Casimir Forces and Metamaterials

Felipe S S Rosa

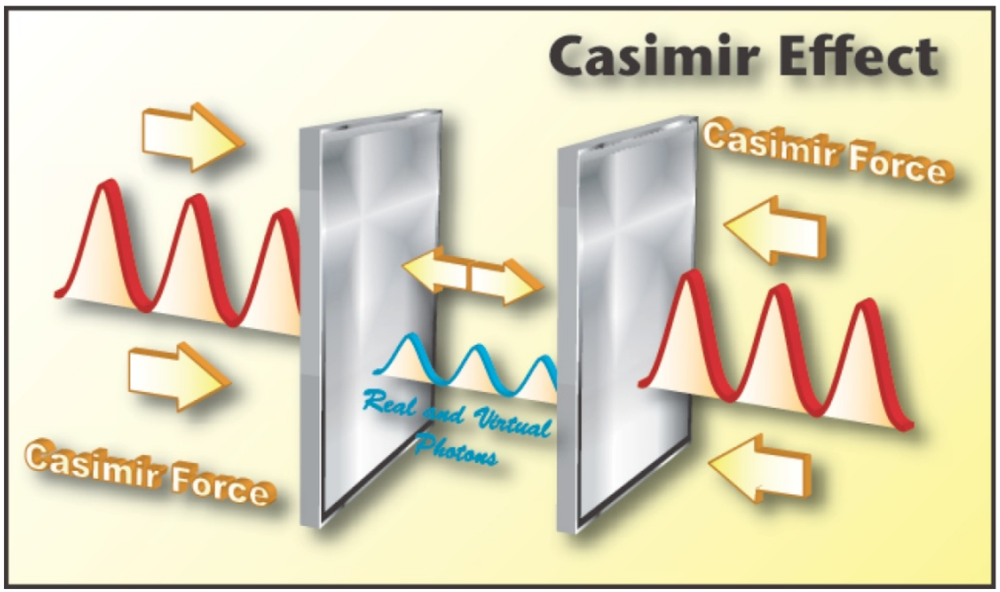
Collaborators: Diego A. R. Dalvit and Peter W. Milonni

Los Alamos NATIONAL LABORATORY

Kavli Institute of Theoretical Physics - KITP

Introduction

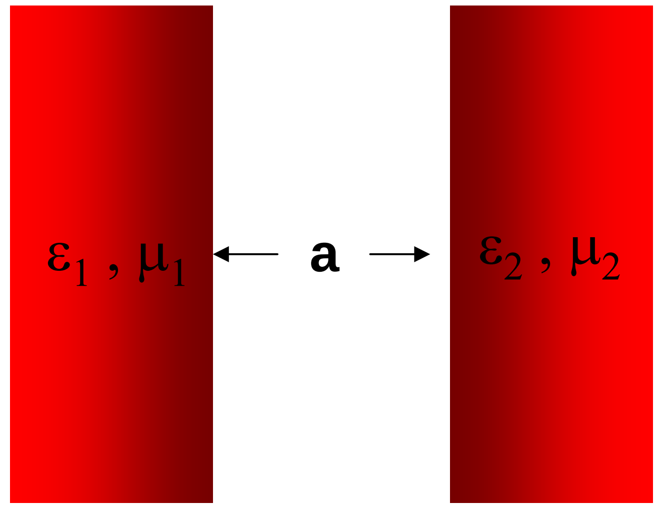
In 1948, H B G Casimir showed that two conducting plates attract each other through the Casimir force.



$$\frac{F_c}{A} = \frac{\hbar c \pi^2}{240 a^4}$$

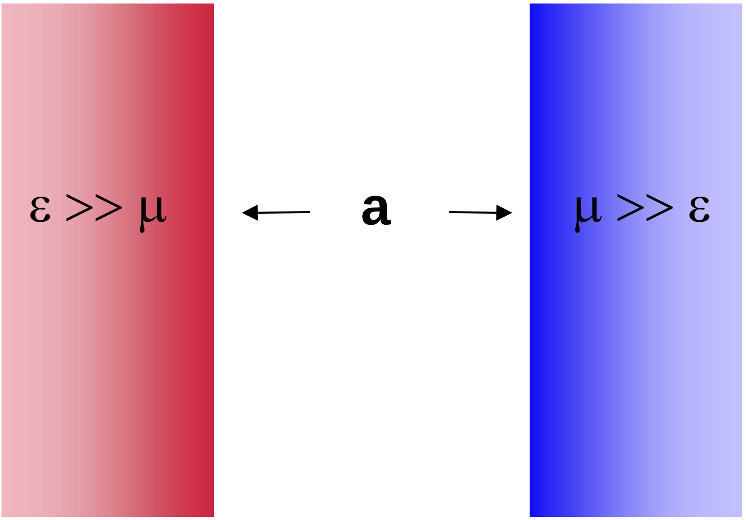
H B G Casimir, Proc. K. Ned. Akad. Wet. **51** 793 (1948)

Generalization for real media



$$\frac{F(a)}{A} = \frac{\hbar}{2\pi^2}\sum_{p=te}^{tm}\int_0^\infty d\xi\, dk\, k\,(k^2+\mu_3\epsilon_3\xi^2/c^2)^{1/2} \times \frac{r_p^{(1)}(\xi,\mathbf{k})\, r_p^{(2)}(\xi,\mathbf{k})\, e^{-2a(k^2+\mu_3\epsilon_3\xi^2/c^2)^{1/2}}}{1-r_p^{(1)}(\xi,\mathbf{k})\, r_p^{(2)}(\xi,\mathbf{k})\; e^{-2a(k^2+\mu_3\epsilon_3\xi^2/c^2)^{1/2}}}$$

One way to reverse the Casimir force is to replace one of the metallic plates by a highly permeable one



In the limit of perfect media, T.H. Boyer showed that

$$F_b = -\frac{7\hbar c\pi^2}{1920 a^4} = -\frac{7}{8} F_c$$

T. H. Boyer, Phys. Rev. A **9**, 2078 (1974).

And why a repulsive force is interesting ?

- **Basically to avoid stiction problems, friction, etc…**

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Courtesy of
SANDIA labs

Telegraph.co.uk

Home | News | Sport | Finance | Comment | Travel | Lifestyle | Culture | Vide

UK | World | Politics | Celebrities | Obituaries | Weird | Earth | Science | Health | Education | T

