Tidal evolution models for exoplanets

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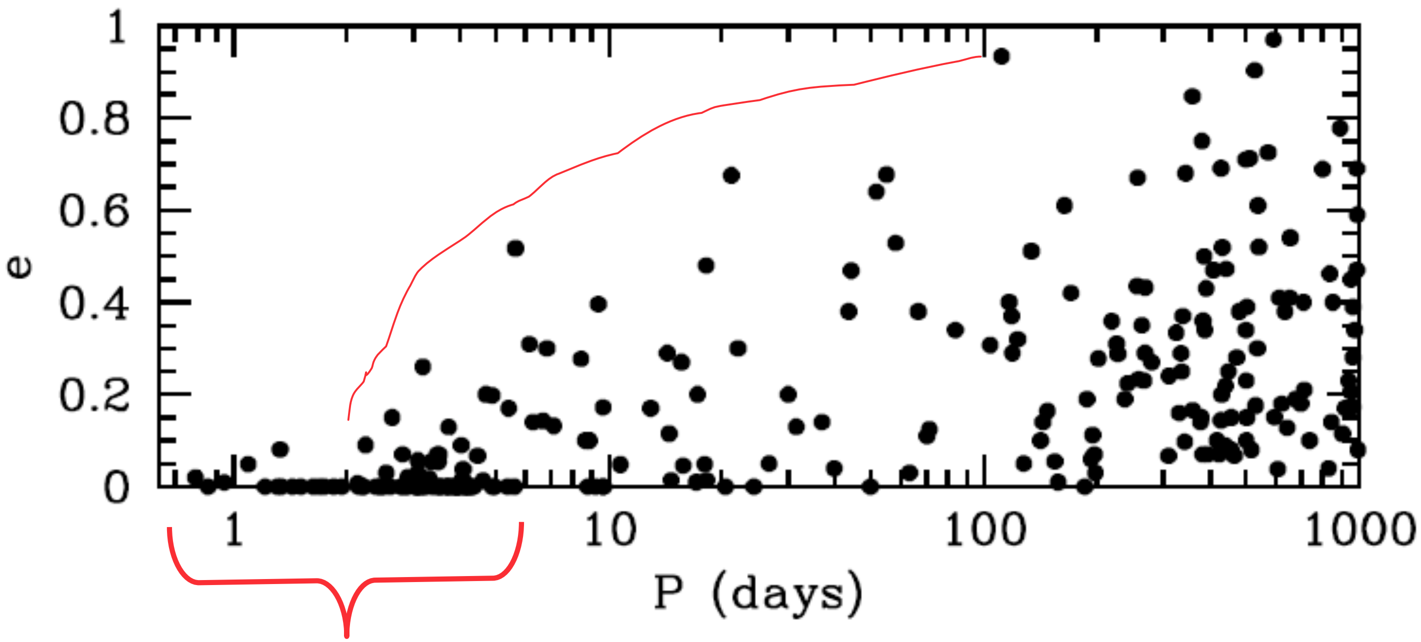
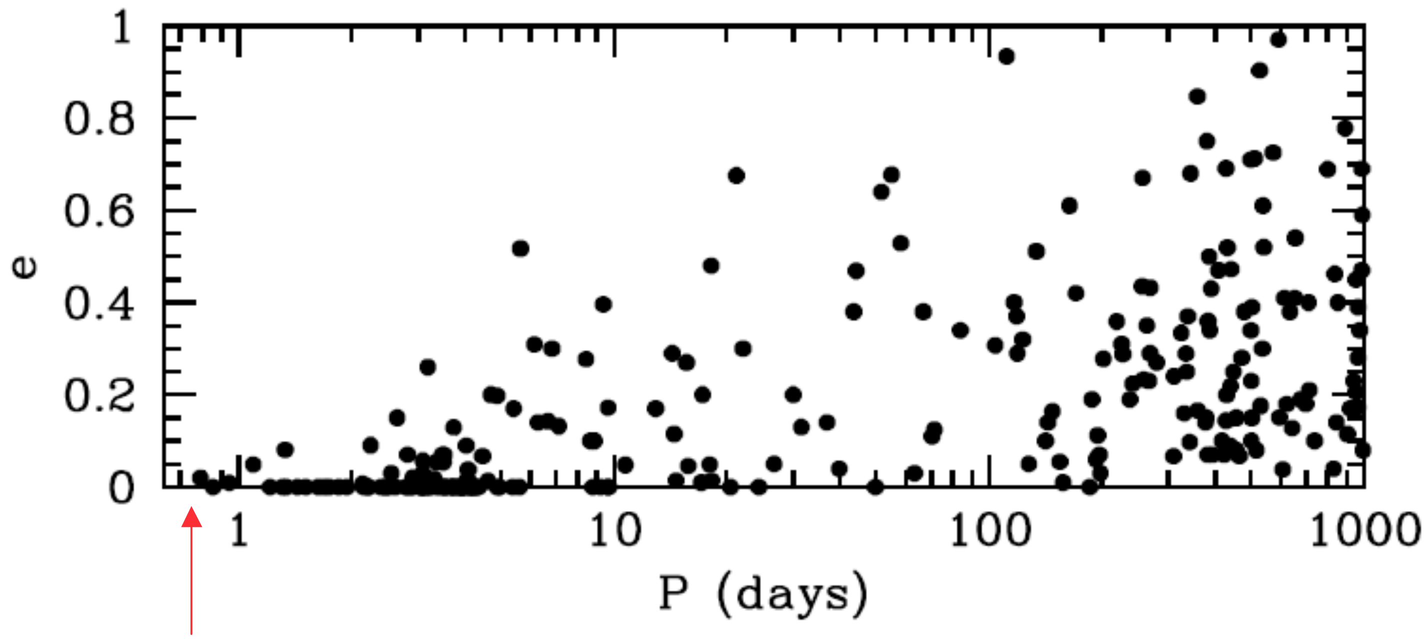
Eccentricity-period distribution



