Towards measuring variations of Casimir Energy by a superconducting cavity

Giuseppe Bimonte
Physics Department
Università di Napoli "Federico II" ITALY
and
INFN-Sezione di Napoli

Recent years have seen an intense experimental work on the Casimir effect.
All experiments so far are force measurements (mostly in the plane-parallel or in the sphere-plate geometry).
No experiments yet exist which probe directly the physical effects of the Casimir energy

It would be interesting to have experimental verifications for:

- Gravitational effects of Casimir energy
- Influence of phase transitions on the Casimir energy/force

Casimir effect & Gravity

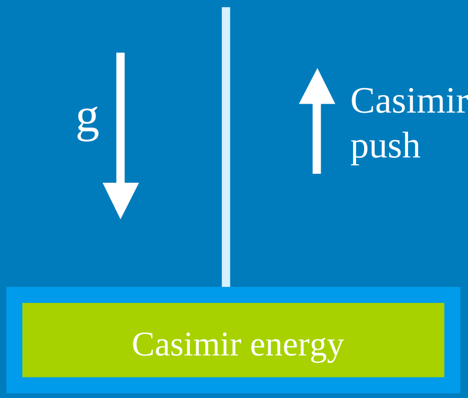
We studied the possibility of testing experimentally if the Equivalence Principle of General Relativity holds for vacuum fluctuations:

"What is the weight of Casimir energy?"

Explicit evaluation of the Maxwell stress tensor in a weak gravitational field gives expected result for the Casimir push W_C:

$$W_C = M_C\, g \qquad M_c = E_c/c^2$$

Since the Casimir energy E_c of a plane parallel cavity is negative, it contributes a negative weight!



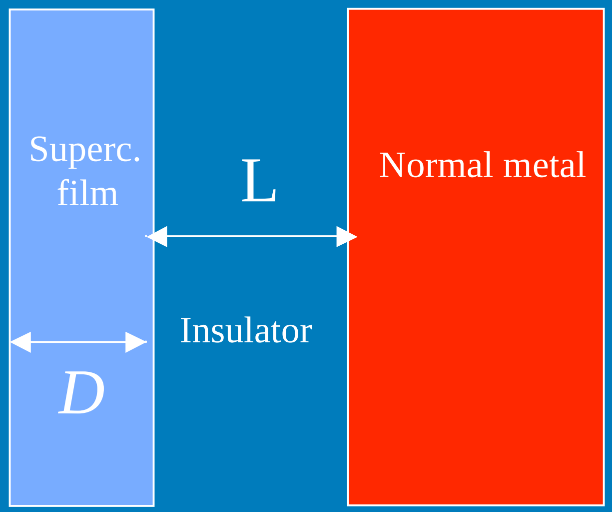
G.Bimonte, E. Calloni, G. Esposito and L. Rosa, Phys.Rev. D74 (2006) 085011; D76 (2007) 025008;
G.Bimonte, G. Esposito and L. Rosa Phys. Rev. D D78 (2008) 024010.
See also S.A. Fulling, K.A.Milton, P.Parashar, A.Romeo,K.V. Shajesh, J Wagner
Phys. Rev.D76 (2007), 025004

Concerning the relation with phase transitions, we proposed an experiment (ALADIN) to probe the influence of the Casimir energy on the superconducting phase transition.

In collaboration with E.Calloni, G. Esposito and L. Rosa

Phys. Rev. Lett. 94 (2005), 180402; Nucl. Phys. B726 (2005) 441.

We consider a Casimir apparatus constituted by a thin superconducting film and a thick normal plate, separated by an insulating layer



D~15 nm

L~10 nm

1. The Casimir free energy stored in the cavity depends on the reflective power of the layers.

2. The optical properties, in the microwave region, of a metal film change drastically when it becomes superconducting.

Therefore:

The Casimir force and free energy change when the state of the film passes from normal to superconducting

Is there a way to observe these effects ?

A standard force measurement would hardly detect any force change across the transition, because the effect on the Casimir force/energy is extremely small (of fractional order 10^{-5} or so).

The reason is easy to understand, because the largest contribution to the Casimir effect comes from modes of energy $\hbar c/L \approx 10$ eV (for L=20 nm), while the transition to superconductivity affects the reflective power only in the microwave region, at the scale $k\,T_c \leq 10^{-4}$ eV (for $T_c \approx 1$ K).