You are here: Home > News

Physicists have 'solved' mystery of levitation

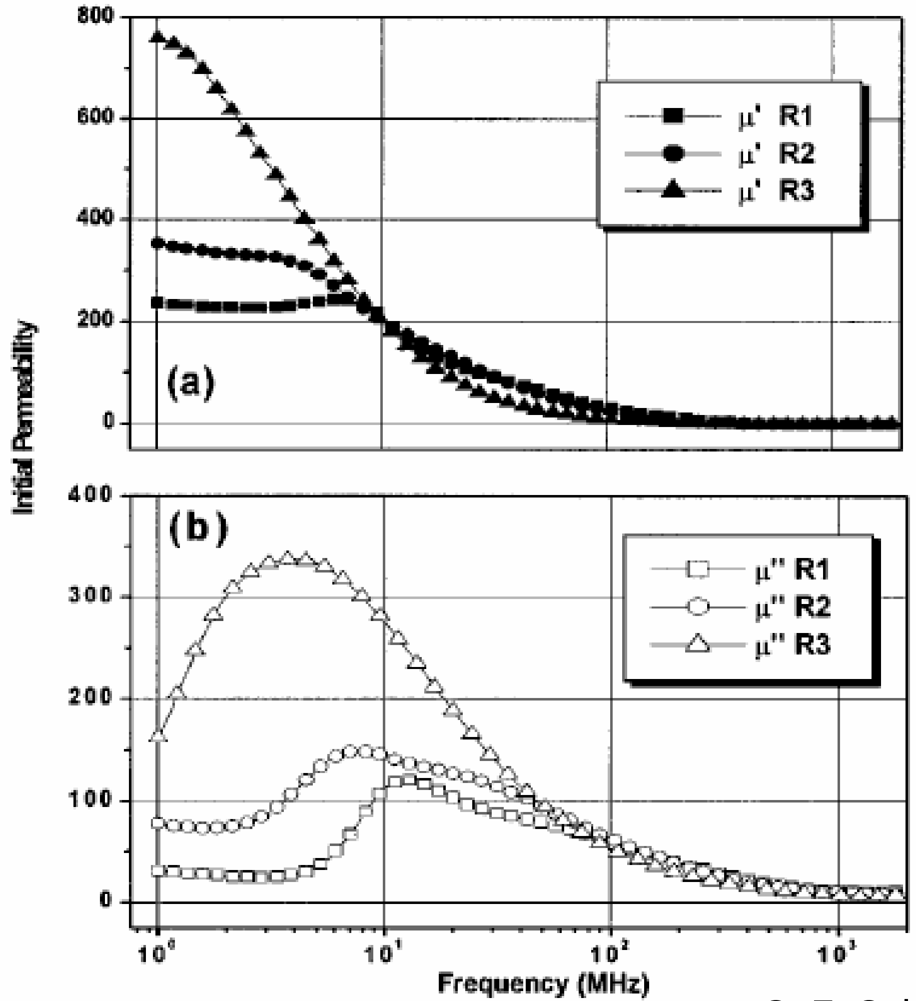
By Roger Highfield, Science Editor
Last Updated: 2:10AM BST 07 Aug 2007

In theory the discovery could be used to levitate a person

Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

"In theory the discovery could be used to levitate a person."

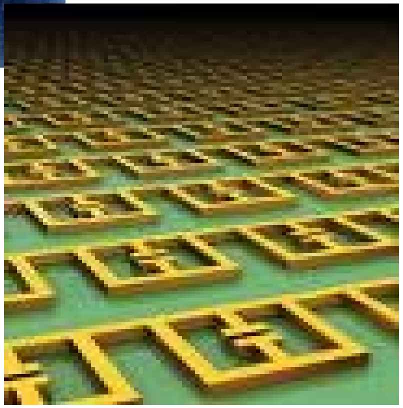
Unfortunately, Nature was not kind enough to provide the magnetic materials needed to change the sign of the force...



O. F. Caltun et al., IEEE Trans. Mag. **37**, 2353 (2001).

Metamaterials

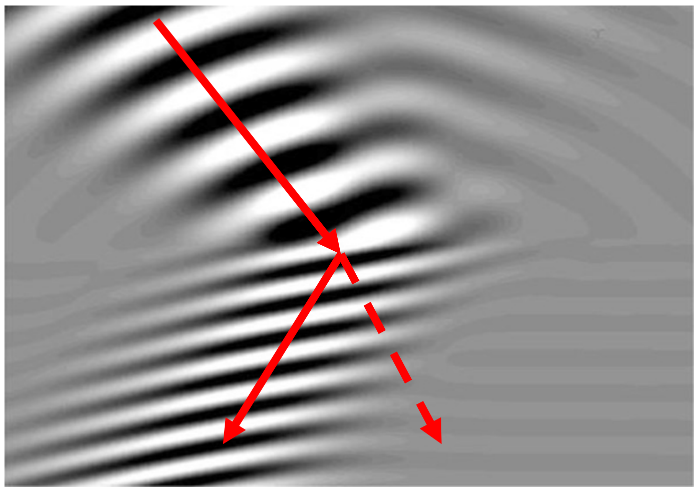


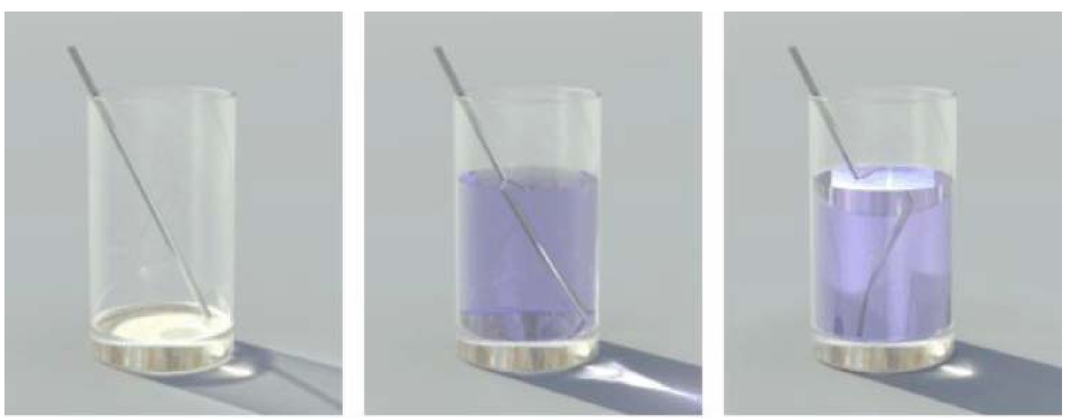



Metamaterials (MMs): artificial materials designed to present some specific electromagnetic property.

Negative index of refraction

Materials with n < 0 present all sorts of new phenomena, like negative refraction

