Magnetosphere, accretion disks, and companions

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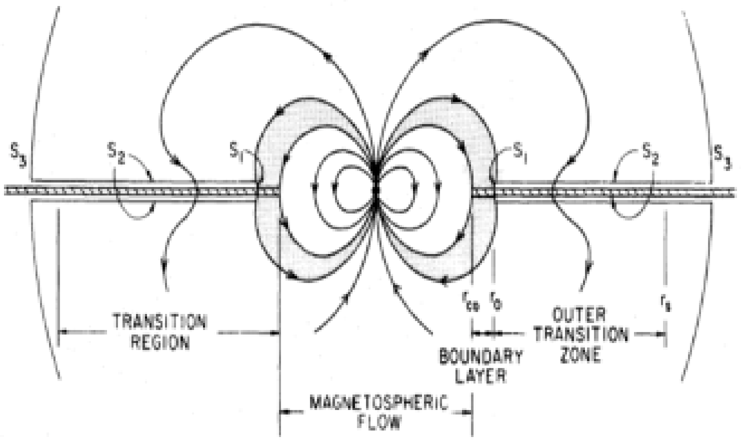
32 slides

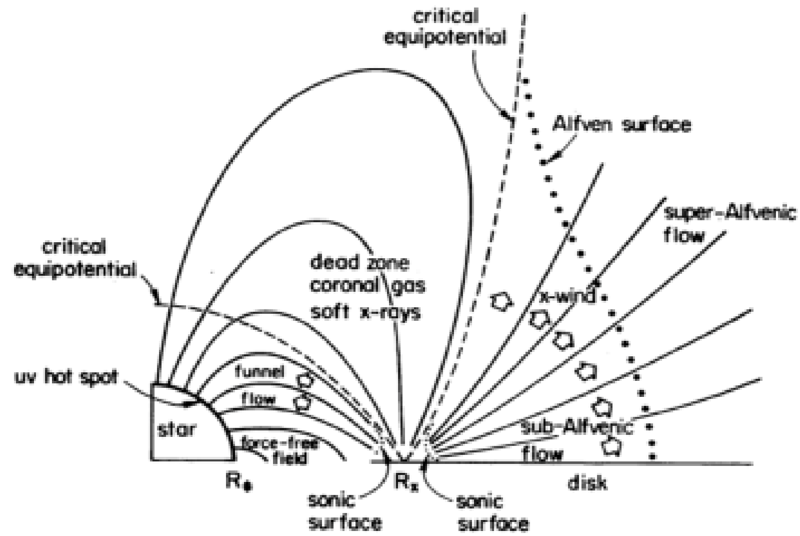
Basic motivations

- Interacting binaries: pulse period, variability, ULX's, neutron star mergers
- T Tauri stars: variability, spin period, accretion vs disks, spin down
- Planet formation: dust retention and accumulation, disk migration, planets' survival
- Satellite formation: planetary spin and satellite retention

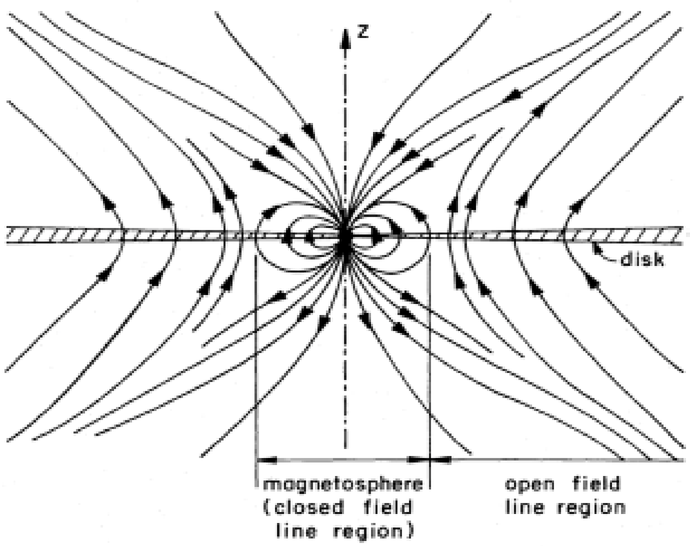
Previous attempts



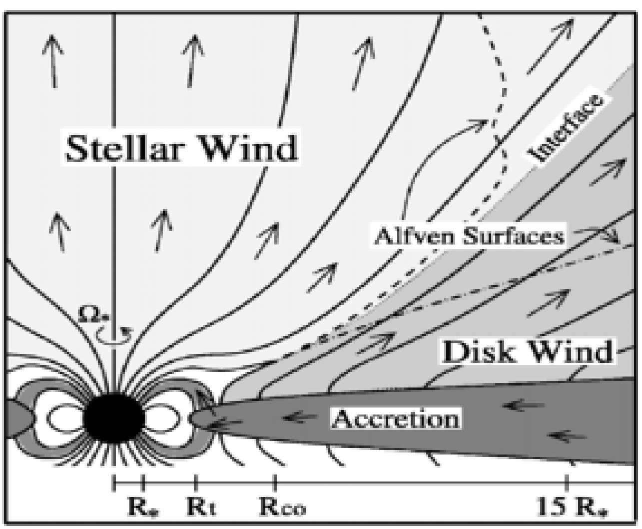
Ghosh & Lamb 1979



Shu et al. 1994

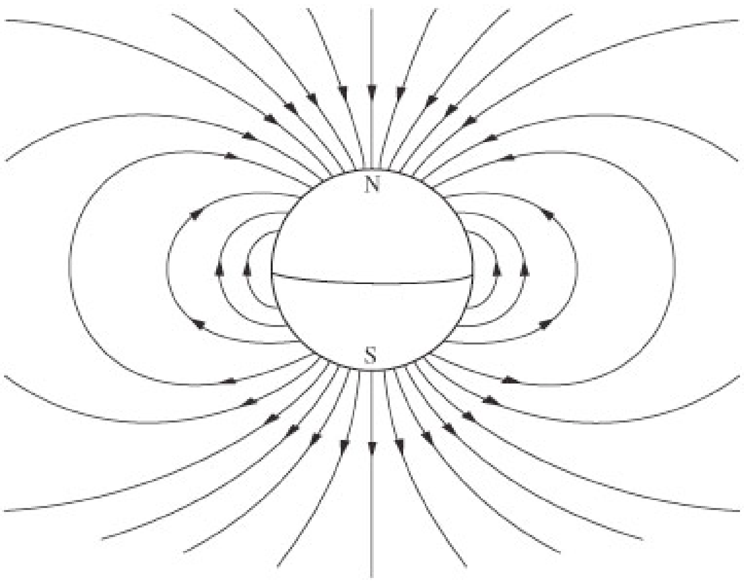


Lovelace et al. 1995



Matt & Pudritz 2005

Magnetized T Tauri stars



Dipole field:
$B=\mu/r^3$

Multipole field: the Sun

Governing equations

Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\rho U_r r + \frac{\partial}{\partial z}\rho U_z = 0,$$

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\Sigma U_r r + \dot{\Sigma}_* + \dot{\Sigma}_d = 0$$

$$\dot{\Sigma}_{*,d} = 2\rho U_z \qquad \dot{M}_d = -2\pi \Sigma U_r r$$

Momentum equation

$$\left(\frac{\partial}{\partial t} + \mathbf{U}\nabla\right)\mathbf{U} = -\frac{\nabla p}{\rho} + \frac{\nabla \sigma_\nu}{\rho} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\rho \mu_0} + \nabla \Psi = -\frac{\nabla p}{\rho} + \frac{\nabla \sigma_\nu}{\rho} + \frac{\mathbf{T}_B - \nabla P_B}{\rho \mu_0} + \nabla \Psi$$

where $\mathbf{T}_B = \mathbf{B}\nabla\mathbf{B}$ is the magnetic tension and $P_B = |B|^2/2$ is the magnetic pressure.

Components of the momentum equation

$$-\Omega^2 r = -\frac{U_\phi^2}{r} = -\frac{c_s^2}{\rho}\frac{\partial\rho}{\partial r} - \frac{GM_*}{r^2} + \frac{1}{\rho\mu_0}\left(B_z\frac{\partial B_r}{\partial z} - B_z\frac{\partial B_z}{\partial r} - \frac{B_\phi}{r}\frac{\partial B_\phi r}{\partial r}\right)$$

$$U_z\frac{\partial U_z}{\partial z} + U_r\frac{\partial U_z}{\partial r} = -\frac{c_s^2}{\rho}\frac{\partial\rho}{\partial z} - \Omega_k^2 z + \frac{1}{\rho\mu_0}\left(T_{Bz} - \frac{\partial P_{Bz}}{\partial z}\right),$$

$$\frac{U_r}{r}\frac{\partial\Omega r^2}{\partial r} = \frac{1}{\rho r^2}\frac{\partial}{\partial r}\rho\nu r^3\frac{\partial\Omega}{\partial r} + \frac{1}{\rho\mu_0}\left(\frac{B_r}{r}\frac{\partial r B_\phi}{\partial r} + B_z\frac{\partial B_\phi}{\partial z}\right) - U_z\frac{\partial\Omega r}{\partial z}$$

Ad hoc assumption 1: effective viscosity

$$\nu = \alpha_\nu c_s^2/\Omega$$

Unperturbed stellar and induced fields

$$\mathbf{B} = \mathbf{B}_* + \mathbf{B}'$$

aligned stellar dipole field

$$B_{\phi *} = 0, \qquad B_{r*} \simeq \frac{3m_* z}{r^4}\left(1 - \frac{5z^2}{2r^2}\right) \qquad B_{z*} \simeq \frac{m_*}{r^3}\left(\frac{9z^2}{2r^2} - 1\right) \qquad \text{at} \quad z << r$$

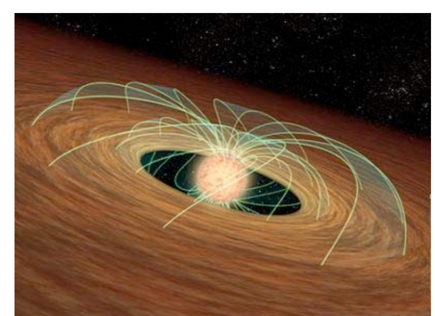
Induction equation for perturbed field in the disk

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \eta_D (\nabla \times \boldsymbol{B})$$

In the limit of negligible diffusion (high conductivity)

$$\frac{\partial B'_\phi}{\partial t} \simeq \frac{\partial(\Omega - \Omega_*) r B_{z*}}{\partial z} + \frac{\partial(\Omega - \Omega_*) r B_{r*}}{\partial r} \sim -\frac{m_*}{r^2}\frac{\partial(\Omega - \Omega_*)}{\partial z} - \frac{9m_*\Omega z}{2r^4}$$

$$\tau_{BI} \simeq \frac{B_{z*}}{\partial B'_\phi / \partial t} \simeq \frac{2r}{9H_T\Omega} \simeq \frac{2r}{9c_s} = \frac{P}{9\pi h} \sim P$$



Induction-diffusion equilibrium

Time independent, axisymmetric state

$$\frac{\partial}{\partial z}\left(U_z B_r - U_r B_z\right) = \frac{\partial}{\partial z}\left[\eta_D\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\right],$$

$$\frac{\partial}{\partial r}\left[r\left(U_z B_r - U_r B_z\right)\right] = \frac{\partial}{\partial r}\left[r\eta_D\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\right],$$

$$\frac{\partial}{\partial z}\left[(\Omega - \Omega_*) r B_z - U_z B_\phi\right] - \frac{\partial}{\partial r}\left[U_r B_\phi - (\Omega - \Omega_*) r B_r\right] = -\frac{\partial}{\partial z}\left(\eta_D \frac{\partial B_\phi}{\partial z}\right) - \frac{\partial}{\partial r}\left(\frac{\eta_D}{r}\frac{\partial r B_\phi}{\partial r}\right)$$

$$\nabla \mathbf{B} = 0 \qquad \text{so that} \qquad \frac{\partial B_z}{\partial z} = -\frac{1}{r}\frac{\partial r B_r}{\partial r}$$

Thin-disk (z < < r) approximation

Ad hoc assumption 2: $\mathbf{B}' < \mathbf{B}_*$

Neglict the r and z dependence in $P_m = \frac{U_r r}{\eta_D}$ (magnetic Prandlt number),

$$\frac{\partial B_z}{\partial z} = \frac{\partial (B_{z*} + B'_z)}{\partial z} = \frac{3(3+P_m)m_*}{r^4}\frac{z}{r} \qquad B_z = B_{z*} + B'_z = \frac{m_*}{r^3}\left(\frac{3z^2}{2r^2}(3+P_m) - 1\right)$$

$$\frac{\partial B_r}{\partial z} = \frac{\partial (B_{r*} + B'_r)}{\partial z} = \frac{m_*}{r^4}\left(3 + P_m - \frac{3z^2}{r^2}\left(\frac{15}{2} + 4P_m + \frac{P_m^2}{2}\right)\right)$$

$$B_r = B_{r*} + B'_r = \frac{m_* z}{r^4}\left[3 + P_m - \frac{z^2}{r^2}\left(\frac{15}{2} + 4P_m + \frac{P_m^2}{2}\right)\right] \simeq \frac{(3+P_m)m_* z}{r^4}$$

$$\frac{\partial B_\phi}{\partial z} = \frac{m_*(\Omega - \Omega_*)}{\eta_D r^2}\left(1 - \frac{3z^2}{2r^2}(3+P_m)\right) - \frac{m_*}{\eta_D}\left[\int \frac{\partial}{\partial r}\left(\frac{\Omega - \Omega_*}{r^3}\right)(3+P_m) z\,dz + \int \frac{dz}{r} r \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\int (\Omega - \Omega_*)\frac{dz}{r}\right].$$

Lowest order terms in z

Lorentz force near the midplane

$$\Omega^2 = \frac{GM_*}{r^3} + \frac{c_s^2}{r^2}\frac{\partial \ln\rho}{\partial \ln r} + \boxed{\frac{m_*^2}{\rho\mu_0 r^8}}\Bigg[\boxed{P_m} - \frac{z^2}{r^2}\left(9P_m + 3P_m^2 - \frac{(\Omega-\Omega_*)^2 r^4}{\eta_D^2}\frac{\partial \ln(\Omega-\Omega_*)/r^2}{\partial \ln r}\right)\Bigg]$$

$$U_r = \frac{1}{\rho\partial\Omega/\partial r}\frac{\partial}{\partial r}\rho\nu r^3\frac{\partial\Omega}{\partial r} - \boxed{\frac{2U_z r}{\Omega}\frac{\partial(\Omega-\Omega_*)}{\partial z}} - \boxed{\frac{2m_*^2}{\mu_0\rho\Omega r^7}\left(\frac{(\Omega-\Omega_*)r^2}{\eta_D}\right)}$$
$$+ \frac{2m_*^2(3+P_m)}{\mu_0\rho\Omega r^7}\left(\frac{(\Omega-\Omega_*)r^2}{\eta_D}\right)\frac{z^2}{r^2}\left(\frac{1}{2} + \frac{\partial\ln(\Omega-\Omega_*)}{\partial \ln r}\right).$$

$$\frac{1}{r}\frac{\partial}{\partial r}\rho U_r r + \frac{\partial}{\partial z}\rho U_z = 0 \quad \text{or} \quad U_z\frac{\partial U_z}{\partial z} = -\frac{U_z^2}{\rho}\frac{\partial\rho}{\partial z} - \frac{U_z}{\rho r}\frac{\partial}{\partial r}\rho U_r r$$

Trans-magneto-sonic flow normal to the disk

$$\frac{(c_s^2 - U_z^2)}{\rho}\frac{\partial\rho}{\partial z} = \frac{U_z^2}{\rho r}\frac{\partial(\rho U_r r/U_z)}{\partial r} - \Omega_k^2 z$$
$$- \boxed{\frac{m_*^2}{\rho\mu_0 r^7}}\left[\boxed{\frac{z}{r}P_m\left(3+P_m\right.} - \left.\frac{z^2}{r^2}(21 + 9P_m + 2P_m^2)\right) + \frac{(\Omega-\Omega_*)r^2}{\eta_D}\frac{r}{\eta_D}\int(\Omega-\Omega_*)dz\right].$$

Magnetic diffusivity

Plasma microscopic diffusivity

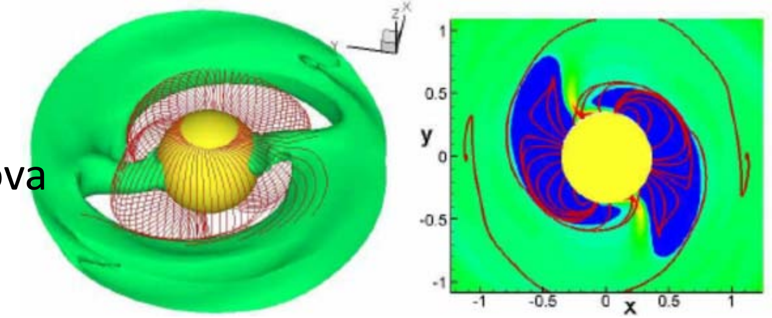
$$\eta_{Dm} = c^2/4\pi\sigma_e \qquad \text{where} \qquad \sigma_e = n_e e^2/m_e \nu_{eN},$$

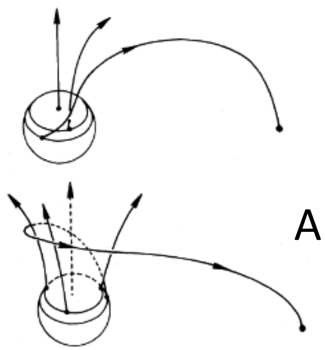
$$\nu_{eN} = n < \sigma v >_{eN} \qquad \text{where} \qquad < \sigma v >_{eN} = 10^{-15}(128kT/9\pi m_e)^{1/2}$$

$$\frac{\tau_{BI}}{\tau_{B\eta}} \sim \frac{0.1}{\xi}\frac{\text{cm}}{r} \qquad \eta_{Dm} \simeq 200\xi^{-1}T^{1/2}\text{cm}^2\text{s}^{-1} \qquad \text{where} \qquad \xi = \frac{n_e}{n}$$

If $\eta_D = \eta_{Dm}$ and $\tau_{B\eta} > \tau_{BI}$, $(\Omega_m - \Omega_*)$ would induce and amplify a toloidal $B_\phi \geq B_{z*}$ before an induction/diffusion equilibrium can be established by microscopic diffusion.

Romanova





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Ad hoc assumption 3: anomalous diffusivity

An ad hoc prescription for an anomalous magnetic diffusivity to establish a $B_\phi \sim B_{z*}$ equilibrium

$$\eta_D \sim \alpha_D H_T^2 \Omega_* (1 + g_c) \sim \frac{\alpha_D H_T r |\Omega - \Omega_*|}{f_c} \qquad \text{where} \qquad g_c \equiv \frac{|\Omega - \Omega_*| r}{\Omega_* H_T}, \qquad f_c \equiv \frac{g_c}{1 + g_c}.$$

Relative magnitude of numbers

$$R_m = \frac{|\Omega - \Omega_*| r H_T}{\eta_D} \sim \frac{f_c}{\alpha_D} \sim 1 \qquad P_{\nu\eta} = \frac{\nu}{\eta_D} \simeq \frac{\alpha_\nu}{\alpha_D(1+g_c)} \leq 1.$$

$$P_m = \frac{f_c U_r}{\alpha_D H_T |\Omega - \Omega_*|} = \frac{U_r r}{\alpha_D H_T^2 \Omega_* (1+g_c)}.$$

$$P_m = \frac{U_r r}{\nu} \frac{\alpha_\nu}{\alpha_D(1+g_c)} \simeq P_{\nu\eta} \qquad \text{viscous dominant region}$$

$$P_m > P_{\nu\eta} \qquad \text{magnetic dominant region}$$

$\eta_D > \eta_{DM}$ if $n_e / n > 10^{-2} / H \sim 10^{-12}$

P_m becomes negligible in the dead zone where $n_e / n << 10^{-12}$

Location of corotation (between magnetosphere and disk gas)

$$R_c = \left(\frac{GM_*}{\Omega_*^2}\right)^{1/3} \qquad \text{Keplerian corotation radius}$$

$$\Omega^2(r,z) \simeq \Omega_k^2 + \frac{c_s^2}{r^2}\frac{\partial \ln\rho}{\partial \ln r} + \boxed{\frac{m_*^2}{\rho(r,z)\mu_0 r^8}\left[P_m\right.} \left. + \frac{f_c^2}{\alpha_D^2}\frac{z^2}{H_T^2}\frac{\partial \ln|\Omega-\Omega_*|/r^2}{\partial \ln r}\right]$$

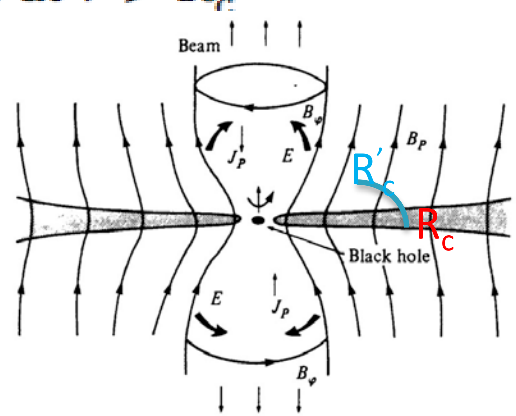
Modified corotation radius R_c' where $\Omega(r,0) = \Omega_*$.

$$S_\Omega = \frac{\Omega-\Omega_*}{|\Omega-\Omega_*|} \qquad S_\Omega = 1 \text{ at } r \le R_c' \qquad S_\Omega = -1 \text{ at } r > R_c'$$

Corotation can occur above the plane at $z = z_{cr}$ where

f_c=g_c= 0

$$\boxed{\rho_{\Omega_*} = \frac{m_*^2 P_m}{\mu_0 r^8 \Omega_*^2 (1 - R_c^3/r^3)}}$$



provided $U_r < 0$ (or $P_m < 0$) at $r < R_c$ or $U_r > 0$ (or $P_m > 0$) at $r > R_c$

Transition from Keplerian flow to corotation

In most regions of a thin disk, $|U_z| < |U_r|$

$$\left(\frac{c_s^2 - U_z^2}{\Omega_k^2}\right)\frac{\partial\rho}{\partial z} \simeq -\rho z - \frac{m_*^2 z[3P_m(3+P_m) + f_c^2 r^2/\alpha_D^2 H_T^2]}{\mu_0 r^8 \Omega_k^2}.$$

Gas is confined by gravity and magnetic pressure. In the range $0 > P_m > -3$, the magnetic tension due to the induced field exerts force away from the mid plane. But, the magnetic pressure dominates over magnetic tension at $z < z_{cr}$ so that $U_z^2 << c_s^2$ and

$$\rho(r,z) \simeq \rho(r,0)\exp\left(-\frac{z^2}{2H_T^2}\right) + \frac{m_*^2}{\mu_0 r^8 \Omega_k^2}\left[3P_m(3+P_m) + \frac{f_c^2 r^2}{\alpha_D^2 H_T^2}\right]\left[\exp\left(\frac{z^2}{2H_T^2}\right) - 1\right].$$

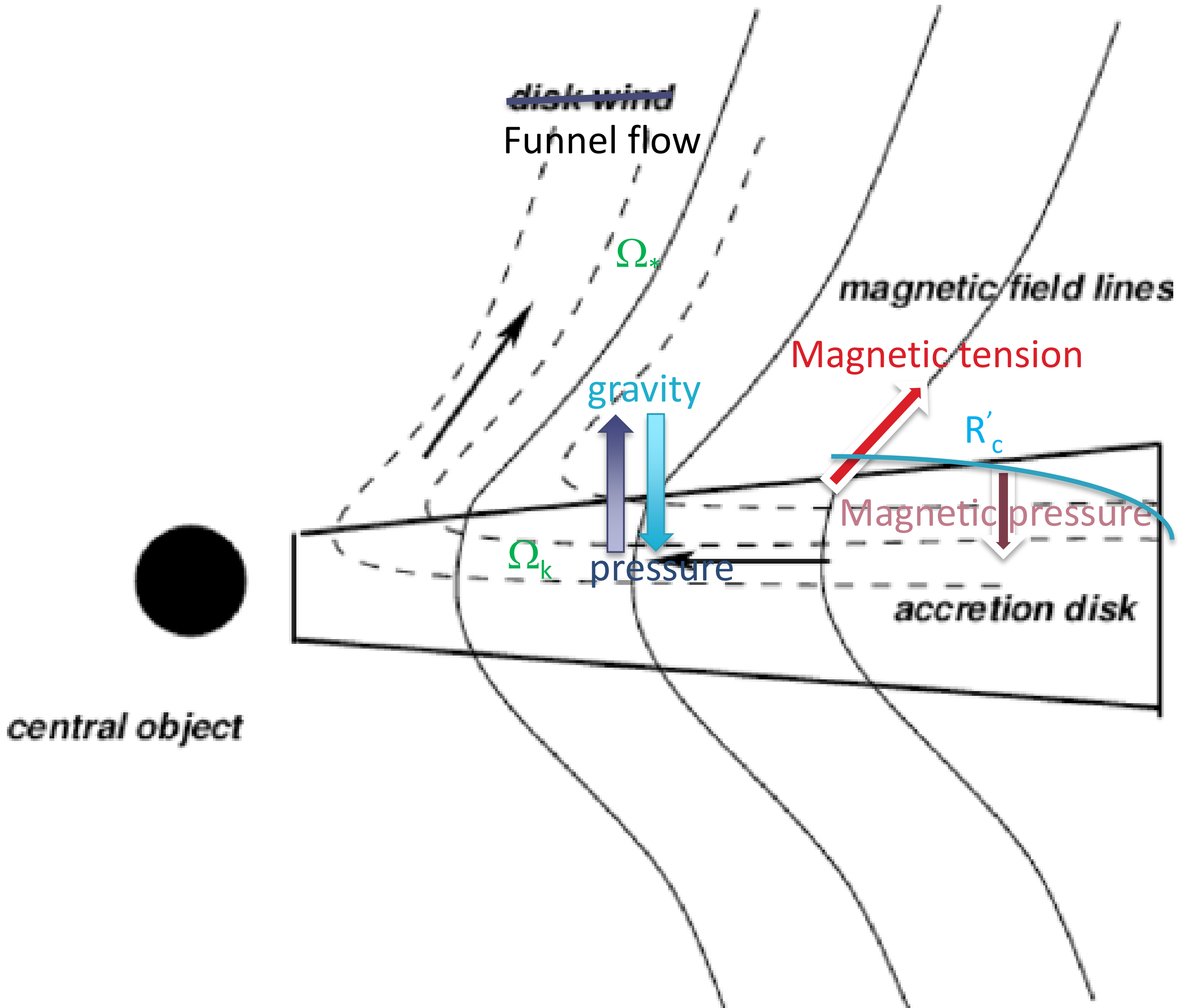
With an isothermal equation state,

$$\rho(r,0) = \frac{\sqrt{2}\pi\Sigma}{H_T} \qquad \text{and} \qquad H_T = \left(\frac{R_g T}{\mu\Omega_k^2}\right)^{1/2} = \frac{c_s}{\Omega_k},$$