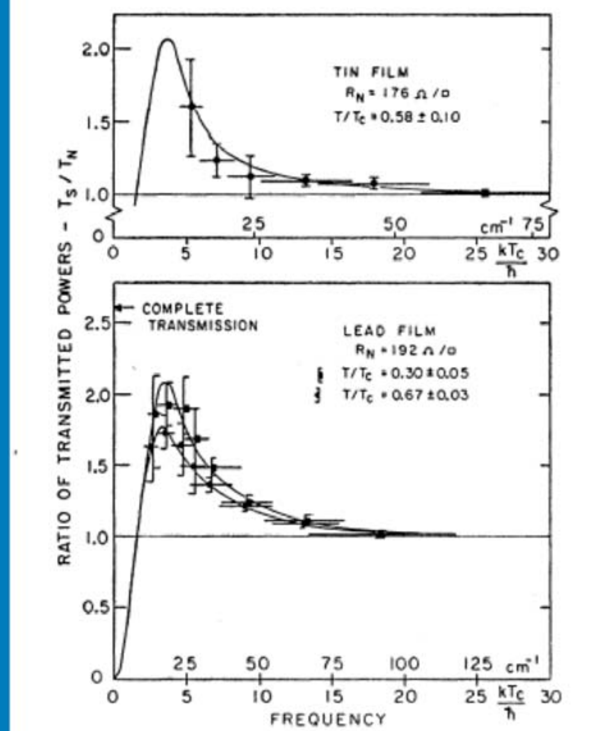


FIG. 2. Experimental ratios of power transmitted through typical lead and tin films in the superconducting and normal states, plotted against frequency. (Throughout the paper we use the term "frequency" to devote $\omega = \Delta E/\hbar$ rather than $\nu = \Delta E/h$.) The frequency uncertainty on each point is the width at half-power of the continuous spectrum used. The vertical error limits are based on the random scatter in the observed signal averaged for one hour. The curves are calculated by using the universal conductivity function for thin superconducting films discussed in Secs. IV and V.

From Glover and Thinkham (1958)

A feasible alternative approach involves directly the variation ΔF_c of Casimir free energy across the transition:

$$\Delta F_c = F_c^{(n)} - F_c^{(s)} \neq 0$$

Indeed ΔF_c is expected to be positive, because, in the superconducting state, the film should be closer to behave as an ideal mirror than in the normal state, and so F_c (s) should be more negative than F_c (n) .

Is there a way to measure ΔF_c?

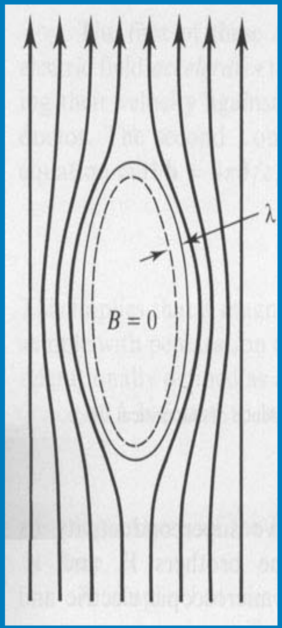
YES: measure the (parallel) critical magnetic field $H_{c\parallel}$ that destroys the superconductivity of the film.

Since the Casimir energy favours the superconducting state, the critical field of a film in a cavity should be a bit LARGER than that of a similar bare film

A CRUCIAL POINT: since the shift of the critical field depends on an energy scale (the film condensation energy E_{cond}) which is orders of magnitude smaller than the Casimir energies F_c, we can achieve huge sensitivities

Magnetic properties of superconductors (s)

- Meissner effect: superconductors show perfect diamagnetsim.
- Superconductivity is destroyed by magnetic fields $H>H_c$.



The magnitude of critical field depends on the shape of the sample and on the direction of the field. For a thick flat slab in a parallel field, it is called thermodynamical field and is denoted as H_c.

The value of H_c is obtained by equating the magnetic work (per unit volume) required to expel the magnetic field with the condensation energy (density) $e_{cond}(T)$ of the superconductor.

$$\frac{H_c^2(T)}{8\pi} = e_{cond}(T)$$

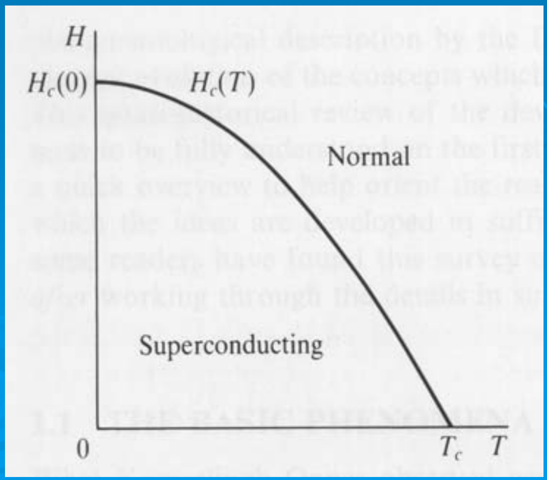
(thick flat slab in parallel field)

$$e_{cond}(T) = f_n(T) - f_s(T)$$

$f_{n/S}(T)$: density of Helmoltz free energy in the n/s state

$H_c(T)$ follows an approximate parabolic law

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$



In a thin films (thickness << penetration depth), expulsion of magnetic field is incomplete, and so magnetic work is smaller. Therefore the critical field is larger than the thermodynamical field H_c by a factor ρ.

