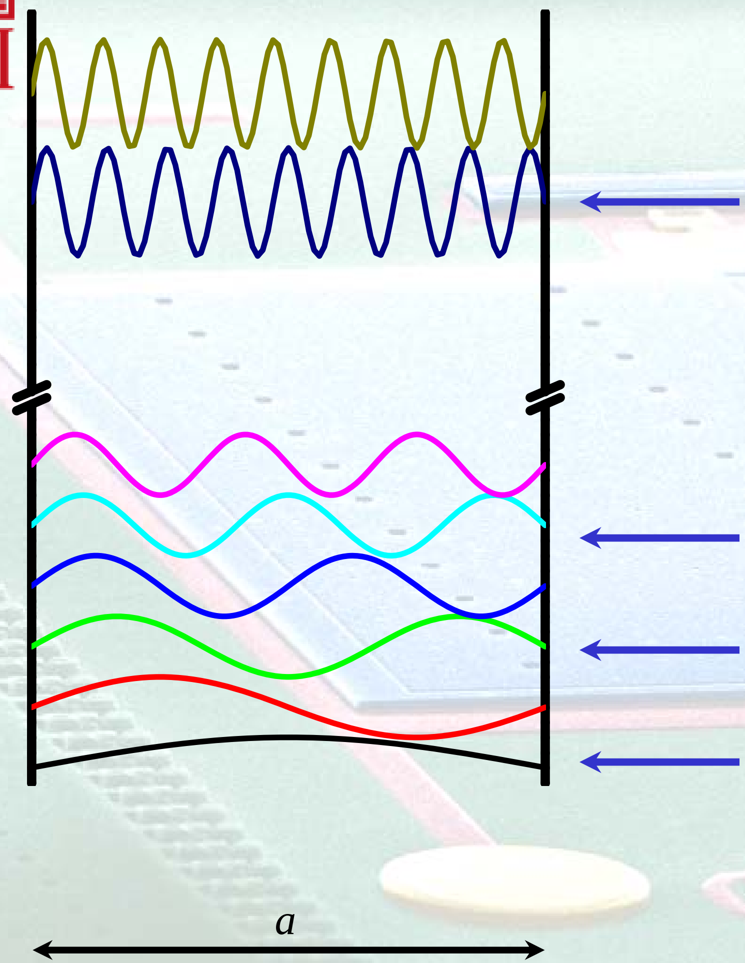
$$<0|\mathbf{E}|0> = <0|\mathbf{B}|0> = 0$$

$$½<0|(\mathbf{E}^2+\mathbf{B}^2|0> = <0|\mathbf{E}^2|0> = \frac{1}{2\Omega}\sum_{\omega,\mathbf{k}}\omega_k$$

$$P_C = -\frac{\pi\hbar c}{480a^4}$$



Vacuum fluctuations plus boundaries

Same result when using the Lifshitz approach

- Dominant electronic force at small (~ 1 nm) separations
- Non-retarded: van der Waals
- Retarded: Casimir

No mode restriction on the outside

Importance of the Casimir effect

- Consequences in nanotechnology (MEMS and NEMS)
  - "Long-range" interaction between moving parts
  - Possibility of controlling the interaction by engineering materials

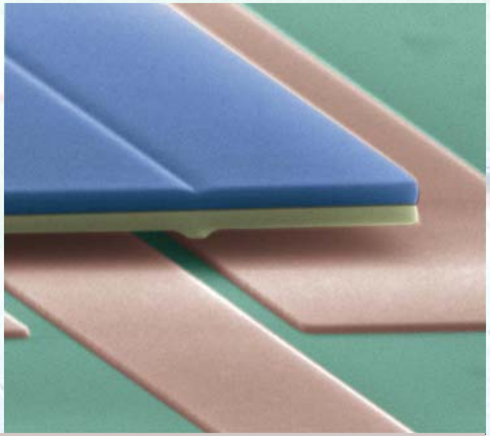
- Consequences in quantum field theory
  - Thermal dependence

- Consequences in gravitation and cosmology
  - Background to measure deviations from Newtonian potential at small separations

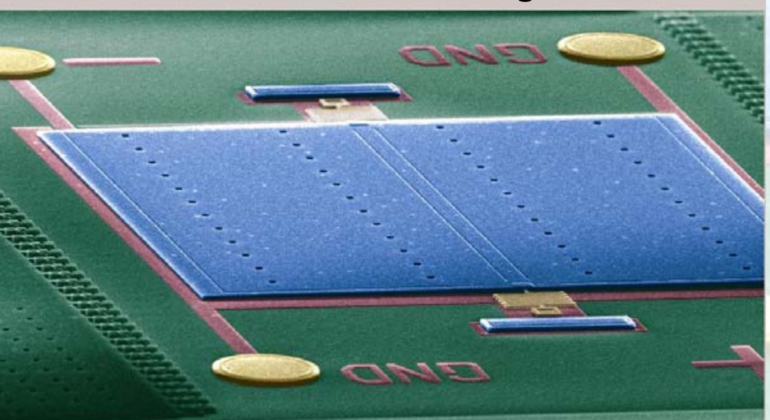
Outline

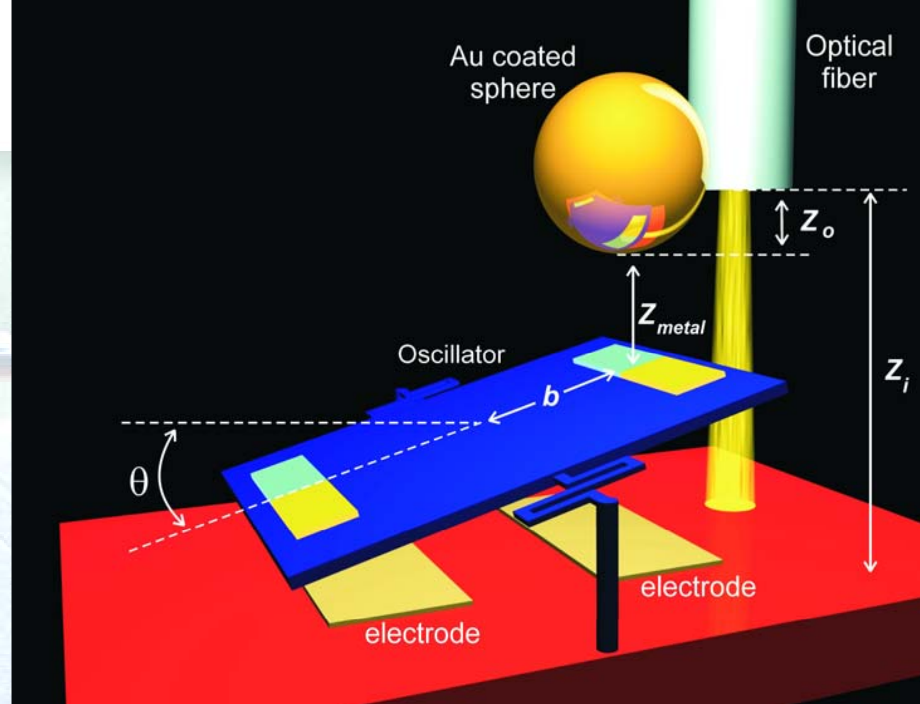
- Precise measurements of the Casimir force
  - -Experimental setup
  - -Minimum detectable force
  - -Position separation
  - -Calibrations
  - -Error budget
  - -Comparison with theory
- Low temperature measurements
  - -Experimental setup
  - -Incomplete results
- Proximity force approximation
  - -An experimentalist view of the PFA
- Summary