Corotation $\Omega(r,z) \simeq \Omega_*$ and $f_c(r, z_{cr}) << 1$ occurs with $\rho = \rho_{\Omega_*} < \rho(r,0)$ at

$$z_{cr}^2 = 2H_T^2 \ln\left(\frac{\rho(r,0) + [3P_m(3+P_m) + f_c(r,0)^2 r^2/H_T^2] m_*^2/\mu_0 r^8 \Omega_k^2}{\rho_{\Omega_*} + 3P_m(3+P_m) m_*^2/\mu_0 r^8 \Omega_k^2}\right).$$

Schematic illustration



Transonic funnel flow

Under corotation, magnetic tension leads to transonic ($U_z = c_s$) flow with

$$\rho_{sonic} = -\frac{m_*^2 3P_m(3+P_m)}{\mu_0 r^8 \Omega_k^2}.$$

In the range of $0 > P_m > -3$, $\rho_{sonic} > 0$ can be attained at

$$z_{sonic}^2 \sim 2H_T^2 \ln\left(\frac{\rho(r,0) + [3P_m(3+P_m) + f_c(r,0)^2 r^2/H_T^2] m_*^2/\mu_0 r^8 \Omega_k^2}{\rho_{sonic} + 3P_m(3+P_m) m_*^2/\mu_0 r^8 \Omega_k^2}\right).$$

So far, we assume $f_c << 1$ (i.e. $\Omega(r,z) \simeq \Omega_*$

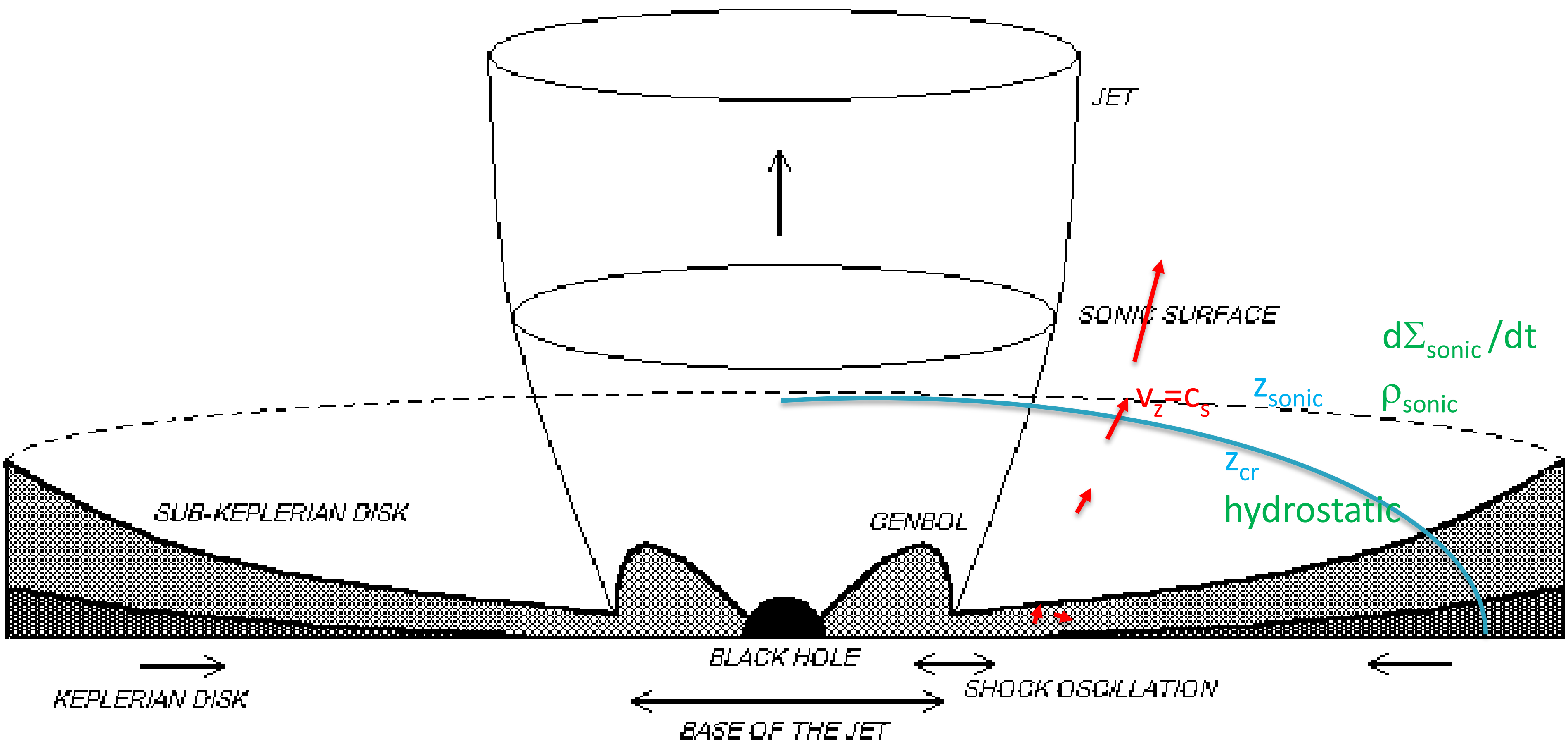
$$\left(\frac{c_s^2 - U_z^2}{\Omega_k^2}\right)\frac{\partial \rho}{\partial z} \simeq -\rho z - \frac{m_*^2 z[3P_m(3+P_m) + \cancel{f_c^2 r^2/\alpha_D^2 H_T^2]}}{\mu_0 r^8 \Omega_k^2}$$

this sufficient condition would be satisfied at z_{sonic} if $z_{sonic} > z_{cr}$ (i.e. $\rho_{sonic} < \rho_{\Omega_*}$).

$$g_\rho(r) \equiv \frac{\rho_{sonic}}{\rho_{\Omega_*}} \simeq 3(3+P_m)\left(1 - \frac{r^3}{R_c^3}\right) \quad \text{and} \quad f_\rho = \frac{g_\rho}{1+g_\rho}$$

If $g_\rho(r) > 1$ (or $f_\rho(r) \sim 1$), $z_{cr} > z_{sonic}$ and the magnetic pressure can only vanish with f_c at $z > z_{cr}$. Thus an outflow is launched at z_{sonic} with the minimum of ρ_{sonic} and ρ_{Ω_*}.

Magneto-sonic transition



Launching a funnel flow

In most regions (with $0 > P_m > -3$), $g_\rho(r) > 1$, $f_\rho(r) \sim 1$, and $z_{cr} > z_{sonic}$, gas flows across the sonic point at z_{cr} with $\rho = \rho_{\Omega_*}$, $U_z = c_s$, and a flux

$$\dot{\Sigma}_{*c} = 2\rho U_z|_{z=z_{cr}} \simeq 2\rho_{\Omega_*} c_s = \left(\frac{2m_*^2 H_T P_m \Omega_k}{\mu_0 r^8 \Omega_*^2 (1 - R_c^3/r^3)} \right).$$

At $R_c > r > [1 - 1/9(3 + P_m)]R_c$, $g_\rho < 1$, $f_\rho \sim g_\rho$ and

$$\dot{\Sigma}_{*s} = 2\rho U_z|_{z=z_{sonic}} \simeq 2\rho_{sonic} c_s \simeq - \left(\frac{6m_*^2 H_T P_m (3 + P_m)\Omega_k}{\mu_0 r^8 \Omega_k^2} \right).$$

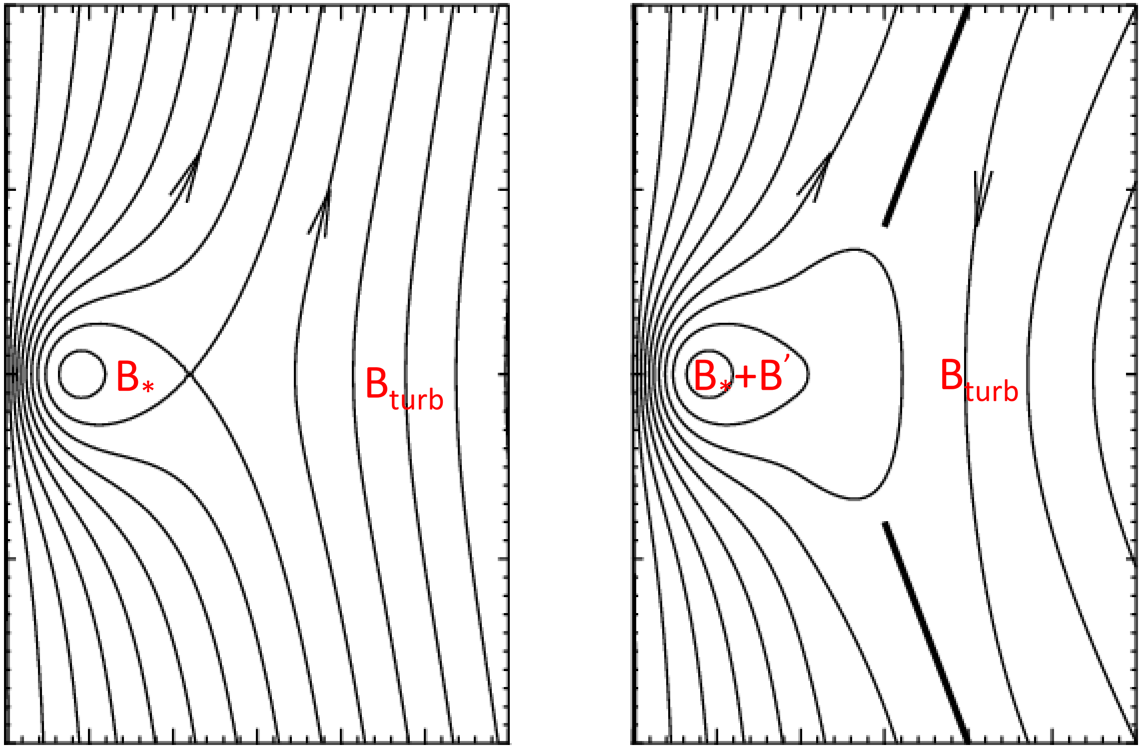
The conditions for flow onto the magnetic funnel is denoted by

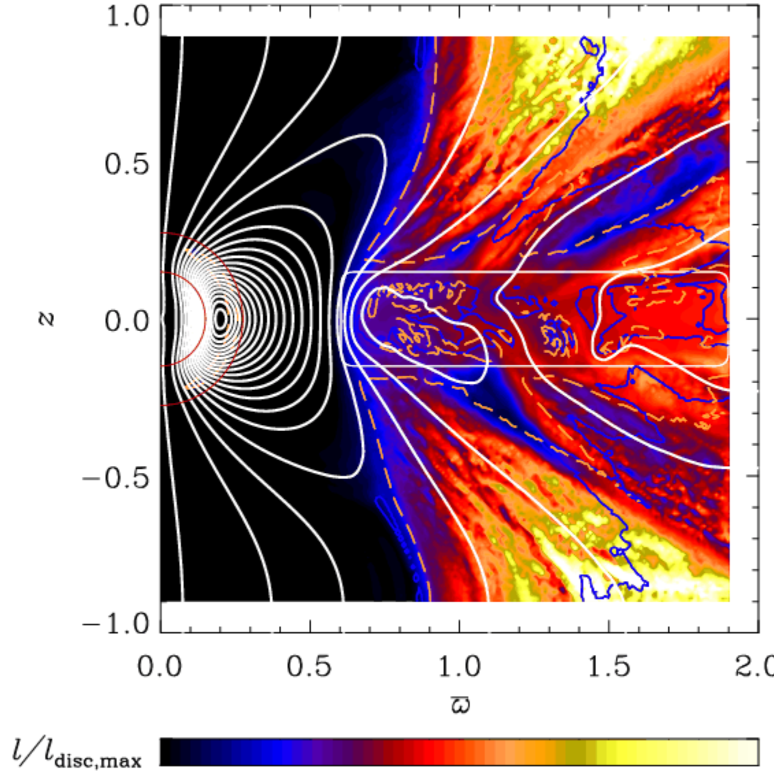
$$S_U = \frac{(1 + S_k)}{4} \left(1 - \frac{U_r}{|U_r|} \right) \qquad \text{and} \qquad S_k = \frac{\Omega_k - \Omega_*}{|\Omega_k - \Omega_*|}$$

such that $S_U = 0$ in all regions except $S_U = 1$ for radially inward flows inside R_c and

$$\dot{\Sigma}_* = f_\rho S_U \dot{\Sigma}_{*c}.$$

Separatrix between stellar and MRI field (R_x)





Von Rekowski & Brandenberg 2004

$$\beta_{turb} = \frac{2\mu_0 p}{B_{turb}^2}, \qquad p = \Sigma H_T \Omega^2, \qquad B_{turb} \simeq B_{*z} \qquad \text{at}$$

$$R_X = \left(\frac{\beta_{turb} B_*^2 R_*^3}{2\mu_0 G M_* \Sigma H_T}\right)^{1/3} R_* = \left(\frac{3\pi\alpha_\nu \beta_{turb} h_X}{2\mu_0}\right)^{2/7} \left(\frac{B_*^4 R_*^5}{G M_* \dot{M}_d^2}\right)^{1/7} R_*$$

Ourside R_x, $$\frac{\partial \Sigma}{\partial t} = -\frac{3}{r}\frac{\partial}{\partial r} r^{1/2} \frac{\partial \Sigma \nu r^{1/2}}{\partial r}.$$ In a steady state,

$$\Sigma = \frac{\dot{M}_d}{3\pi\alpha_\nu h^2 \Omega r^2} = \Sigma_X \left(\frac{r}{R_X}\right)^{-1/2} \qquad \text{and} \qquad \Sigma_X = \frac{\dot{M}_*}{3\pi\alpha_\nu h_X^2 \sqrt{G M_* R_X}}$$

Vertically average governing equations

Funnel flow

$$\boxed{\frac{\partial \Sigma}{\partial t} = -\frac{\partial \Sigma U_r r}{r \partial r}} - \boxed{\frac{f_\rho f_c}{\alpha_D \mu_0} \frac{m_*^2 (U_r/\Omega_{k*} r)}{r^{11/2}\sqrt{GM_*}} \frac{2S_U}{[(r/R_c)^3 - 1]} \left(\frac{\Omega_{k*}}{|\Omega - \Omega_*|} \right)}$$

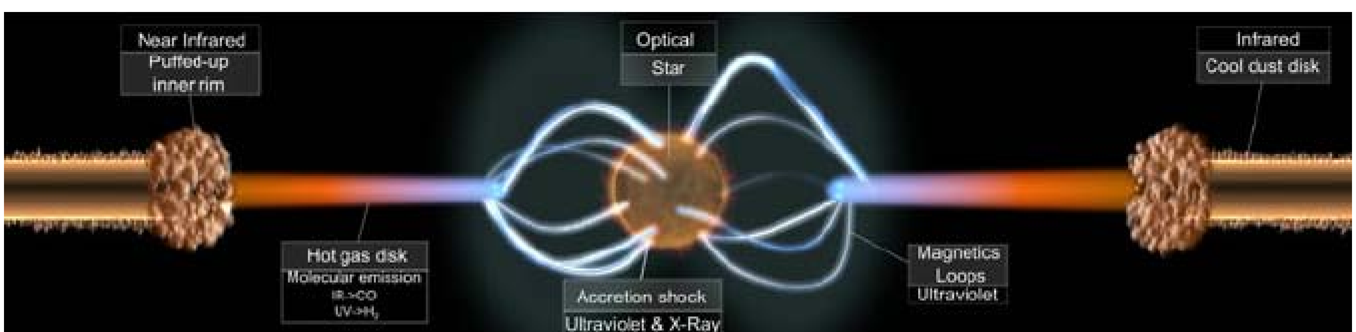
Viscous Keplerian flow

$$\boxed{\Omega^2(r,z) \simeq \Omega_k^2} + \boxed{\frac{m_*^2 P_m H_T}{\Sigma \mu_0 r^8}}$$

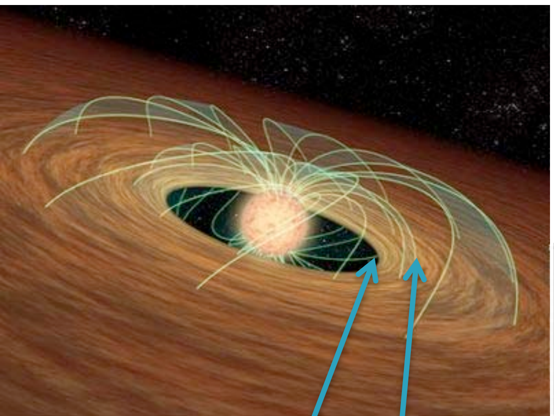
Magnetic pressure
Dependence on P_m

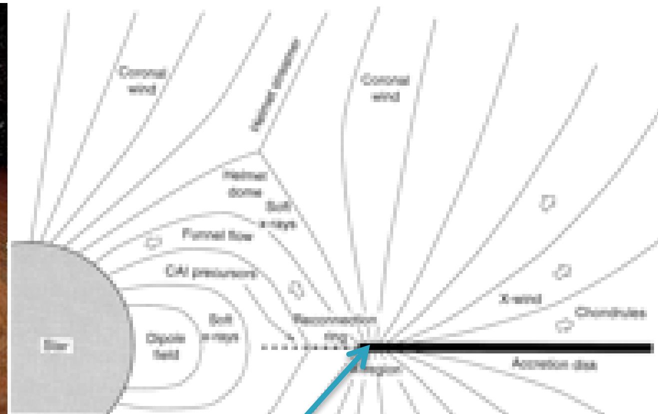
$$\boxed{\Sigma U_r r \frac{\partial \Omega r^2}{\partial r} \simeq \frac{\partial}{\partial r} \frac{\Sigma \alpha_\nu r^5}{2} \left(\frac{H_T}{r} \right)^2 \frac{\partial \Omega^2}{\partial r}} - \boxed{\frac{(f_c U_r/\Omega_{k*} r)}{(r^3/R_c^3 - 1)} \frac{f_\rho 2 S_U \Omega_{k*}}{|\Omega - \Omega_*|}} + \boxed{\left[\frac{\sqrt{2} f_c S_\Omega}{\pi} \right] \frac{m_*^2}{\mu_0 \alpha_D r^4}}.$$

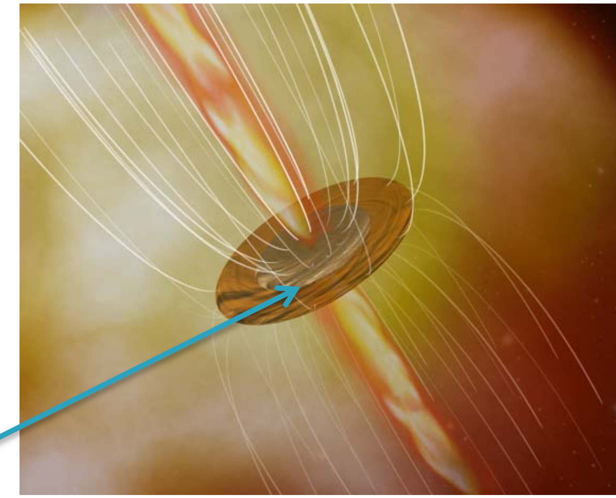
Magnetic tension
Dependence on S_Ω

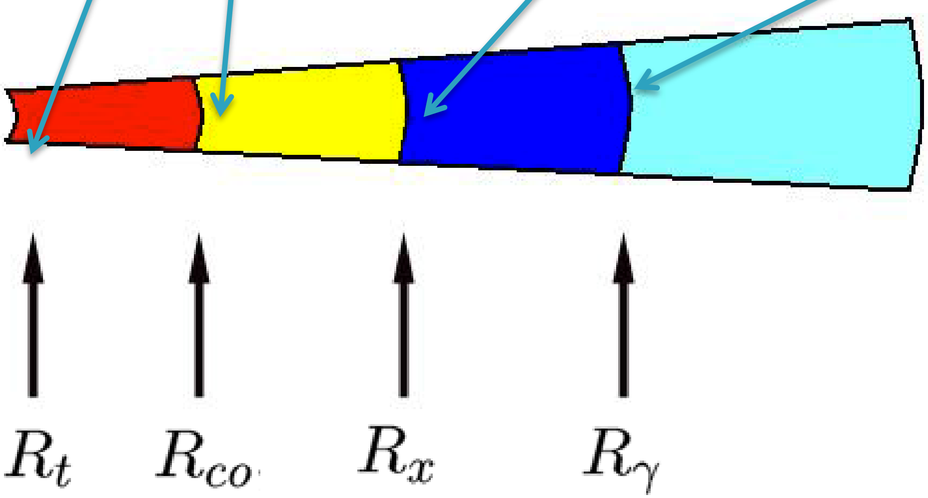


Mass loading and angular momentum flow



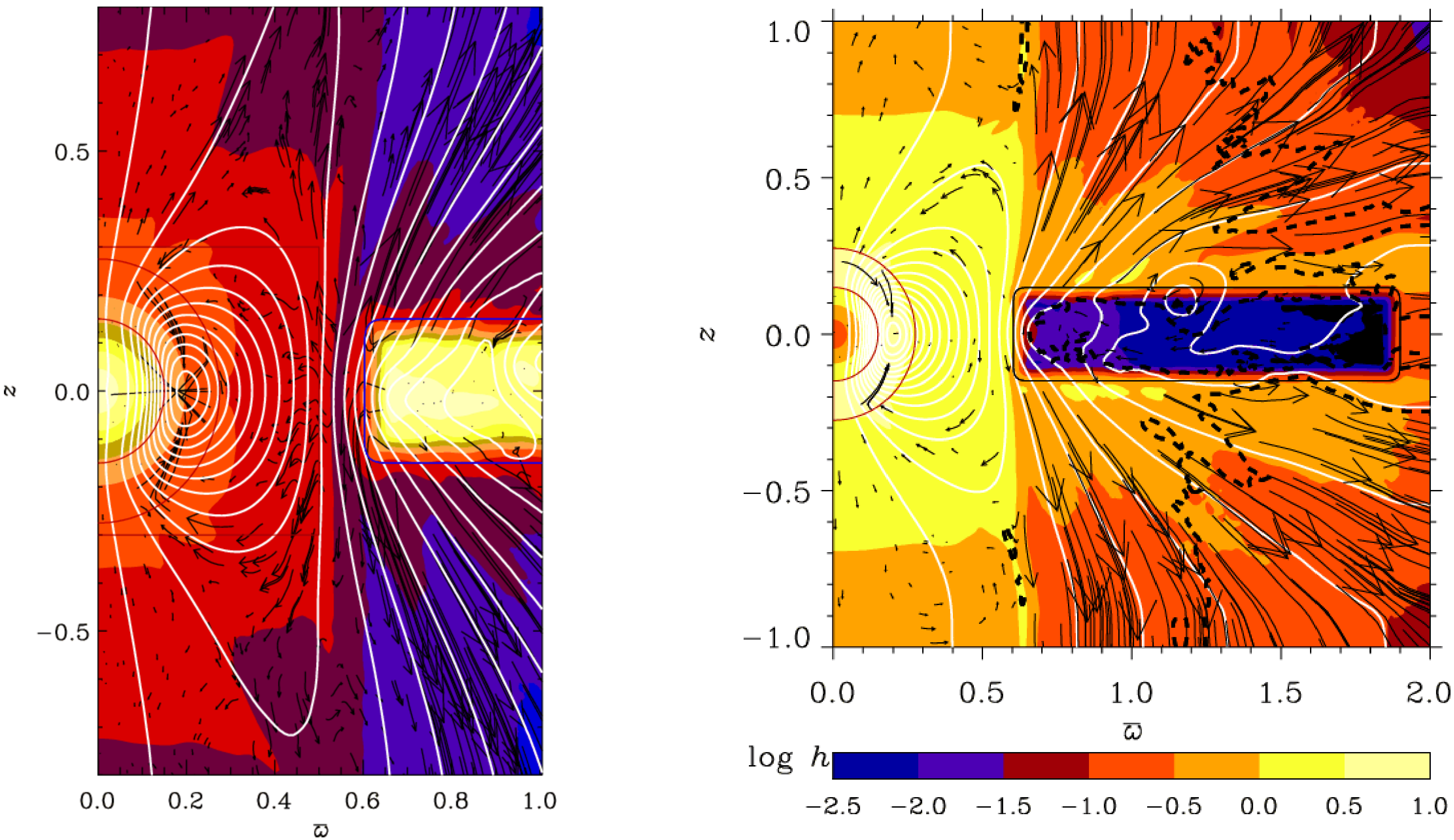






Rui Xu & Wendy Ju

Unstable flows in numerical simulations



Von Rekowski & Brandenberg 2004

With dynamo $$\frac{\partial A}{\partial t} = \mathbf{v}\mathbf{x}\mathbf{B} + \alpha_{dyn}\mathbf{B} - \eta_D \nabla\mathbf{x}\mathbf{B}$$

Normalized parameters

$$l \equiv \frac{r}{R_*}, \qquad l_c \equiv \frac{R_c}{R_*}, \qquad \tau \equiv t\Omega_{k*}, \qquad \Omega_{k*} \equiv \left(\frac{GM_*}{R_*^3}\right)^{1/2},$$

$$\omega_\phi \equiv \frac{\Omega_m}{\Omega_{k*}}, \qquad v_r \equiv \frac{U_r}{\Omega_{k*}R_*}, \qquad R_c \equiv \left(\frac{GM_*}{\Omega_*^2}\right)^{1/3},$$

$$P_m = \frac{f_c v_r}{\alpha_D h l |\omega_\phi - l_c^{-3/2}|}, \qquad \boxed{A \equiv \frac{m_*^2}{\dot{M}_* \sqrt{GM_* R_*^7}},}$$

$$\dot{m}(r) \equiv \frac{\dot{M}_d}{\dot{M}_*} = -\frac{2\pi\Sigma U_r r}{\dot{M}_*} = -2\pi A\sigma v_r l, \qquad \sigma \equiv \frac{\Sigma G M_* R_*^4}{m_*^2},$$

$$f_c = \frac{g_c}{1+g_c}, \quad g_c = \frac{|\omega_\phi l_c^{3/2} - 1|}{h}, \quad f_\rho = \frac{g_\rho}{1+g_\rho}, \quad g_\rho = 3(3+P_m)\left(1 - \frac{l^3}{l_c^3}\right)$$

$$S_\Omega = \frac{\omega_\phi l_c^{3/2} - 1}{|\omega_\phi l_c^{3/2} - 1|}, \qquad S_U = \frac{(1+S_k)}{4}\left(1 - \frac{v_r}{|v_r|}\right), \qquad \text{and} \qquad S_k = \frac{[1-(l/l_c)^{3/2}]}{|1-(l/l_c)^{3/2}|}$$

Normalized governing equations

$$\omega_\phi^2 = \frac{1}{l^3} + \frac{(f_c/\sqrt{2}\pi\alpha_D\mu_0)}{|\omega_\phi - l_c^{-3/2}|}\frac{v_r(l)}{\sigma l^8}$$

$$-\frac{\dot{m}}{2\pi}\frac{\partial l^2\omega_\phi}{\partial l} = \frac{\alpha_\nu A}{2}\frac{\partial}{\partial l}\left(h^2 l^5 \sigma \frac{\partial \omega_\phi^2}{\partial l}\right) - \frac{A f_c}{\alpha_D \mu_0 l^4}\left(\frac{\sqrt{2}S_\Omega}{\pi} + \frac{2 S_U f_\rho v_r}{|\omega_\phi - l_c^{-3/2}|[(l/l_c)^3 - 1]}\right)$$

$$A\frac{\partial \sigma}{\partial \tau} = \frac{1}{2\pi l}\frac{\partial \dot{m}}{\partial l} - \frac{2 f_c}{\alpha_D \mu_0}\frac{S_U f_\rho A v_r}{l^{13/2}|\omega_\phi - l_c^{-3/2}|[(l/l_c)^3 - 1]}$$

Physical parameters: R_*, M_*, Ω_*, m_*, and $\dot{M}_*$. Model Parameters: α_ν, α_D, and h.