$$H_{c\parallel} = \rho H_c$$

Landau-Ginzburg theory gives for ρ the value (λ >> D) :

$$\rho = \sqrt{24}\,\frac{\lambda}{D}\left(1 + \frac{9D^2}{\pi^6 \xi^2}\right)$$

ξ= correlation length,
λ = penetration depth

Typically, for classic pure superconductors λ ≅ 500 Å ξ ≅ 3000 Å

Then, by recalling the previous equation relating H_c to e(T), we obtain

$$\frac{V}{8\pi}\left(\frac{H_{c\parallel}(T)}{\rho}\right)^2 = E_{cond}(T)$$

where V is the volume occupied by the film and $E_{cond}(T) = V\, e(T)$ is its condensation energy

Superconducting film in a Casimir cavity

When the superconducting film is placed inside the cavity, the condensation energy E_{cond} of the film has to be augmented by the difference ΔF_c among the Casimir free energies

$$\frac{V}{8\pi}\left(\frac{H_{c\parallel}(T)}{\rho}\right)^2 = E_{cond}(T) \longrightarrow \frac{1}{8\pi}\left(\frac{H_{c\parallel}^{(cav)}(T)}{\rho}\right)^2 V = E_{cond} + \Delta F_c$$

ΔF_c causes a shift of critical field δH_c:

$$\frac{\delta H_c}{H_{c\parallel}} \approx \frac{1}{2}\frac{\Delta F_c}{E_{cond}}$$

We can achieve high sensitivities because E_{cond} is orders of magnitude smaller than typical Casimir free energies F_c .

For a Be film with a T_c= 0.5 K
A=1 cm^2 , D=5 nm, T/T_c=0.97

$$E_{cond} = 3.5\times 10^{-8}\ erg$$

For a plane parallel cavity with
A=1 cm^2 L=10 nm

$$F_c \approx E_c = -\frac{\pi^2}{720}\frac{\hbar c A}{L^3} = -0.43\ erg$$

F_c is 10 million times larger than E_{cond}!

So even a tiny fractional change of F_c can be large compared with E_{cond}, and cause a measurable shift of critical field.

Time varying fields in superconductors

While dc currents flow in superconductors with no resistance, superconductors show finite dissipation when traversed by time-varying e.m. fields.

At frequencies $\omega << \Delta(T) / \hbar$ the qualitative behavior of superconductors is described by the Casimir-Gorter two-fluid model, which assumes that the total electron density n is divided in two parts: the density n_s of superconducting electrons and the density n_n of normal electrons.

$$\sigma'(\omega) = (\pi n_s e^2/2m)\,\delta(\omega) + (n_n e^2 \tau_n/m)\,(1+\omega^2\tau_n^2)^{-1} ,$$

The presence of normal electrons is the cause of dissipation. The δ–function is a dc contribution accounting for superconductive electrons

The model assumes: $n_n \propto (T/T_c)^4, \quad n_s \propto 1-(T/T_c)^4$

As T is decreased below T_c there are less and less normal electrons and so there is less and less dissipation

An accurate description, valid for frequencies upto a few times Δ(T), is provided by the Mattis-Bardeen permittivity of BCS theory.

BCS conductivity in the local limit

$$\sigma(\omega) = \kappa\delta(\omega) + \hat{\sigma}_s(\omega)$$

$$\sigma(\omega) = \sigma'(\omega) + i\sigma''(\omega)$$

$$\hat{\sigma}'_s(\omega) = \frac{\hbar n e^2}{2m\omega\tau_n}\left[\int_{\Delta}^{\infty} dE\ J_T + \theta(\hbar\omega - 2\Delta)\int_{\Delta-\hbar\omega}^{-\Delta} dE\ J_D\right]$$

$$J_T\ :=\ g(\omega,\tau_n,E)\left[\tanh\frac{E+\hbar\omega}{2kT} - \tanh\frac{E}{2kT}\right]$$

$$J_D\ :=\ -g(\omega,\tau_n,E)\ \tanh\left(\frac{E}{2kT}\right)$$

$\Delta(T)$ is the BCS gap.