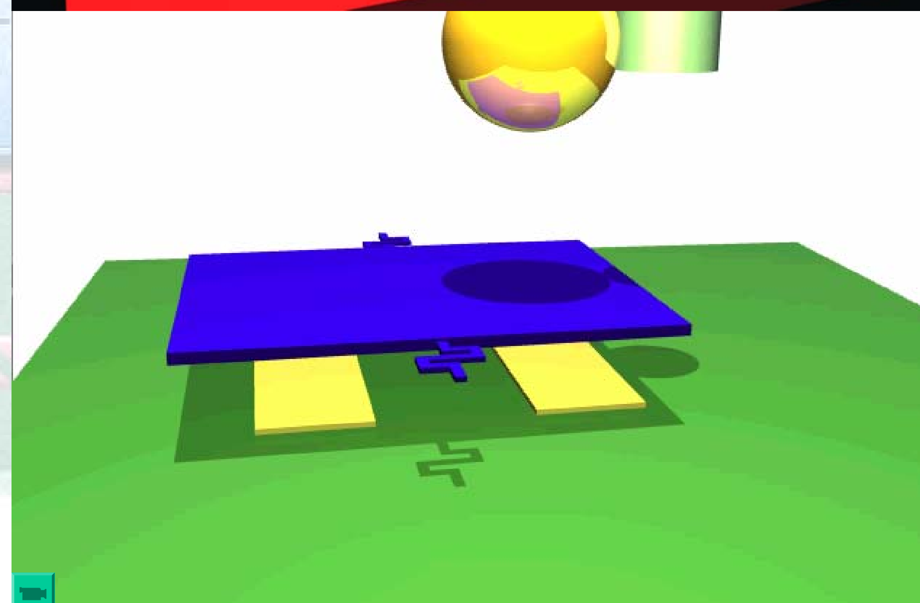
Experimental setup



$$z_{metal} = z_i - z_o - z_g - b\Theta$$

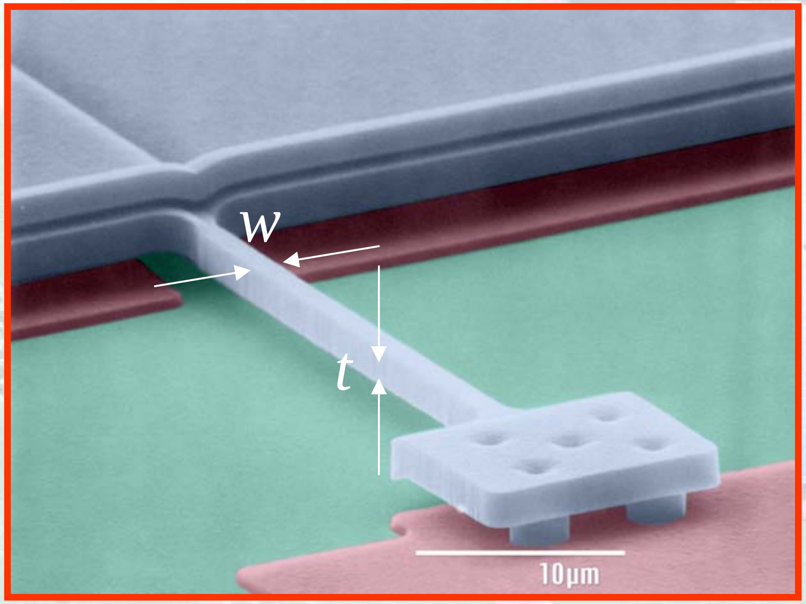




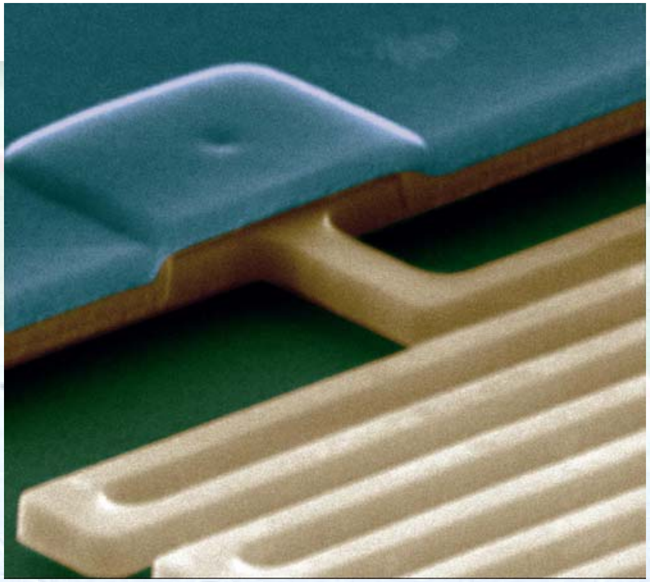


Experimental setup

$$\delta F = \sqrt{\frac{(2k_B T\kappa)}{\pi f_r Q}}\frac{1}{b}$$



w, t = 2µm

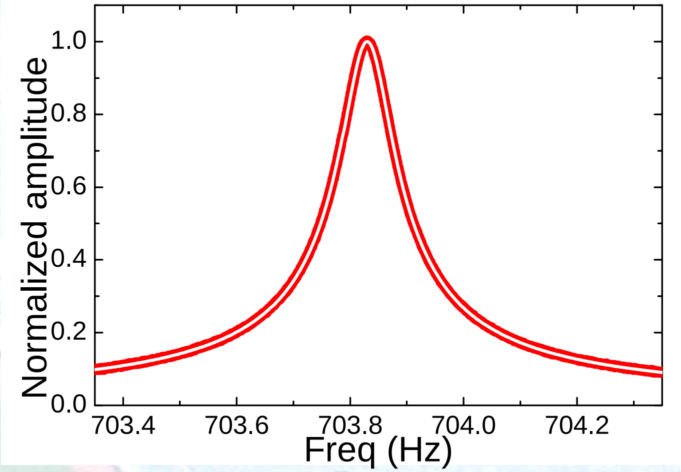


$$\kappa_s = \frac{wt^3 E_{Si}}{6L_{serp}} \sim 9\times 10^{-10}\ \frac{Nm}{rad}$$

$$\kappa_r \approx \frac{2w^3 tE_{Si}}{3L}$$

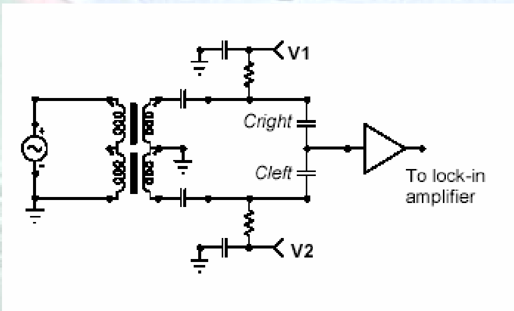
$$\kappa_s \approx \kappa_r / 40$$

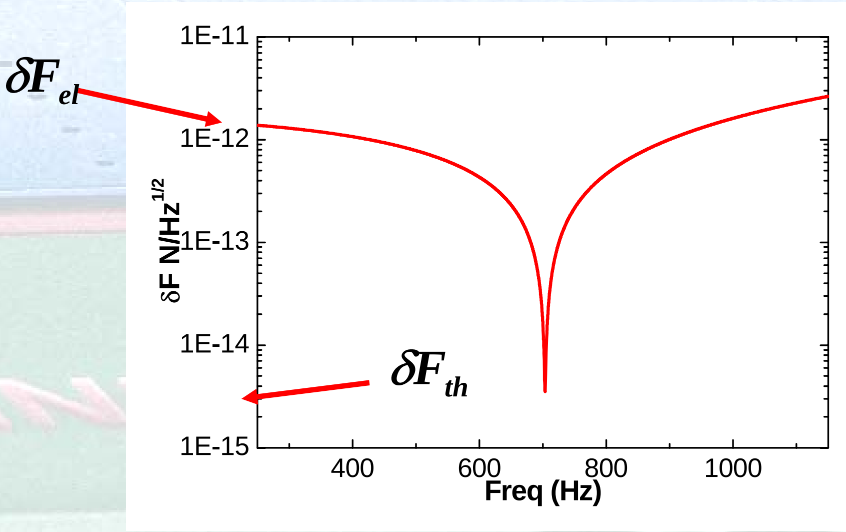
Minimum detectable force



$$\delta F = \frac{1}{b}\sqrt{\left(\frac{4k_B T\kappa}{\omega_o Q}\right)+\left(\frac{\delta\theta_{el}\kappa}{A(\omega)}\right)^2}$$

$$= \frac{1}{b}\sqrt{\delta F_{th}^{\ 2}+\delta F_{el}^{\ 2}} \sim 4\ fN/\sqrt{Hz}$$



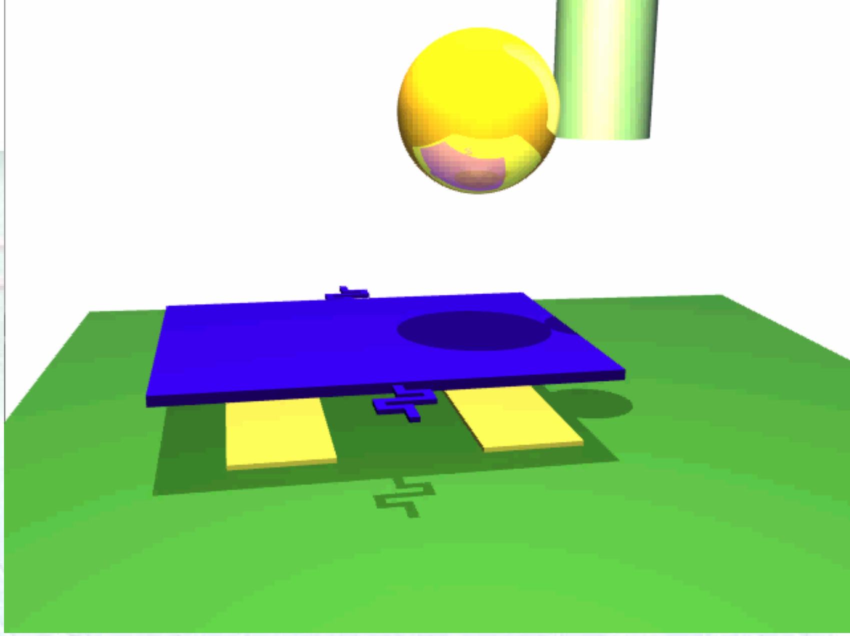


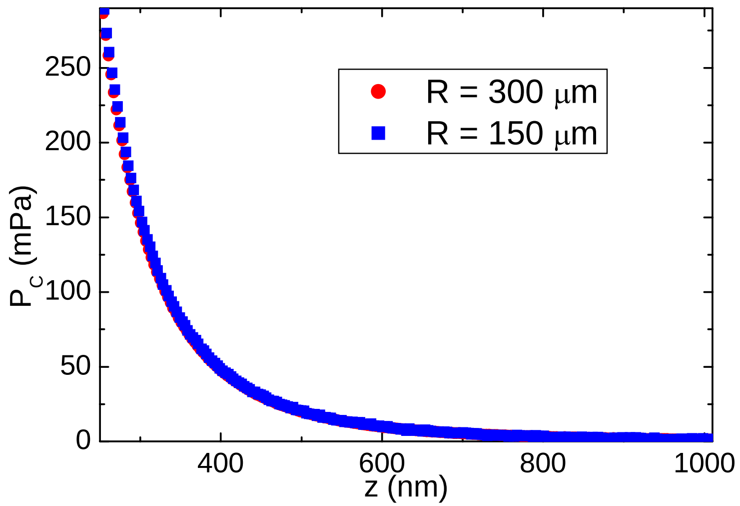
$$\delta\theta \approx 10^{-9}\ \mathrm{rad}/\sqrt{\mathrm{Hz}}$$

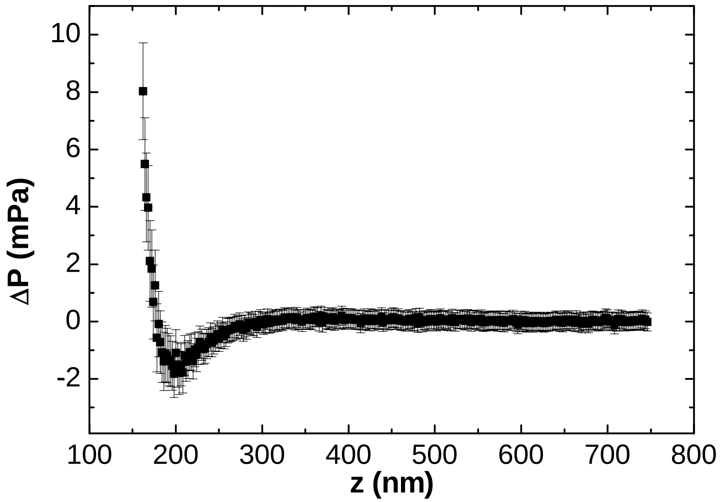
Dynamic measurements

$$\omega_r^2 = \omega_o^2 \left(1 - \frac{b^2}{I\omega_o^2} \frac{\partial F_C}{\partial z} \right)$$

$$F_C = 2\pi R \times E_C \Rightarrow \frac{\partial F_C}{\partial z} = 2\pi R \times P_C$$