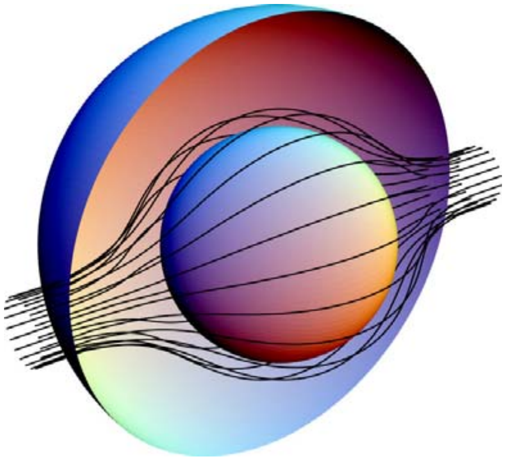


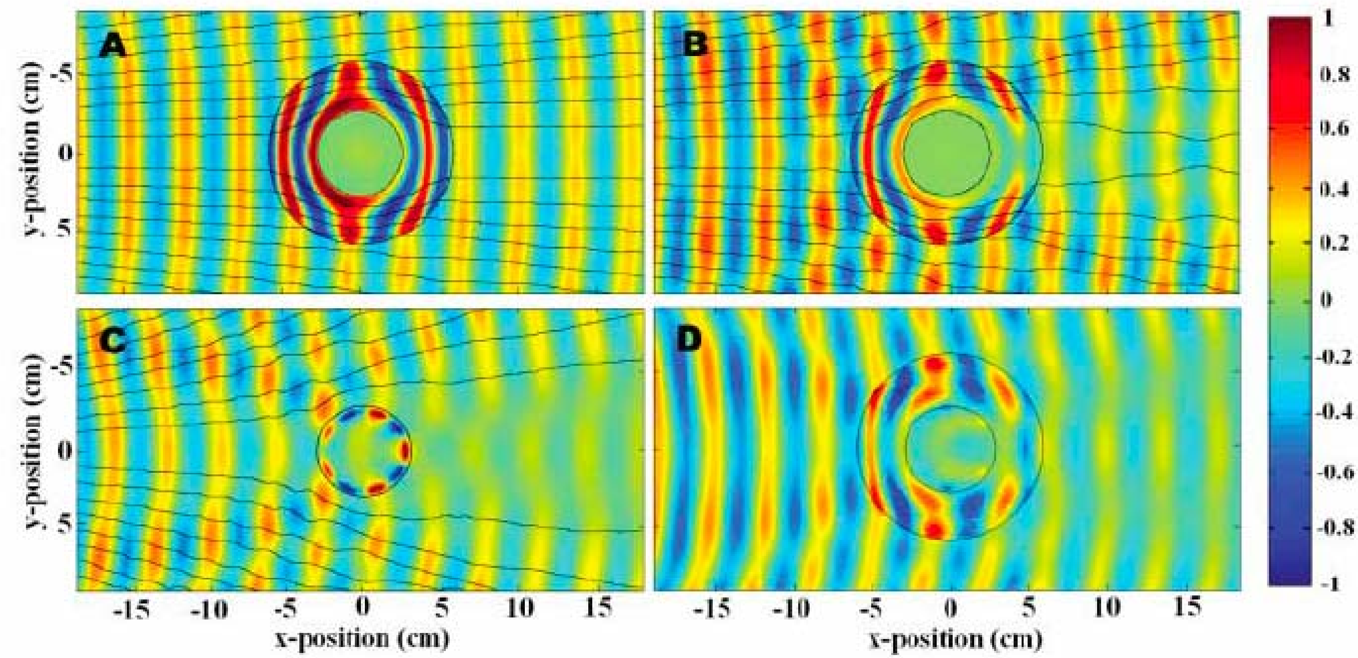


(a) (b) (c)

Invisibility Cloaks

By careful engineering processes, it is possible to control the flow of electromagnetic waves as they go through the MM

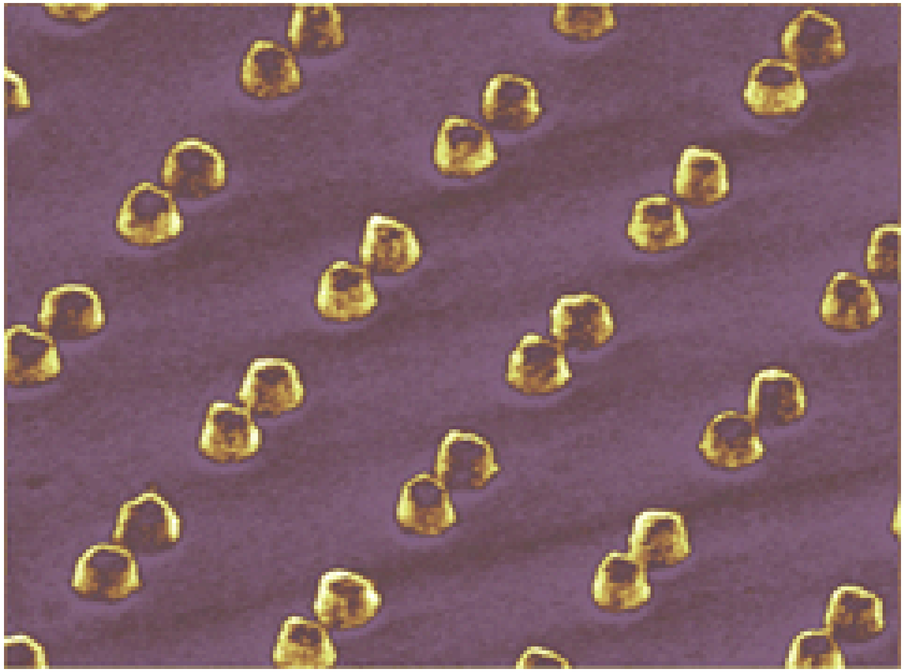




D. Schurig et al., Nature **314**, 997 (2006).

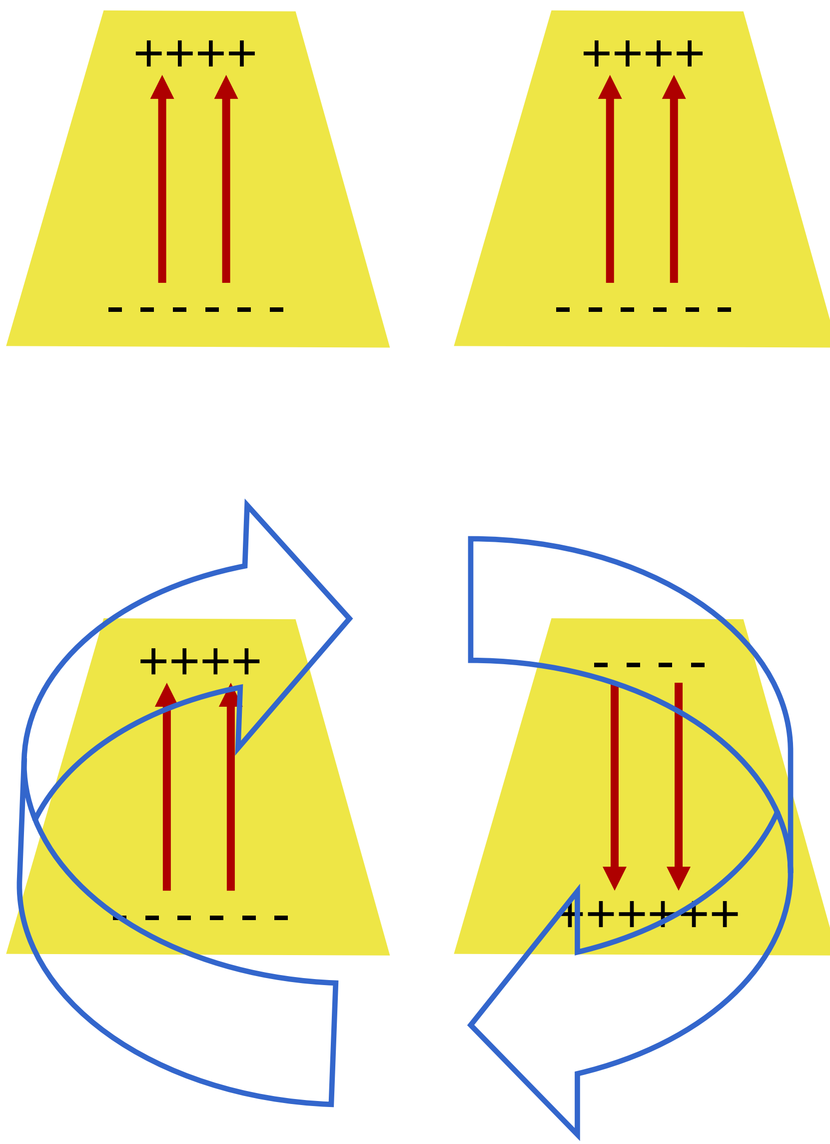
Metamaterials Cont'd

Let us consider the following example of a MM



golden nanopillars on top of a substrate

A.N. Grigorenko et al., Nature **438**, 335 (2005).