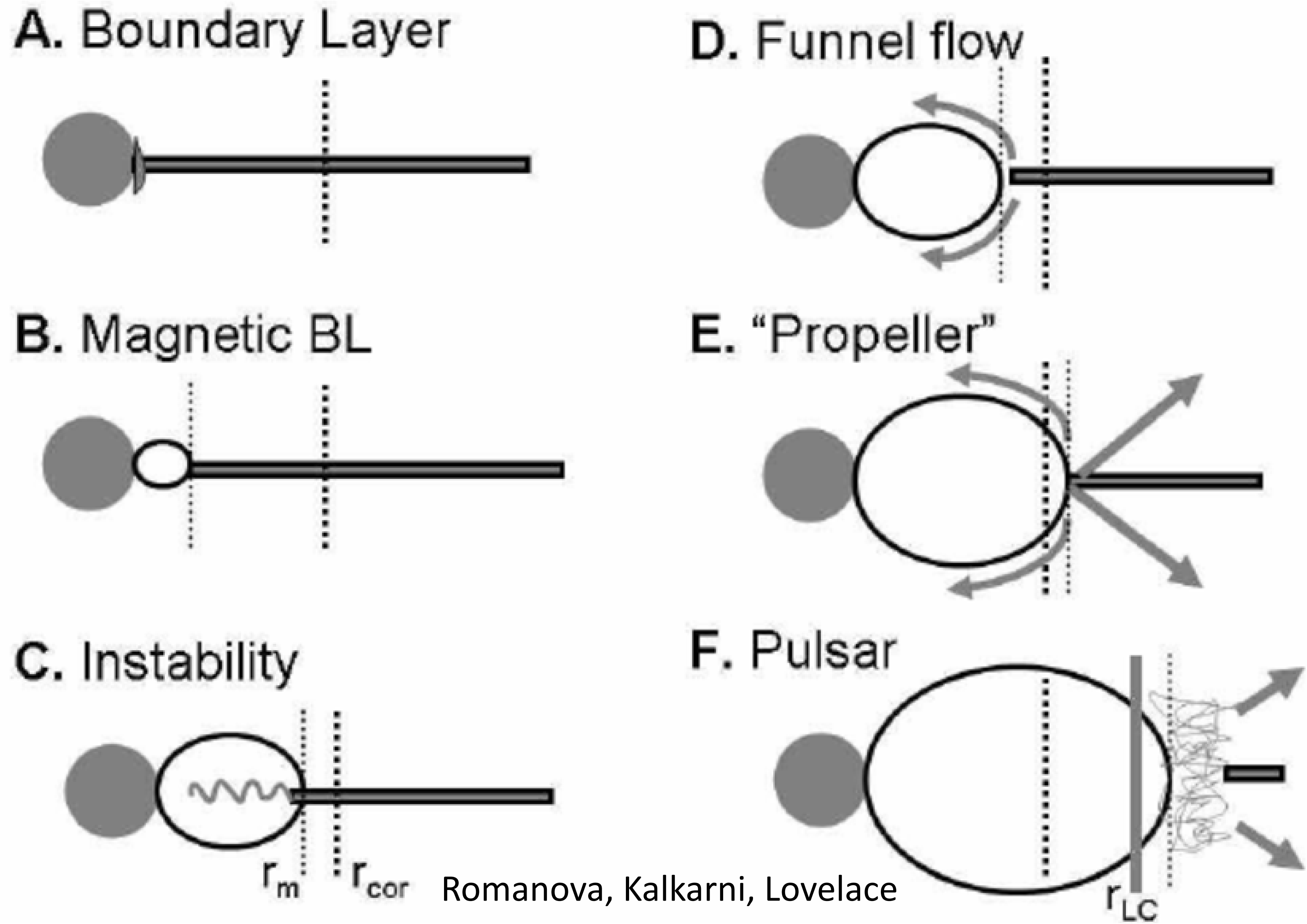
Critical radii: No singularity

a) Keplerian corotation radius l_c,

b) Modified corotation radius $\omega_\phi = l_c^{-3/2}$,

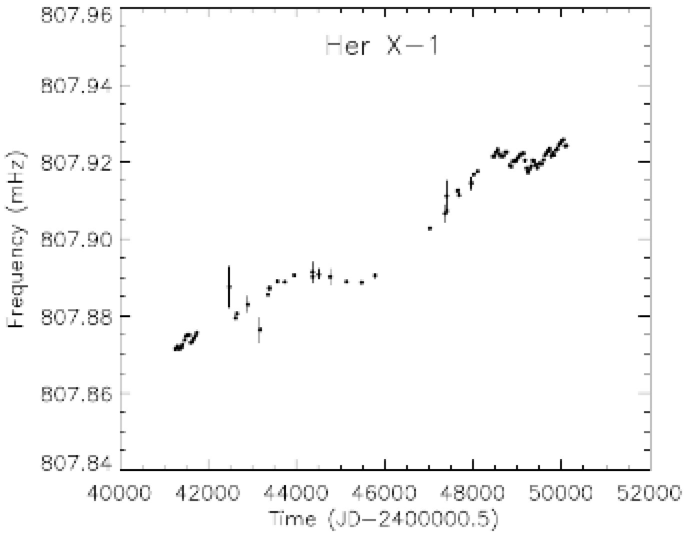
c) Truncation radius: $l \sim A^{2/7}$.

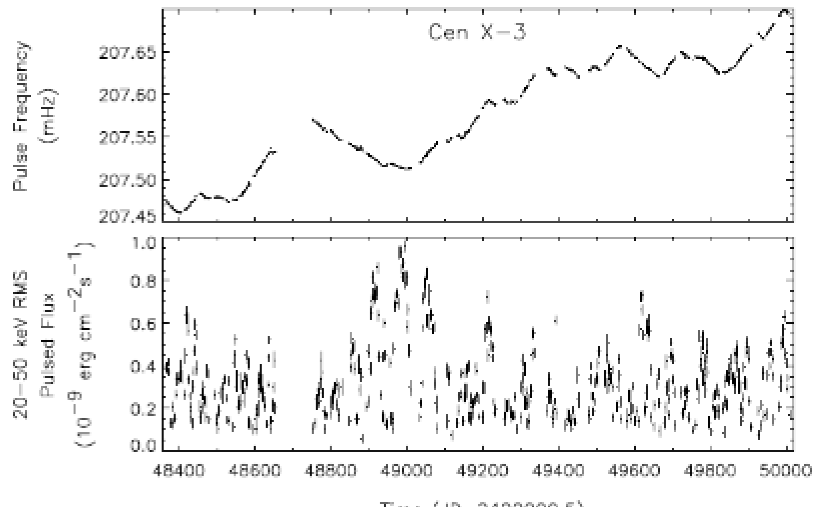
Schematic models

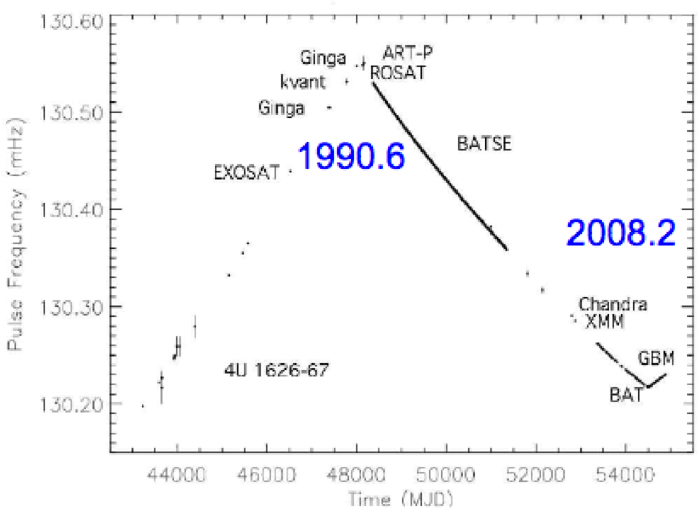


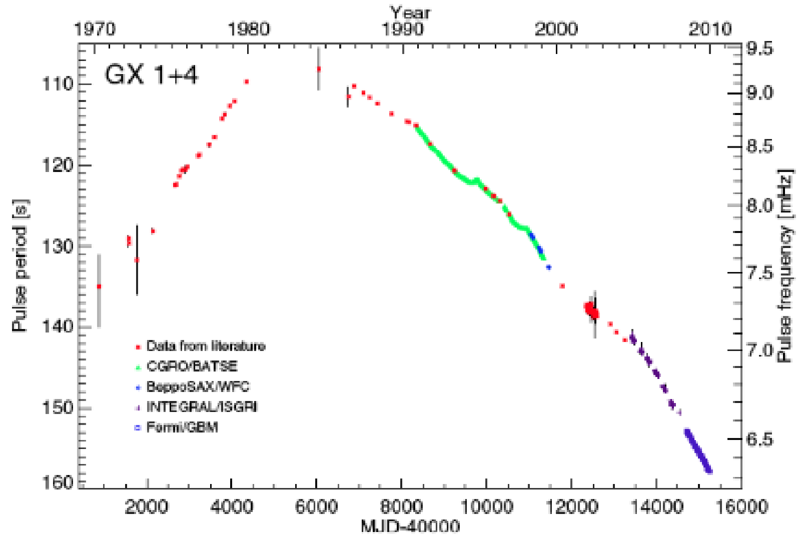
Romanova, Kalkarni, Lovelace

Spin evolution of X-ray pulsars

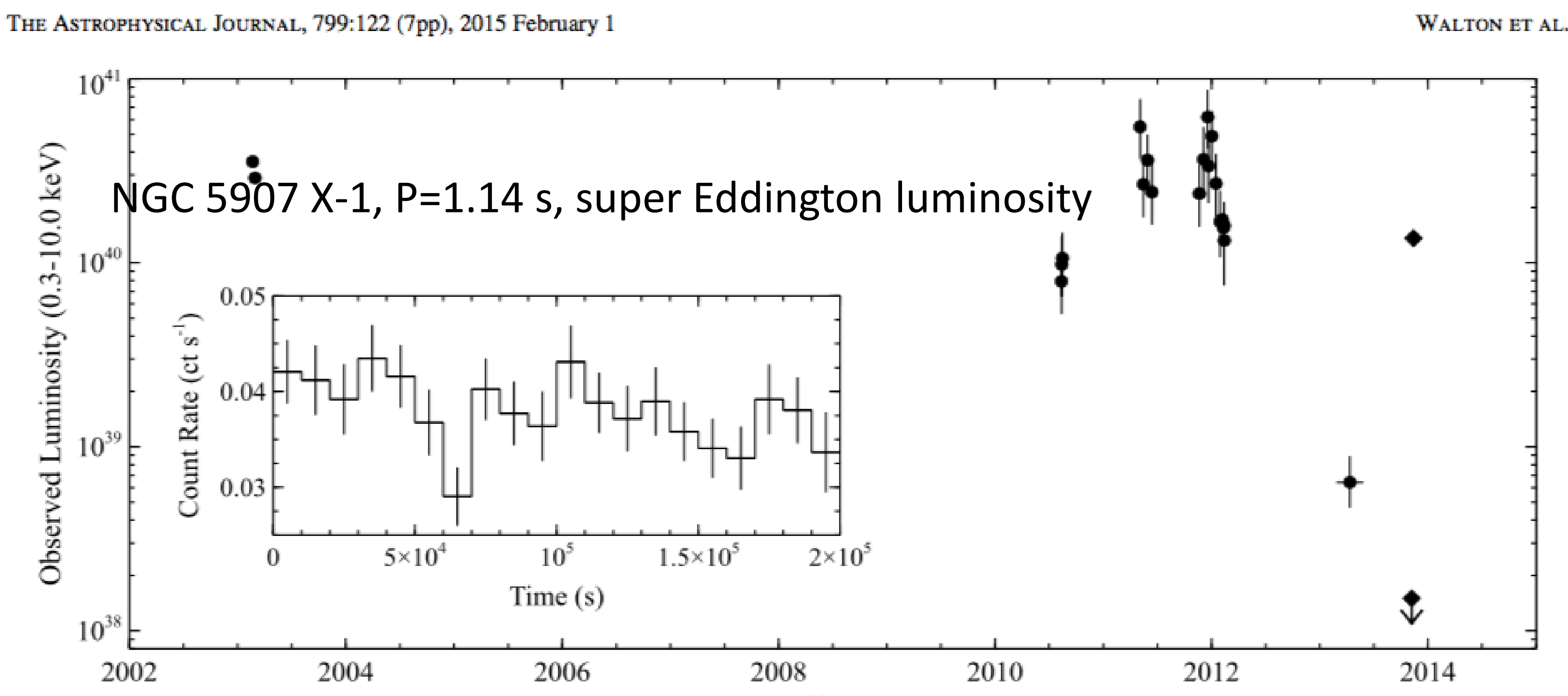






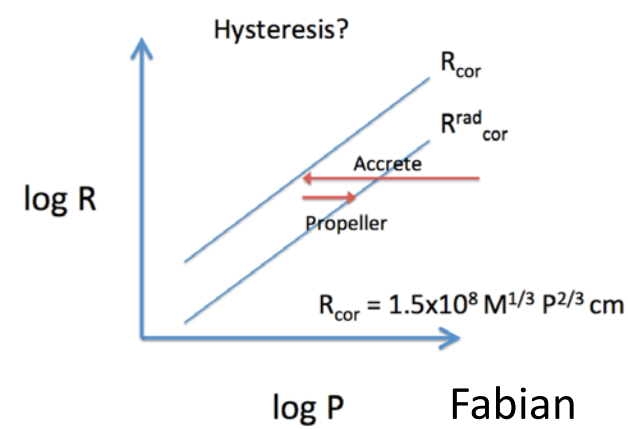


Ultra-luminous X-ray sources



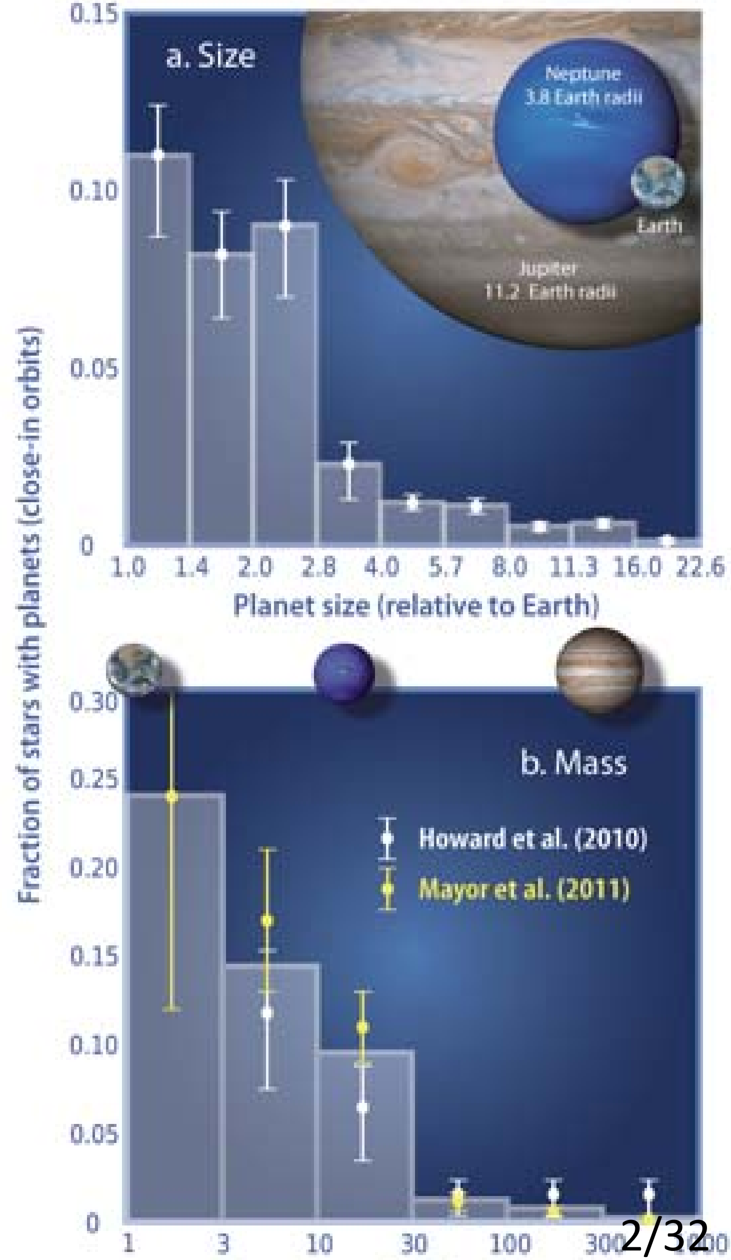
$$\omega_\phi^2 = \frac{1-\lambda}{l^3} + \frac{(f_c/\sqrt{2}\pi\alpha_D\mu_0)}{|\omega_\phi - l_c^{-3/2}|}\frac{v_r(l)}{\sigma l^8}$$

$$\lambda = \frac{\epsilon L}{L_{edd}}, \qquad L = \frac{GM_* \dot{M}_z}{R_*}, \qquad L_{edd} = \frac{4\pi G M_* m_p c}{\sigma_T}$$

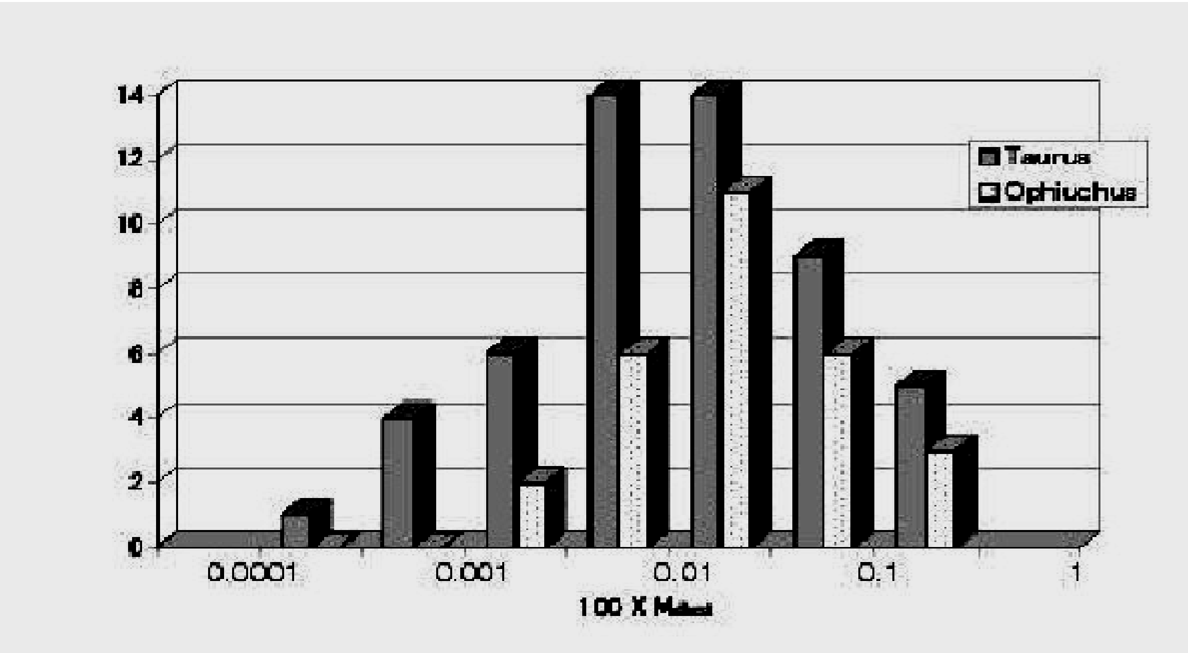


Fabian

Observed Properties of Extrasolar Planets
Howard (2013)



Weak mass constraint: Available planet building material



Mm dust (Beckwith & Sargent)

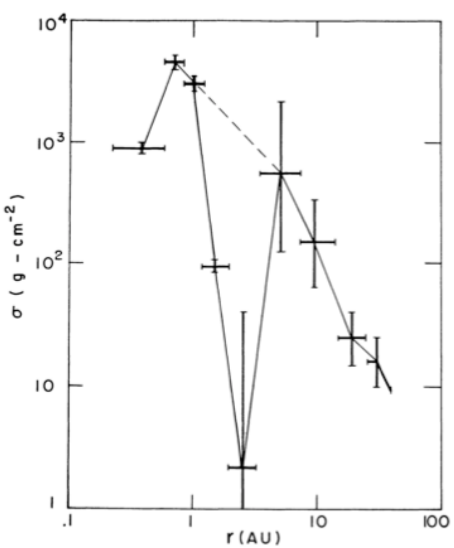


Fig. 1. Surface densities, σ, obtained by restoring the planets to solar composition and spreading the resulting masses through contiguous zones surrounding their orbits. The meaning of the 'error

Minimum Mass Nebula model

Wilder

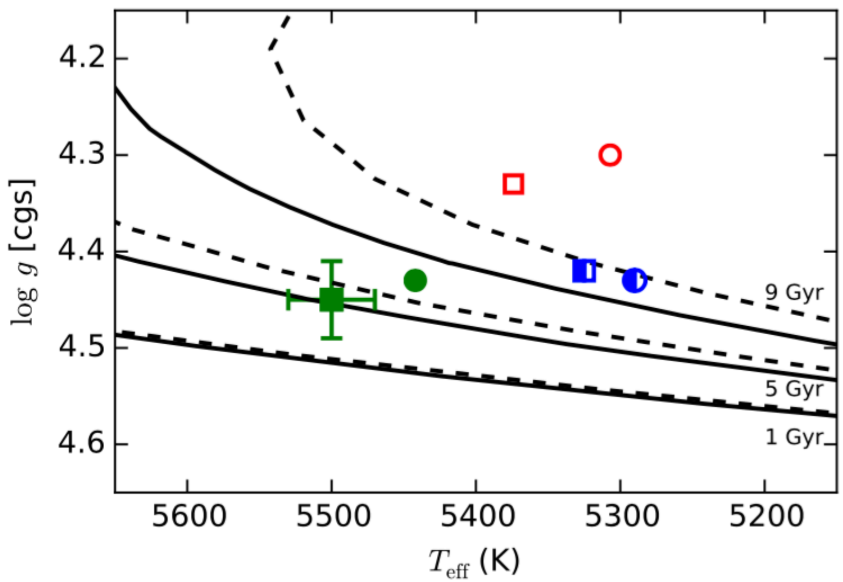
TABLE 5
SME MEAN ABUNDANCES FOR PLEIADES STARS

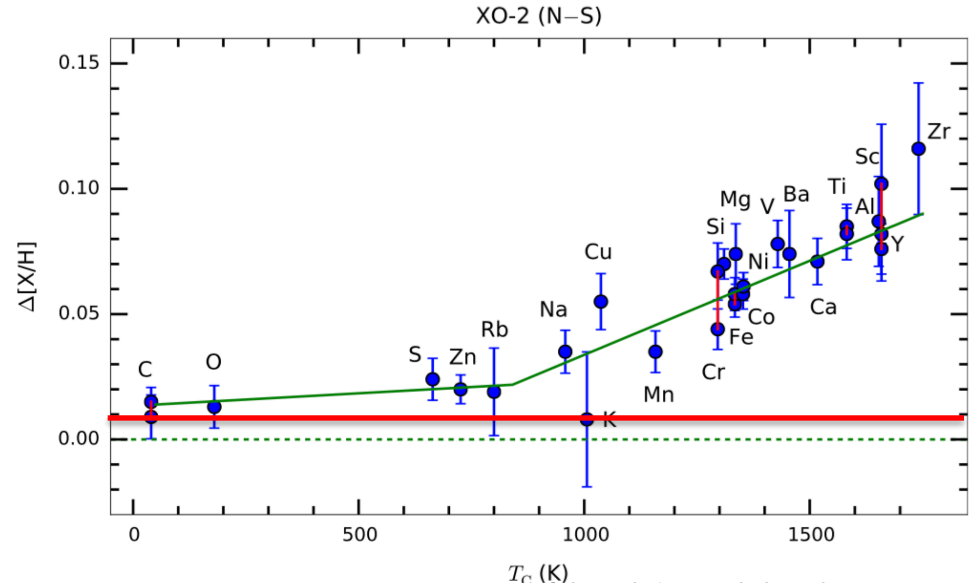
ELEMENT	MEAN log N^a (SME)	log N^b Solar	log N^b Meteoritic
Li...	2.51 ± 0.513	1.16	3.31
Na...	6.23 ± 0.042	6.33	6.31
Si...	7.54 ± 0.054	7.55	7.55
Ca...	6.33 ± 0.025	6.36	6.34
Sc...	3.00 ± 0.094	3.10	3.09
Ti...	4.93 ± 0.044	4.99	4.93
V...	4.02 ± 0.038	4.00	4.02
Cr...	5.61 ± 0.037	5.67	5.68
Fe...	7.44 ± 0.021	7.54	7.51
Co...	4.81 ± 0.051	4.92	4.91
Ni...	6.13 ± 0.031	6.25	6.25