Separation measurement

z_g = (2172.8 ± 0.1) nm, interferometer

z_i = ~(12000.0 ± 0.2) absolute interferometer

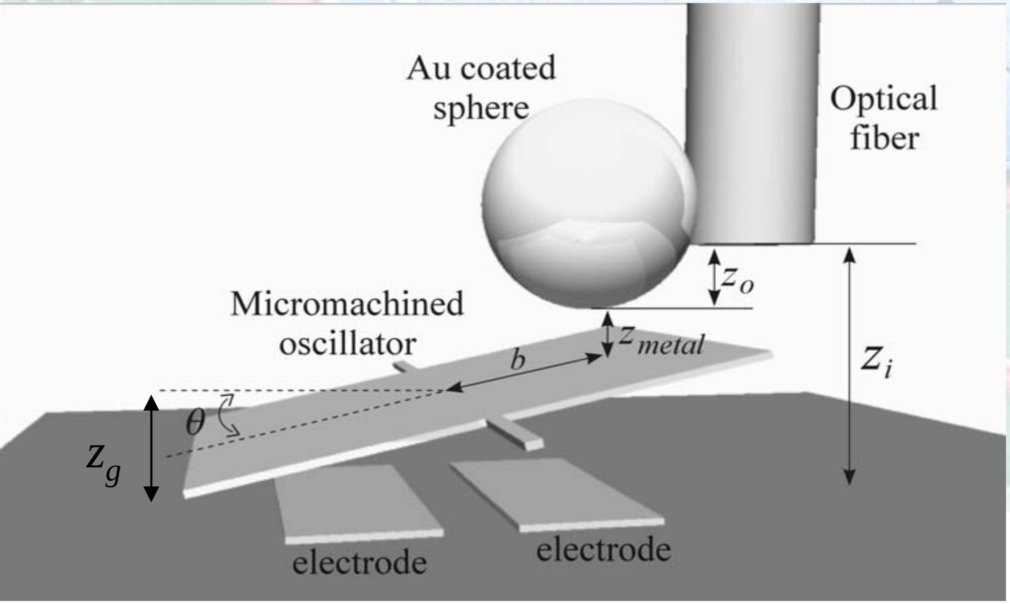
z_o is determined using a known interaction

z_o = (8162.3 ± 0.5) nm, electrostatic calibration

z_i, Θ are measured for each position

b = (207 ± 2) μm, optical microscope

Θ = ~(1.000 ± 0.001) μrad



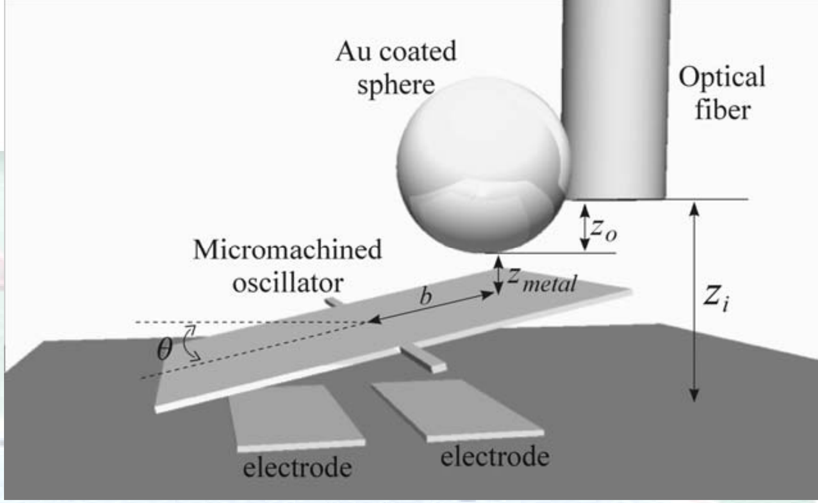
Separation measurement

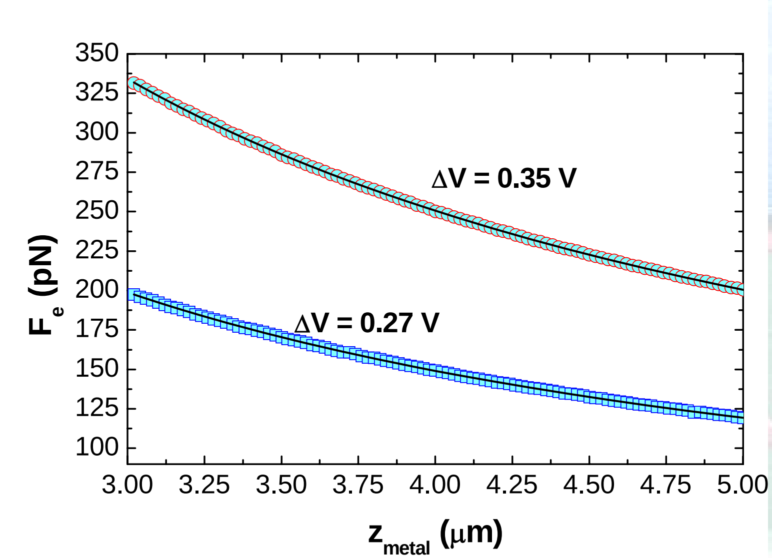
Electrostatic force calibration

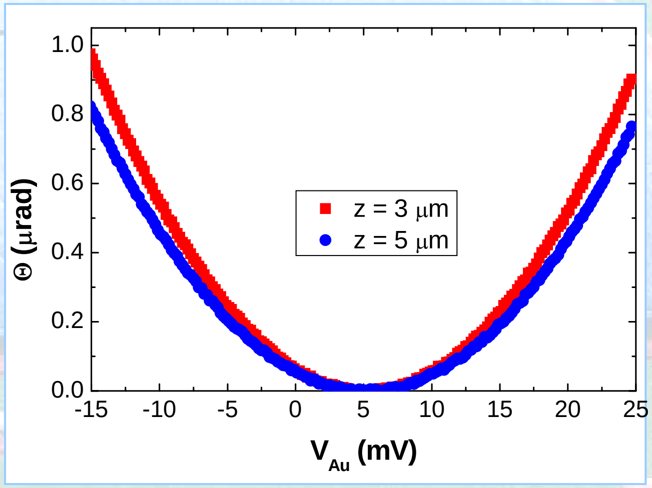
$$F_e = -2\pi\varepsilon_o (V - V_{Au})^2 \sum_n \frac{\coth u - n\coth nu}{\sinh nu} \sim$$

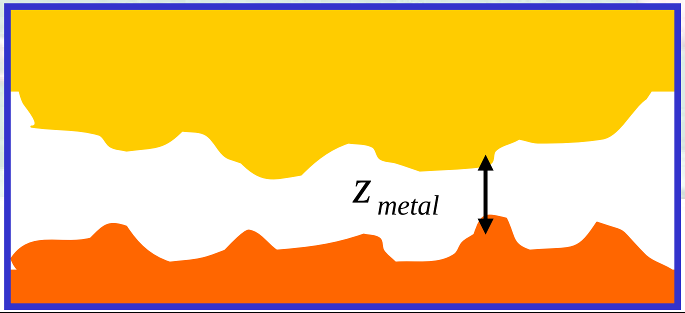
$$-2\pi\varepsilon_o (V - V_{Au})^2 \sum_{i=0} A_i \left(\frac{R}{(z_{metal} + \bar{z})} \right)^{1-i}$$

$$u = 1 + \frac{(z_{metal} + \bar{z})}{R}$$



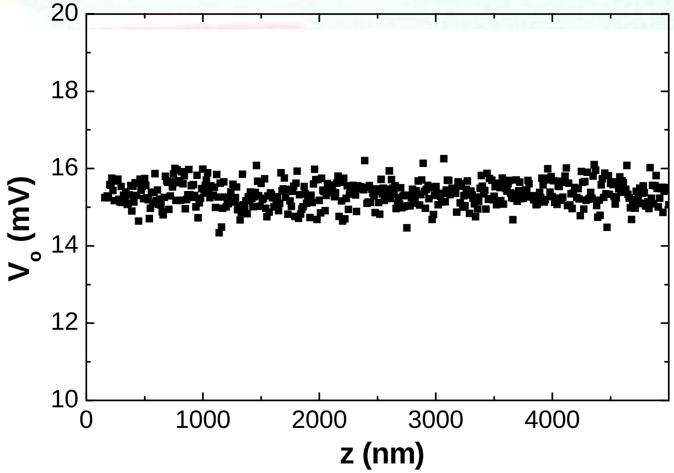




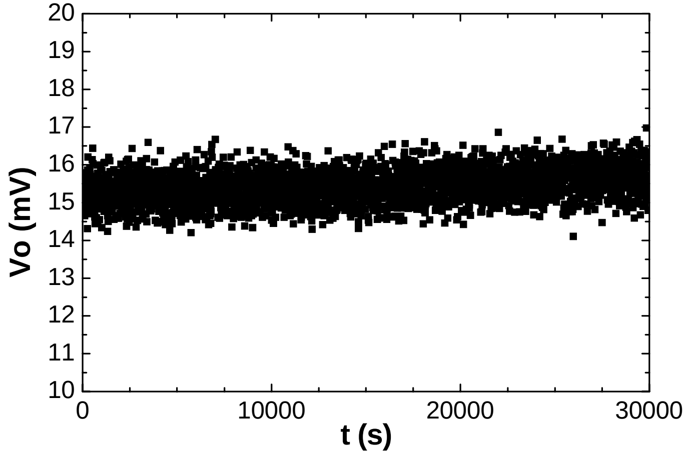


Separation measurement

Electrostatic force calibration



… and time



V_o **must** be constant as a function of separation…

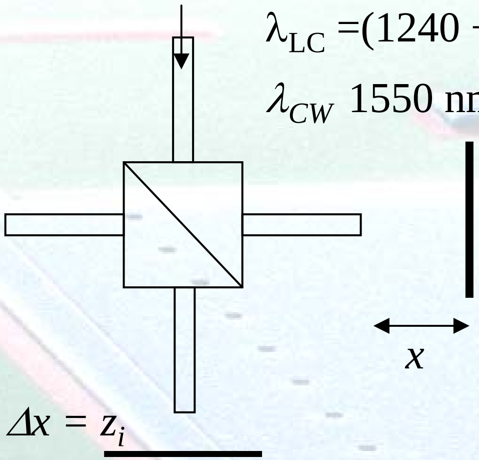
Otherwise, V_o needs to be determined at each point