Electric dipole

↓

Contribution to ε_{MM}

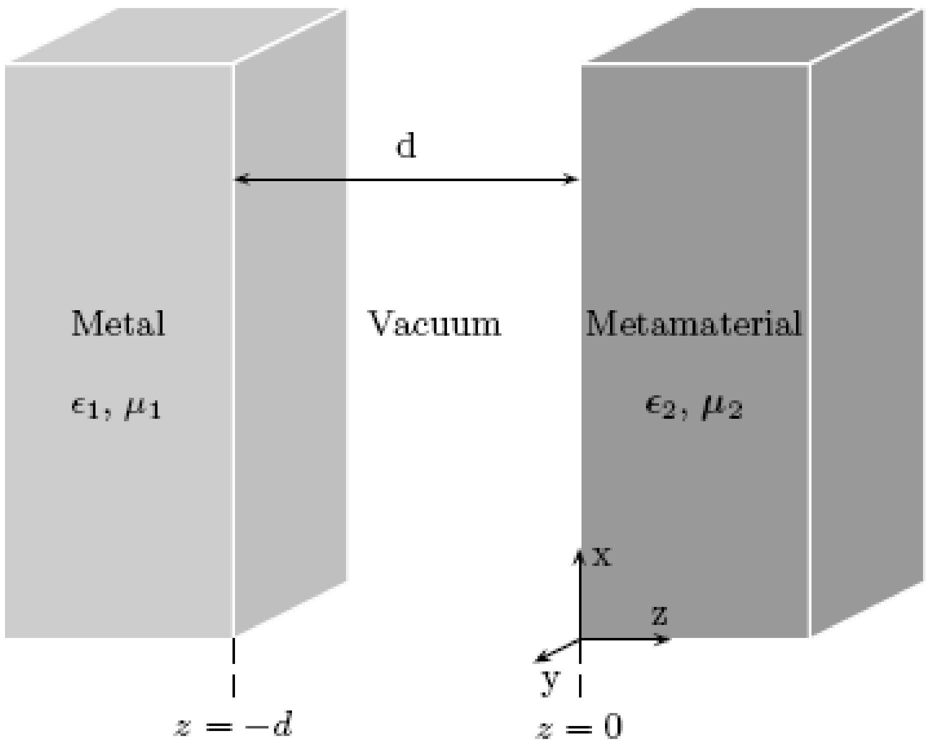
Magnetic dipole

↓

Contribution to μ_{MM}

MMs + Casimir Effect

In order to fix ideas, let us consider a metallic half-space facing a magnetodielectric half-space



Assuming the metal is reasonably described by a Drude response, we have

$$\epsilon_1(\omega) = 1 - \frac{\Omega_e^2}{\omega^2 + i\gamma_e\omega} \qquad \mu_1 = 1$$

The simplest model for a metamaterial based on resonant effects consists on Drude-Lorentz formulas

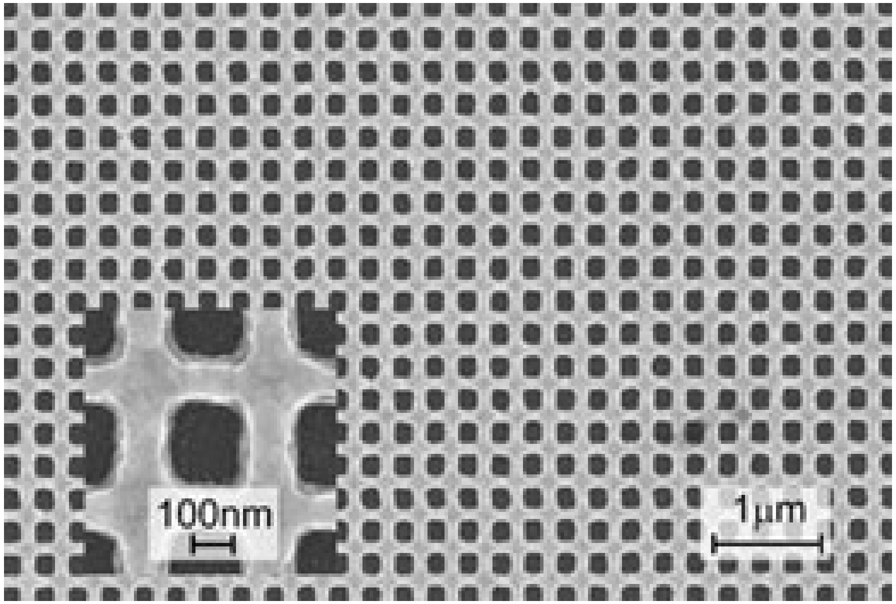
$$\epsilon_2(\omega) = 1 - \frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega}$$

$$\mu_2(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}$$

but for some MMs this may be not good enough…

1. Drude Backgound

In some metallic-based metamaterials, there is a net conductivity due to the metallic structure



G. Dolling et al., Science **312**, 892 (2006).

So, instead of of having simply a resonant contribution

$$\epsilon_2(\omega) = 1 - \frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega}$$