$$\begin{aligned} g\ :=\ &\left[1 + \frac{E(E+\hbar\omega)+\Delta^2}{P_1P_2}\right]\frac{1}{(P_1-P_2)^2+(\hbar/\tau_n)^2} \\ &- \left[1 - \frac{E(E+\hbar\omega)+\Delta^2}{P_1P_2}\right]\frac{1}{(P_1-P_2)^2+(\hbar/\tau_n)^2} \end{aligned}$$

$$P_1 := \sqrt{(E+\hbar\omega)^2-\Delta^2}\,,\quad P_2 := \sqrt{E^2-\Delta^2}$$

The coefficient κ of the δ-function is fixed such as to satisfy the oscillator sum rule.

$$\int_0^{\infty} d\omega\sigma'(\omega) = \frac{\pi n e^2}{2m}$$

For $T \rightarrow T_c$ the BCS conductivity reduces to the Drude model

An important quantity is the so-called impurity parameter

$$y = \frac{h}{2\tau_n\Delta(T)}$$

For y »1 (dirty limit) non-local corrections are small. In ultrathin films typically y »1 .

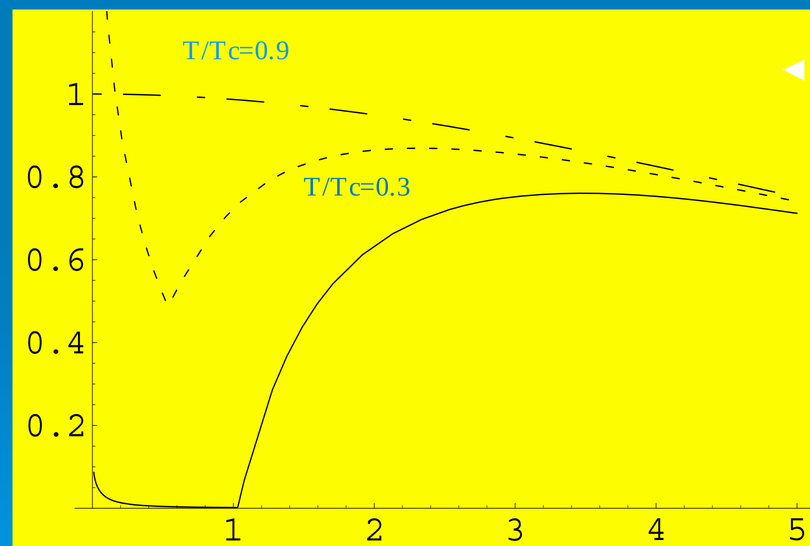


FIG. 8: Plots of $m\,\sigma'_s(\omega)/(ne^2\tau_n)$, for $T/T_c = 0.3$ (solid line), $T/T_c = 0.9$ (dashed line) and $T = T_c$ (point-dashed line). On the abscissa, the frequency ω is in reduced units $x_0 = \hbar\omega/(2\Delta(0))$, and $y_0 = 2\Delta(0)/\tau_n \simeq 8.7$.

Computation of ΔF_c: the Casimir effect in real materials.

The theory for dispersive media was developed by **Lifshitz (1956).** He computed the Casimir energy F_c in a dielectric cavity, by evaluating the v.e.v. of the e.m. stress-energy tensor in the empty space between the plates.
Alternative methods exist today (summation on evanescent modes, Green functions etc).

Lifshitz assumed local electrodynamics: i.e. that one can describe the propagation of e.m. waves by a complex permittivity $\varepsilon(\omega)$, depending only on the frequency ω, and not on the wave-vector **q**.

Today we know that Lifshitz formula for the Casimir free energy is valid also in spatially dispersive Media, provided one uses the proper expression for the reflection coefficients of the slabs.

As the optical properties of a superconducting film differ from those of a normal film only for photon energies smaller than a few times $k_B T_c$, the computation of ΔF_c **involves modes with characteristic wavelengths of order**

$$\lambda_{char} = \hbar c / k_B T_c \approx 2 \text{ mm} \qquad \text{for } T_c = 1 \text{ K}$$

which belong to the microwave domain, where normal metals show an anomalous skin effect. At cryogenic temperatures the anomalous region may further extend, because of longer electrons mean-free paths.

In the superconducting state of the film, non-local effects may be even more important. Indeed, because of the very small skin depth δ of e.m. fields in superconductors , the anomalous skin effect is observed, in clean superconductors, even inside the frequency domain characteristic of the normal skin effect in the normal state (extreme anomalous skin effect) .

However in ultrathin films:

thickness $D \ll$ skin depth δ

non-local effects are much less important than in bulk samples.

A recent study (R. Esquivel-Sirvent and V.B. Svetovoy (2005)) of the Casimir effect for ultrathin (normal) metallic films shows that non-local effects give negligible corrections (percent level or less). This holds especially for TE modes, which we found are the relevant ones for ΔF_C.