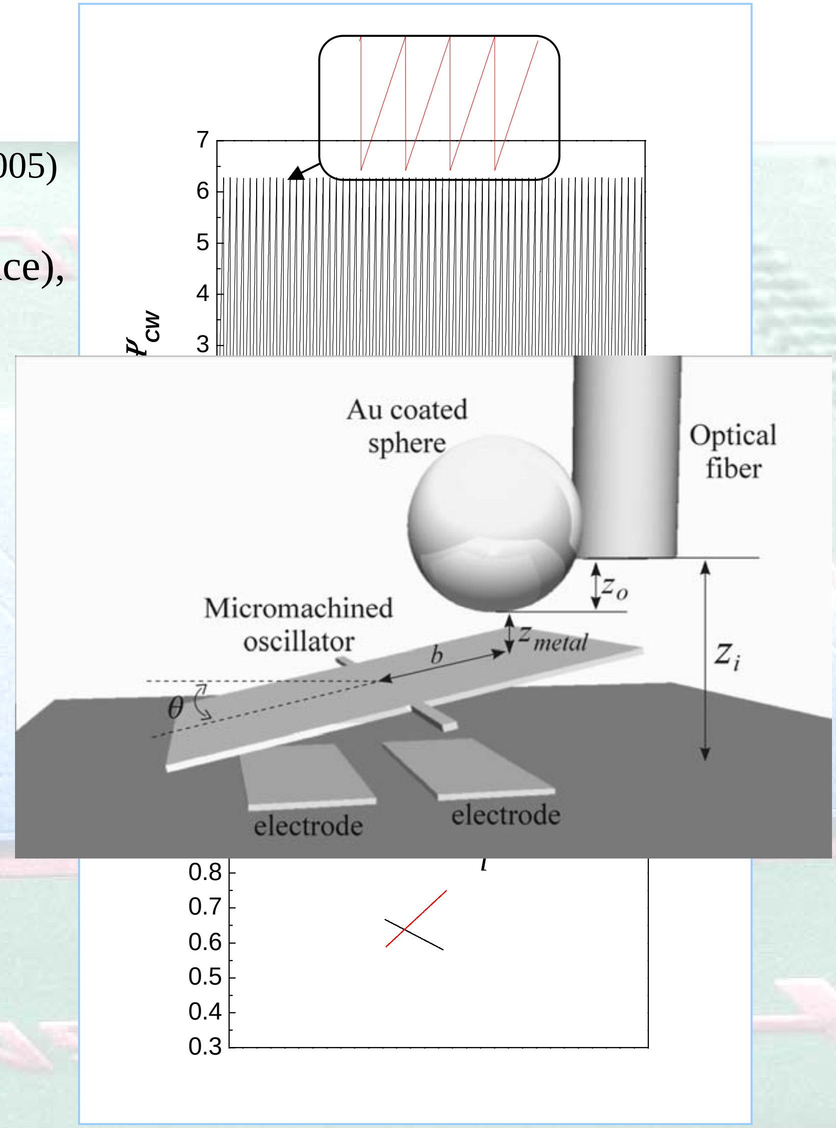
Distance measurement

Interferometer (Yang *et al.*, Opt. Lett. **27**, 77 (2005))

λ_{LC} =(1240 +/- Δ) nm (low coherence),

λ_{CW} 1550 nm (high coherence) in





$$\psi_D = \psi_{LC} - \tfrac{4}{5}\psi_{CW}$$

$$z_i = \frac{\lambda_{CW}}{4}\left[\mathrm{int}\left(S_{fringe}\right) + S_{phase}\right]$$

$$S_{phase} = \left(\psi_D(x_1) - \psi_D(x_2)\right)(\mathrm{mod}\, 2\pi)$$

-Phases obtained doing a Hilbert transform of the amplitude

-Changes in Δ (about 2 nm) give different curves.

Intersections provide Δx

-Quite insensitive to jitter. Only $2\Delta\Delta x'/(\lambda_{CW})^2$

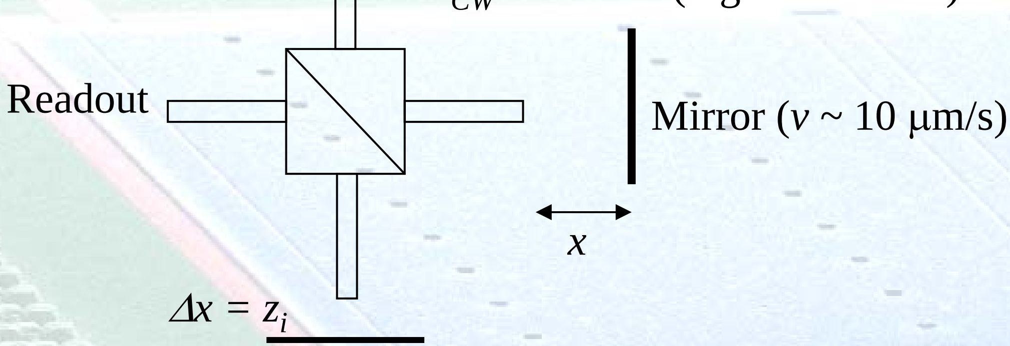
Instead of $2\Delta x'/\lambda_{CW}$

Distance measurement

Interferometer (Yang *et al.*, Opt. Lett. **27**, 77 (2005)

λ_{LC} =(1240 +/- Δ) nm (low coherence),

λ_{CW} 1550 nm (high coherence) in



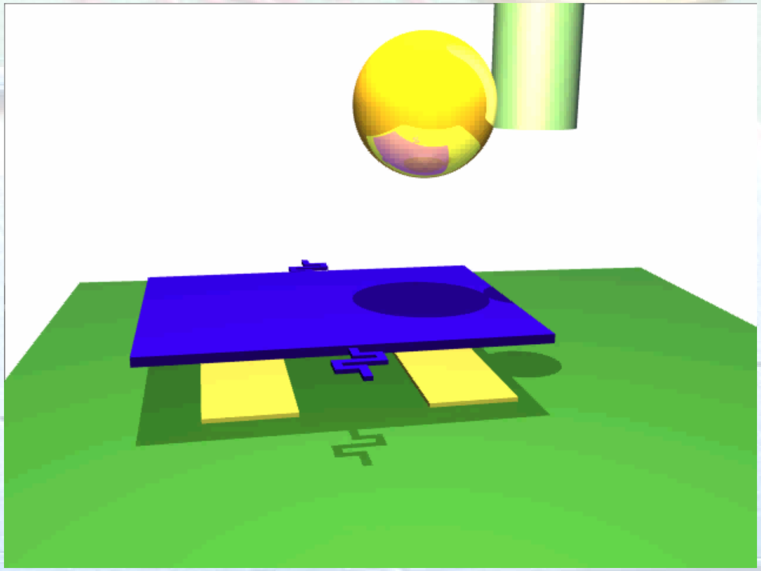
-Problems in lack of parallelism (curvature of wavefronts) are compensated when subtracting the two phases

-Gouy phase effect is ~ $\phi_G(\omega) \cong \arctan\left(\frac{\Delta f}{NA \cdot f}\right)$, and gives an error much smaller than the random one

Pressure determination

Three effects:

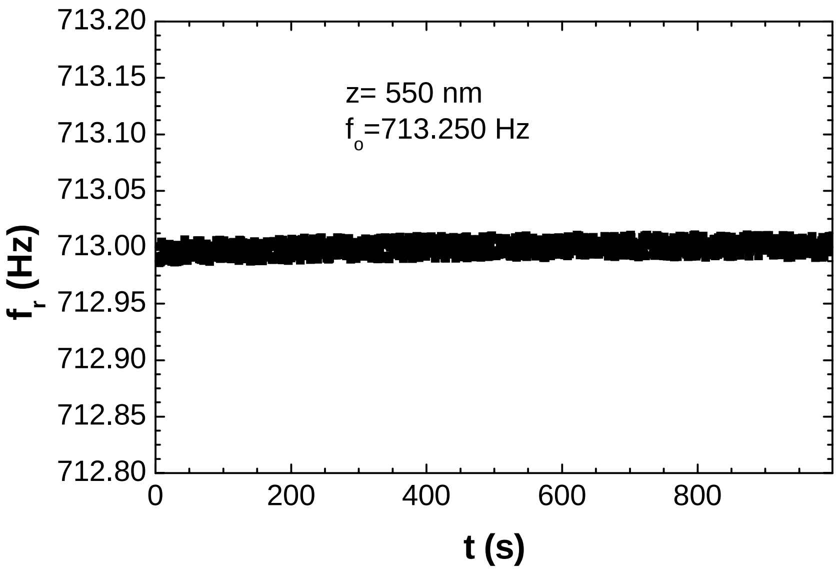
-New equilibrium position
-Softer spring
-Non-linear effects



Pressure determination

$$\omega_r^2 = \omega_o^2\left(1 - \frac{b^2}{I\omega_o^2}\frac{\partial F_C}{\partial z}\right)$$

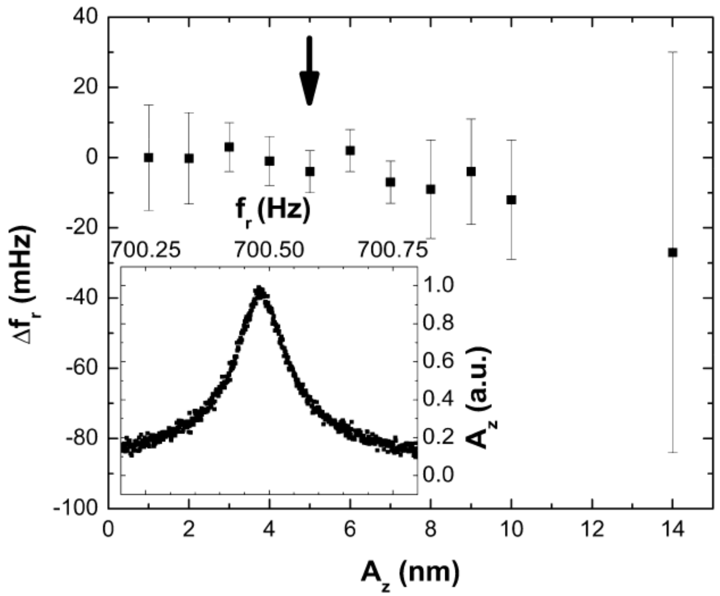
$$F_C = 2\pi R \times E_C \Rightarrow \frac{\partial F_C}{\partial z} = 2\pi R \times P_C$$