Evidence for tidal circularization

Eccentricity-period distribution



record 0.79 days WASP-19b.

WASP-12b - overflowing its Roche lobe?

$m_p = 1.15 M_J$

Reasons for getting your tidal evolution model right

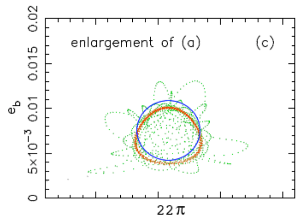
- Understanding the efficiency of the tidal dissipation process helps us to constrain aspects of the theory of planet formation and subsequent dynamical evolution
  - timing: did the planet arrive at its present location before or after the PMS?
    - what role did the proto-stellar spin play? (Dobbs-Dixen et al 2004)
  - disk migration or something else (Kozai ???)
  - what role did/do the magnetic fields of both the star and planet play?
  - are some planets swallowed by the star ?
    - are we seeing ``the last of the Mohecans" (al la DNC Lin: WASP-12b)
  - how much has the original eccentricity distribution been modified by tides?

Reasons for getting your tidal evolution model right

- has Kozai forcing by companion (star or planet) + tides played a significant role in shaping period distribtion of short-period planets?
    - at what stage in its orbital evolution is a Kozai-forced system like HD 80606? (e=0.93)
- what role do tides play in planet inflation ? (other mechanisms: eg. magnetic fields, stellar insolation)
- for non-circular short-period high-density planets, what can we say about their internal structure?
    - what about water worlds?
    - in between (rocky + oceans)
- how do tides influence the existence or otherwise of moons ?

Reasons for getting your tidal evolution model right

- what can we deduce about the internal structure of short-period planets with companions (one needs a low enough planet Q-value for fixed point to be reached)
  - do planets form with or without cores?
    - some do and some don't ?



  - The HAT-P-13 system - the Rossetta stone for internal structure ? (G. Laughlin)
    - not if it is significantly inclined (probably it is not…)

- can we use the same theory to guide our search for low-mass companion planets?
  - why does GJ 436 (a Neptune-sized object) have such a large eccentricity ?
    - Does it simply have a large Q-value? (a=0.029 AU, e=0.15, a/R_p=160)
    - Or does it have a low-mass companion hiding under an inclined rock?

Reasons for getting your tidal evolution model right

- how do known exoplanets differ from Solar System planets of similar mass ?
- are there Mercury analogues (eg 3:2 spin-orbit resonance) - most surely there are!
- is there evidence for tides ``breaking" mean-motion resonances between more distant companions?
- what can we deduce about the structure and damping efficiency of host stars?
    - Is this consistent with what we know about binary pairs?

Equilibrium tide models

Equilibrium tide: assumes hydrostatic equilibrium (only really true for circular, synchronous spin-aligned systems but reasonable for modest eccentricities)

In the beginning…

Phil. Trans. Roy. Soc. Lon.

XX. *On the Secular Changes in the Elements of the Orbit of a Satellite revolving about a Tidally distorted Planet.*

By G. H. DARWIN, *F.R.S.*

Received December 8,—Read December 18, 1879.

Equilibrium tide models

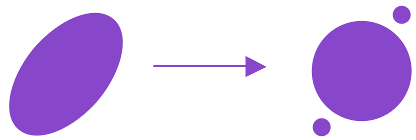
bulge potential + Lagrange
planetary equations