In ultrathin films, non local effects are negligible also in the superconducting state, because the electron mean-free path does not become large

For example, in pure Be superconducting films ($D \approx 4.2$ nm, $T_c \approx 0.6$ K)
$l \approx 64$ nm ($\tau_n = l/v_F = 3 \cdot 10^{-14}$ s for $v_F = 2.2 \cdot 10^6$ m/s) (P.W. Adams et al. PRB R2952 (1998))

Therefore, one is in the so-called dirty case, where local electrodynamics remains valid.

This is confirmed in experiments (Glover and Thinkham, 1958) showing that the film conductivity is independent on film thickness, for small thicknesses.

Lifshitz formula for ΔF_c

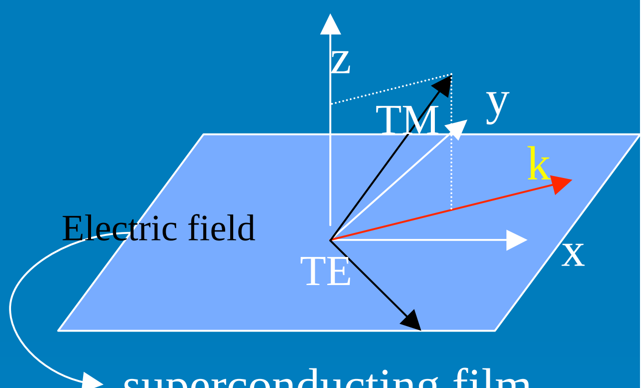
At T=0, the unrenormalized difference of Casimir energies $\Delta E_0^{(C)}$ can be written as a sum over cavity modes:

$$\Delta E_0^{(C)}(L,D) = A\frac{\hbar}{2}\int\frac{dk_1 dk_2}{(2\pi)^2}\left\{\sum_p\left(\omega^{(n,TM)}_{\mathbf{k}_\perp,p}+\omega^{(n,TE)}_{\mathbf{k}_\perp,p}\right) - \sum_p\left(\omega^{(s,TM)}_{\mathbf{k}_\perp,p}+\omega^{(s,TE)}_{\mathbf{k}_\perp,p}\right)\right\}$$

$k=(k_x,k_y)$ is the two-dimensional wave-vector in the plane of the film

$\omega^{(n/s,TM)}_{\mathbf{k}_\perp,p}$ $\left(\omega^{(n/s,TE)}_{\mathbf{k}_\perp,p}\right)$ proper frequencies of the TE (TM) modes in the n/s states of the film

(TE (TM) modes: electric field perpendicular (parallel) to the plane formed by k and the normal to the film).



superconducting film

By use of the argument theorem, the renormalized $\Delta_E^{(C)}$ can be written as an integral over imaginary frequencies ζ

$$\Delta E^{(C)} = \frac{\hbar A}{4\pi^2 c^2}\int_1^\infty p\,dp\int_0^\infty d\zeta\,\zeta^2\left(\log\frac{Q_n^{TE}}{Q_s^{TE}}+\log\frac{Q_n^{TM}}{Q_s^{TM}}\right)$$

$$k_\perp^2 = (p^2-1)\varsigma^2/c^2$$

The coefficients $Q_{n/s}^{TE/TM}$ are functions of the reflection coefficients $\Delta_{ij}^{TE/TM}$ for the i-j interface

$$Q_I^{TE/TM}(\zeta,p)=\frac{(1-\Delta_{1I}^{TE/TM}\Delta_{12}^{TE/TM}e^{-2\zeta pL/c})^2-(\Delta_{1I}^{TE/TM}-\Delta_{12}^{TE/TM}e^{-2\zeta pL/c})^2e^{-2\zeta K_I D/c}}{1-(\Delta_{1I}^{TE/TM})^2e^{-2\zeta K_I D/c}},$$

$$\Delta_{jl}^{TE}=\frac{K_j-K_l}{K_j+K_l},\quad \Delta_{jl}^{TM}=\frac{K_j\epsilon_l(i\zeta)-K_l\epsilon_j(i\zeta)}{K_j\epsilon_l(i\zeta)+K_l\epsilon_j(i\zeta)},\quad K_j=\sqrt{\epsilon_j(i\zeta)-1+p^2},\quad I=n,s\ ;\ j,l=1,2,n,s.$$

At finite T the integral over complex frequencies is replaced a sum over the Matsubara modes $\zeta_l = 2\pi\, l\, k_B T / \hbar$

$$\Delta F_C=\frac{Ak_BT}{4\pi c^2}\sum_{l=-\infty}^{\infty}\varsigma_l^2\int_1^{\infty}p\,dp\left(Log\frac{Q_n^{TE}}{Q_s^{TE}}+Log\frac{Q_n^{TM}}{Q_s^{TM}}\right)(\varsigma_l,p)$$

The ratios $Q_n^{TE/TM}/Q_s^{TE/TM}$ approach one fast for $\zeta_l > 2\Delta/\hbar$ and thus the range of summation is effectively restricted to modes with energy less than a few times $2\Delta(T)$.