Homogenety ΔF_e<5%
G dwarfs in Pleiaides stars (100 Myr old)

Planet-hosting twin binary stars

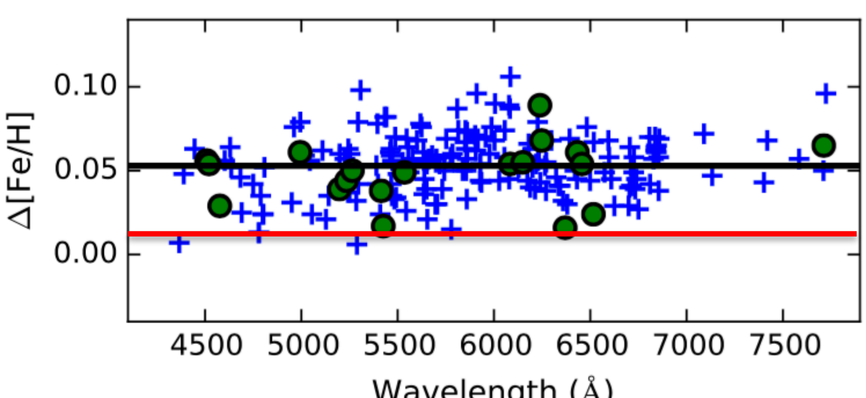
THE ASTROPHYSICAL JOURNAL, 808:13 (14pp), 2015 July 20

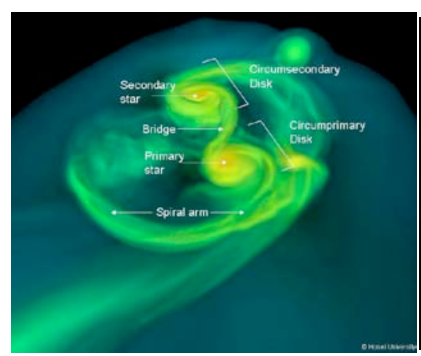




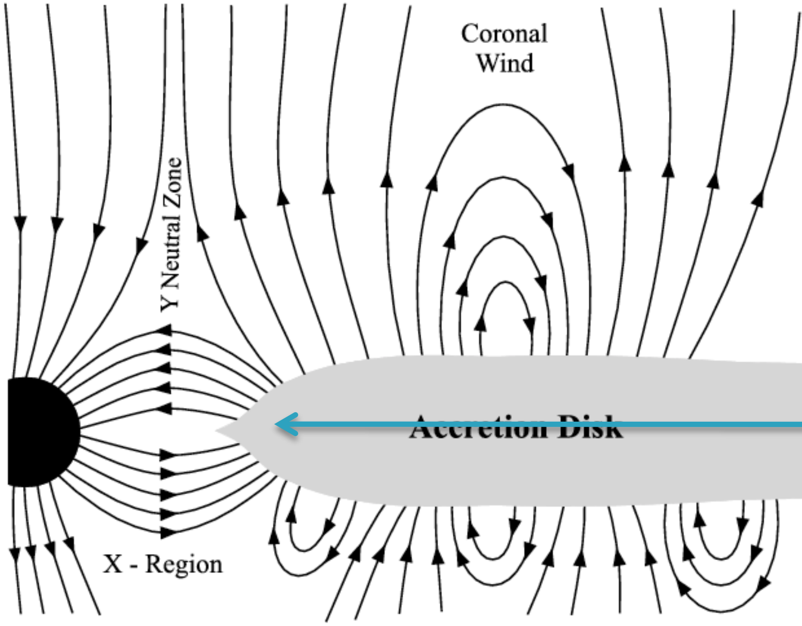
Maximum amount of building block is
<2 MMNS in volatile ices and
<10 MMNS in refractories (if well mixed)

Differential spectral analysis: XO-2
Ramirez et al 2015





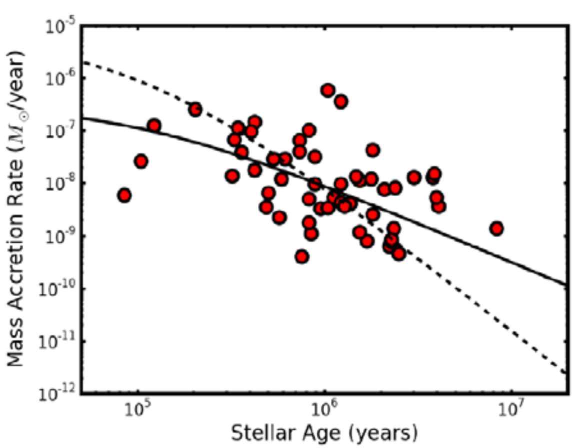
Disk truncation radius



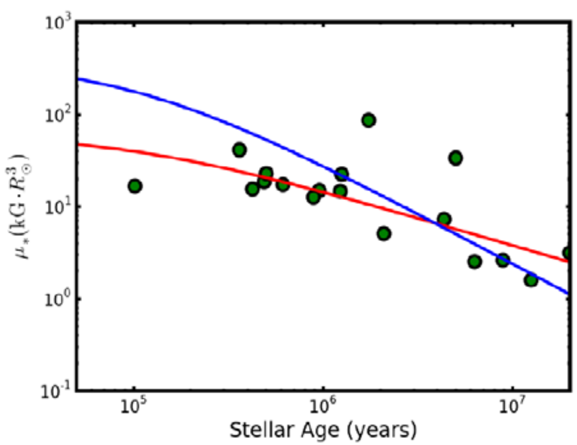
$$B^2/8\pi = \rho v^2 = \rho v R^2 \frac{v}{R^2} = \dot{M}_D \frac{(GM)^{\frac{1}{2}}}{R^{\frac{5}{2}}}.$$

$$R_t = \left(\frac{4\pi f_c}{\alpha_D \mu_0}\right)^{2/7} \left(\frac{B_*^4 R_*^5}{G M_* \dot{M}_d^2}\right)^{\frac{1}{7}} R_*.$$

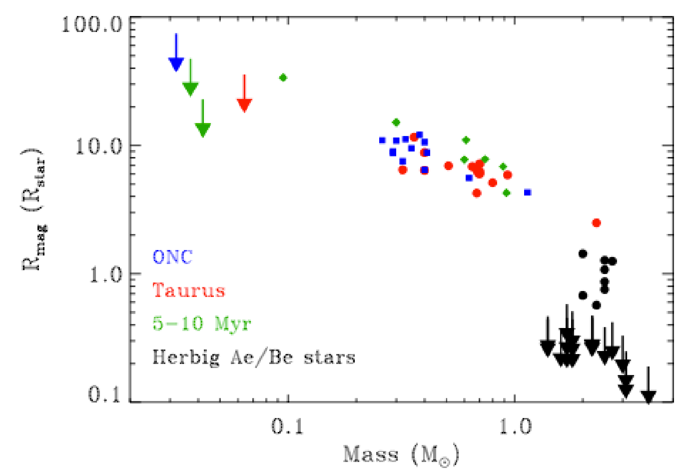
$R_t < R_{sublimation}$ when the host stars accreted most of its mass



Mass Accretion Rate



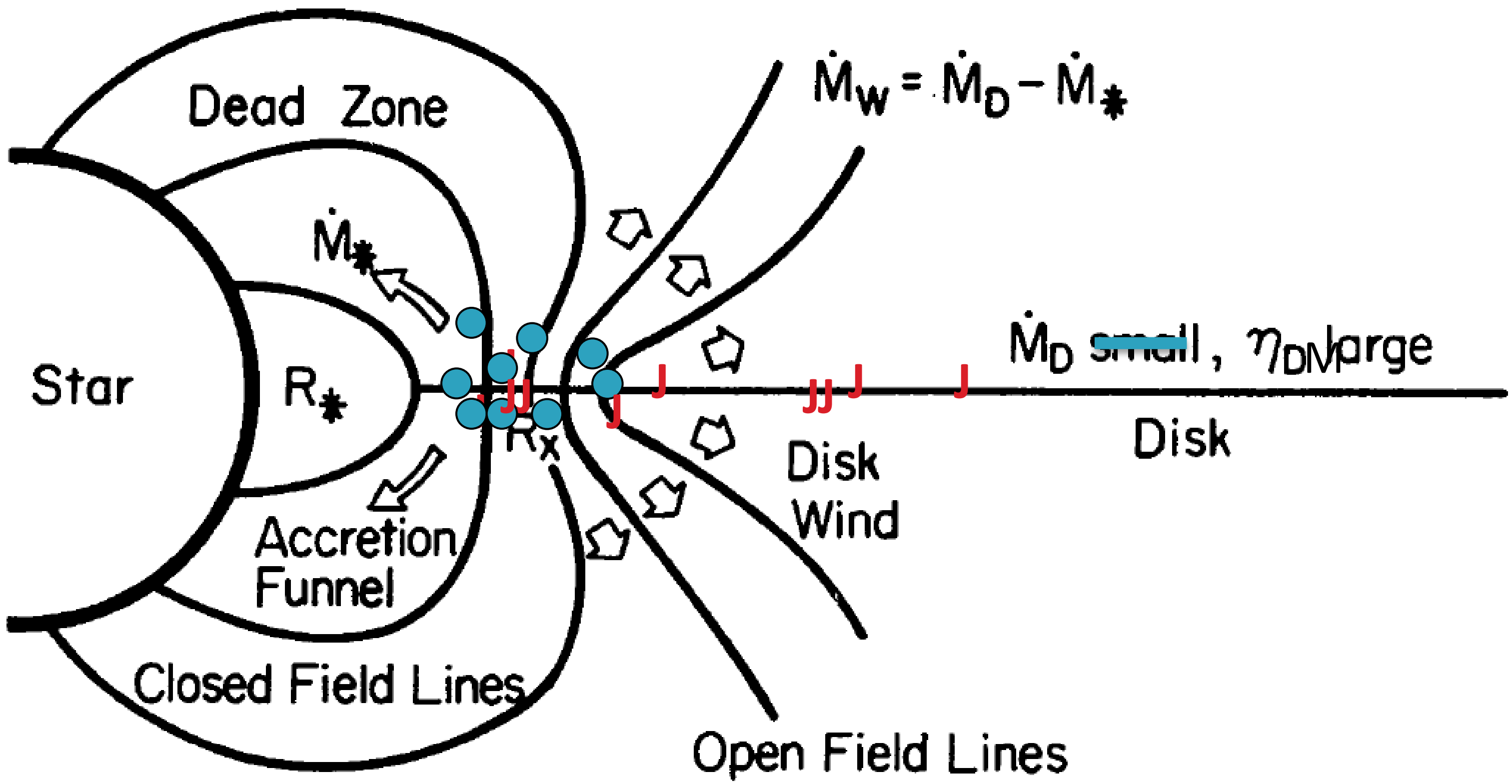
Stellar Dipole Moment



Magnetosphere radius at 1-3 Myr

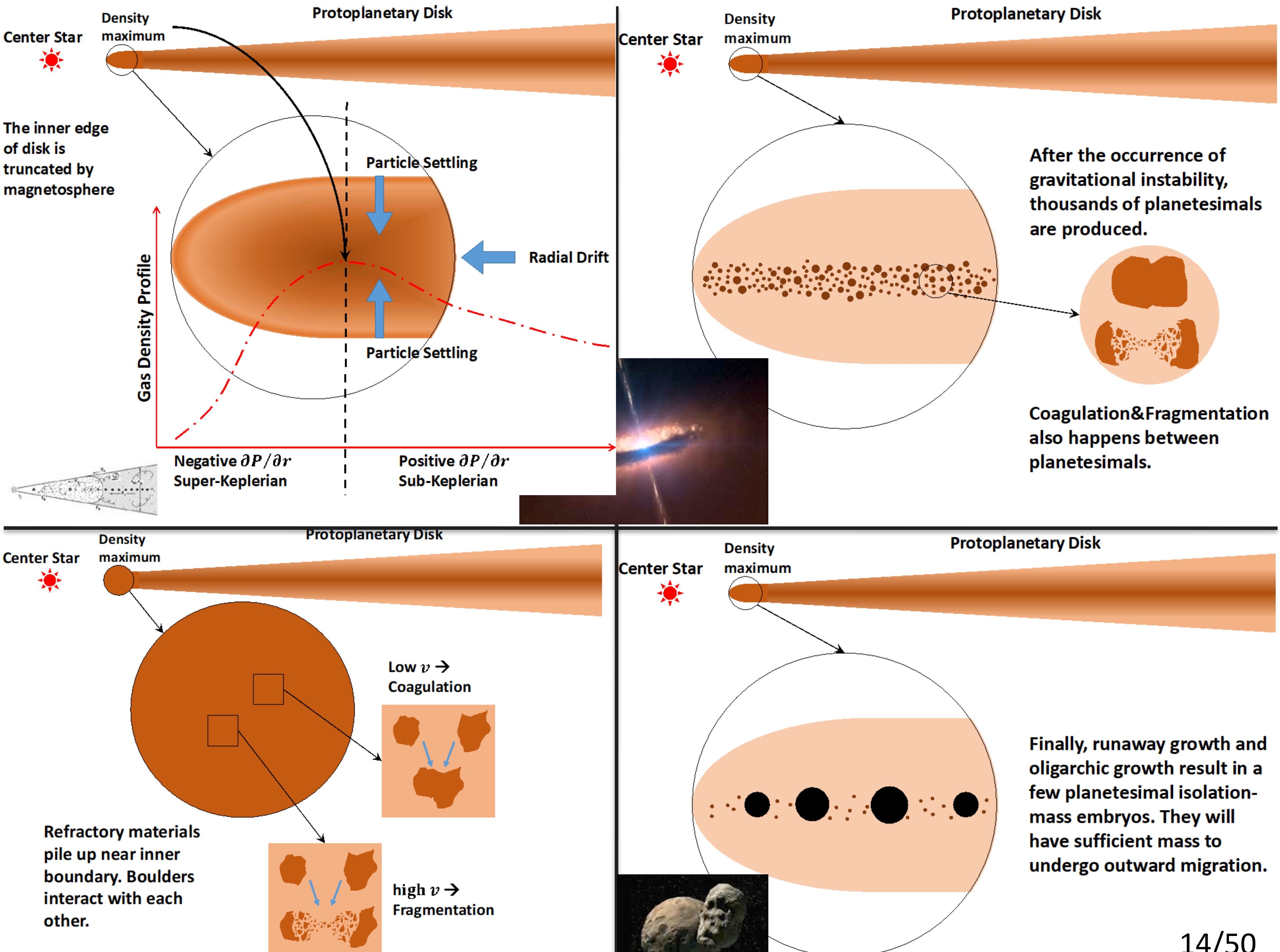
Herczeg

Mass loading and angular momentum flow



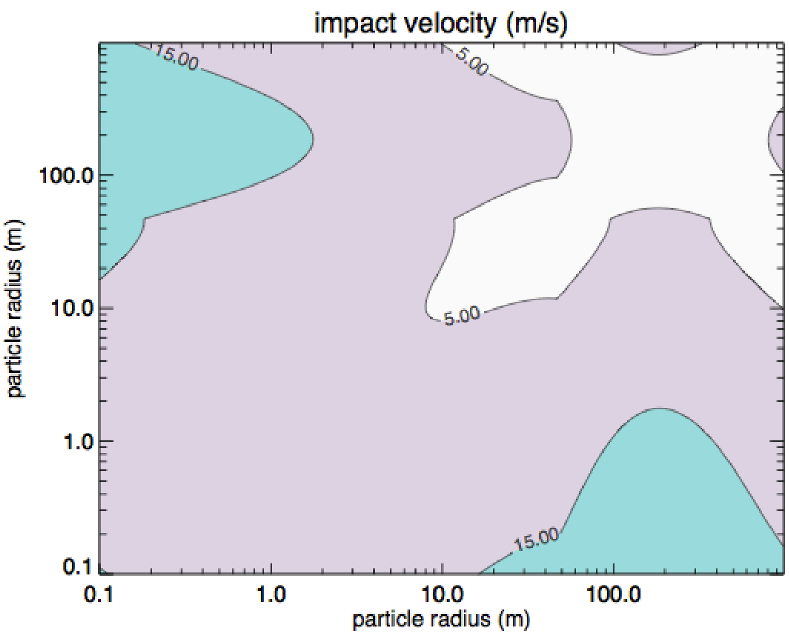
Wendy Ju & Datung Zhang

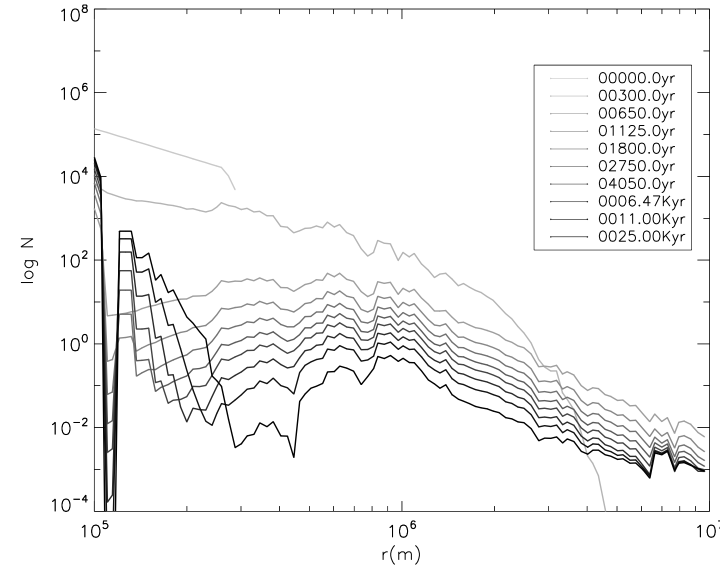
$$\eta_D \sim \alpha_D \left(r H_T |\Omega_m - \Omega_*| + H_T^2 \Omega_* \right) = \alpha_D H_T^2 \Omega_* (1 + g_c)$$

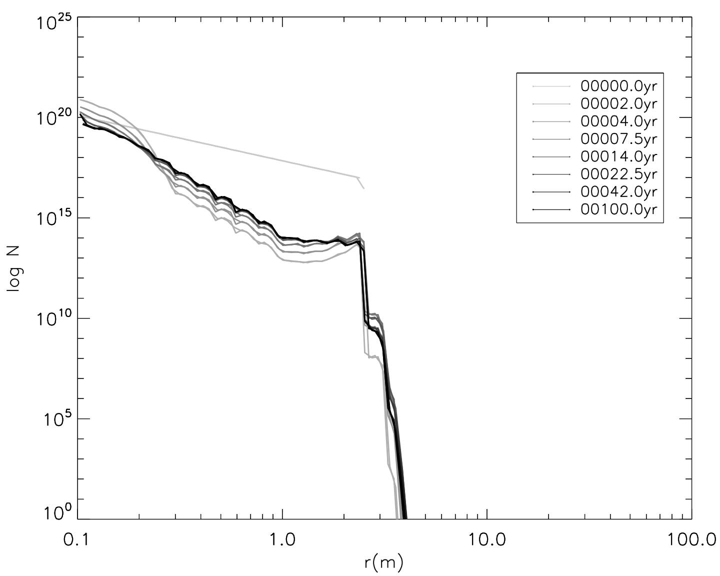


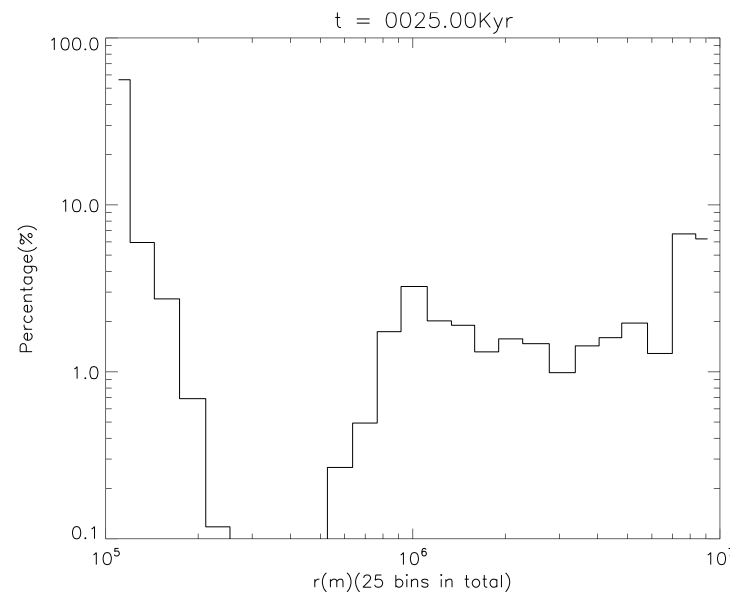
Center Star
Density maximum
Protoplanetary Disk
The inner edge of disk is truncated by magnetosphere
Particle Settling
Radial Drift
Particle Settling
Gas Density Profile
Negative $\partial P/\partial r$
Super-Keplerian
Positive $\partial P/\partial r$
Sub-Keplerian
Center Star
Density maximum
Protoplanetary Disk
After the occurrence of gravitational instability, thousands of planetesimals are produced.
Coagulation&Fragmentation also happens between planetesimals.
Center Star
Density maximum
Protoplanetary Disk
Low v → Coagulation
Refractory materials pile up near inner boundary. Boulders interact with each other.
high v → Fragmentation
Center Star
Density maximum
Protoplanetary Disk
Finally, runaway growth and oligarchic growth result in a few planetesimal isolation-mass embryos. They will have sufficient mass to undergo outward migration.

Planetesimal growth in a trap



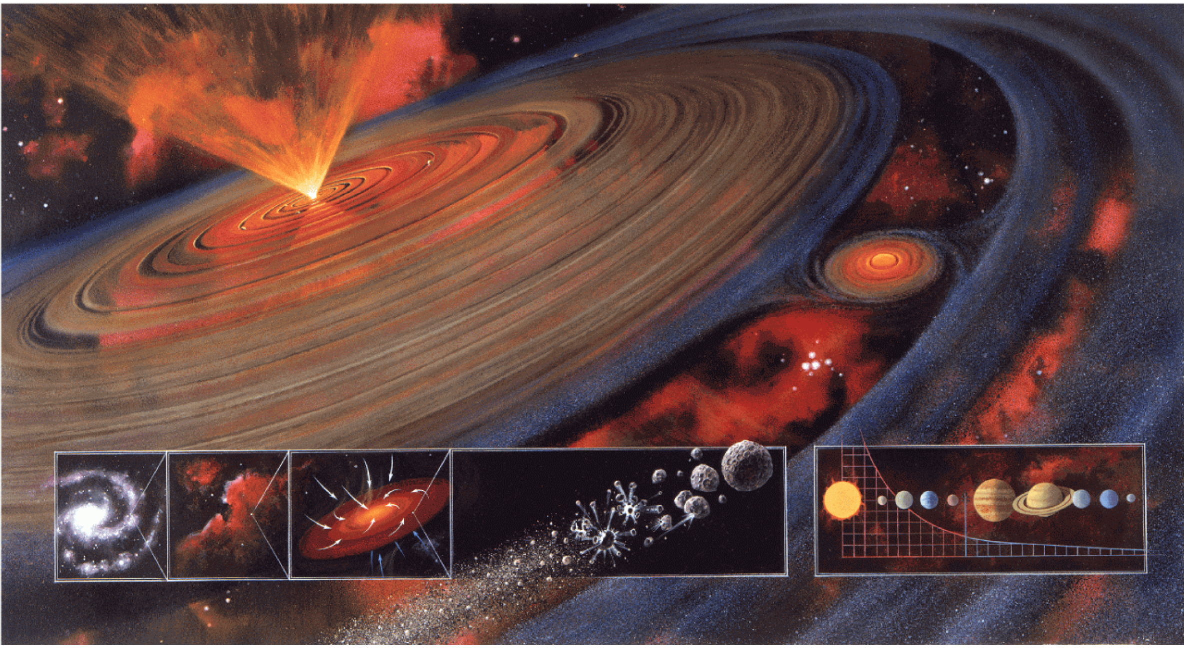


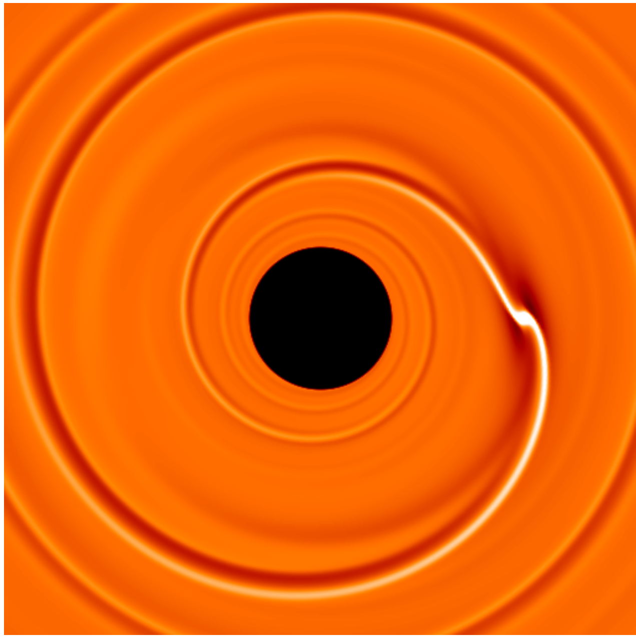




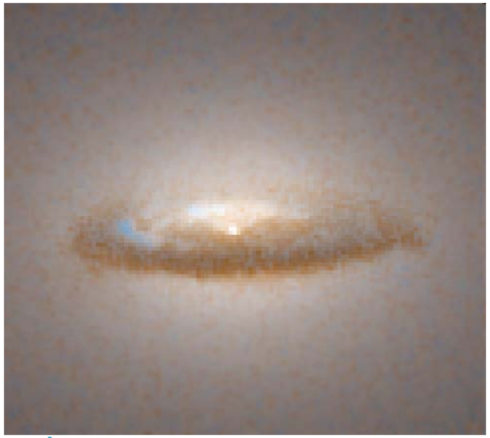
Rixin Li, Yuan Zhang, Bili Dong

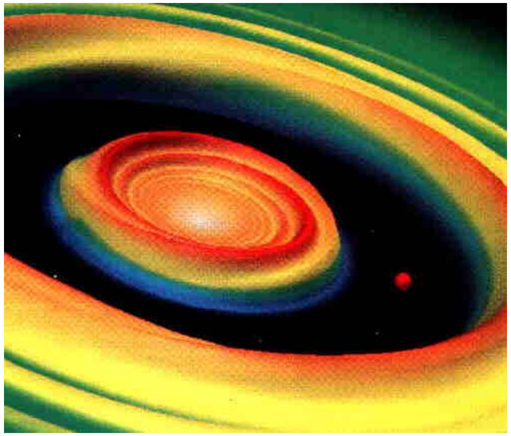
Current Paradigm: **Core** formation & **planet-disk interaction**