first-order in eccentricity

DARWIN 1879

constant lag time

HUT 1981



Jeffreys 1961, Goldreich 1963

constant lag angle

Energy, angular momentum
→ work done, torque

Closed-form in eccentricity,
spin, first-order in inclination

Equilibrium tide models

ALEXANDER 1973

EGGLETON (et al) 1998

fluid-dynamical description

Fluid shear in rotating frame ≡ constant lag time

Closed-form in eccentricity,
spin, AND inclination

Equilibrium tide models

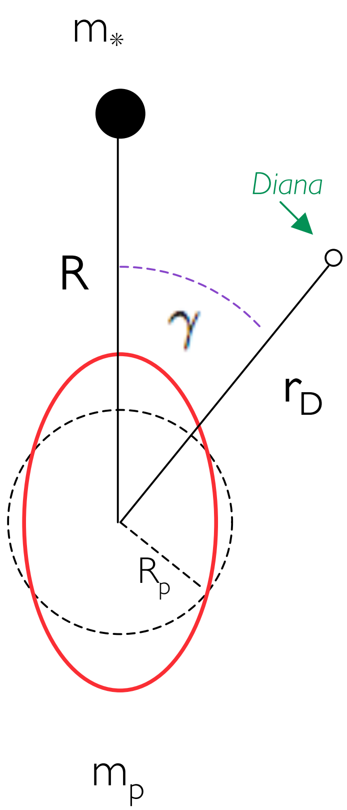
Constant lag angle versus constant lag time…

these days: Goldreich vs Hut / Eggleton.

And then there's that hybrid model many people use…

Equilibrium tide models

Constant lag angle versus constant lag time: DARWIN



Potential due to bulge:

$$\frac{\Phi_p(\mathbf{r}_\mathcal{D})}{Gm_p/R_p} = k_{2p}\left(\frac{m_*}{m_p}\right)\left(\frac{R_p}{R}\right)^3\left(\frac{R_p}{r_D}\right)^3 P_2(\cos\gamma)$$

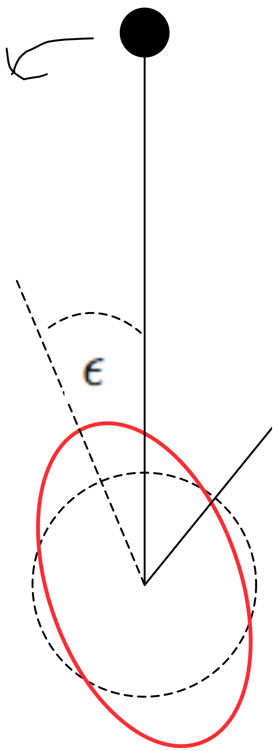
$$= k_{2p}\left(\frac{m_*}{m_p}\right)\left(\frac{R_p}{a_p}\right)^3\left(\frac{R_p}{a_\mathcal{D}}\right)^3 \sum_{jkm} \mathcal{C}_{jkm}(e_p, e_\mathcal{D}) \cos\left[j\,M_p + k\,M_\mathcal{D} + m\,\varpi_\mathcal{D}\right]$$

$$M_p = n\,t, \quad M_\mathcal{D} = \lambda_\mathcal{D} - \varpi_\mathcal{D}$$

$$j,\ k\ = \text{Fourier indices}, \quad m = \text{spherical harmonic order} = 0 \text{ or } 2$$

Equilibrium tide models

Constant lag angle versus constant lag time: DARWIN



Potential due to bulge with each tidal component lagged:

$$\frac{\Phi_p(\mathbf{r}_\mathcal{D})}{Gm_p/R_p} = k_{2p}\left(\frac{m_*}{m_p}\right)\left(\frac{R_p}{a_p}\right)^3\left(\frac{R_p}{a_\mathcal{D}}\right)^3\sum_{jkm}\mathcal{C}_{jkm}(e_p,e_\mathcal{D})\cos\left[j\,(M_p+\epsilon_{jm})+k\,M_\mathcal{D}+m\,\varpi_\mathcal{D}\right]$$

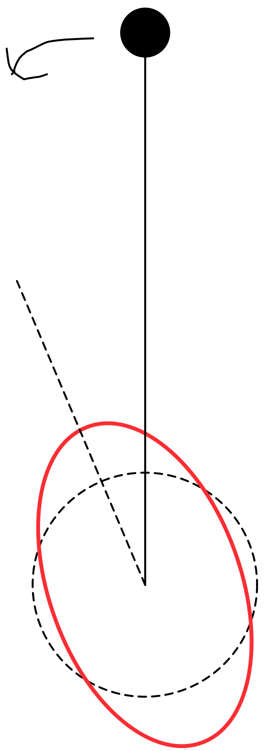
$$= k_{2p}\left(\frac{m_*}{m_p}\right)\left(\frac{R_p}{a_p}\right)^3\left(\frac{R_p}{a_\mathcal{D}}\right)^3\sum_{jkm}\mathcal{C}_{jkm}(e_p,e_\mathcal{D})\left\{\cos\left[j\,M_p+k\,M_\mathcal{D}+m\,\varpi_\mathcal{D}\right]\right.$$

$$\left.+\epsilon_{jm}\sin\left[j\,M_p+k\,M_\mathcal{D}+m\,\varpi_\mathcal{D}\right]+\mathcal{O}(\epsilon_{jm}^2)\right\}$$