Choices of permittivities for the layers

1) We took the permittivity $\varepsilon_1(\omega)$ of the insulating layers as constants.

2) For the outer plates and for the film in the normal state, we used the Drude model. This is a valid approximation in the relevant microwave region.

$$\varepsilon(i\varsigma) = 1 + \frac{\Omega_P^2}{\varsigma(\varsigma+\gamma)}$$

$\gamma=1/\tau$ τ = relaxation time
Ω_p plasma frequency

3) For the film in the superconducting state we used the BCS expression for the conductivity (analytically continued to the imaginary axis)

Results of numerical computations of ΔF_c

The contribution of the TM modes to ΔF_c is negligible with respect to the TE modes (by three orders of magnitude).

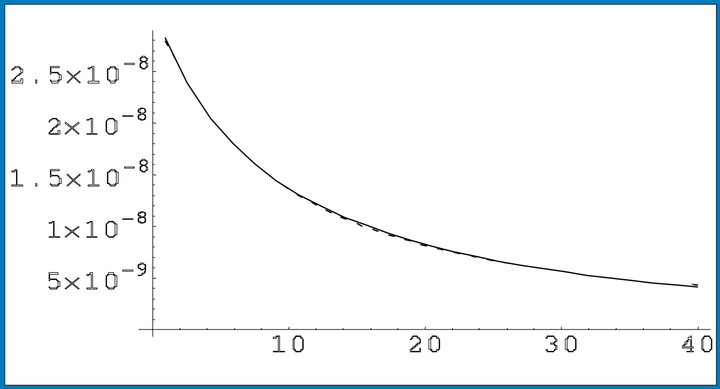
ΔF_c is practically independent (to better than four digits) on the value of the dielectric constant of the insulating gaps.

ΔF_c increases with the film thickness D, and saturates for $D \cong c/\Omega_p \cong 10$ nm.

4) ΔF_c increases when the gap separation L decreases, and approaches a finite limit for $L \to 0$.

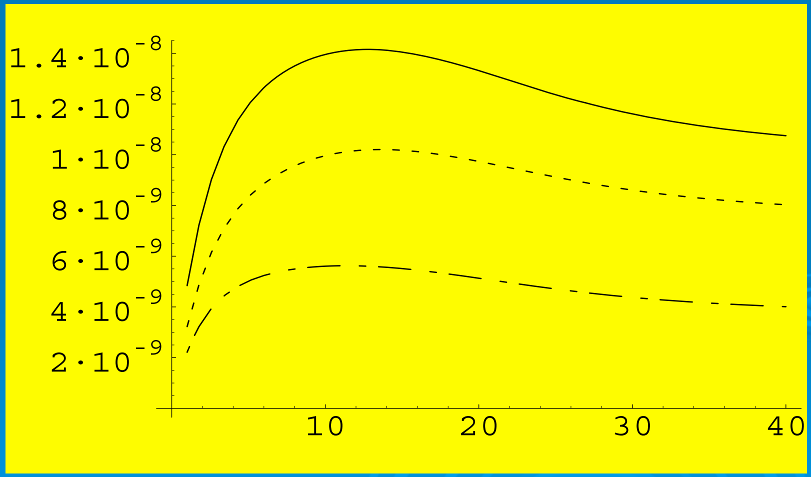
5) ΔF_c increases significantly with the plasma frequencies of the film (Ω_n) and of the outer normal plates (Ω_2).

6) ΔF_c has a maximum for values around 10 of the impurity parameter y



ΔF_c (in erg) as a function of L (in nm) . Also shown (solid line) is a fit of equation:

$$\Delta F_c \propto \frac{1}{1+(L/L_0)^{\alpha}}$$



ΔF_c (in erg) as a function of y, for $\Omega_n=\Omega_2=18.9$ eV (solid line), $\Omega_n=18.9$ eV, $\Omega_2=12$ eV (dashed line) and $\Omega_n=\Omega_2=12$ eV (point-dashed line)

The TE zero mode contribution

The contribution of the TE zero mode to the Matsubara sum involves the quantity C