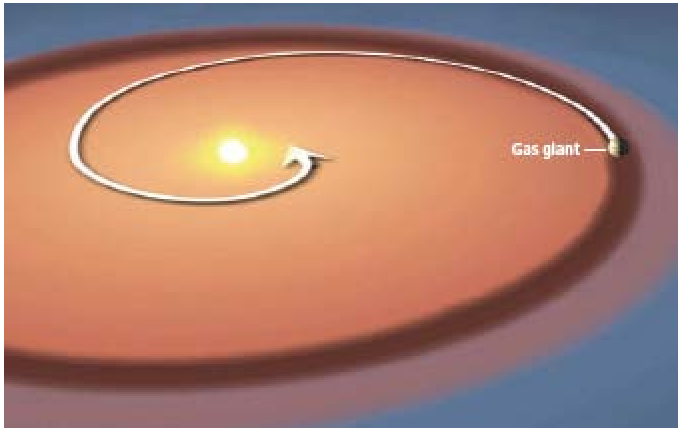


Critical core mass





Planetary **migration**



Gap formation

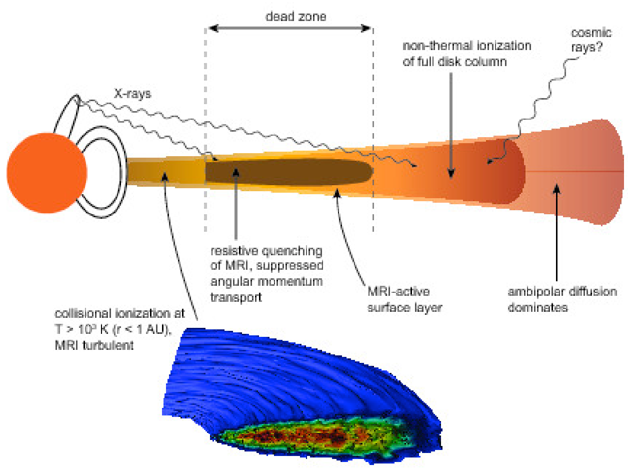
Planet-disk tidal interaction

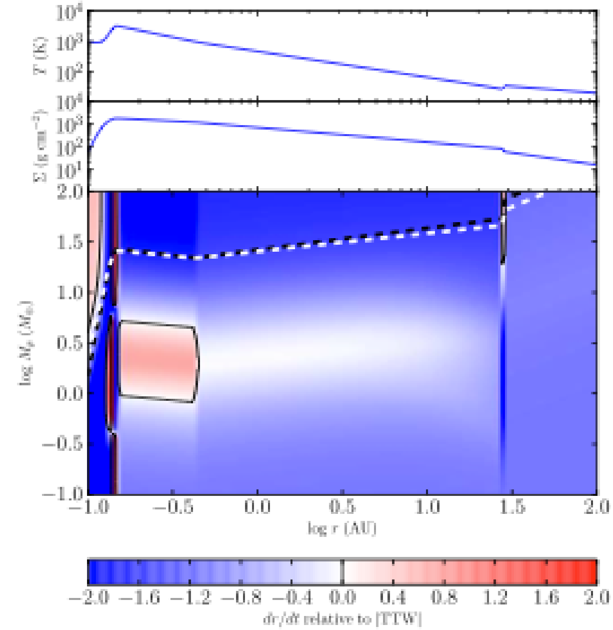
Total tidal torque:

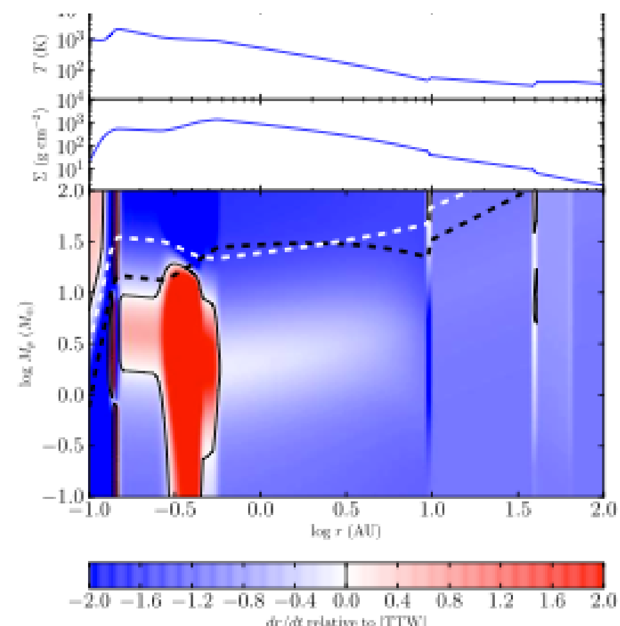
$$\Gamma = \Gamma_L + \Gamma_c \quad = f(p,q,p_\nu,q_\nu,p_\kappa,q_\kappa, B_*)\Gamma_0$$

$$\Gamma_0 = (q/h)^2 \Sigma_p r_p^4 \Omega_p^2,$$

p and q depend on disk structure &
p_ν,q_ν,p_κ, and q_κ also depend on m_p







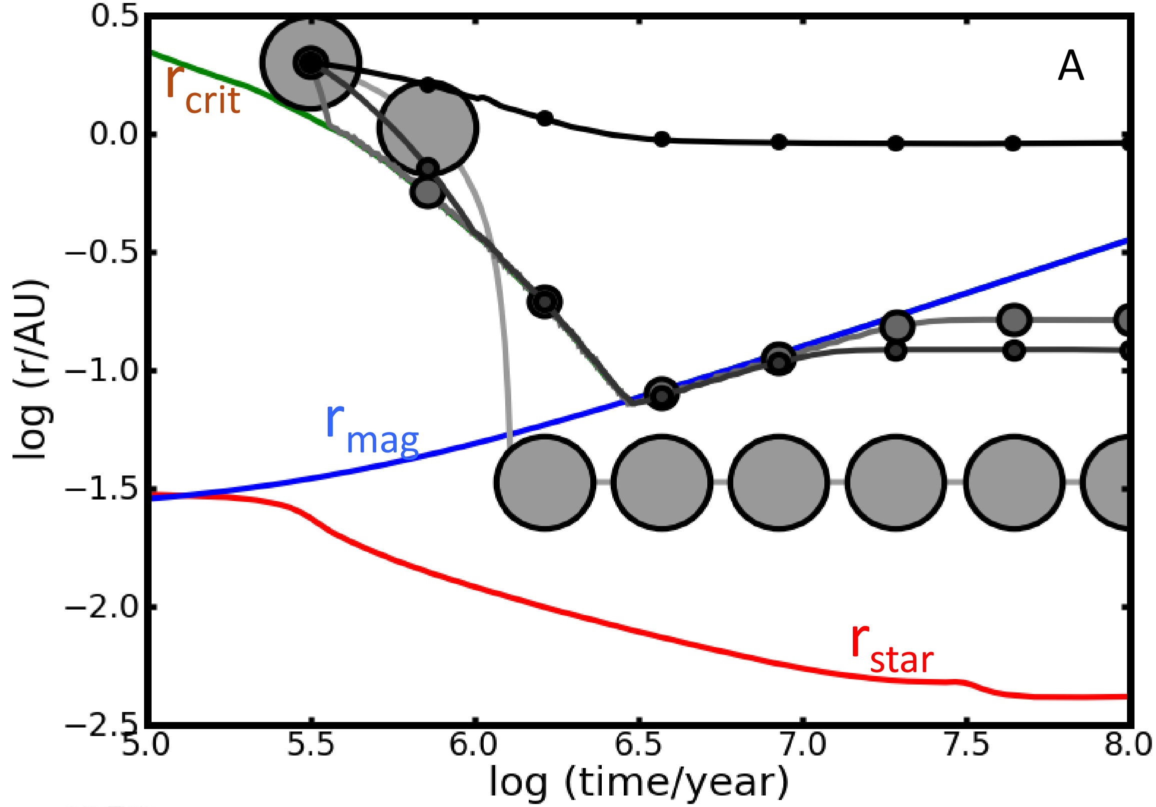
$$\frac{dr}{dt} = f(p,q,p_\nu,p_\chi)\frac{M_p}{M_*}\frac{\Sigma r^2}{M_*}\left(\frac{r\Omega_K}{c_s}\right)^2 r\Omega_K$$

$$(1/e)de/dt = (a/H)^4\,(M_p\Sigma a^2/M_*^2)\,\Omega$$

Garaud, Kretke

Super Earths: some key issues

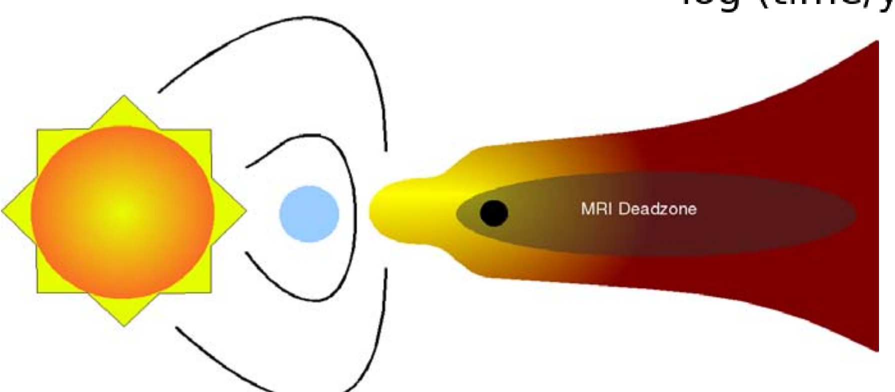
- How to differentiate type I and II migration?



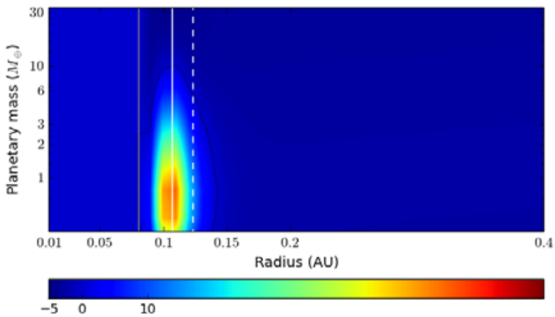
Sub/warm Earths ½ Earth

Neptunes 5 Earth
SuperEarths 2 Earth

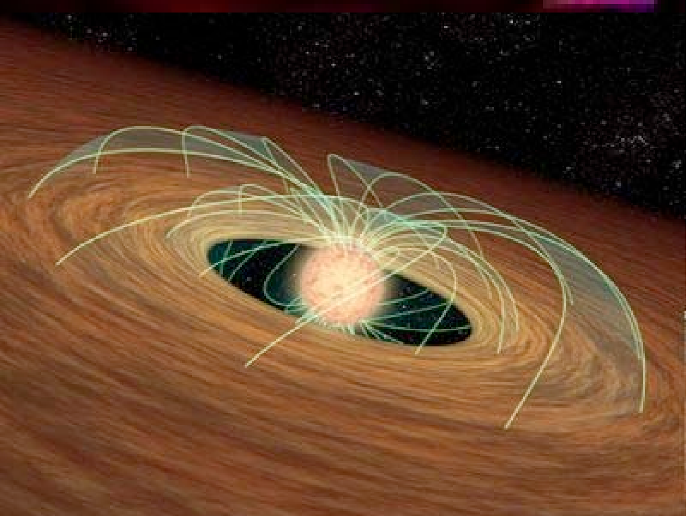
Gas giants Jupiter

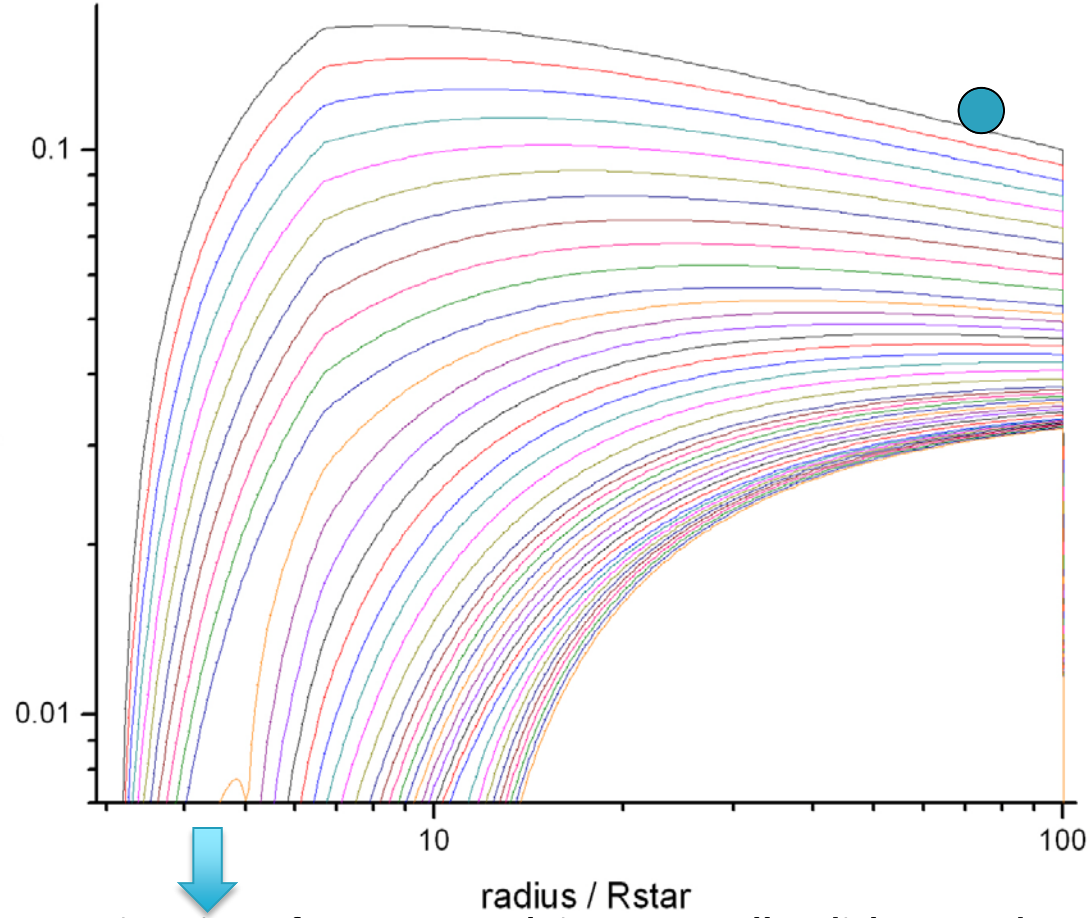


Hot Jupiters park Closer than Super Earths



Kretke 14/32





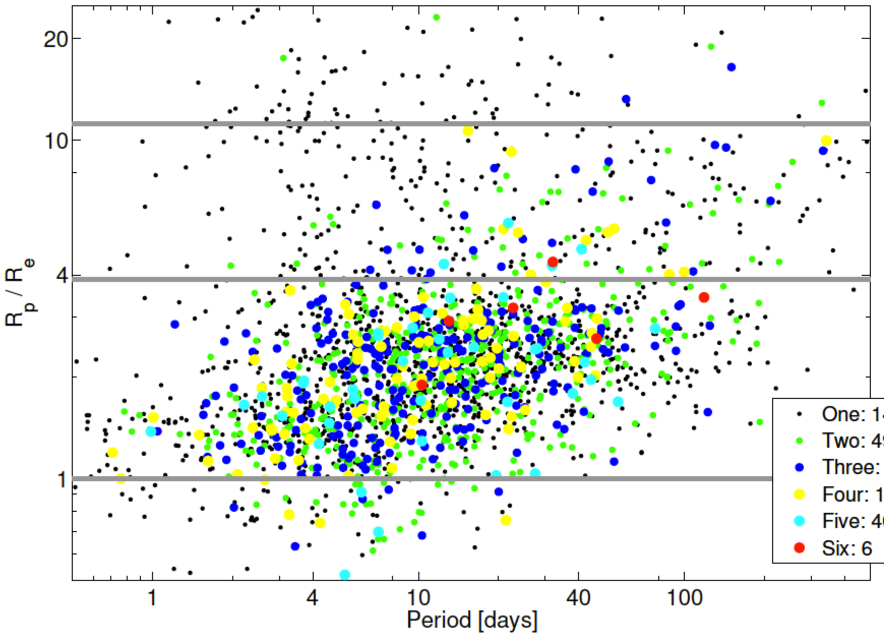
Migration of a Super Earth in protostellar disk around a magnetized T Tauri star. The Super Earth: (a) grows & migrate inward to inner-edge; (b) migrates slightly outwards with the expanding disk inner edge; (c) halts migrating after gas is mostly depleted. (Xu et al 2015 in preparation)

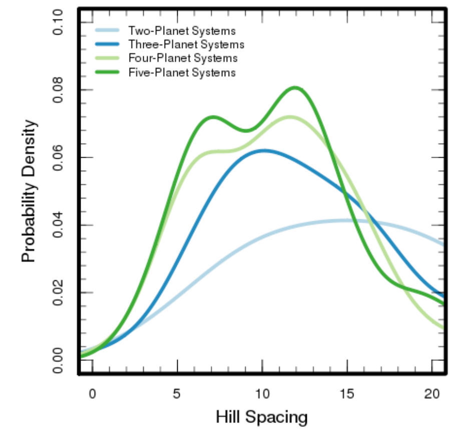
To model P distribution of Kepler's new-found planetary candidates.

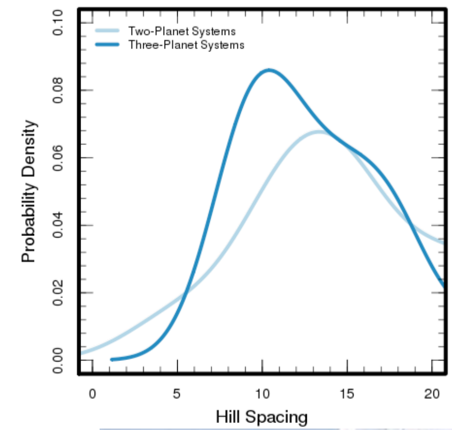
KIAA undergraduate student Ju Wenhua (Princeton) and Xu Rui (CfA, Harvard)

New Candidate Catalog (Batalha et al. 2012)

What can we learn from Multiple systems !!!





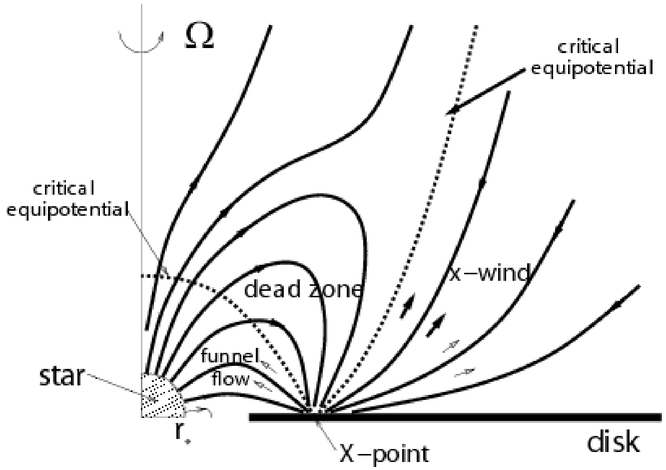


How compact can multiple systems be?

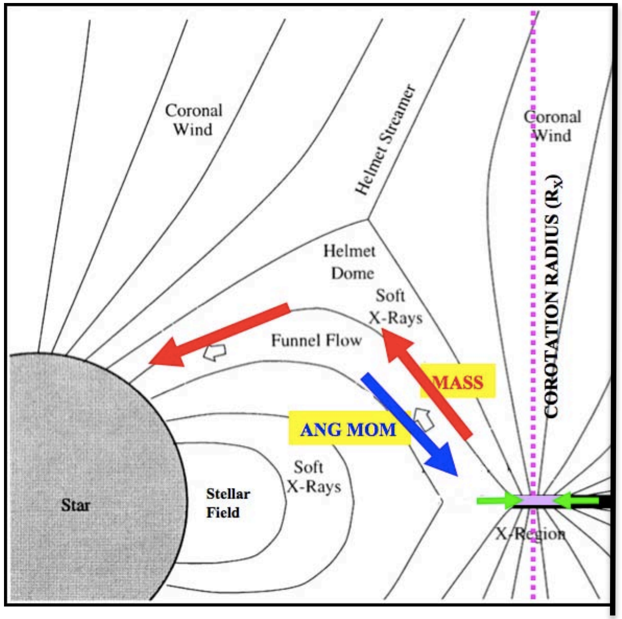
Stability and coplanarity

Kevin Schlaufman
Xiaojia Zheng

Star-disk angular momentum transfer



X-wind model



Mass flow rate from the disk to the magnetic funnel

$$\dot{M}_z = 2\pi \int_{R_{in}}^{R_X} S_U \dot{\Sigma}_* r dr = \frac{4\pi m_*^2/\alpha_D}{\mu_0\sqrt{GM_*}} \int_{R_{in}}^{R_X} \frac{(U_r/\Omega_{k*}r)S_U}{[(r/R_c)^3-1]} \frac{f_c f_\rho \Omega_{k*} dr}{r^{9/2}|\Omega-\Omega_*|}$$

$$\dot{J}_* = \boxed{\int_{R_{in}}^{R_X} \left(\frac{4\sqrt{2}S_\Omega f_c m_*^2}{\mu_0 \alpha_D r^4}\right) dr} + \boxed{2\pi \int_{R_{in}}^{R_X} S_U \dot{\Sigma}_* \Omega_* r^3 dr - \dot{M}_z \Omega_* R_*^2.}$$

Funnel flow

Magnetic tension

Dependence on S_Ω

$$\dot{\Omega}_* = \dot{J}_*/I_* = \dot{J}/\gamma_* M_* R_*^2 \qquad \text{and} \qquad \tau_{cr} = \Omega_*/\dot{\Omega}_*$$

$$\dot{\Omega}_* = \frac{AB\dot{M}_*\Omega_{k*}}{\gamma_* M_*} \quad \text{where} \quad B \equiv \int_{l_{in}}^{l_X} \frac{f_c dl}{\alpha_D \mu_0}\left[\frac{\sqrt{32}S_\Omega}{l^4} + \frac{4\pi S_U(l^{-7/2}-l^{-11/2})f_\rho}{[(l/l_c)^3-1]|\omega_\phi - l_c^{-3/2}|}\frac{\Omega_*}{\Omega_{k*}}\right].$$

Advanced protostellar-disk evolution

C. F. Manara et al.: X-shooter st

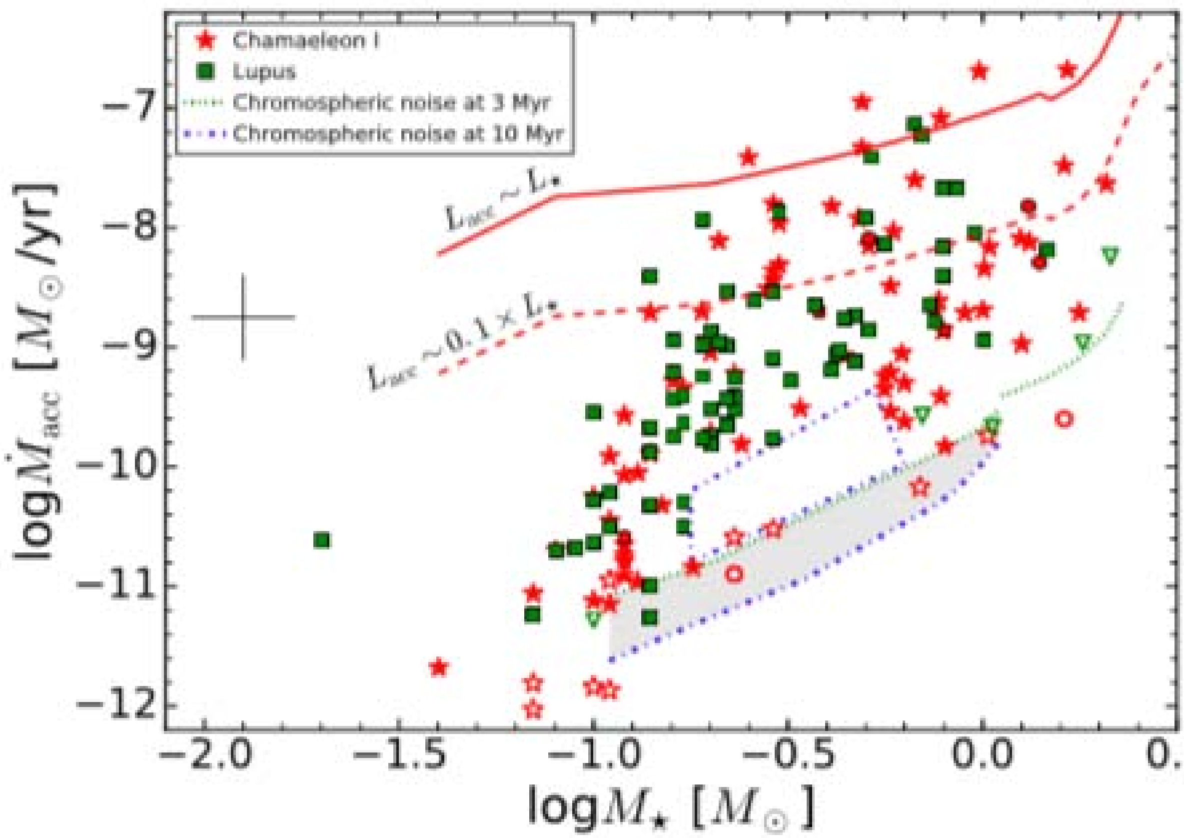
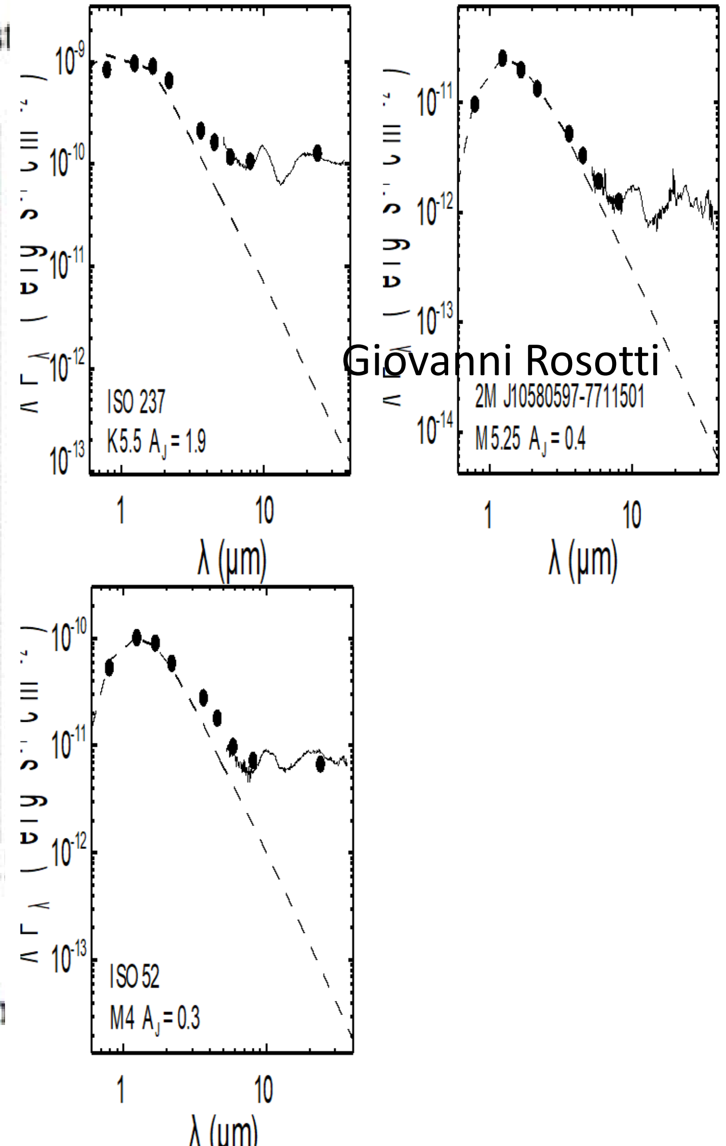