Spin of distorted body faster than orbital motion: tidal bulge LEADs line of centres: *orbit is torqued*

Equilibrium tide models

Constant lag angle: GOLDREICH



Using Lagrange's planetary equations to write down rates of change of orbital elements, then putting $e_M = e_D$, $a_M = a_D$ and averaging over the orbital period, gives

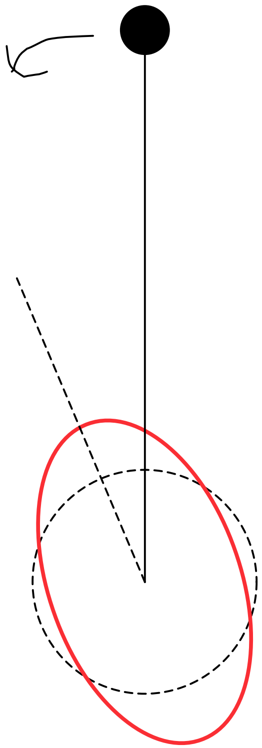
$$\frac{1}{e}\frac{de}{dt} = \frac{3}{2} n\, k_{2p} \left(\frac{m_*}{m_p}\right) \left(\frac{R_p}{a}\right)^5 \left[\epsilon_0 - \frac{49}{4}\epsilon_1 + \frac{1}{4}\epsilon_2 + \frac{3}{2}\epsilon_3\right]$$

and related expressions for semimajor axis and apsidal angle.

Spin of distorted body faster than orbital motion: tidal bulge LEADs line of centres: *orbit is torqued*

Equilibrium tide models

Constant lag angle: GOLDREICH 1963



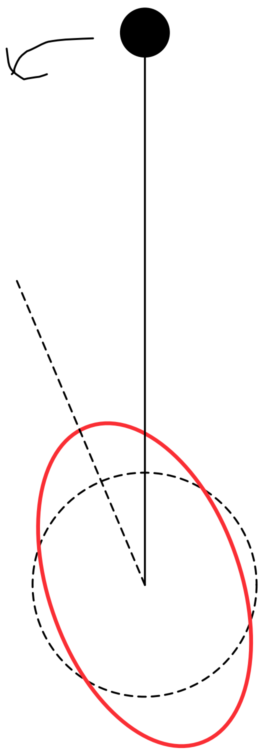
Using Lagrange's planetary equations to write down rates of change of orbital elements, then putting $e_M=e_D$, $a_M=a_D$, gives

$$\frac{1}{e}\frac{de}{dt} = \frac{3}{2} n\, k_{2p} \left(\frac{m_*}{m_p}\right)\left(\frac{R_p}{a}\right)^5 \left[\epsilon_0 - \frac{49}{4}\epsilon_1 + \frac{1}{4}\epsilon_2 + \frac{3}{2}\epsilon_3\right]$$

lag angles

Equilibrium tide models

Q-value: Goldreich & Soter 1966:



the tidal dissipation function 1/Q defined by

$$Q^{-1} = \frac{1}{2\pi E_0} \oint \left(-\frac{dE}{dt}\right) dt \simeq 2\epsilon$$

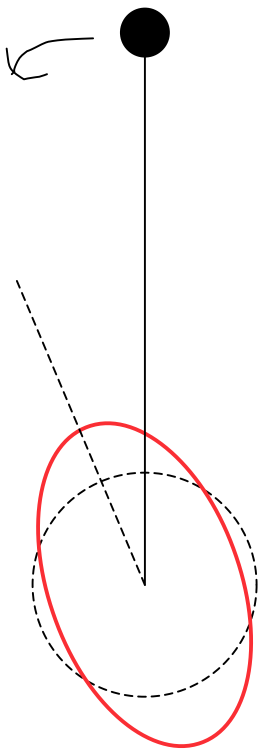
where E_0 is the maximum energy stored in the tidal distortion and -dE/dt is the energy lost during one complete cycle.

Equal lag angles: $\epsilon_0 = \epsilon_1 = \epsilon_2 = \epsilon_3$ ⟶ a single Q-value

Goldreich's argument for equal lag angles was based on the fact that for the Earth, Q varies by less than a factor of four over a range of one cycle per second to one cycle per year (Goldreich 1963)

Equilibrium tide models

Q-value: Goldreich & Soter 1966:



$$\frac{1}{e}\frac{de}{dt} = \frac{3}{2} n\, k_{2p} \left(\frac{m_*}{m_p}\right) \left(\frac{R_p}{a}\right)^5 \left[\epsilon_0 - \tfrac{49}{4}\epsilon_1 + \tfrac{1}{4}\epsilon_2 + \tfrac{3}{2}\epsilon_3\right]$$

$$= \frac{171}{16} n \left(\frac{2}{3}\frac{k_{2p}}{Q_p}\right) \left(\frac{m_*}{m_p}\right) \left(\frac{R_p}{a}\right)^5 \sigma$$