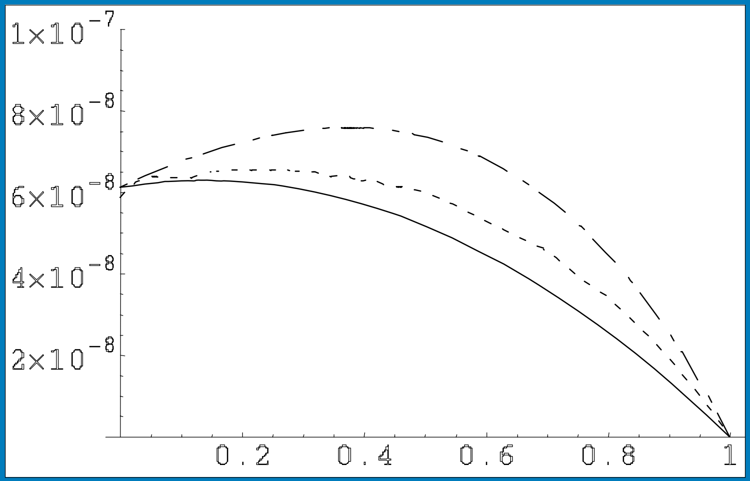
$$C = \lim_{\varsigma \to 0} (\varsigma^2 \varepsilon(i\varsigma))$$

If one uses the Drude function, C=0, and one gets no contribution from the zero mode.
If however one uses the plasma model

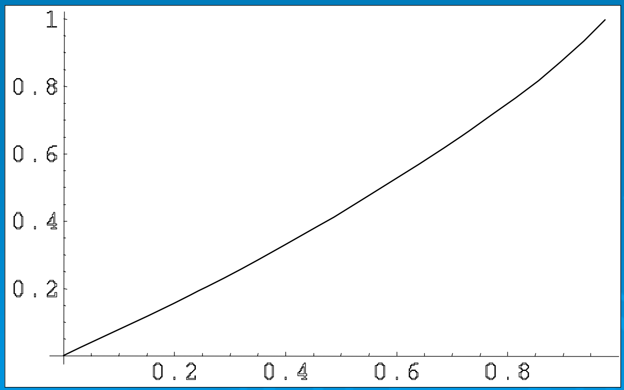
$$\varepsilon(i\varsigma) = 1 + \frac{\Omega_p^2}{\varsigma^2}$$

$C=\Omega_p^2$ and the zero mode contributes.

The relative weight of the zero mode becomes larger near T_c, because the nunber of Matsubara modes contributing to ΔF_c is of the order of a few times T_c/T.



ΔF_c (in erg) as a function of T/T_c.
a) Drude model for the outer plates (solid line)
b) Plasma model for the outer plates (point-dashed line)
c) T→0 approximation for the Matsubara series (solid line)



Relative weight of the TE zero mode, as a function of T/Tc.

Shift of critical field

$$\frac{\delta H_c}{H_c} \approx \frac{1}{2}\frac{\Delta F_c}{\mathrm{E}_{\mathrm{cond}}}$$

Larger shifts are obtained for superconductors with small E_{cond}, namely small critical fields.

The effect becomes larger close to T_c, because

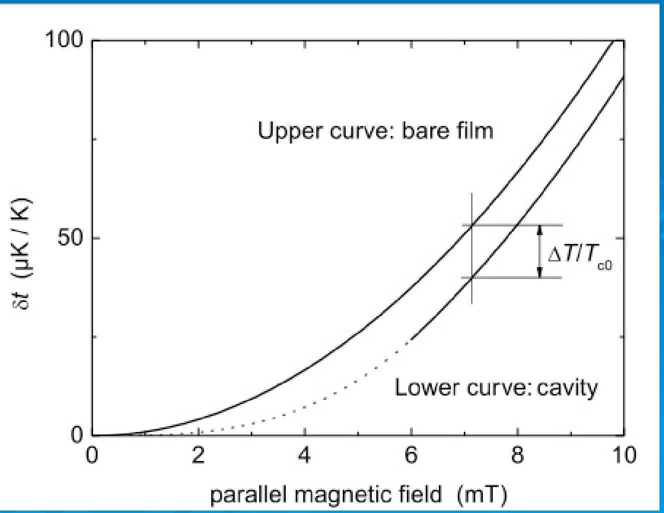
$$\Delta F_c \propto (1 - T/T_c)$$

$$E_{cond} \propto (1 - T/T_c)^2$$

The best material should be Be. In bulk form Tc=24 mK, $H_c(0)$=1 Oe. Ultrathin Be films have higher T_c than bulk samples. For example, for D=5 nm Adams et al. (Phys. Rev.B58 (1998) R2952) report T_c=.5 K. As a rule, E_{cond} scales like T_c^2