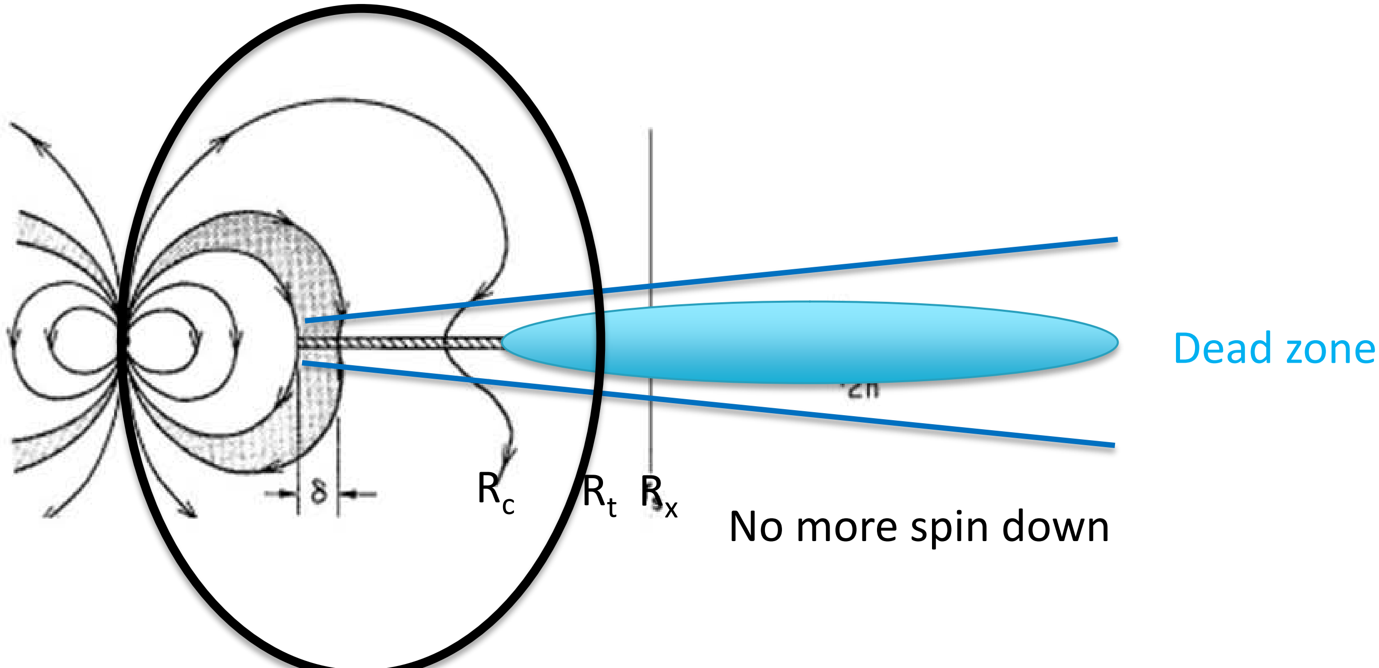
Fig. 5. Accretion rate vs. stellar mass for the objects with disks in th



Giovanni Rosotti

Late stage of disk depletion

large $R_t = \left(\frac{4\pi f_c}{\alpha_D \mu_0}\right)^{2/7} \left(\frac{B_*^4 R_*^5}{GM_* \dot{M}_d^2}\right)^{\frac{1}{7}} R_*$. and R_X

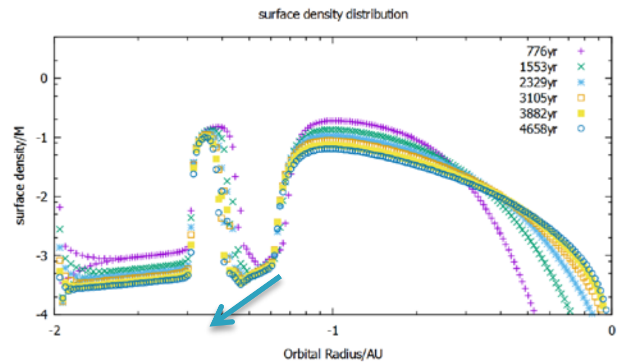


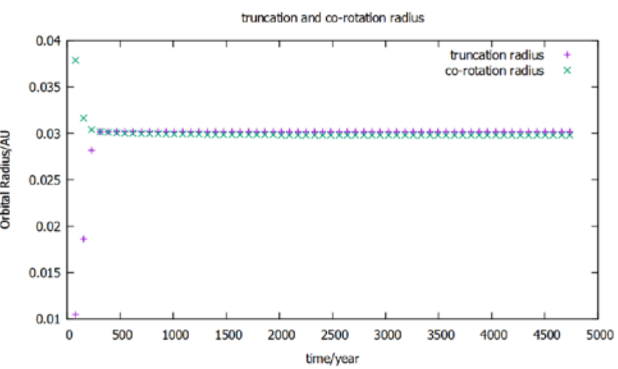
α_D >>1 in the dead zone. => reduce effective R_t and R_c

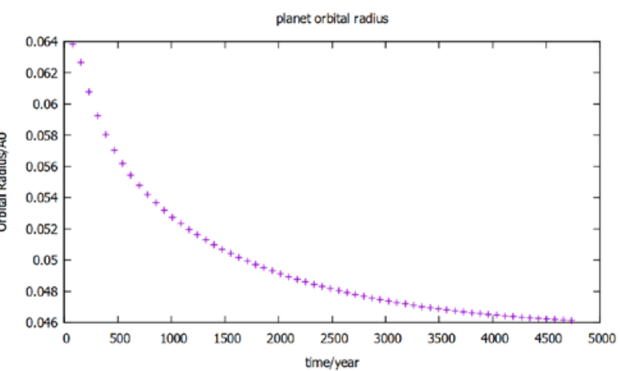
$$\dot{M}_z = 2\pi \int_{R_{in}}^{R_X} S_U \dot{\Sigma}_* r dr = \frac{4\pi m_*^2 / \alpha_D}{\mu_0 \sqrt{GM_*}} \int_{R_{in}}^{R_X} \frac{(U_r/\Omega_{k*} r) S_U}{[(r/R_c)^3 - 1]} \frac{f_c f_\rho \Omega_{k*} dr}{r^{9/2} |\Omega - \Omega_*|}$$

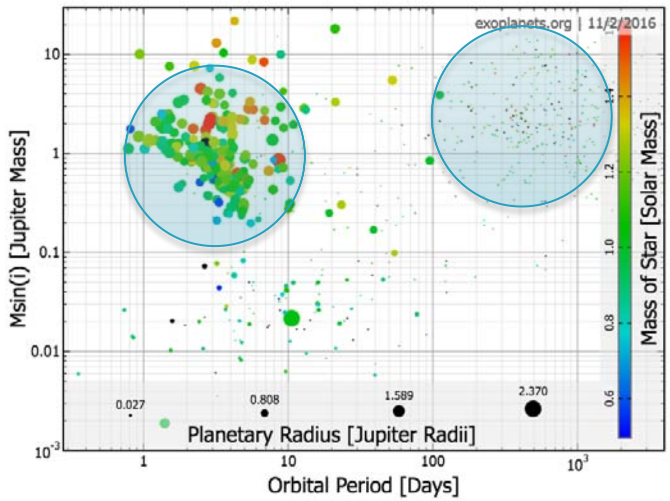
$$\dot{J}_* = \int_{R_{in}}^{R_X} \left(\frac{4\sqrt{2} S_\Omega f_c m_*^2}{\mu_0 \alpha_D r^4}\right) dr + 2\pi \int_{R_{in}}^{R_X} S_U \dot{\Sigma}_* \Omega_* r^3 dr - \dot{M}_z \Omega_* R_*^2.$$

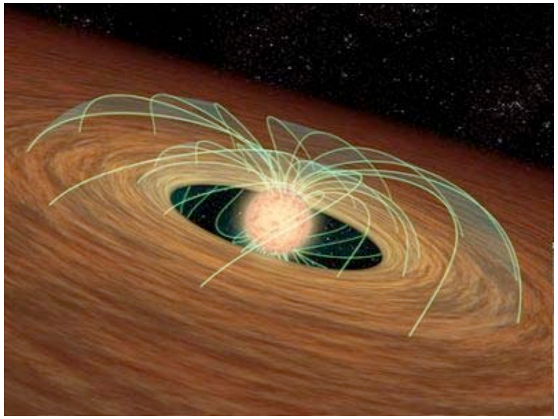
Disk depletion, magnetic gate, quench flow onto star & stall type II migration