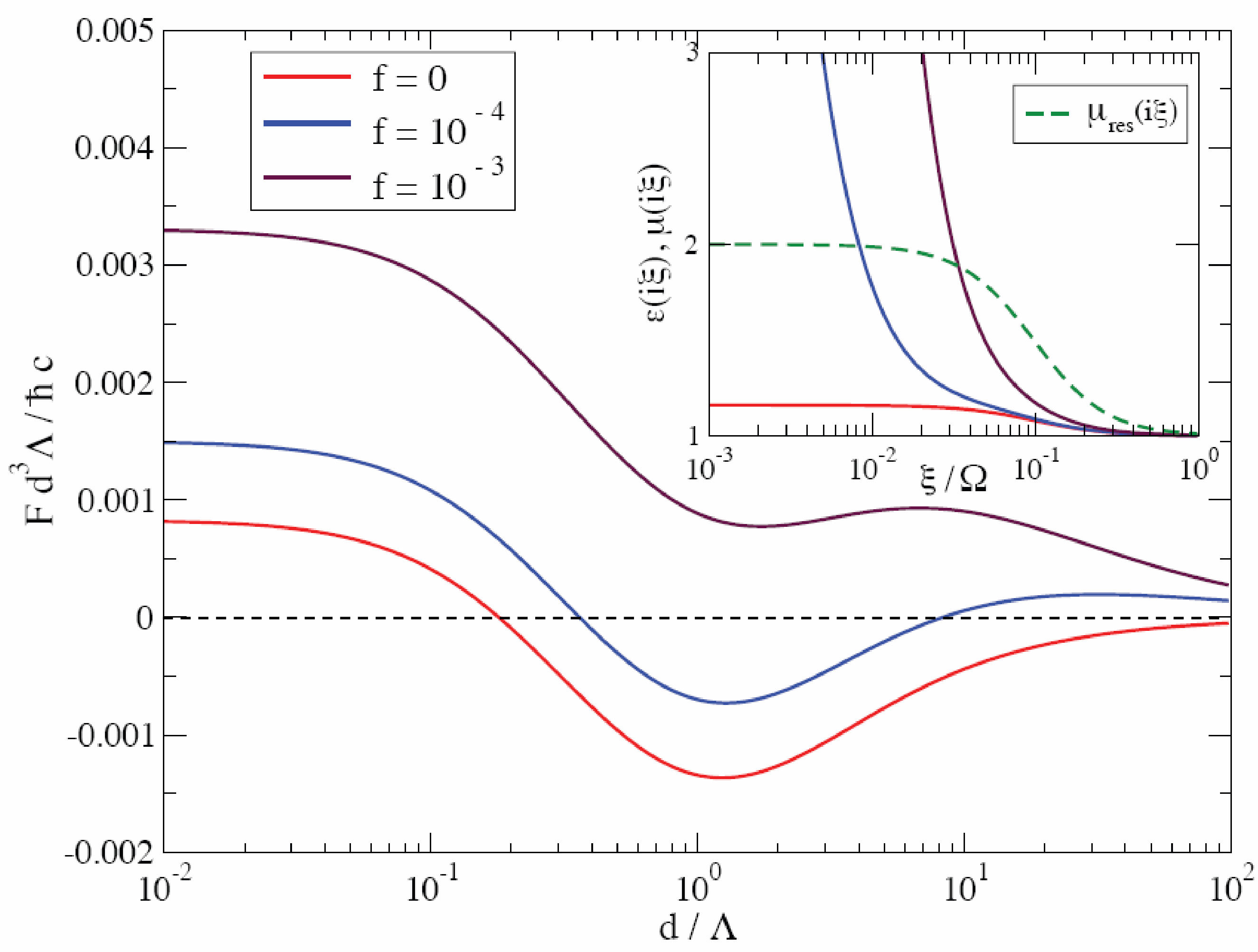
we have also a metallic background

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

or, at imaginary frequencies

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Plot for different filling factors



This effect is readily understood through our previous reasoning, meaning that

$$r_{te}^{(i)} = \frac{\mu_i \sqrt{k^2 + \mu_3 \epsilon_3 \frac{\xi^2}{c^2}} - \mu_3 \sqrt{k^2 + \mu_i \epsilon_i \frac{\xi^2}{c^2}}}{\mu_i \sqrt{k^2 + \mu_3 \epsilon_3 \frac{\xi^2}{c^2}} + \mu_3 \sqrt{k^2 + \mu_i \epsilon_i \frac{\xi^2}{c^2}}}$$

does not present a positive sign for a sufficiently large frequency range.

2. Anisotropy

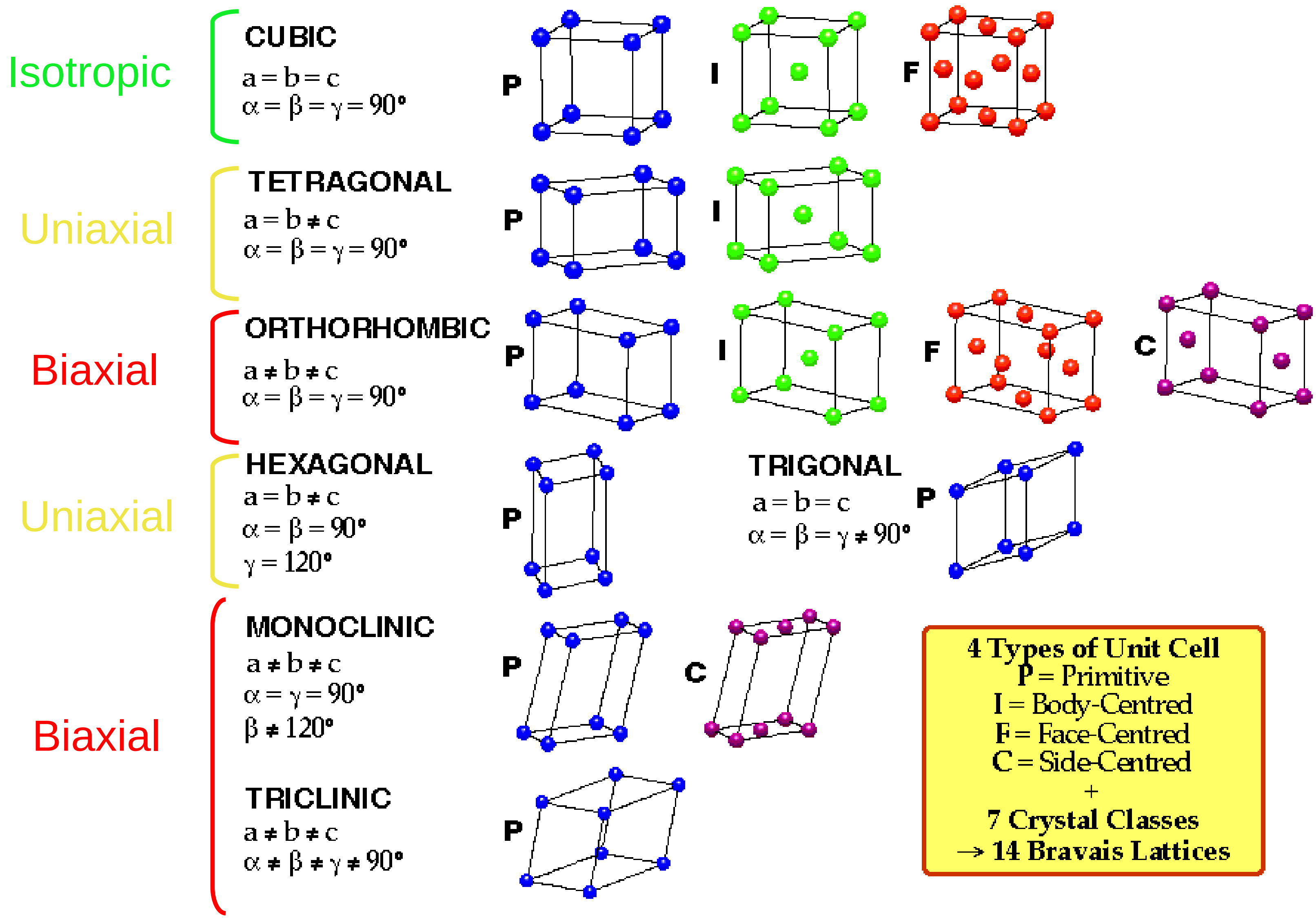
In an anisotropic medium,the constitutive relations between E, D, B and H are more involved

$$\mathbf{D} = \epsilon \cdot \mathbf{E} \qquad \mathbf{H} = \mu^{-1} \cdot \mathbf{B}$$