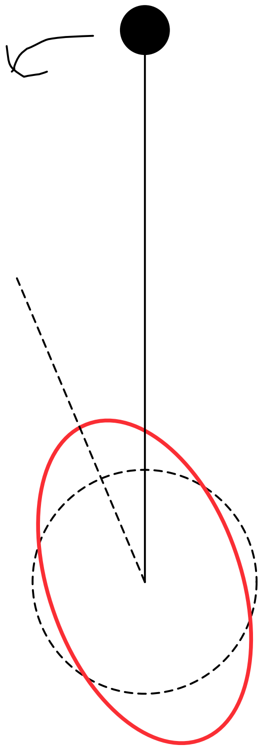
$\left(\frac{2}{3}\frac{k_{2p}}{Q_p}\right)$ = $1/Q'$

$$\sigma = \mathrm{sign}(2\Omega - 3n)$$

= sign of term with largest coeff

Equilibrium tide models

Q-value: Goldreich & Soter 1966:



$$\frac{1}{e}\frac{de}{dt} = \frac{3}{2} n\, k_{2p} \left(\frac{m_*}{m_p}\right) \left(\frac{R_p}{a}\right)^5 \left[\epsilon_0 - \tfrac{49}{4}\epsilon_1 + \tfrac{1}{4}\epsilon_2 + \tfrac{3}{2}\epsilon_3\right]$$

$$= \frac{171}{16} n \left(\frac{2}{3}\frac{k_{2p}}{Q_p}\right) \left(\frac{m_*}{m_p}\right) \left(\frac{R_p}{a}\right)^5 \sigma$$

$1/Q'$

-63/4 for synchronized spin

$$\sigma = \mathrm{sign}(2\Omega - 3n)$$

= sign of term with largest coeff

Equilibrium tide models

HUT 1981

$$\frac{1}{e}\frac{de}{dt} = -27\,n\,\frac{k_{2p}}{T}q(1+q)\left(\frac{R_p}{a}\right)^8\cdot f(e,\Omega/n)$$

$q=m_2/m_1$

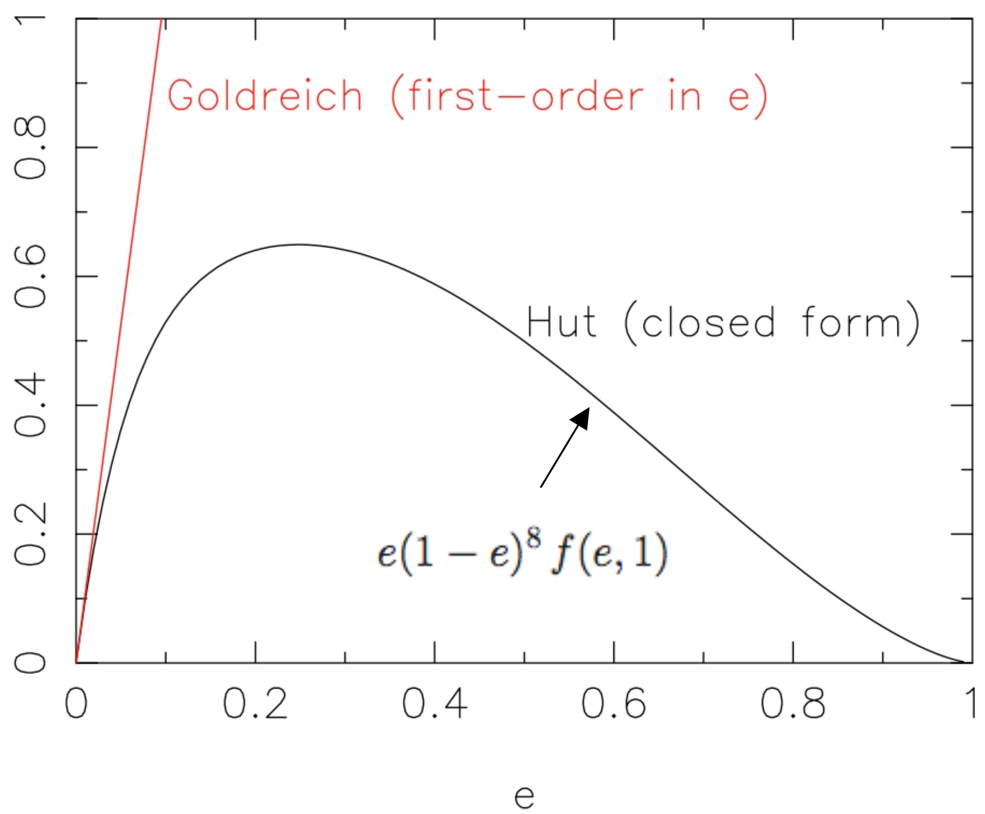
$$= -27\,n\,k_{2p}(n\tau)\left(\frac{m_*}{m_p}\right)\left(\frac{R_p}{a}\right)^5\cdot f(e,\Omega/n)$$