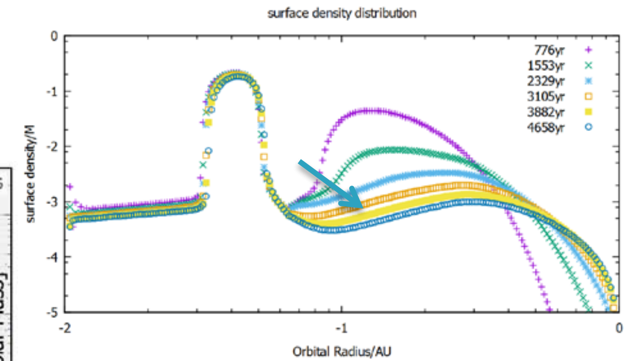


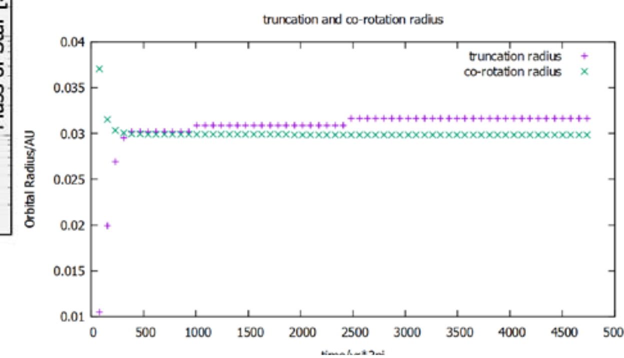


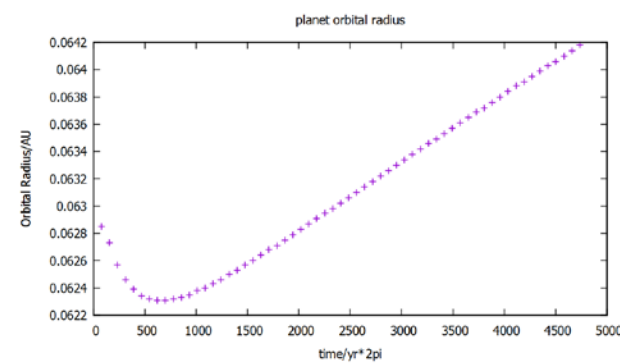






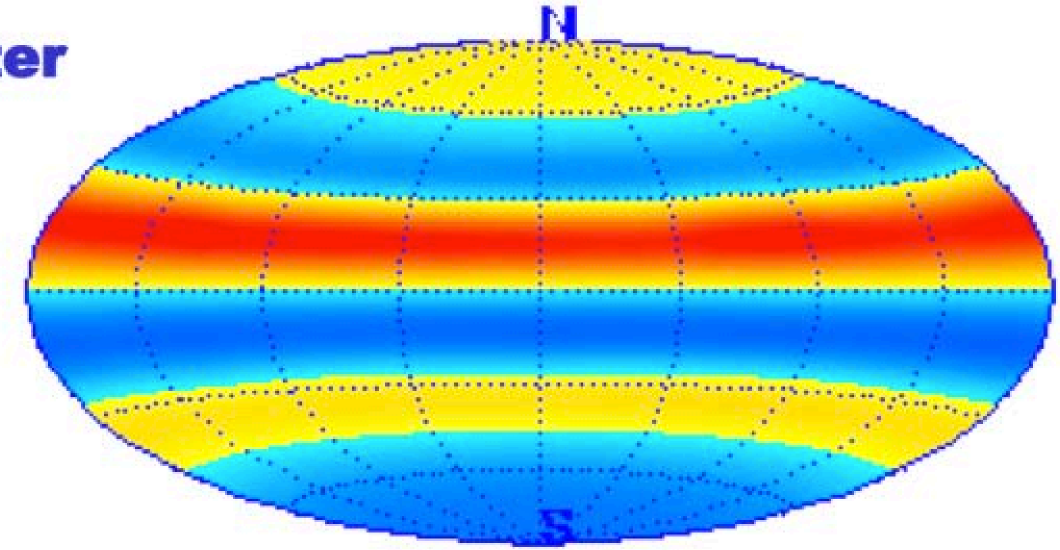




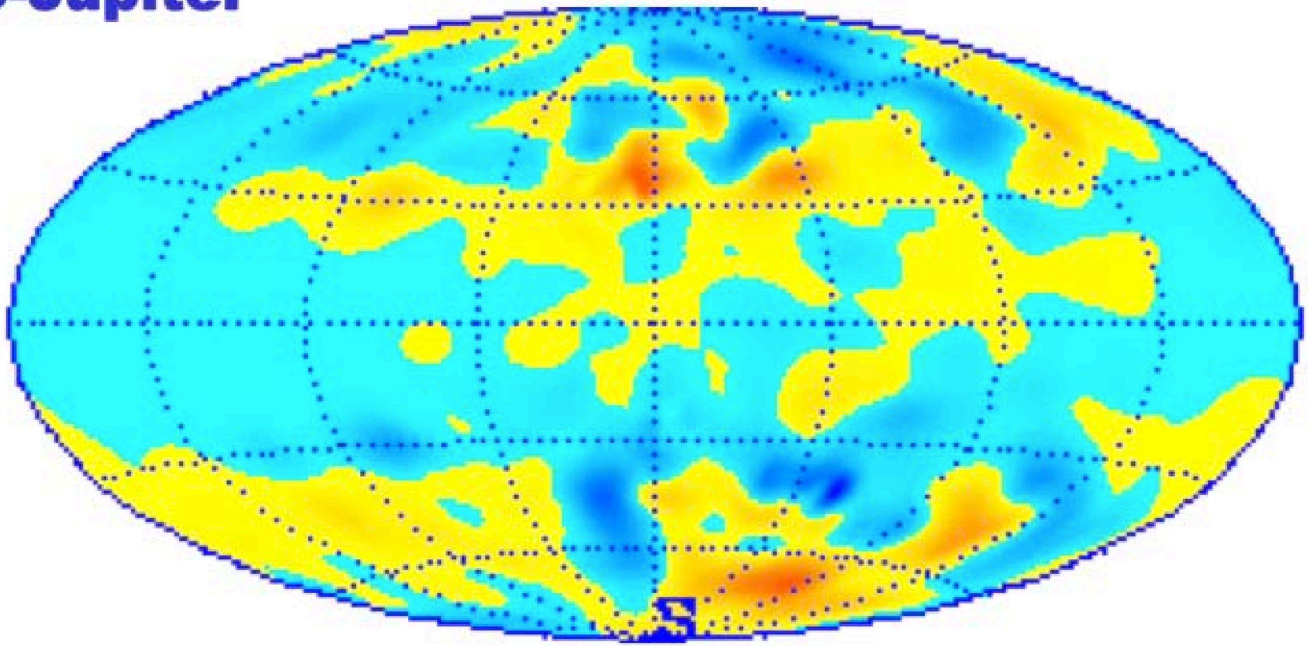


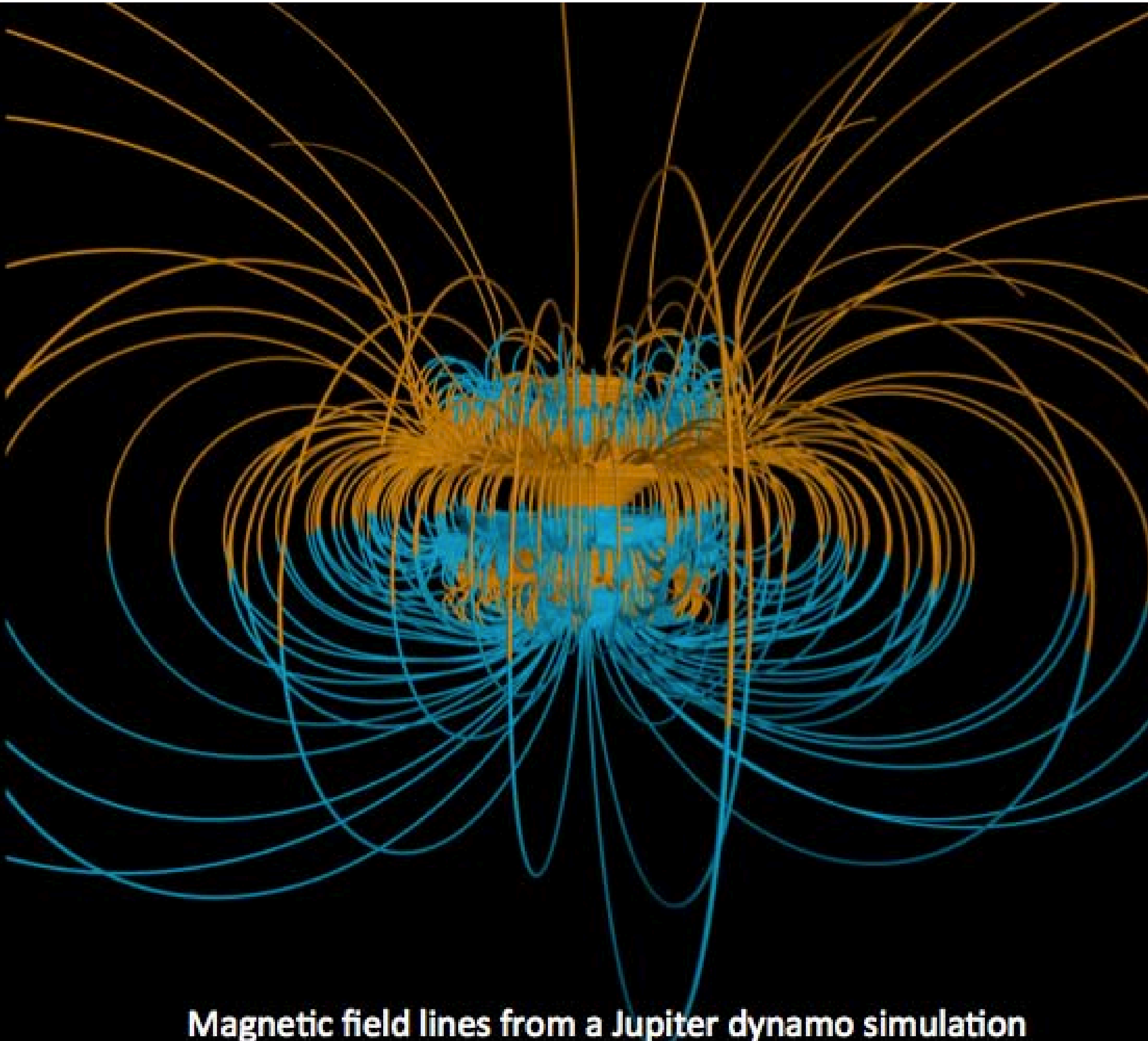
Yu Yan

Jupiter



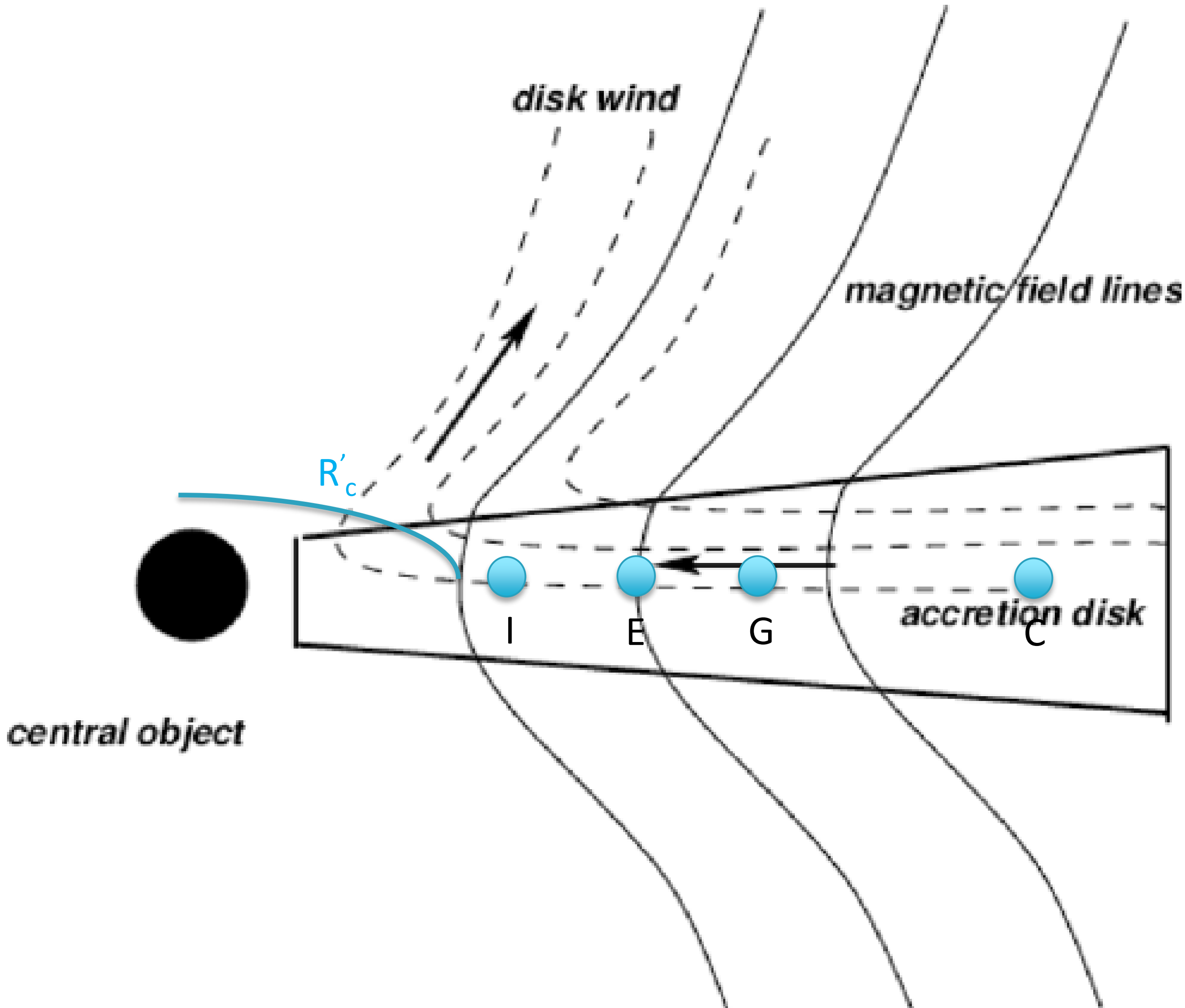
proto-Jupiter



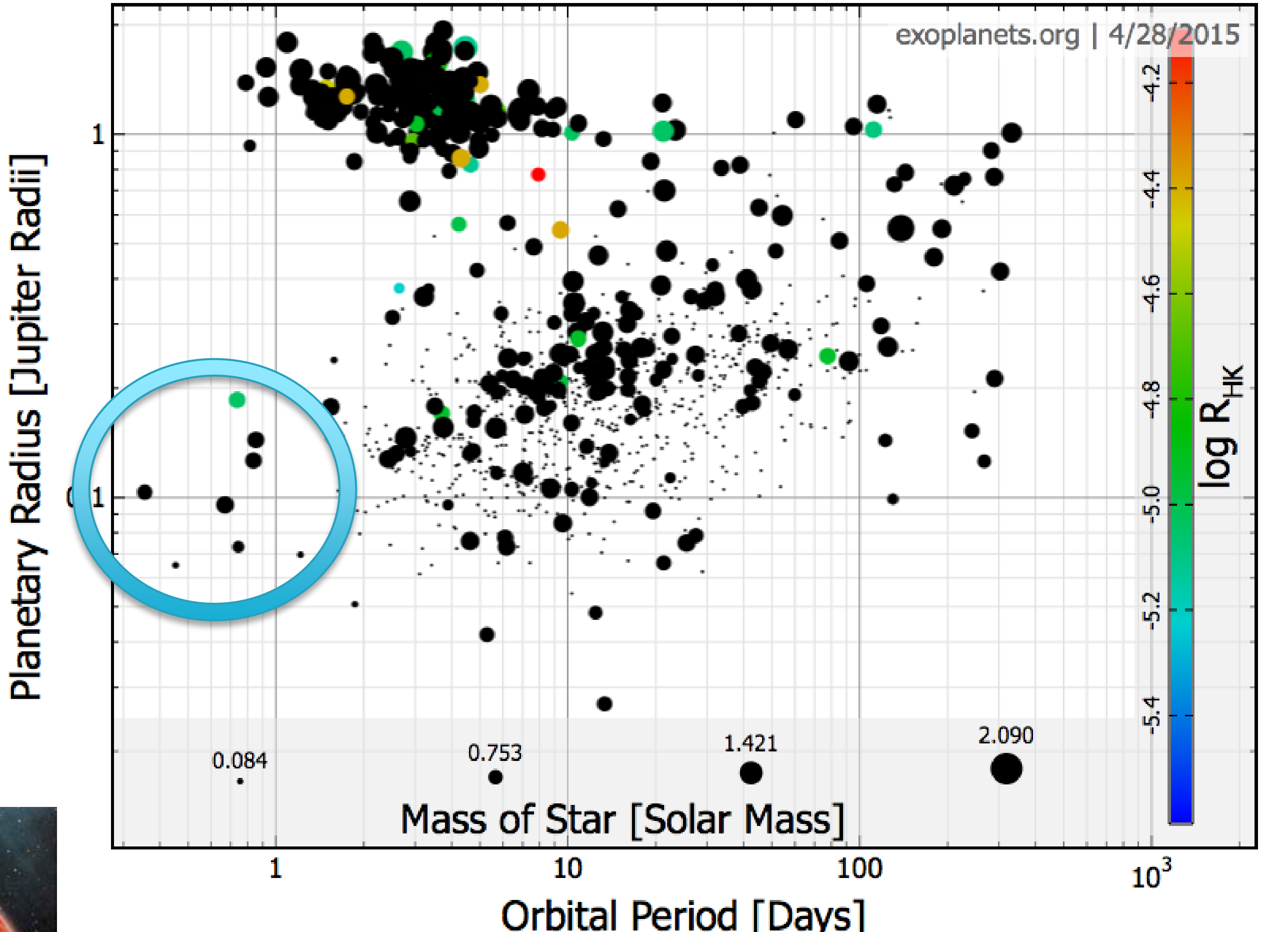


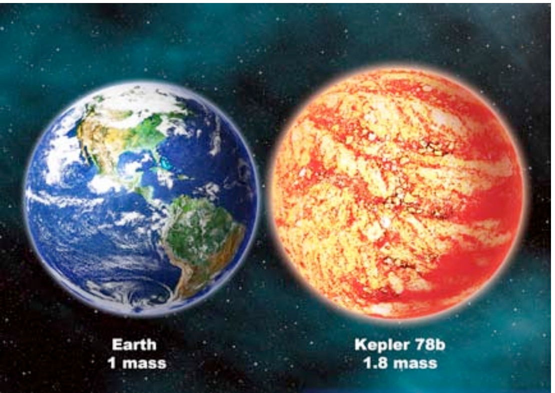
Gary Glatzmaier

Proto-Jupiter's spin down and moons

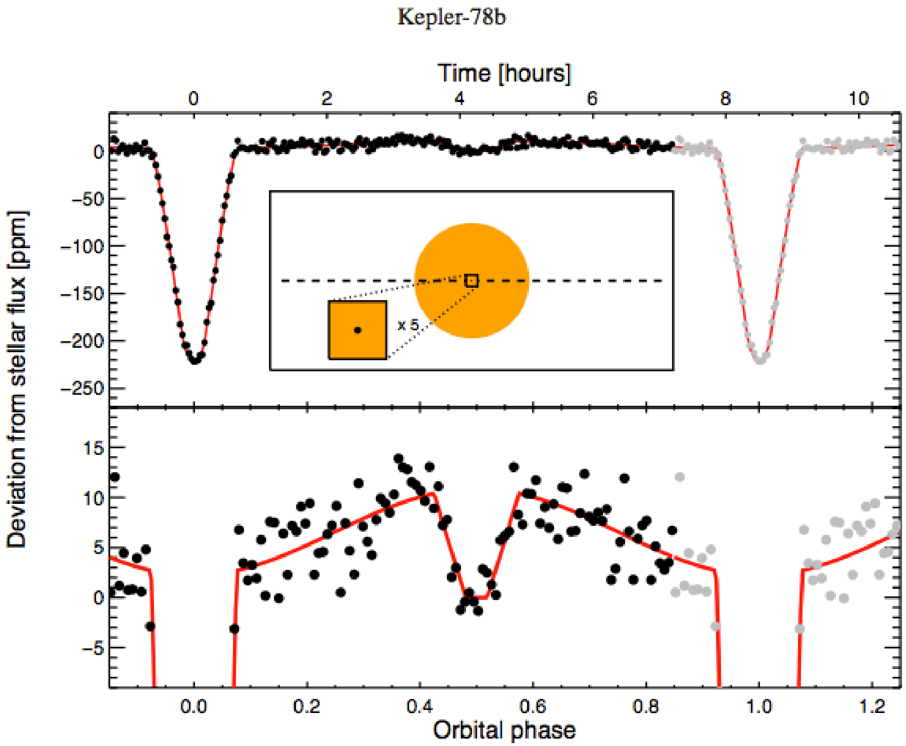


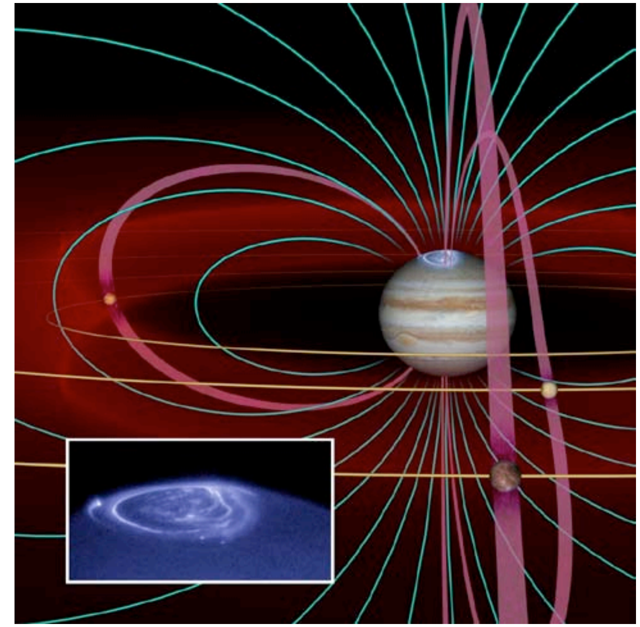
Close-in super Earths

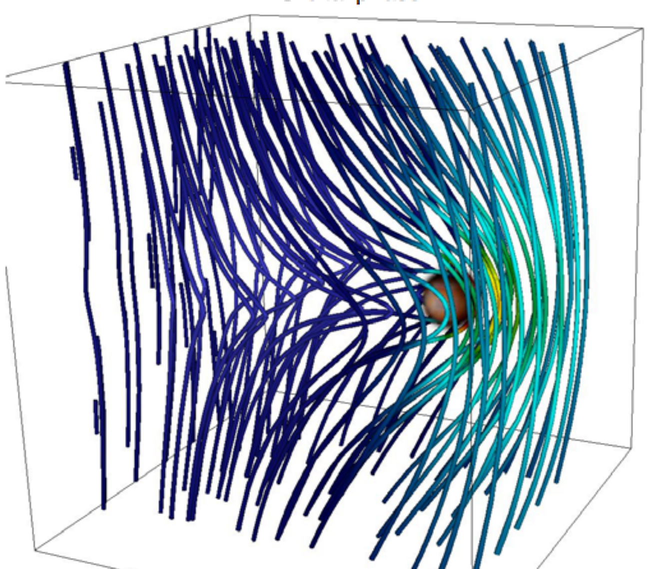


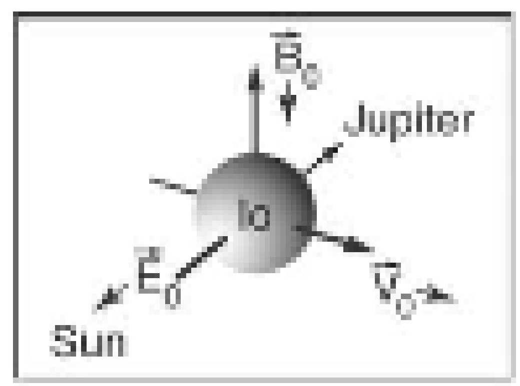


Inside the stellar magnetosphere









Laine, De Colle, Lin Mengxiang

Unipolar induction

$$U_0 = 2R_p a(\omega_p - \omega_*)\frac{\mu_0 m}{4\pi a^3}$$

$$U_p = U_0\frac{\mathcal{R}_p}{\mathcal{R}_p + \mathcal{R}_*}$$

$$U_* = U_0\frac{\mathcal{R}_*}{\mathcal{R}_p + \mathcal{R}_*}$$

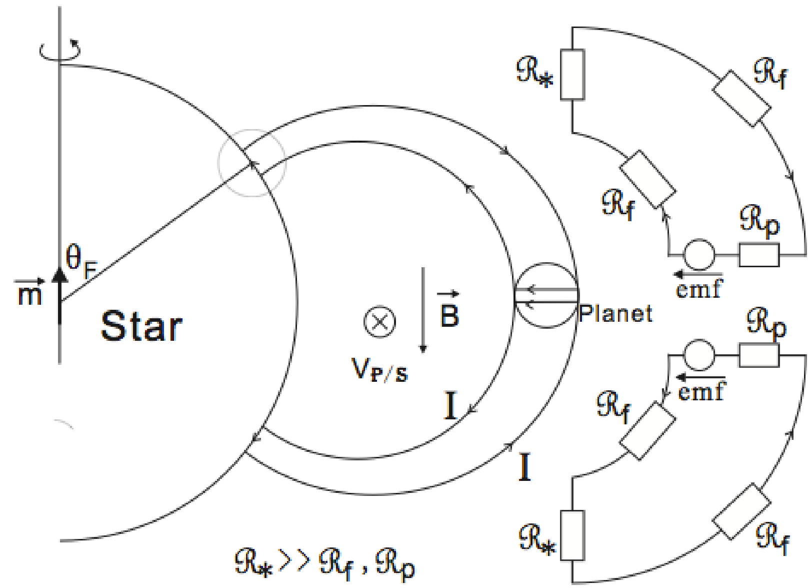
$$I = \frac{U_0}{\mathcal{R}_p + \mathcal{R}_*}$$

$$\mathcal{P}_{Tot} = U_0 I = \frac{U_0^2}{\mathcal{R}_p + \mathcal{R}_*}$$

$$\mathcal{P}_p = U_p I = \mathcal{P}_{Tot}\frac{\mathcal{R}_p}{\mathcal{R}_p + \mathcal{R}_*}$$

$$\mathcal{P}_* = U_* I = \mathcal{P}_{Tot}\frac{\mathcal{R}_*}{\mathcal{R}_p + \mathcal{R}_*}$$

$$|\mathcal{T}_p| = |\mathcal{T}_*| = \frac{\mathcal{P}_{Tot}}{|\omega_p - \omega_*|} = \mathcal{T},$$



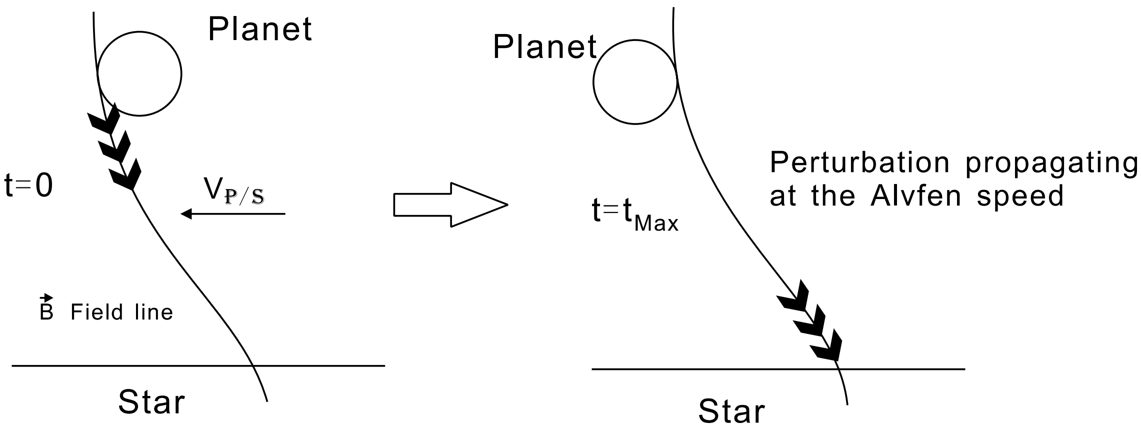
$$\mathcal{T}_{tide} = \frac{3}{2}\frac{k_*}{Q_*}\frac{GM_p^2}{a^6}R_*^5$$

$$\mathcal{T}_L = 8R_p^2(\omega_p - \omega_*)a^2\left(\frac{R_*}{a}\right)^6\frac{\mathcal{B}_*^2(R_*)}{\mathcal{R}_{Tot}}.$$

$$\tau_a = \frac{a}{\dot{a}} = \frac{L_p}{2\mathcal{T}} > 4\times 10^6 yrs$$

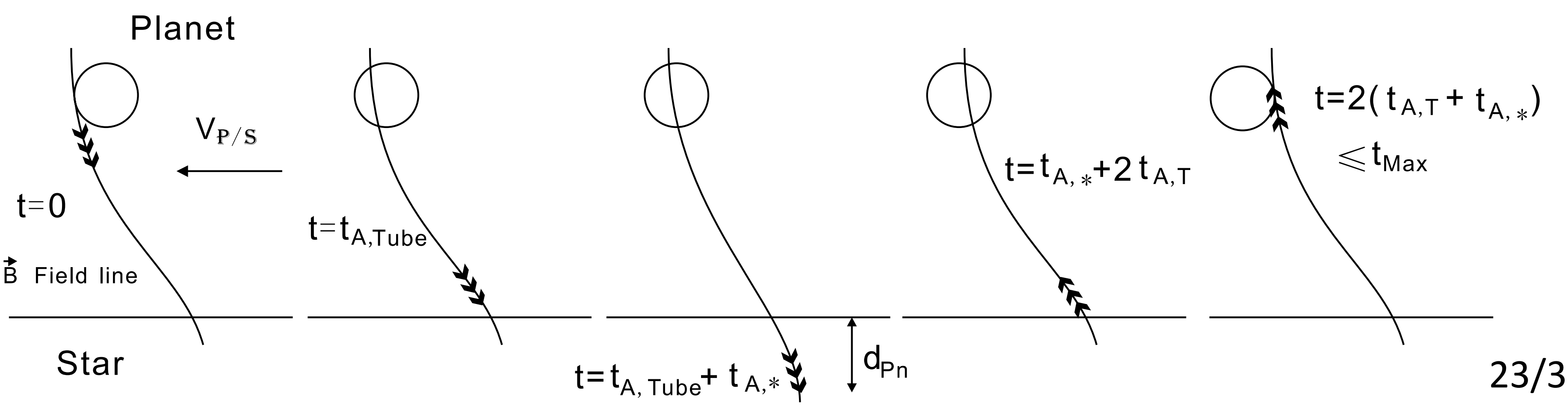
Closure of flux tube

Scenario 1: the planet is too far away. $t_{A,\ tube} > t_{Max}$



$$
\begin{aligned}
t_A &< t_{max} \\
t_{max} &= \frac{2R_p}{v_{p/*}} \frac{\mathcal{R}_{Tot}}{\mathcal{R}_p} = t_0 \left(1 + \frac{\mathcal{R}_*}{\mathcal{R}_p}\right) \\
t_A &= 2(t_{A,FT} + t_{A,*}),
\end{aligned}
$$

Scenario 2: the planet is sufficiently close

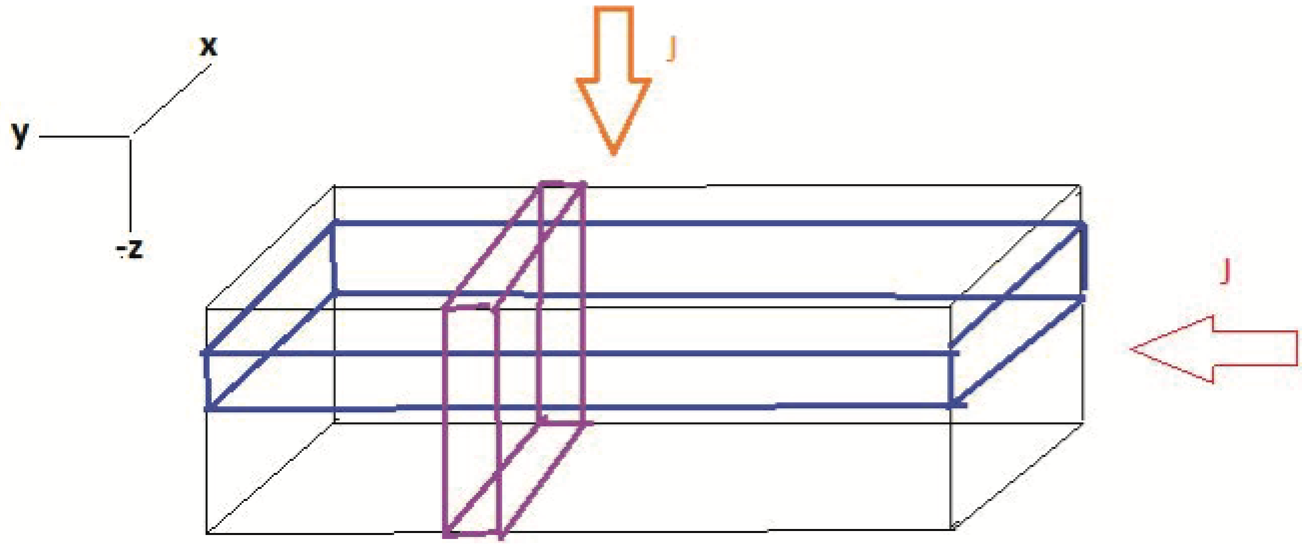


Stellar and planetary conductivity

the stellar electric conductivities in the footprint σ_0 and σ_p thus reduce to

$$\sigma_0 = A_4 \frac{T^{3/4}}{\sqrt{P}} \exp\left(\frac{-E_H}{2kT}\right)\left[1 + \sqrt{f_K}\exp\left(\frac{E_H - E_K}{2kT}\right)\right]$$

$$\sigma_p = \frac{\sigma_0}{1 + A_3^2 \frac{T}{P^2} \mathcal{B}_*^2(r)} \approx \frac{\sigma_0}{A_3^2 \frac{T}{P^2} \mathcal{B}_*^2(r)},$$

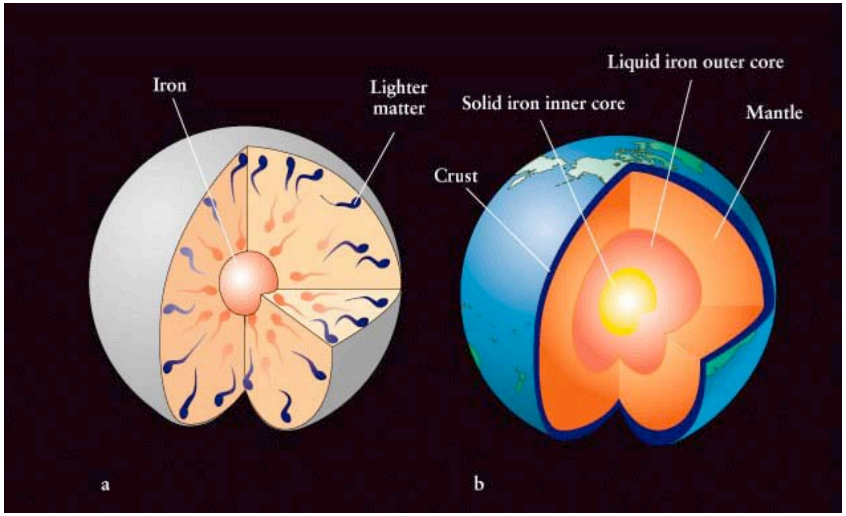


$$\delta\mathcal{R}_{yz} = \int_x \delta\mathcal{R}_{xyz} = 2\int_{x=0}^{\sqrt{R_{max}^2 - z^2 - y^2}} \frac{1}{\sigma_p(r)} \frac{dx}{dydz}$$

$$\delta\mathcal{R}_z^{-1} = \int_y \delta\mathcal{R}_{yz}^{-1} = 2\int_{y=0}^{\sqrt{R_{max}^2 - z^2}} dydz \left(2\int_{x=0}^{\sqrt{R_{max}^2 - z^2 - y^2}} \frac{dx}{\sigma_p(r)}\right)^{-1}$$

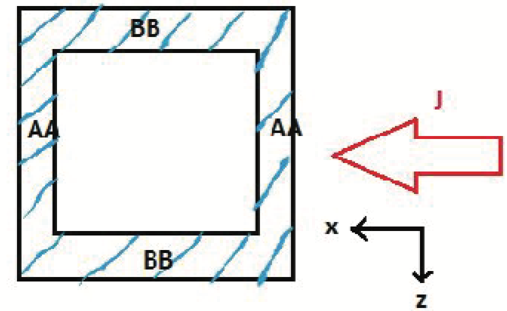
$$\mathcal{R}^{-1} = \int_z \delta\mathcal{R}_z^{-1} = \int_{z=0}^{R_{max}} \int_{y=0}^{\sqrt{R_{max}^2 - z^2}} dydz \left(\int_{x=0}^{\sqrt{R_{max}^2 - z^2 - y^2}} \frac{dx}{\sigma_p(r)}\right)^{-1}$$

Geology and conductivity

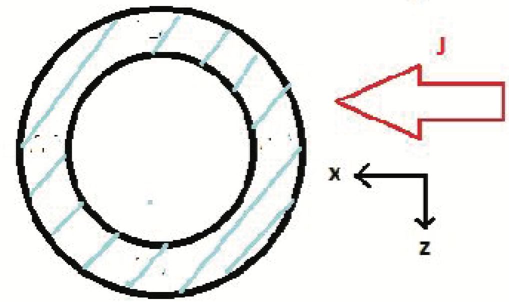


High conductivity region

High resistivity region



Electric conductivity depends only on height, i.e. on x in AA and on z in BB



Electric conductivity depends only on radius

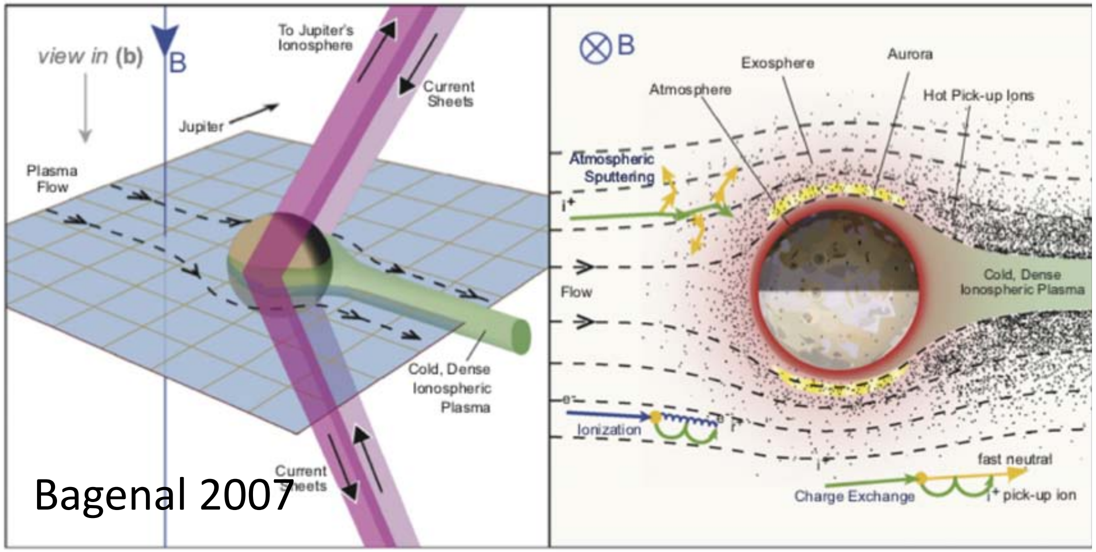
$$\mathcal{R}_p^{-1} = \int_{z=0}^{R_{max}} \int_{y=0}^{\sqrt{R_{max}^2 - z^2}} dydz \left(\int_{x=0}^{\sqrt{R_{max}^2 - z^2 - y^2}} \frac{dx}{\sigma_p(r)} \right)^{-1}$$

$$\mathcal{R}_\perp = \left([2s] \int_z \sigma(z) dz \right)^{-1}$$

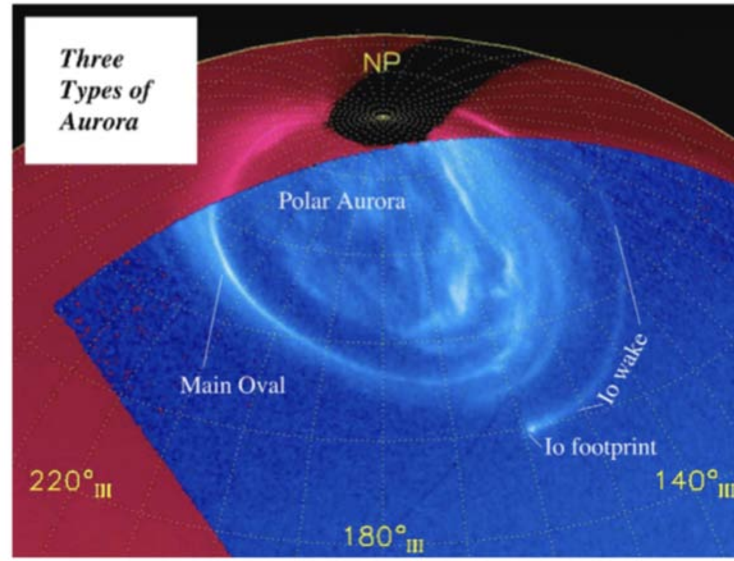
$$\mathcal{R}_\parallel = \frac{1}{R_p^2} \int_z \frac{dz}{\sigma(z)},$$

$$\mathcal{R}_\perp = (2/R_p)\, 1/[\sigma(inner) + \sigma(outer)] \text{ and } \mathcal{R}_\parallel = (1/2R_p)[1/\sigma(inner) + 1/\sigma(outer)].$$