due to the tensorial nature of the permittivity and the permeability

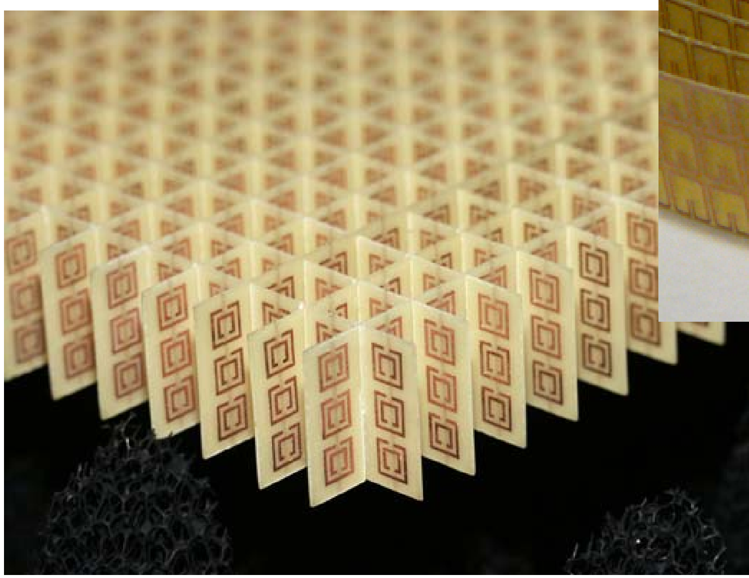
$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}$$

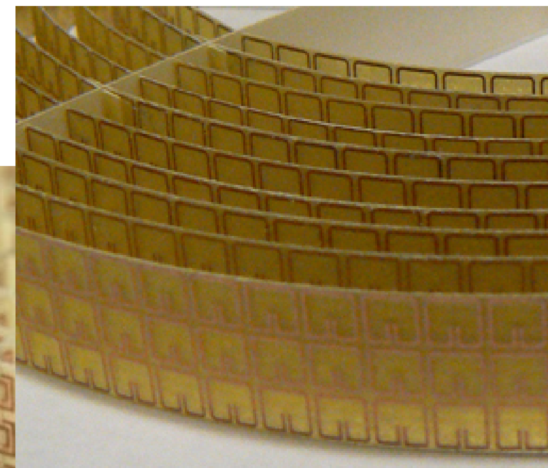
The Bravais lattices



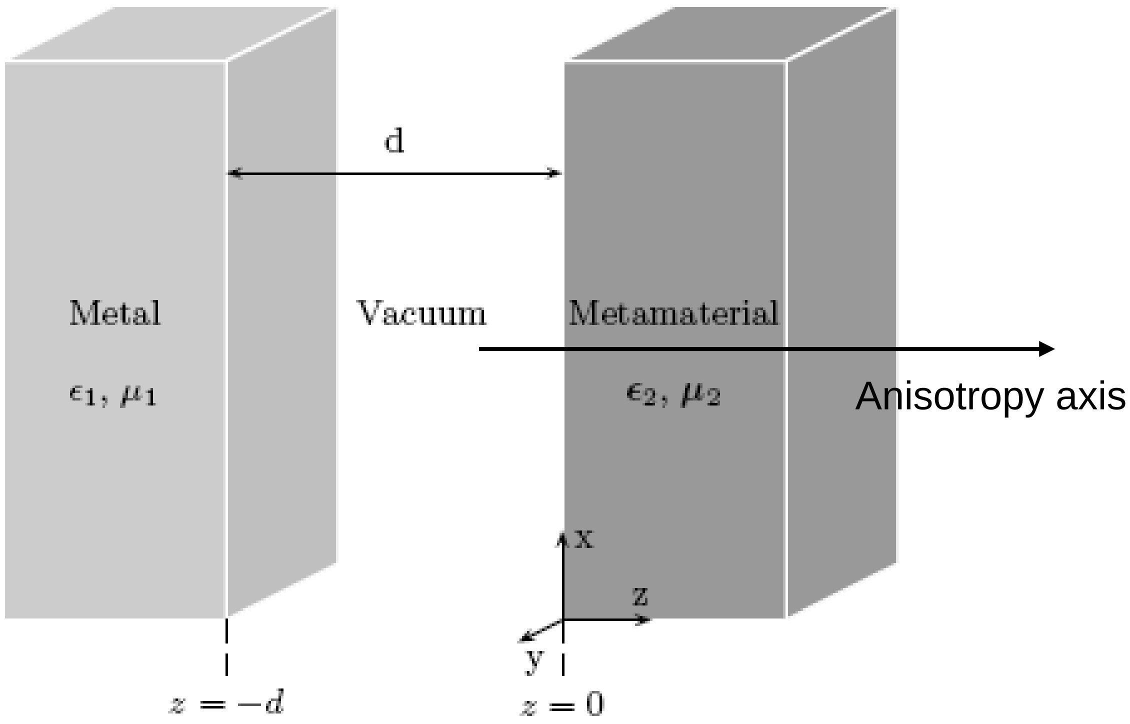
2.1. Uniaxial Anisotropy

The uniaxial anisotropy is very relevant in the context of metamaterials, since there are several designs based in stacks of different materials.





Let us consider the case where the anisotropy is along the z-axis



The electromagnetic tensors then become

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \; ; \; \mu_{ij} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$

where

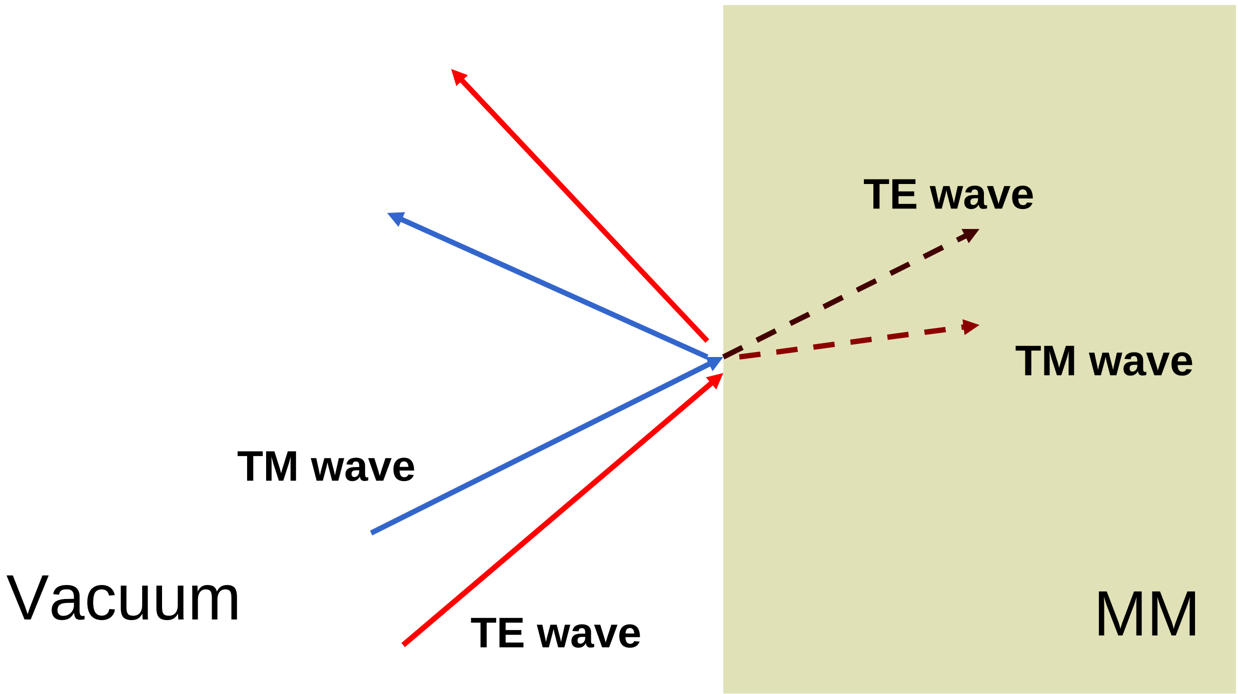
$$\epsilon_{xx}(\omega) = \epsilon_{yy}(\omega) = 1 - (1 - f_x)\frac{\Omega^2_{e,x}}{\omega^2 - \omega^2_{e,x} + i\gamma_{e,x}\omega} - f_x\frac{\Omega^2_{D,x}}{\omega^2 + i\gamma_{D,x}\omega},$$