The Mardling & Lin swindle

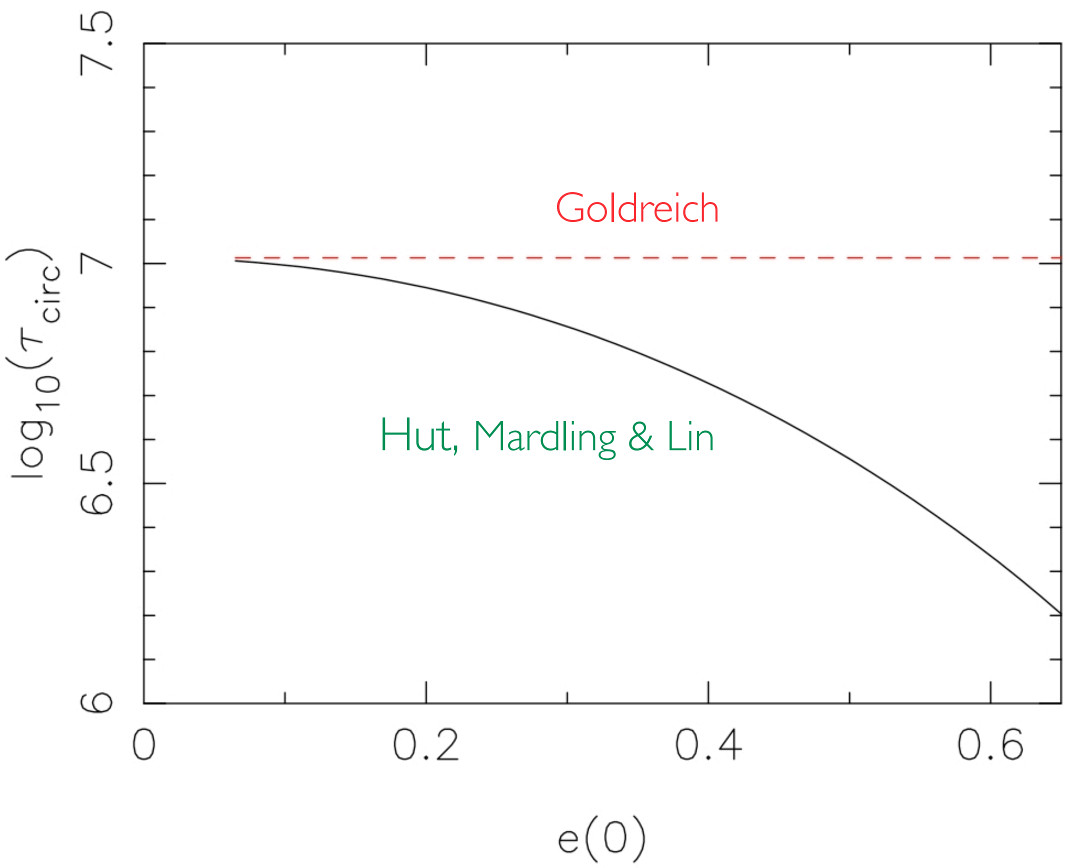
$$= -27\,n\,\frac{k_{2p}}{Q}\left(\frac{m_*}{m_p}\right)\left(\frac{R_p}{a}\right)^5\cdot f(e,\Omega/n)$$

(``swindle": a fraudulent scheme or action)

Equilibrium tide models



Equilibrium tide models



See also Leconte, Chabrier, Baraffe, Levrard 2010…

Here I have taken constant semi=0.03 AU and synchronous rotation, with

for Goldreich:

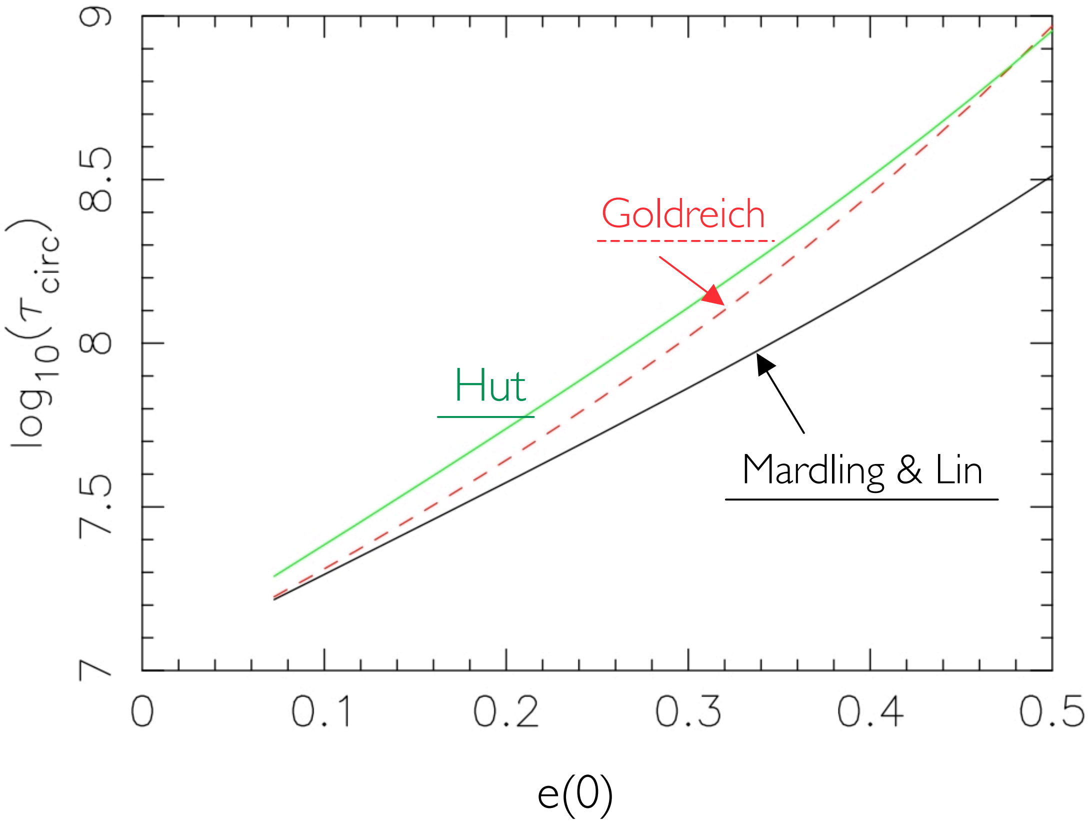
$$\tau_{circ} = \frac{e}{\dot{e}} = \frac{2}{21}\left(\frac{Q_p}{k_{2p}}\right)\left(\frac{m_p}{m_*}\right)\left(\frac{a}{R_p}\right)^5$$

For Hut and M&L, τ_{circ} determined numerically (time when e=e(0)/2.718), with

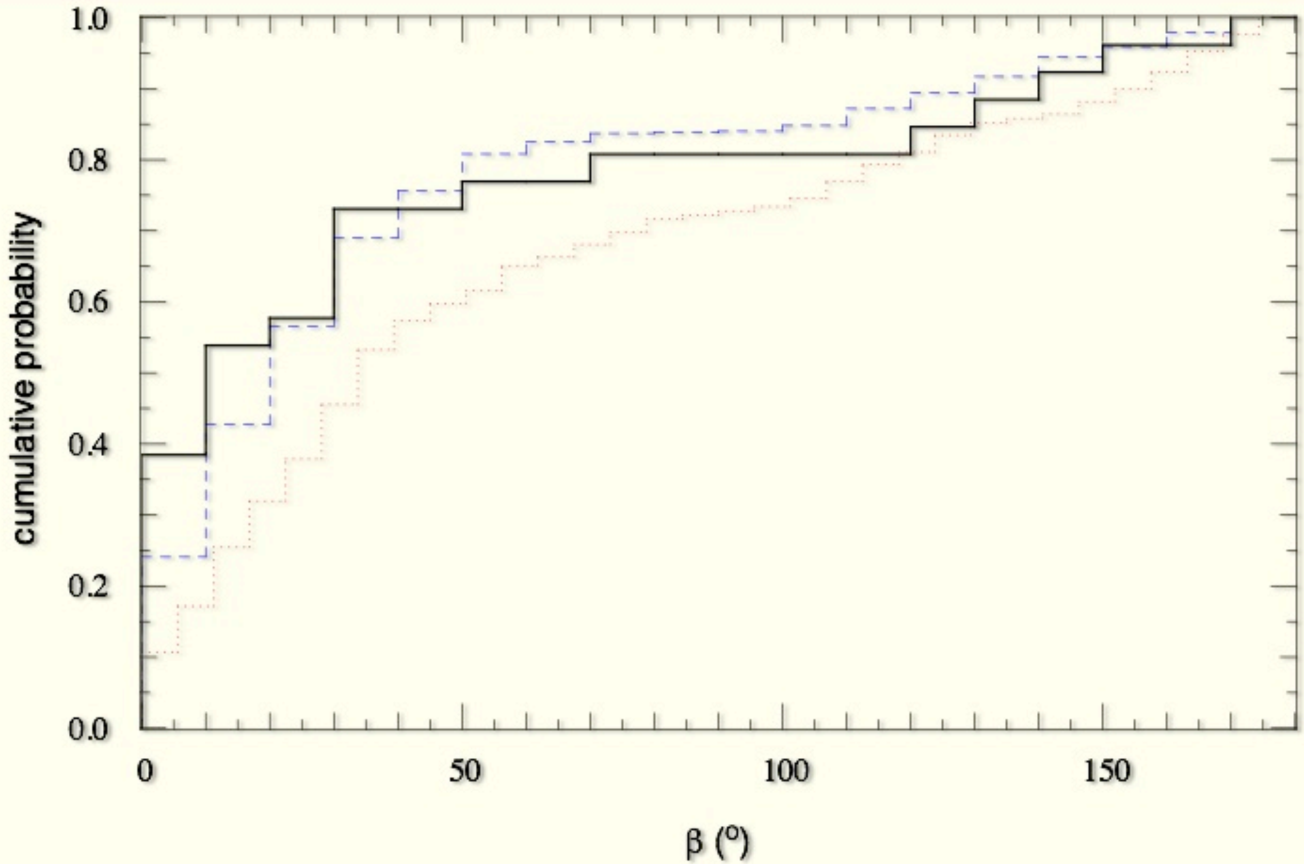
$$\tau_{Hut} = \frac{1}{(\Omega_{Jupiter} - n_{Io}) \cdot Q_{Jupiter}}$$