Expected signal for an Al-Au system: Al thickness D=14 nm, gap L=6 nm

reduced temperature $t(H)=T_c(H) / T_c$



The Aladin experiment

- D. Born, F. Tafuri - Seconda Università di Napoli and INFN

- E. Il'ichev, U. Huebner - Institute for photonic Technology , Jena (Germany)

- G. Bimonte, E. Calloni, G. Esposito, L. Rosa and R.Vaglio - Università di Napoli Federico II and INFN

Most recent results in the Proceedings Jssue of QFEXT07 (Leipzig) G.Bimonte et al. J. Phys. A 41 (2008) 164023

Preliminary results
Al-Al_2O_3-Au system

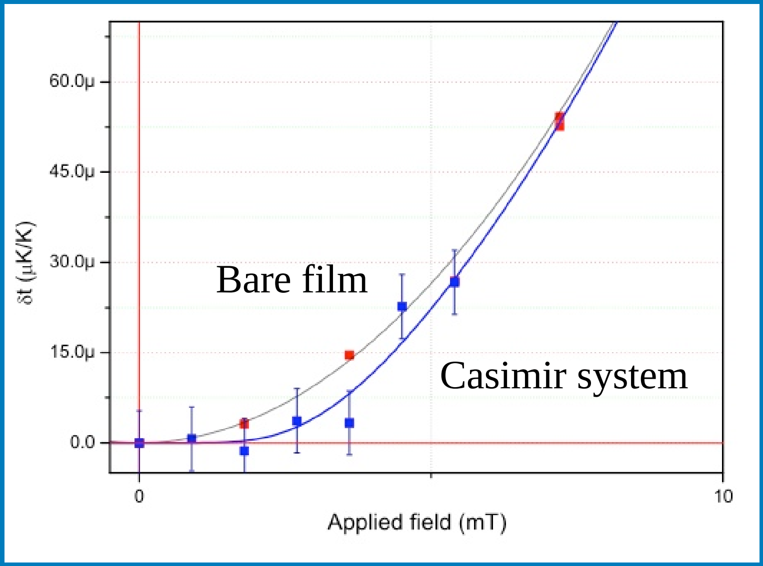
The bare film and the cavity are deposited at close distance on the same chip, to ensure that both feel the same H field

In our measurements we fix H and slowly cool the system. For each H, we measure the critical temperatures $T_c(H)$ for the Casimir system and the bare film.

We compare the respective reduced temperatures

$t(H)=T_c(H) / T_c$

where T_c is the critical temperature in zero field



Error bars are 6 μK/K

zero-point case.

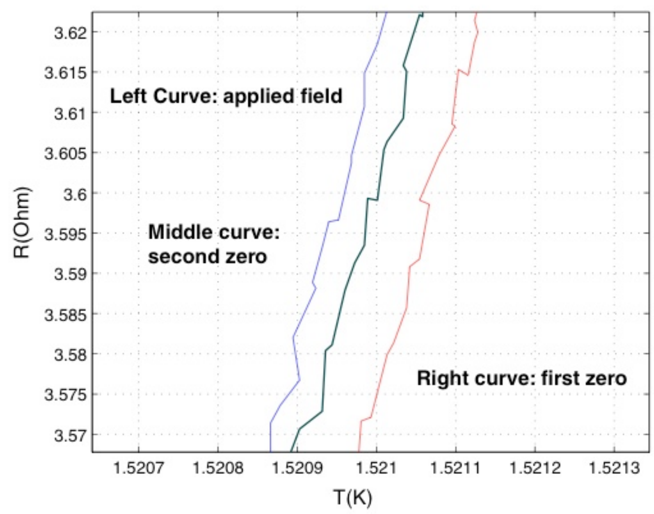


Figure 5. Example of a measurement triplet: a zoom of the resistance versus temperature for two transitions in zero field and one with $\mu_0 H = 7.2$ mT. The mean shift is 80 μK.

More measurements are needed:

1) we plan to check dependence on gap width L
2) we plan to use films with lower T_c (Zn) to get larger signal

References

G. Bimonte, E. Calloni, G. Esposito, L. Milano and L.Rosa, Phys. Rev. Lett. 94 (2005) 180402.

G.Bimonte, E. Calloni, G. Esposito and L.Rosa, Nucl. Phys. B 726 (2005) 441.

G.Bimonte, D. Born, E.Calloni, G. Esposito, U.Huebner,E.Il'ichev,L.Rosa, F. Tafuri and R.Vaglio,
J. Phys. A: Math. Theor. 41 (2008) 164023

Conclusions

- We propose to use superconducting cavities to measure variations of Casimir energy across phase transitions. This is a novel approach, since all experiments so far are force measurements.
- Use of rigid cavities allows realization of many different geometries.
- Possibility of clarifying a current controvery about the contribution of the TE zero mode to the Casimir energy.