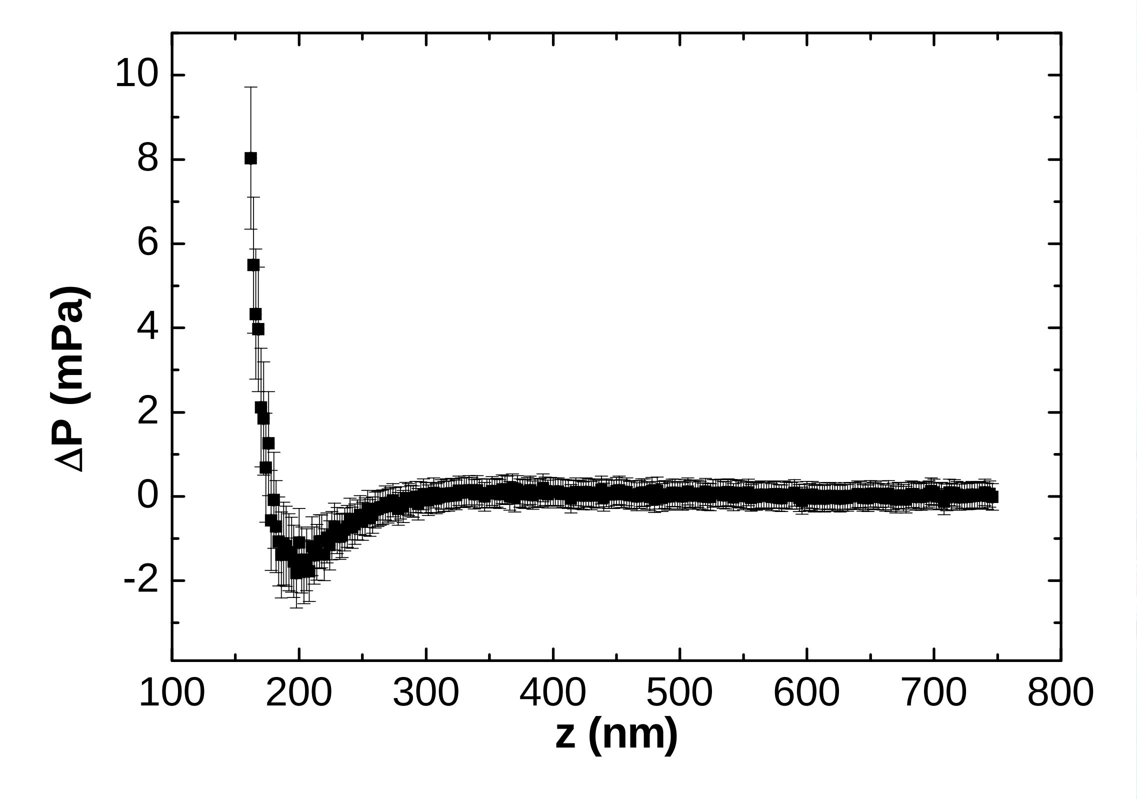
Determined by:

-looking into the response of the oscillator in the thermal bath

-Inducing a time dependent separation between the plate and the sphere

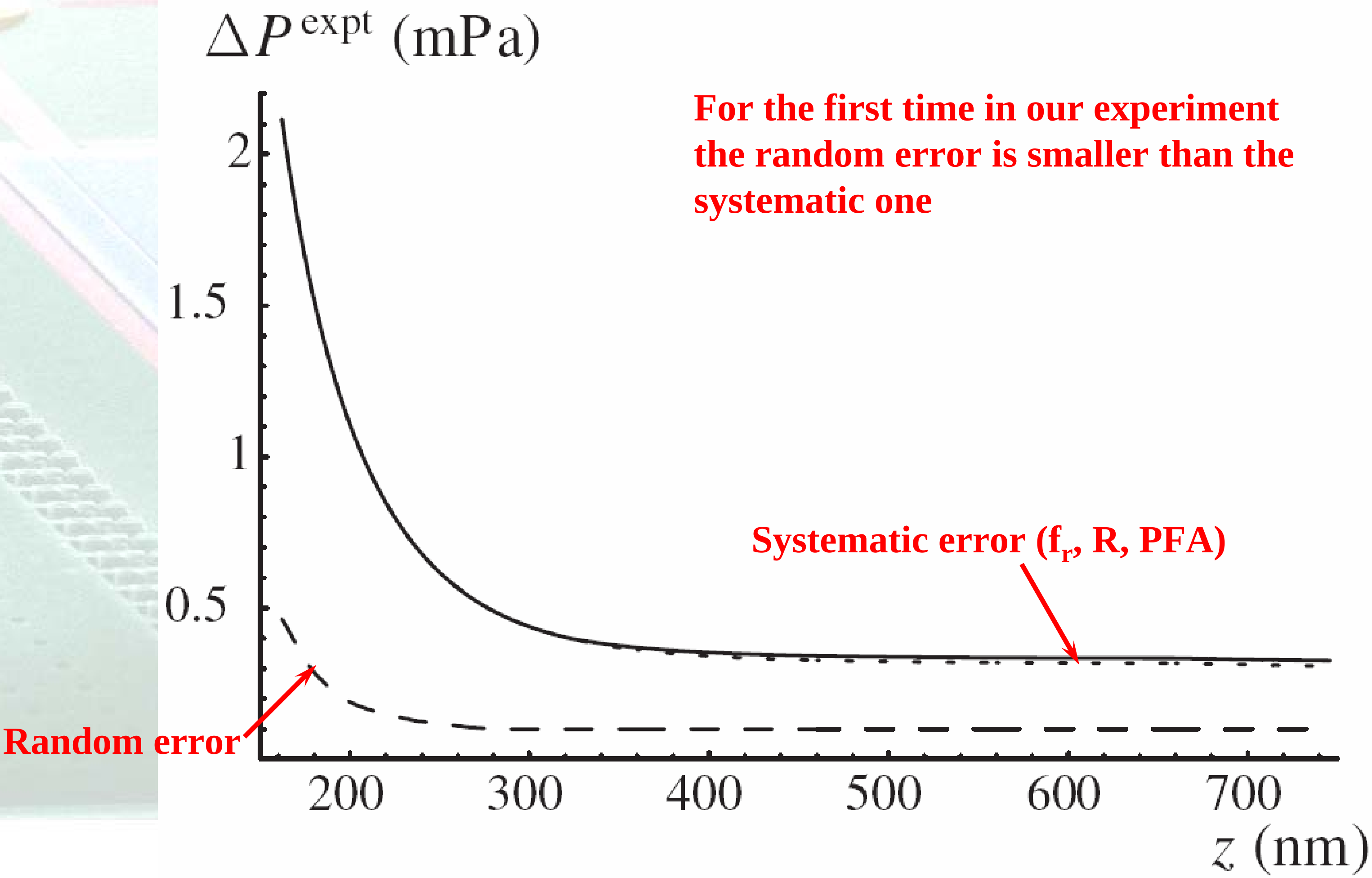


Equivalent P_C measurement



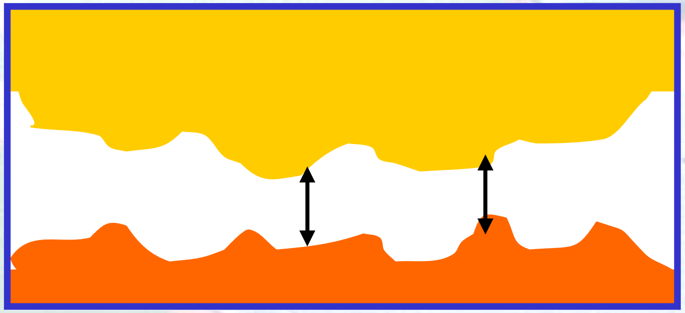
Error budget

For the first time in our experiment the random error is smaller than the systematic one



Comparison with theory

Roughness corrections

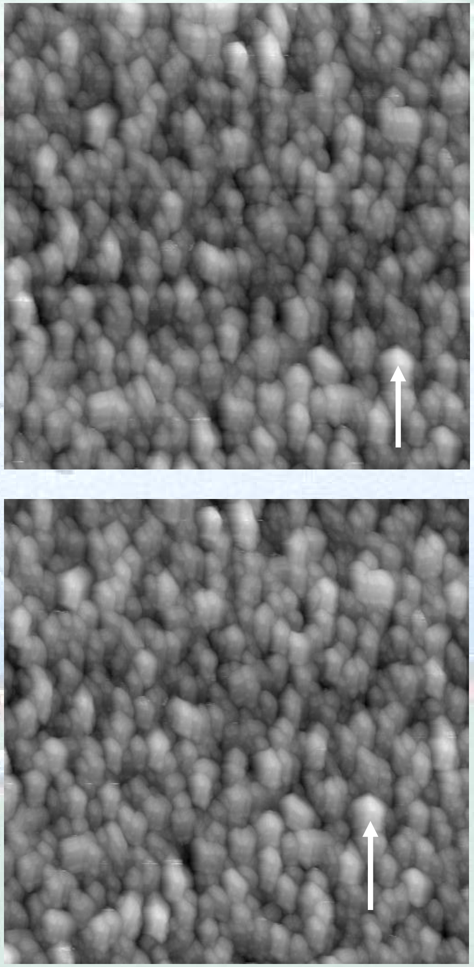


$$F_C = \sum_i v_i F_{CS}(z)$$

v_i: Fraction of the sample at separation z_i

Roughness corrections are ~0.5% to the Casimir force at 160 nm

AFM image of the Au plane



(10x10 μm^2)
~ 20 nm$_{pp}$

Comparison with theory

Finite conductivity and finite temperature

$$P(z) = -\frac{k_B T}{\pi} \sum_{l=0}^{\infty}{}' \int_0^{\infty} k_\perp \, dk_\perp q_l \times \left\{ \left[r_\parallel^{-2}(\xi_l, k_\perp) e^{2 q_l z} - 1 \right]^{-1} + \left[r_\perp^{-2}(\xi_l, k_\perp) e^{2 q_l z} - 1 \right]^{-1} \right\}$$

$$r_{\parallel,L}^{-2}(\xi_l, k_\perp) = \left[\frac{k_l + \varepsilon(i\xi_l) q_l}{k_l - \varepsilon(i\xi_l) q_l} \right]^2, \quad r_{\perp,L}^{-2}(\xi_l, k_\perp) = \left[\frac{k_l + q_l}{k_l - q_l} \right]^2$$