n_{Io}=1.8 day, $Q_J=3.6\times10^4$

Equilibrium tide models

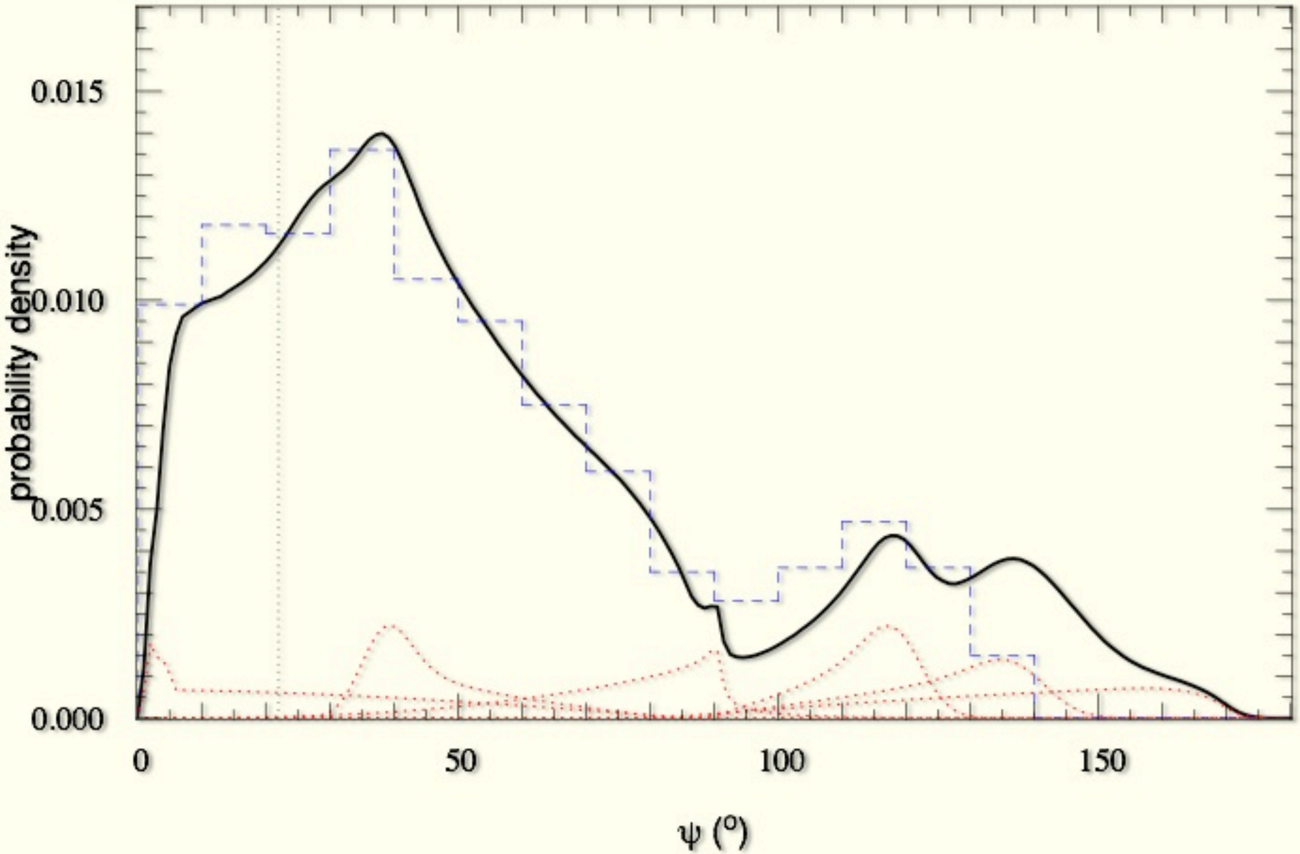


constant peri - 0.03 AU



$$\sin\beta \simeq \sin\psi \sin\alpha$$

Transform theoretical ψ to β



$$\cos\psi = \cos I \cos i + \sin I \sin i \cos\beta$$

Transform from β to ψ

TRIAUD, CAMERON, QUELOZ ET AL. SUBMITTED