$$\epsilon_{zz}(\omega) = 1 - (1 - f_z)\frac{\Omega^2_{e,z}}{\omega^2 - \omega^2_{e,z} + i\gamma_{e,z}\omega} - f_z\frac{\Omega^2_{D,z}}{\omega^2 + i\gamma_{D,z}\omega},$$

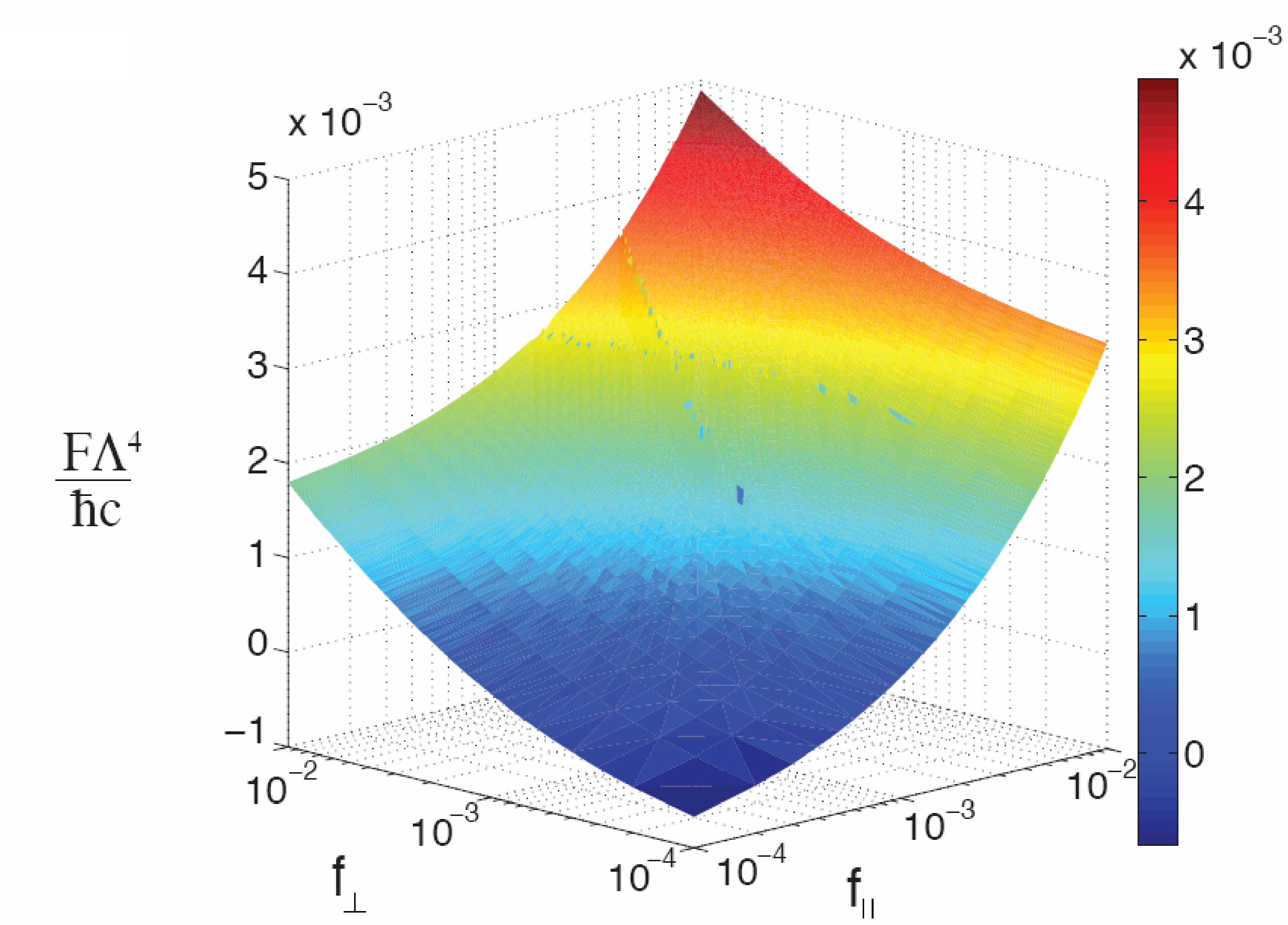
$$\mu_{xx}(\omega) = \mu_{yy}(\omega) = 1 - \frac{\Omega^2_{m,x}}{\omega^2 - \omega^2_{m,x} + i\gamma_{m,x}\omega},$$

$$\mu_{zz}(\omega) = 1 - \frac{\Omega^2_{m,z}}{\omega^2 - \omega^2_{m,z} + i\gamma_{m,z}\omega}$$

This case is not much harder than the isotropic one, since the MM supports TE and TM waves



If the anisotroopy is completely coded in the filling factors, we have



FSSR, D.A.R. Dalvit, P.W. Milonni, PRL **100**, 183602

3. Magnetic activity

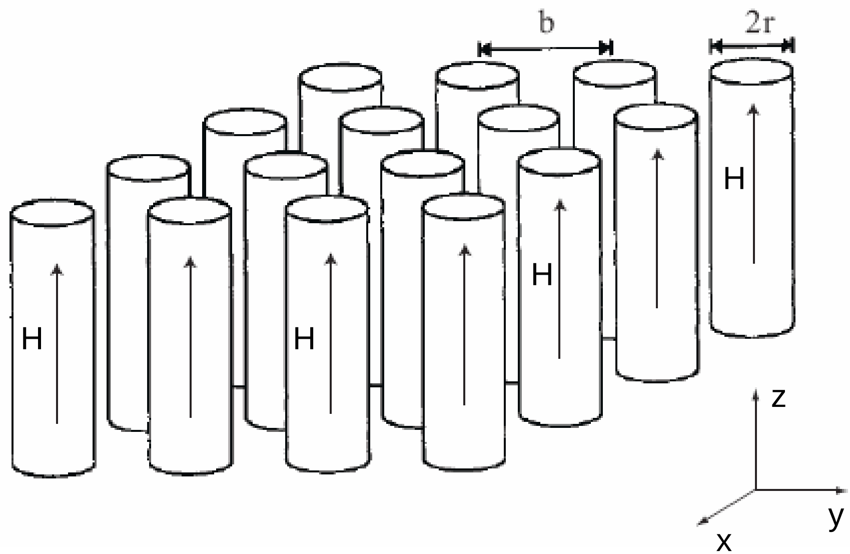
We have been assuming that a Drude-Lorentz formula for the permeability

$$\mu_2(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}$$

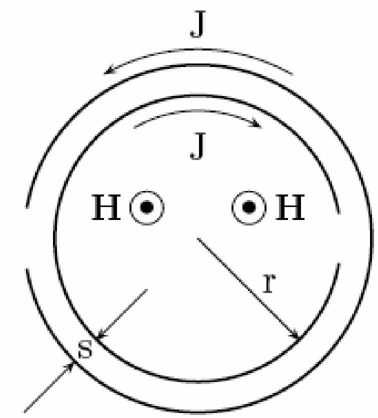
holds throughout the whole spectrum. But typical measurements are made only in the vicinities of resonances, so it is impossible to be sure…