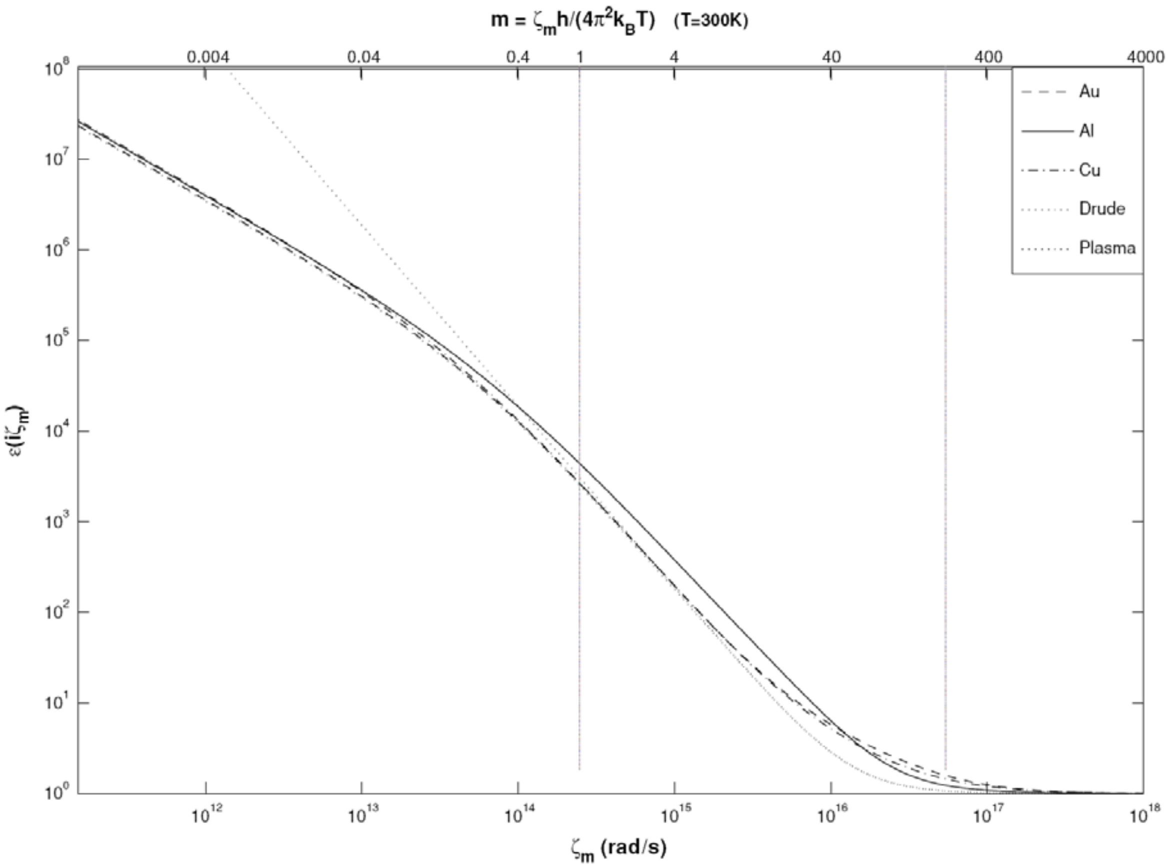
$$q_l^{\,2} = k_\perp^{\,2} + \left(\frac{4\pi^2 k_B T l}{hc} \right)^2$$

$$\varepsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^{\infty} \frac{\omega \, \mathrm{Im}\, \varepsilon(\omega)}{\omega^2 + \xi^2} \, d\omega$$

$$k_l^2 = k_\perp^2 + \varepsilon(i\xi_l) \xi_l^2 / c^2$$

Comparison with theory

$$\varepsilon(\mathrm{i}\xi) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega\,\mathrm{Im}\,\varepsilon(\omega)}{\omega^2+\xi^2}\,\mathrm{d}\omega$$



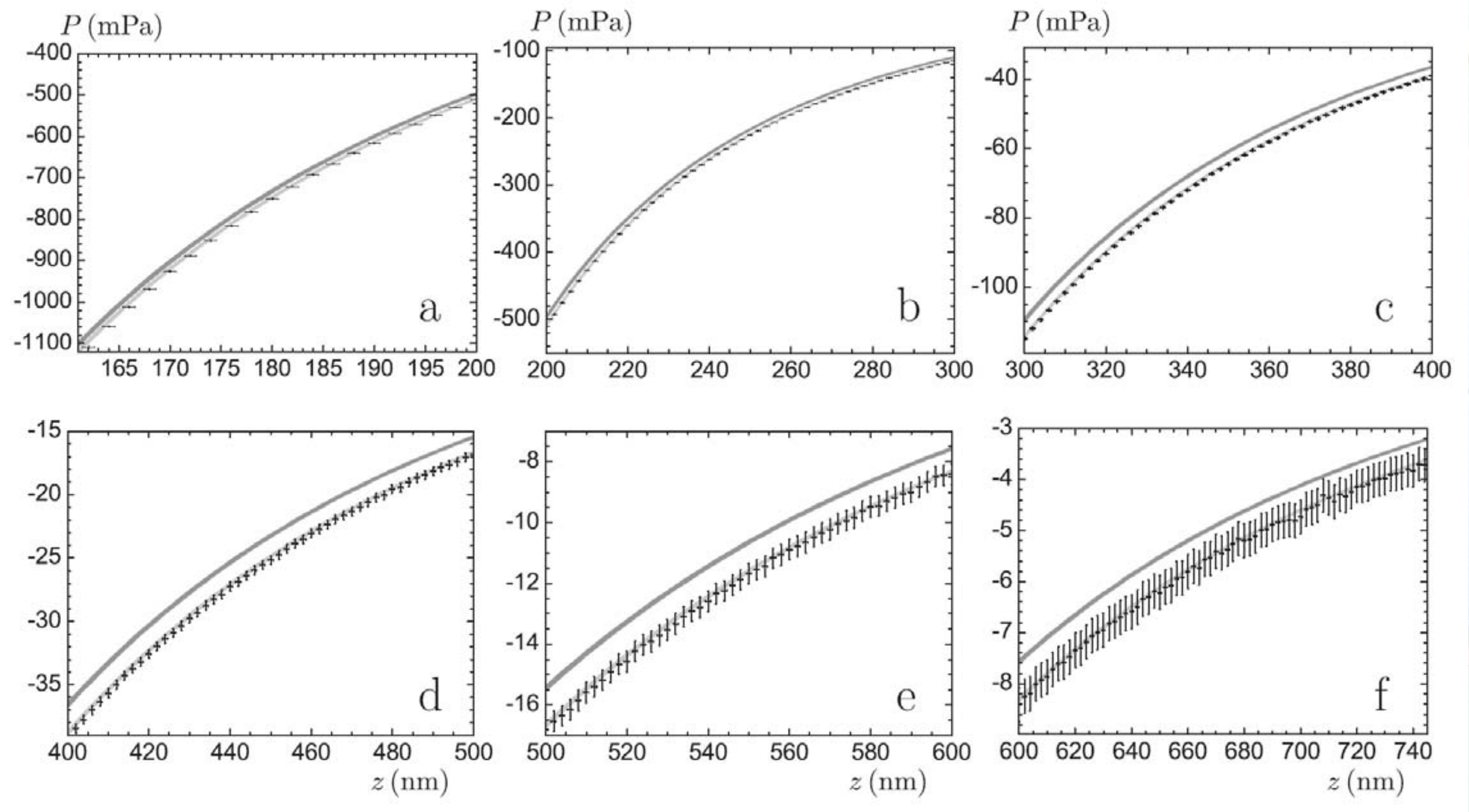
$$\varepsilon(\omega) = 1 - \frac{\omega_p^{\,2}}{\omega(\omega + i\gamma)}$$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^{\,2}}{\omega^2}$$

Bentsen et al., J. Phys. A (2005)