Non closure: Alfven Wing model



Bagenal 2007



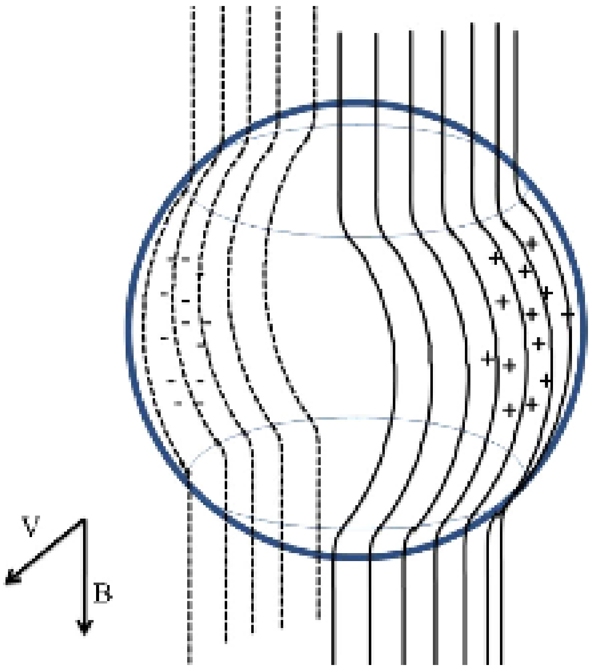
$$\mathcal{R}_{Tot} = \mathcal{R}_p + \mathcal{R}_{wing}$$

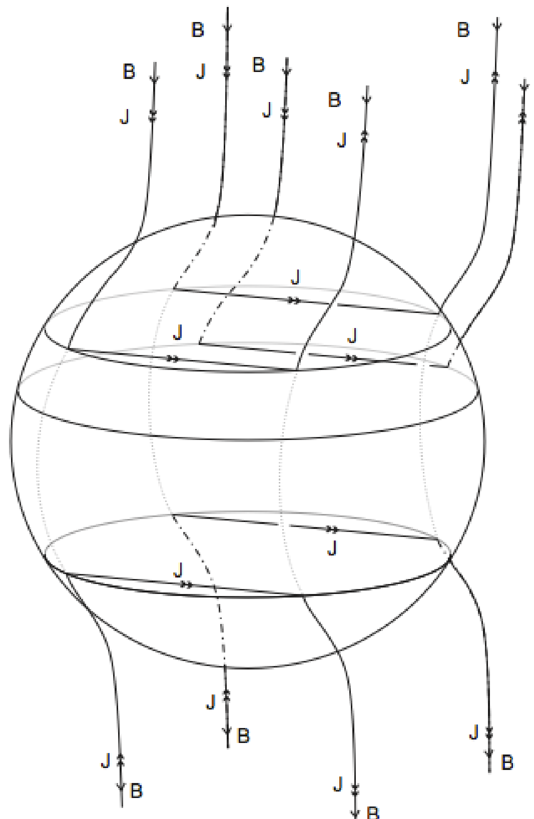
$$\mathcal{R}_{wing} = (\Sigma_A)^{-1} = \mu_0 v_A$$

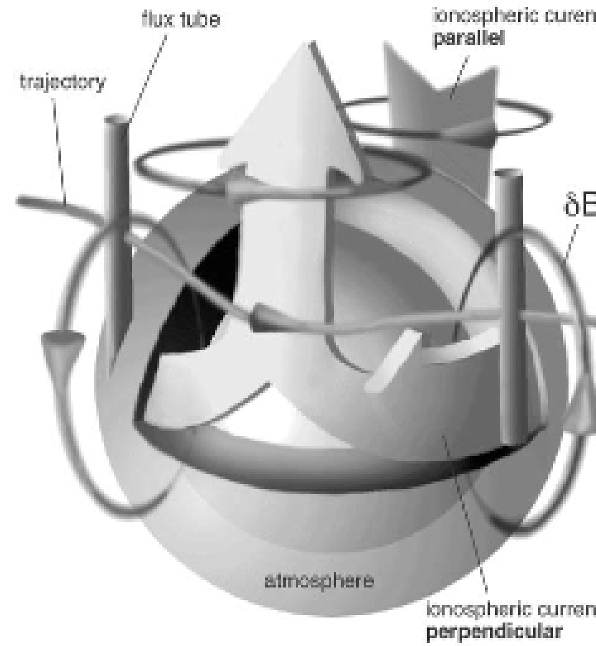
$$\mathcal{R}_{wing} \geq \mathcal{R}_* \geq \mathcal{R}_p \qquad \mathcal{I} \approx U_0/\mathcal{R}_{wing}$$

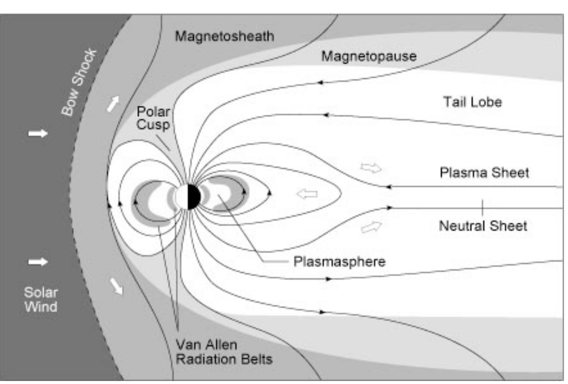
P_p would be reduced by several orders of magnitude

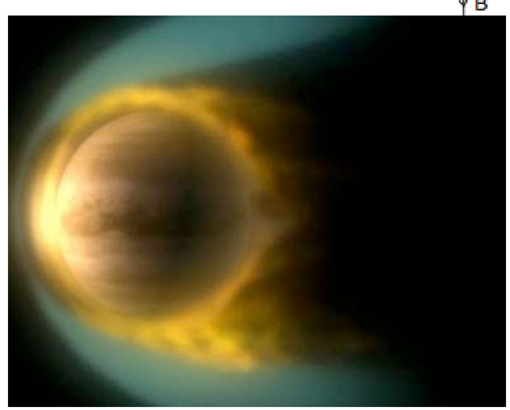
Inside the super Earths

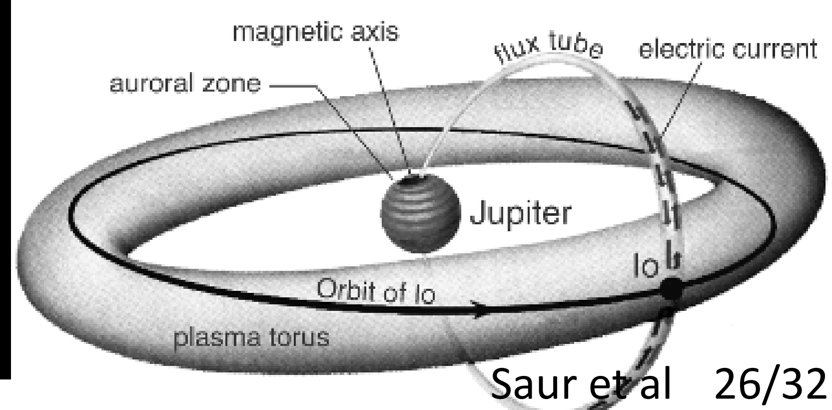










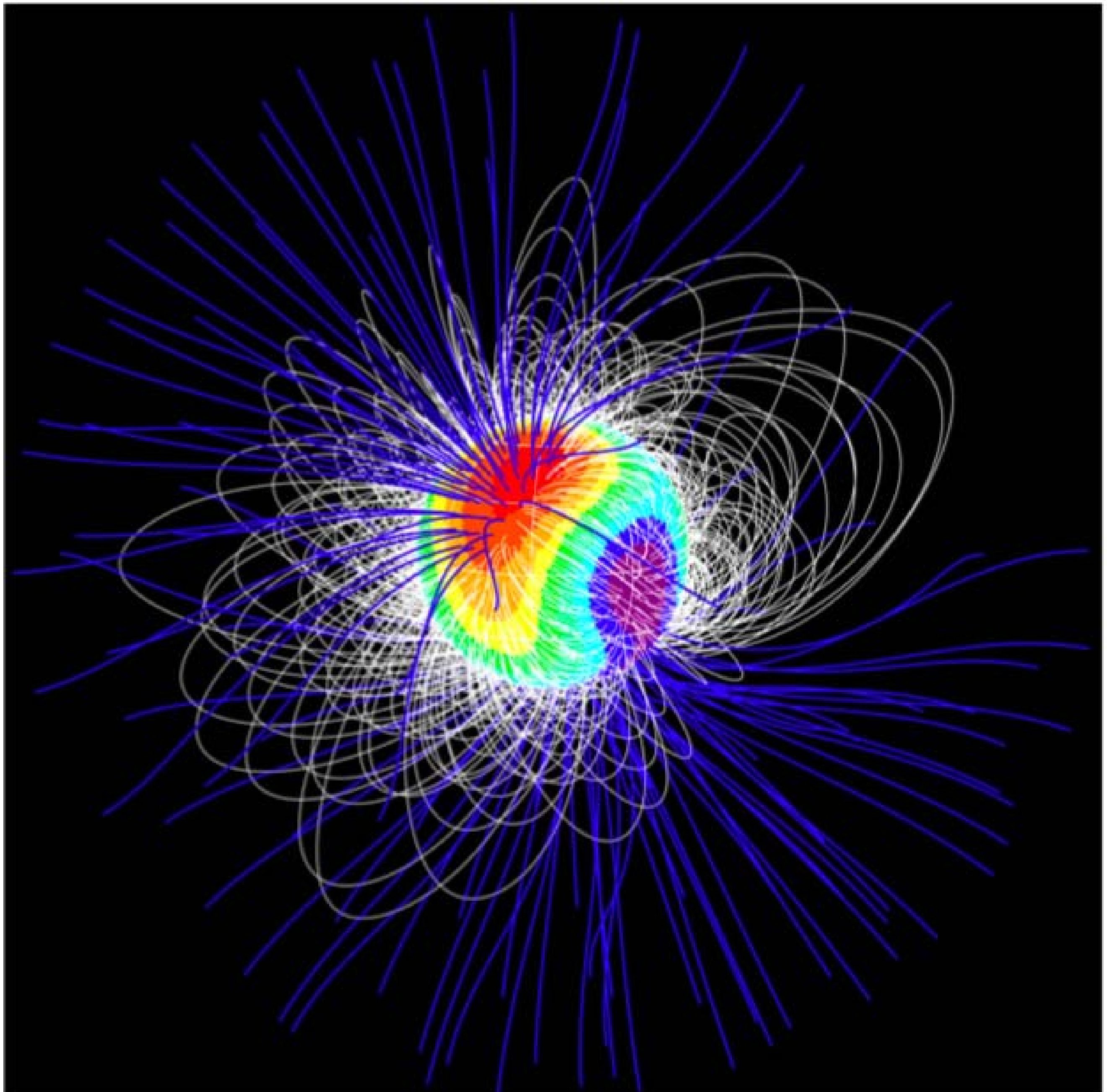


R. Laine

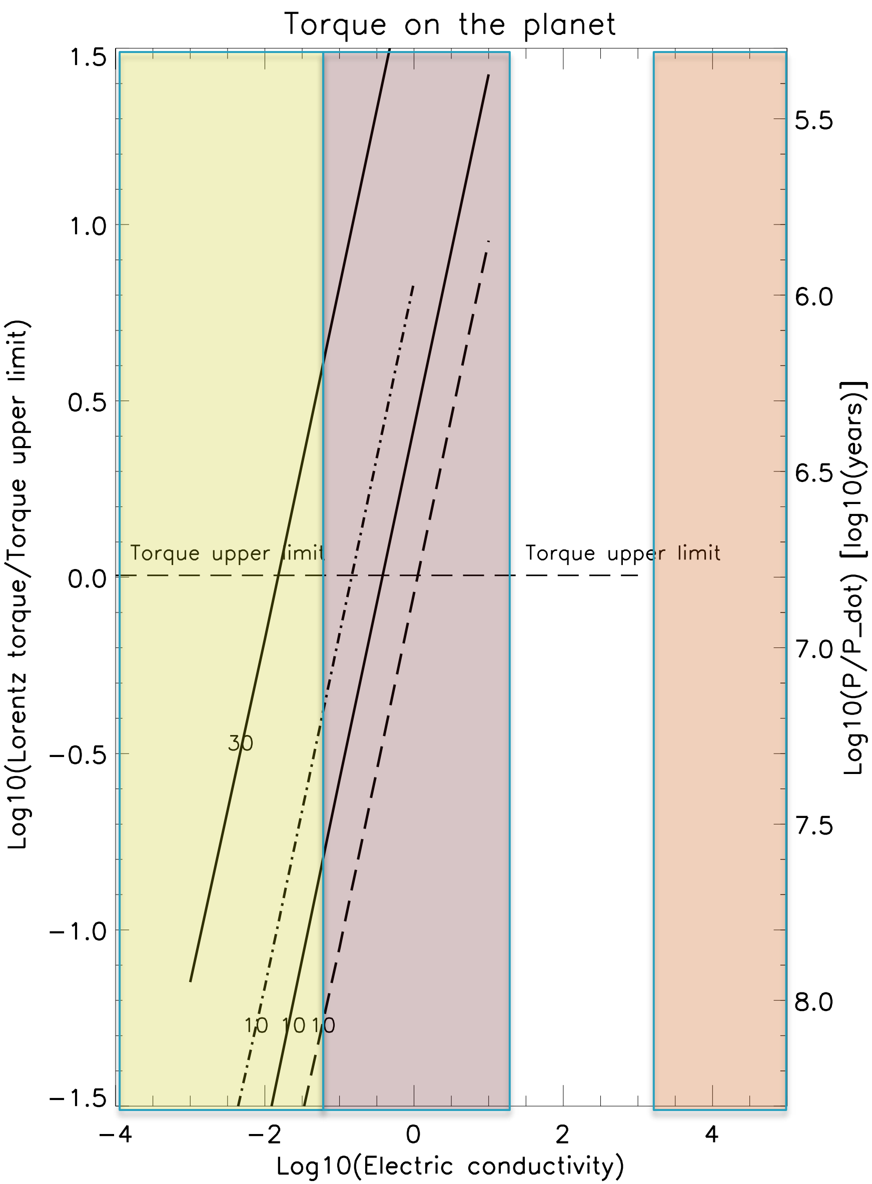
Saur et al

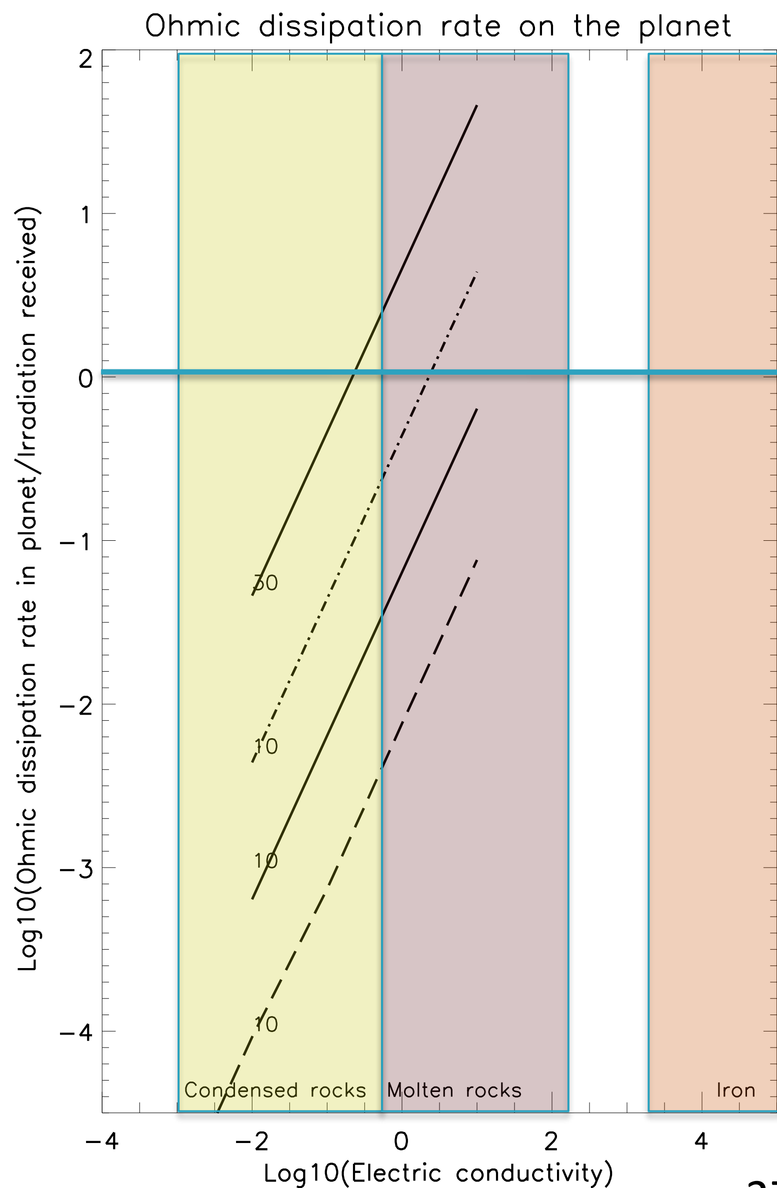
Moutou 2016

The magnetic properties of Kepler-78

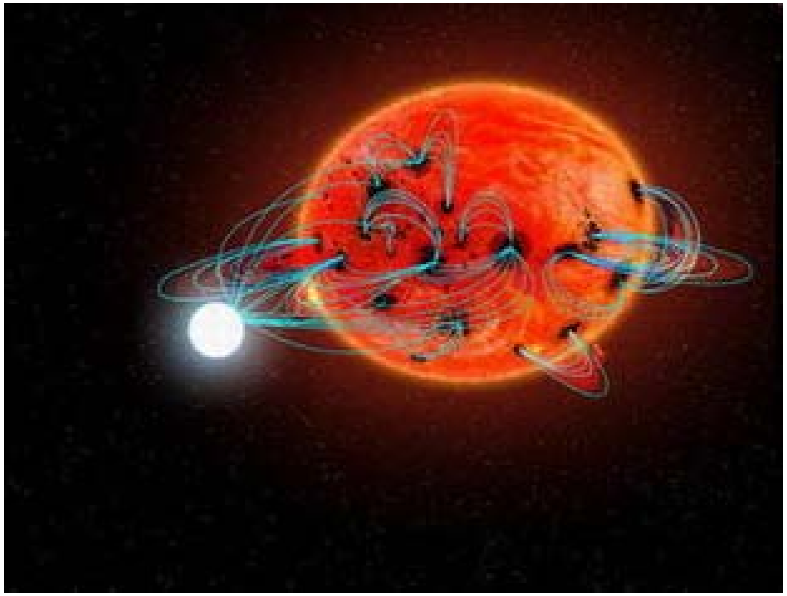


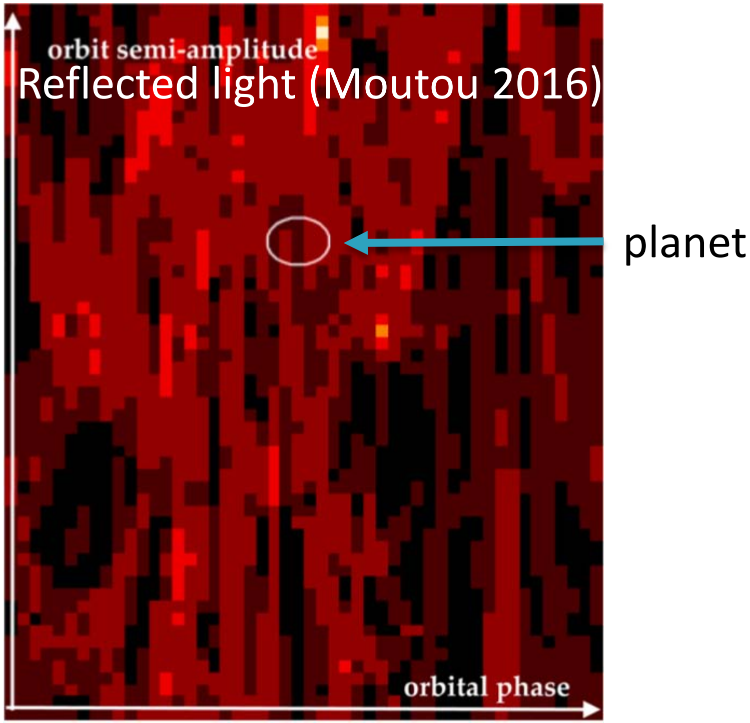
Torque and power

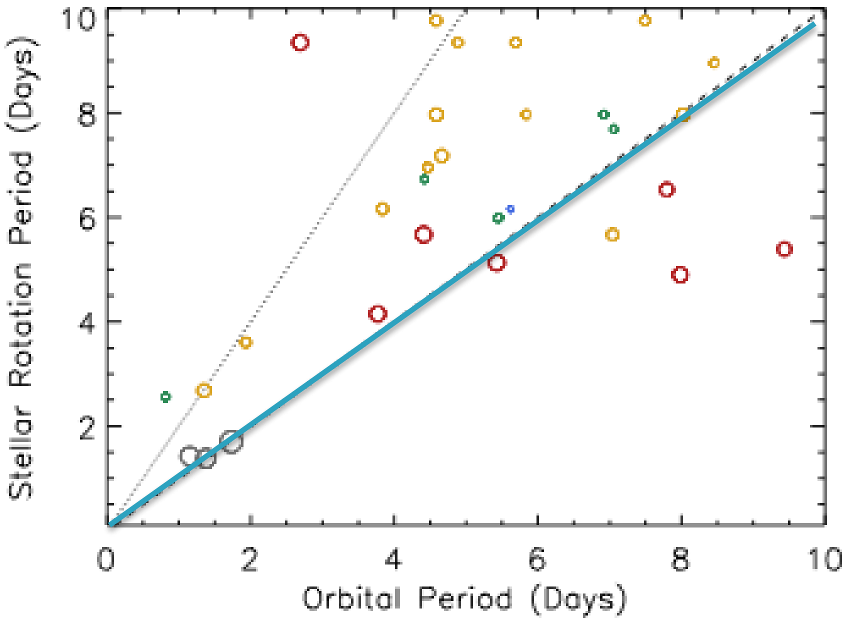




Foot prints and stellar spots



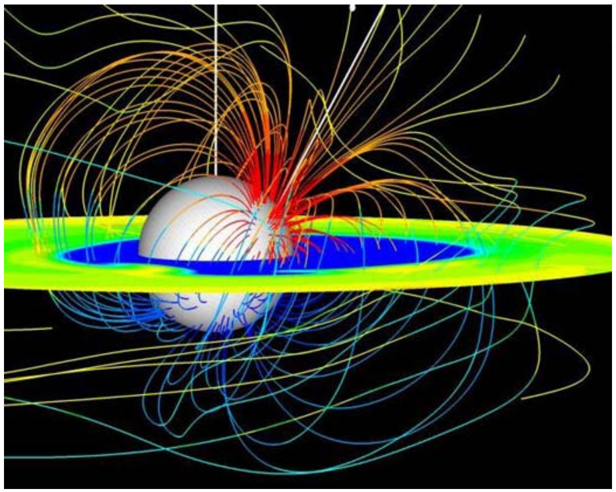


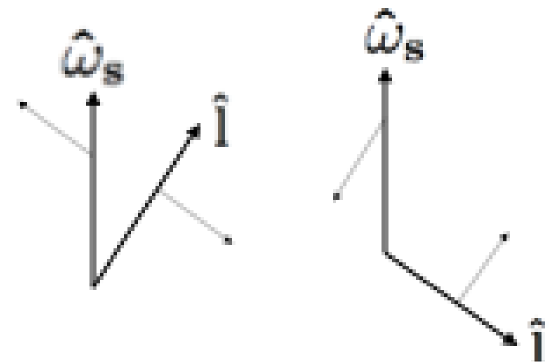


Summary

- Magnetosphere-disk interaction commonly occur in many astrophysical context
- There is a general feedback tendency toward a state of corotation
- Sensitive feedback mechanisms can regulate accretion rate and direction of angular momentum transfer
- T Tauri stars' magnetosphere provides a location for dust accumulation, robust planetesimal formation, and destination of migrating planets
- The spin of post T Tauri stars is determined by the rate of disk depletion
- The retention of Galilean moons and Jupiter's spin may also be due to magnetosphere-disk interaction
- Stellar magnetosphere continue to interact with close-in planets through unipolar induction

Misaligned magnetosphere





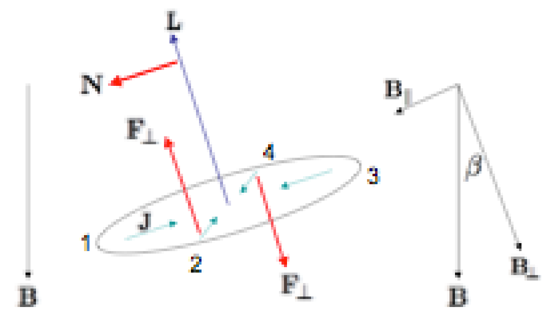
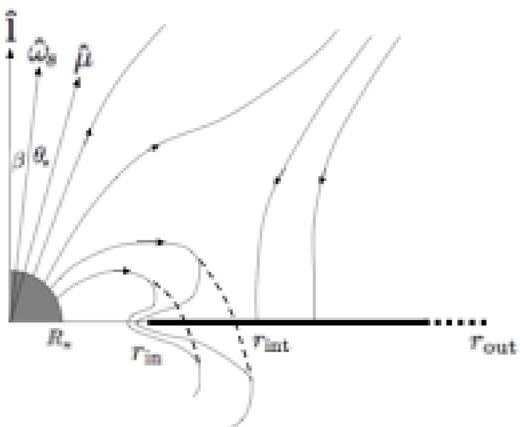


Figure 3. A toy model for understanding the origin of the warping torque. A tilted rotating metal plate (with angular momentum $\mathbf{L}$) in an external magnetic field $\mathbf{B}$ experiences a vertical magnetic force around region 2 and 4 due to the interaction between the induced current $\mathbf{J}$ and the external $\mathbf{B}_\parallel$, resulting in a torque $\mathbf{N}$ which further increases the tilt angle β.

Observable tests



Lai, Foucart, & Lin

Similar effects in close-in planets: additional Torque and dissipation avenues

Cumming & Lin