For a MM consisting of an array of double-layered cylinder sheets

Lateral view



Top view



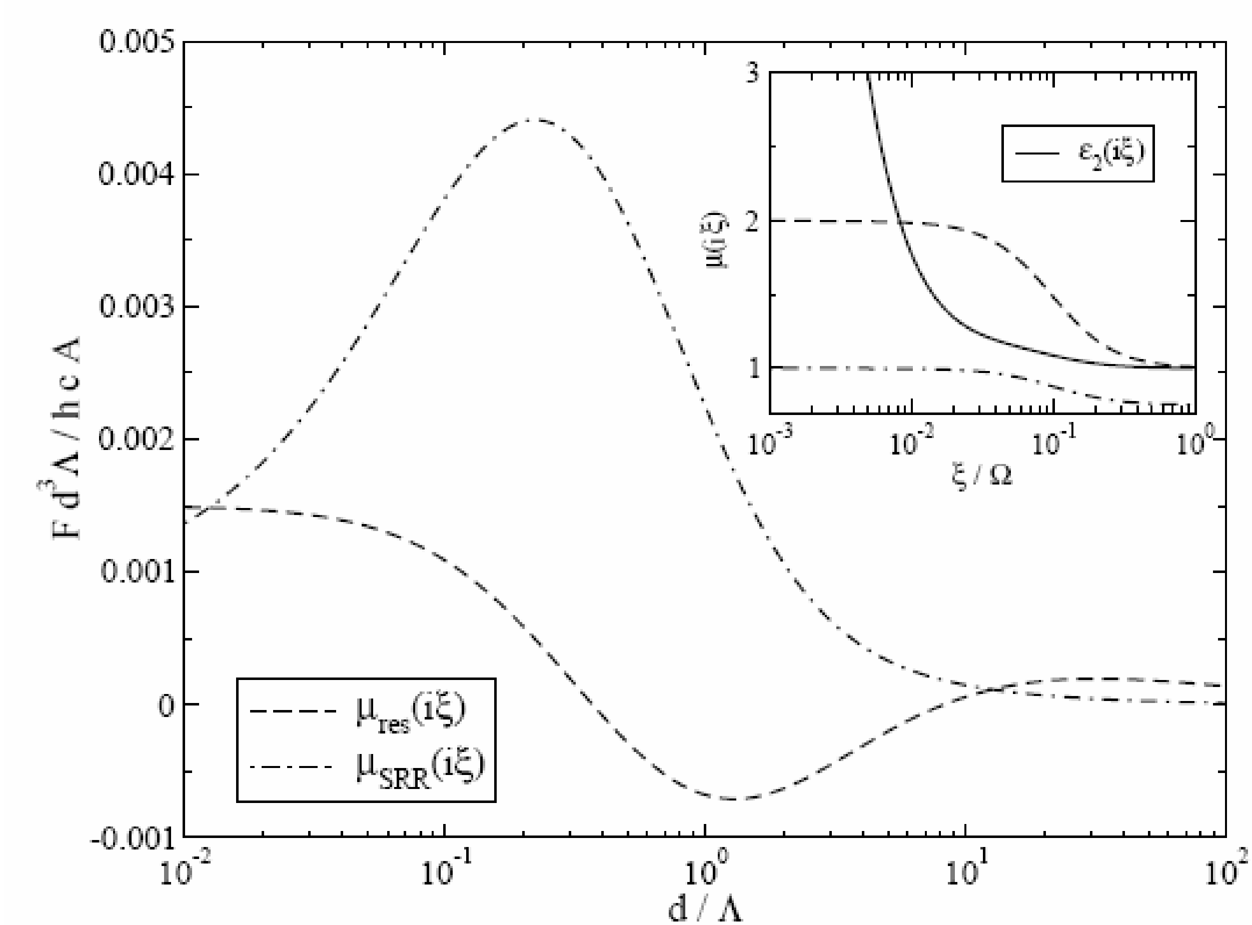
it is possible to derive the following permeability

$$\mu_{srr}(\omega) = 1 - \frac{C\omega^2}{\omega^2 - \omega_{i,e}^2 + \gamma_{i,e}\omega}$$

In the imaginary axis, we have

$$\mu_2(i\xi) = 1 - \frac{C\xi^2}{\xi^2 + \omega_m^2 + \gamma_m \xi} \quad \Rightarrow \quad \epsilon_2(i\xi) > \mu_2(i\xi)$$

and the results are



Discussion

- Everything discussed so far is based on the assumption that an effective medium approximation holds.

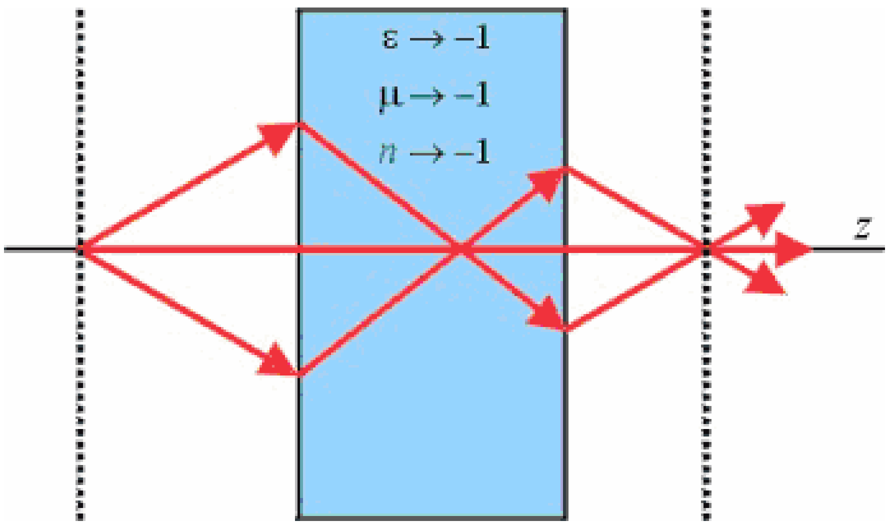
- Temperature may increase or decrease Casimir repulsion.

- Could the experiments measure at least some reducecd attraction?

And not only stacks, but the fishnet shown before is also (approximately) uniaxial

- Plane (and perfect) lenses
- Reverse Doppler effect
- Etc

Plane lens



Reverse Doppler effect