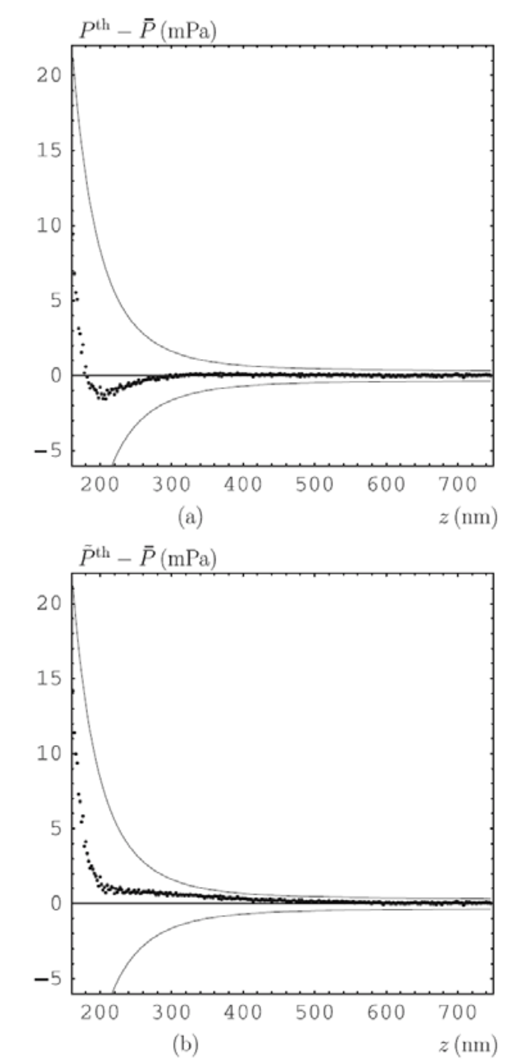
Pressure determination

PRD **75**, 077101

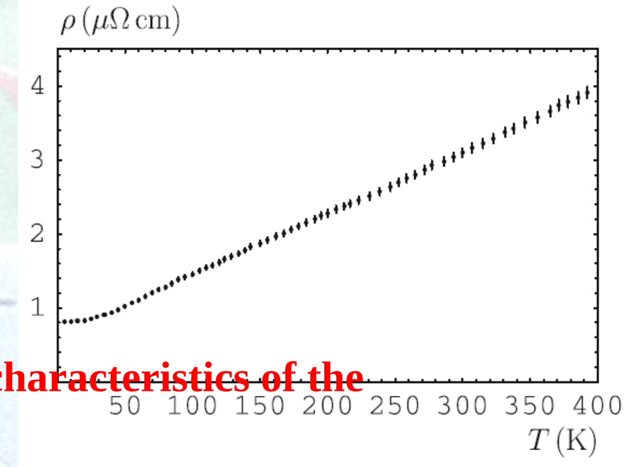


-Dark grey, Drude model approach
-Light grey, impedance approach

Pressure determination



Plasma model



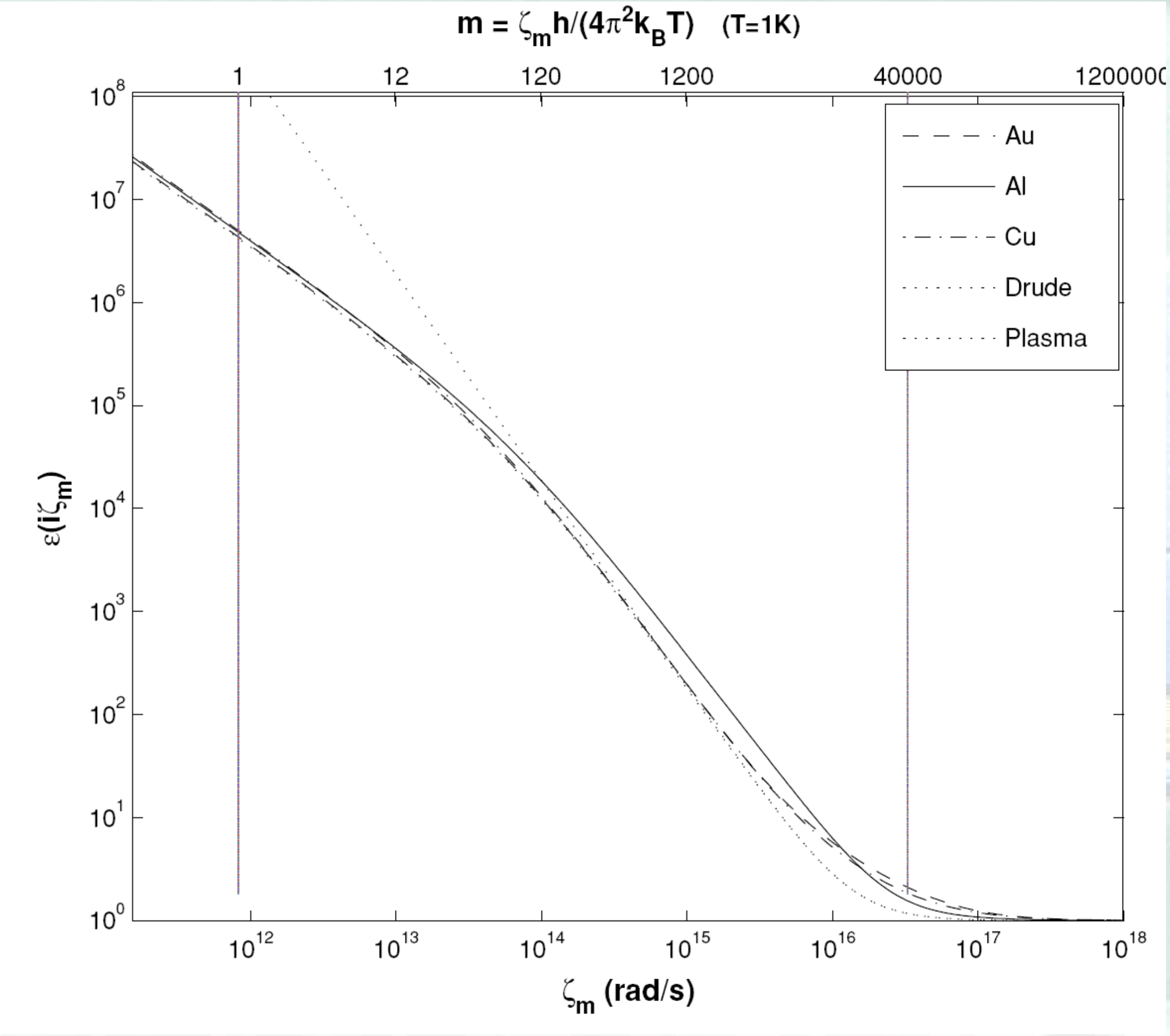
What are the characteristics of the Au used?

$$\rho(T) = \frac{4\pi}{\omega_p^2 \cdot \tau(T)} \propto \frac{T}{\omega_p^{3/2}}$$

Leontovich impedance

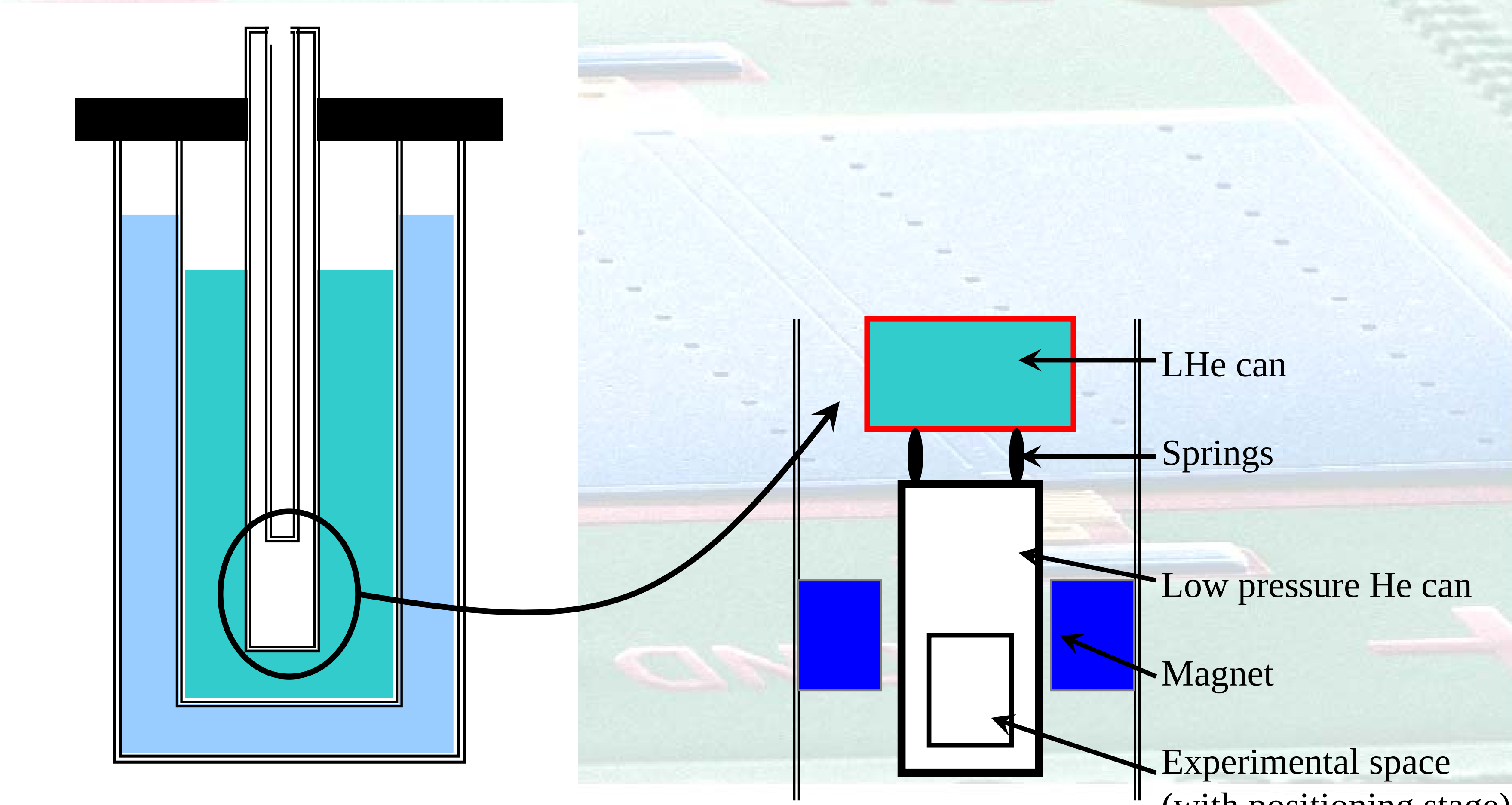
$$\omega_p = (8.9 \pm 0.1)\mathrm{eV}$$

Low temperature measurement



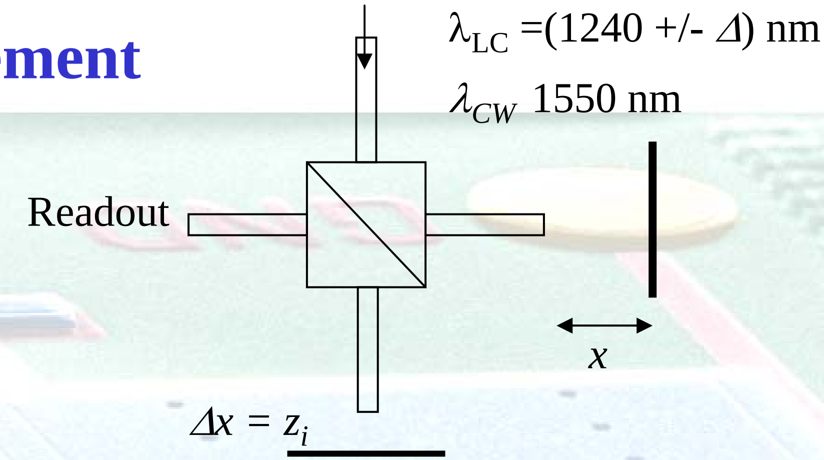
Low temperature measurement

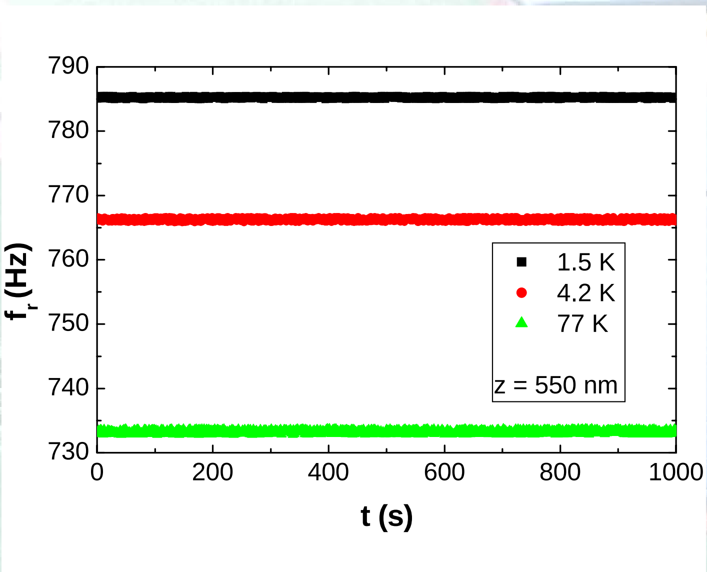
Setup schematic



Low temperature measurement

Characterization





Measured error is ~ 5 nm.

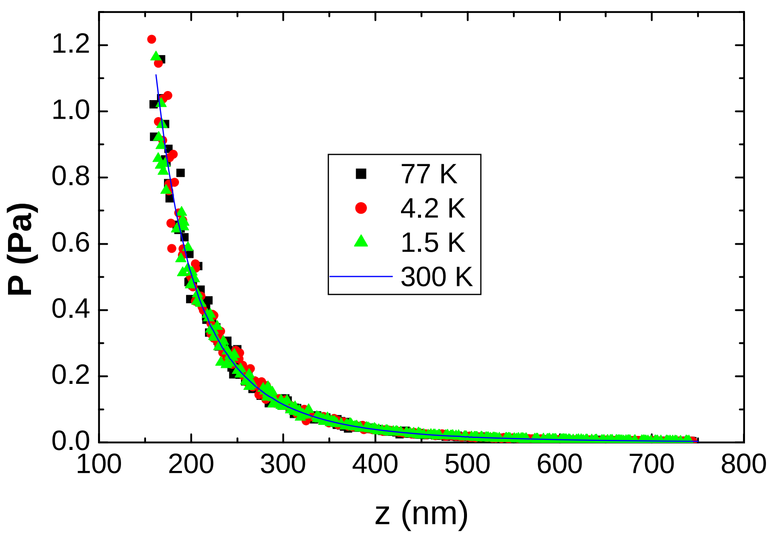
Mechanical vibrations

and problems with the interferometer

When compared with previous measurements,
the error in frequency is ~ 30 times larger at 1.5 K
and ~ 40 times larger at 4.2 K and 77 K, yielding an
increased error in P_C

Low temperature measurement

Results



Measurements at 1.5 K have the lowest noise

All data seem to coincide with the room temperature measurements within the larger Experimental errors

The error on the low T measurement, e_l(400 nm) = 5 mPa is larger than the difference between the Drude and plasma models of 2.4 mPa

This statement holds true at all temperatures and separations investigated

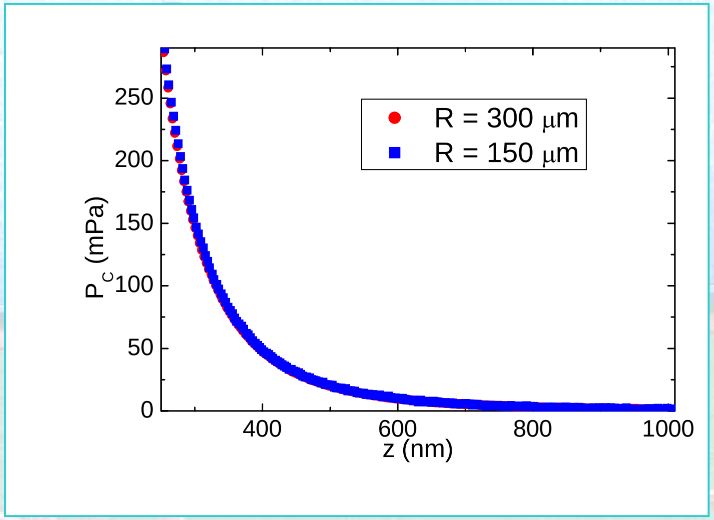
Proximity force approximation

$$F_C(z,R) = 2\pi R \times \varepsilon^{pp}_{Casimir}(z)\left[1+\beta\frac{z}{R}+\ldots\right]$$

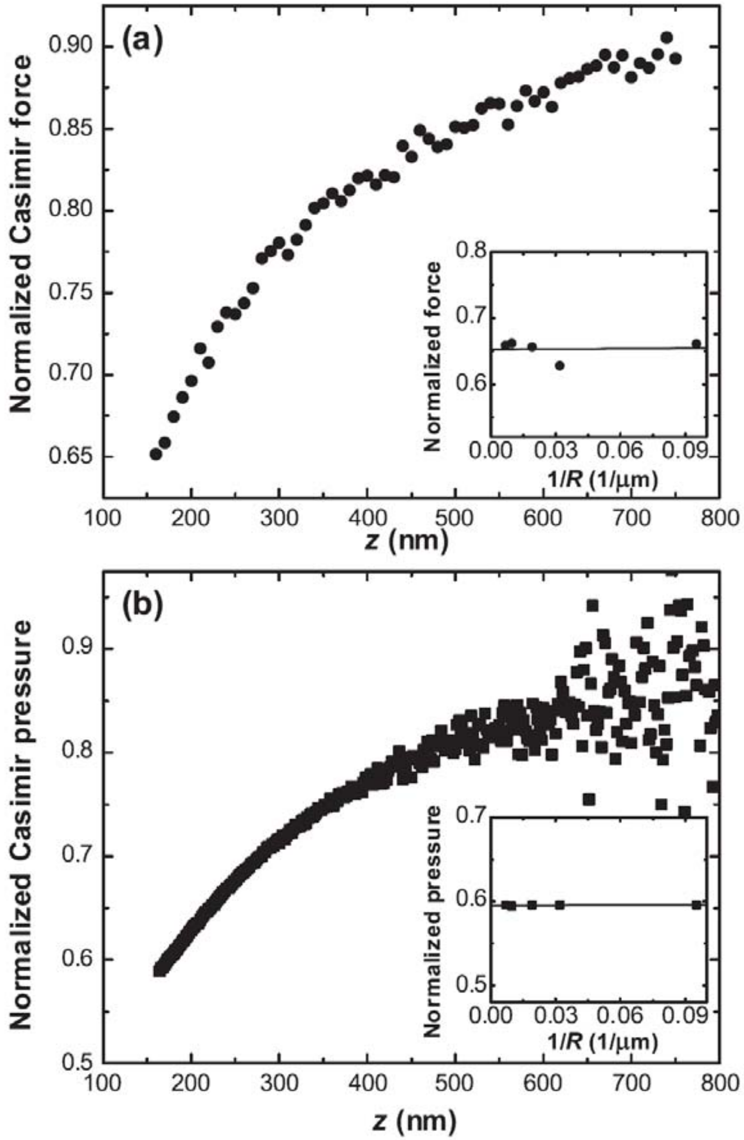
$$P^{eff}(z,R) = P^{pp}(z)\left[1+\beta'\frac{z}{R}+\ldots\right] = -\frac{1}{2\pi R}\frac{\partial F_C}{\partial z}$$

Measurements as a function of *R*

10.5 μm
31.4 μm
52.3 μm
102.8 μm
148.2 μm



Proximity force approximation



Insets:
Fits at z =170 nm

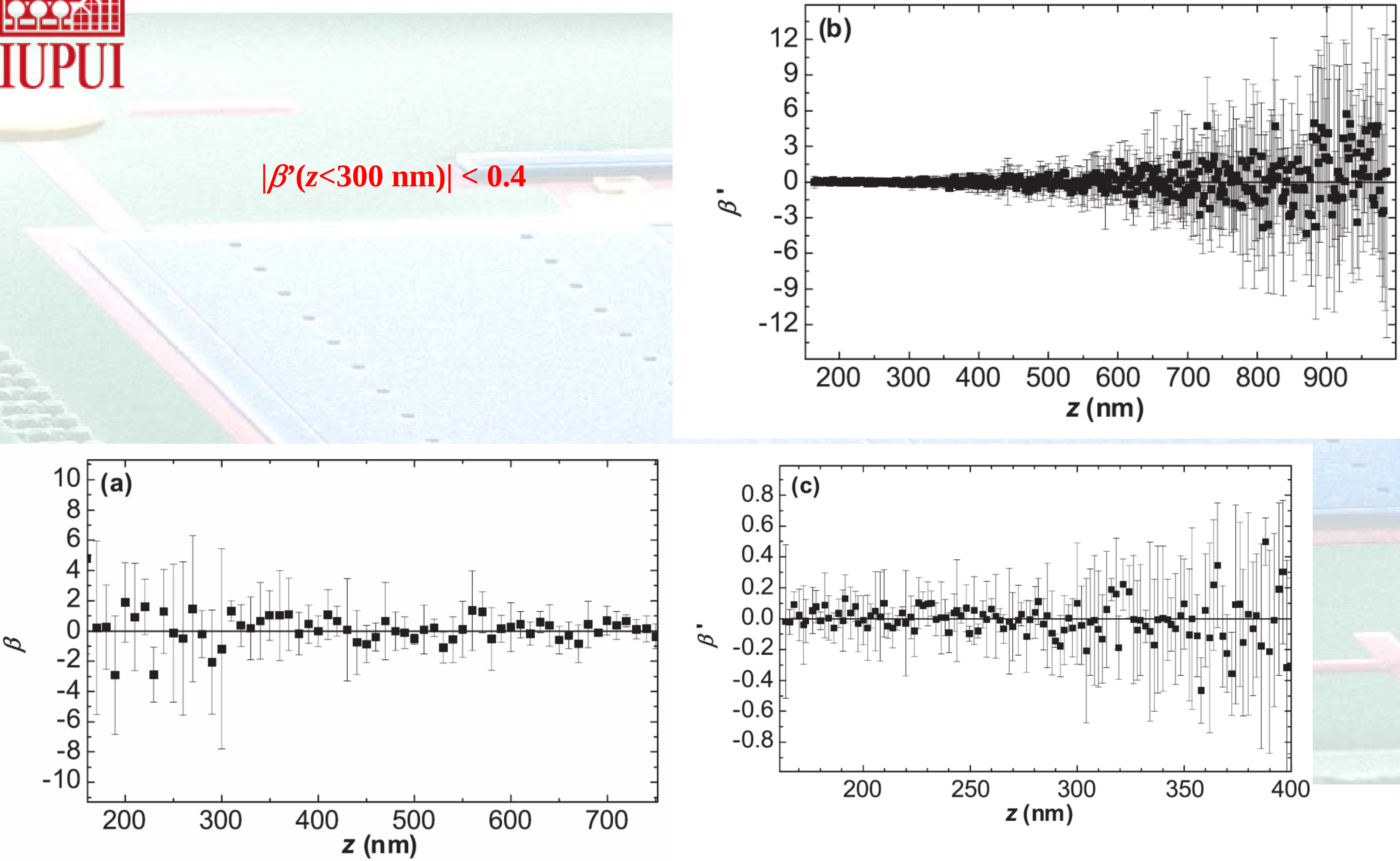
$$F_C(z,R) = 2\pi R \times \varepsilon^{pp}_{Casimir}(z)\left[1+\beta\frac{z}{R}+\ldots\right]$$

$$P^{eff}(z,R) = P^{pp}(z)\left[1+\beta'\frac{z}{R}+\ldots\right] = -\frac{1}{2\pi R}\frac{\partial F_C}{\partial z}$$

Yield the slopes β, β'

Proximity force approximation

$|\beta'(z<300\text{ nm})| < 0.4$



Summary

• Precise measurement of the Casimir force, with random errors smaller than systematic ones

This system cannot be used as is to improve on the sensitivity in P_C

• Good agreement with plasma model

Differences with Drude model **cannot** be explained as a problem in the separation measurement. It appears that any model with a finite relaxation time will give discrepancies when comparing with the Casimir force.

The Casimir measurements are an effective measurement of the Casimir interaction dependence on T.

• Preliminary data on temperature dependence

Significant noise sources. Main suspects: Mechanical coupling and interferometer variations. In consequence the data agree within the experimental error with both the Drude and generalized plasma models.

• Proximity force approximation

Experimentally determined the coefficients for the first order in the *z/R* expansions