GRAVITY AND ENTANGLEMENT

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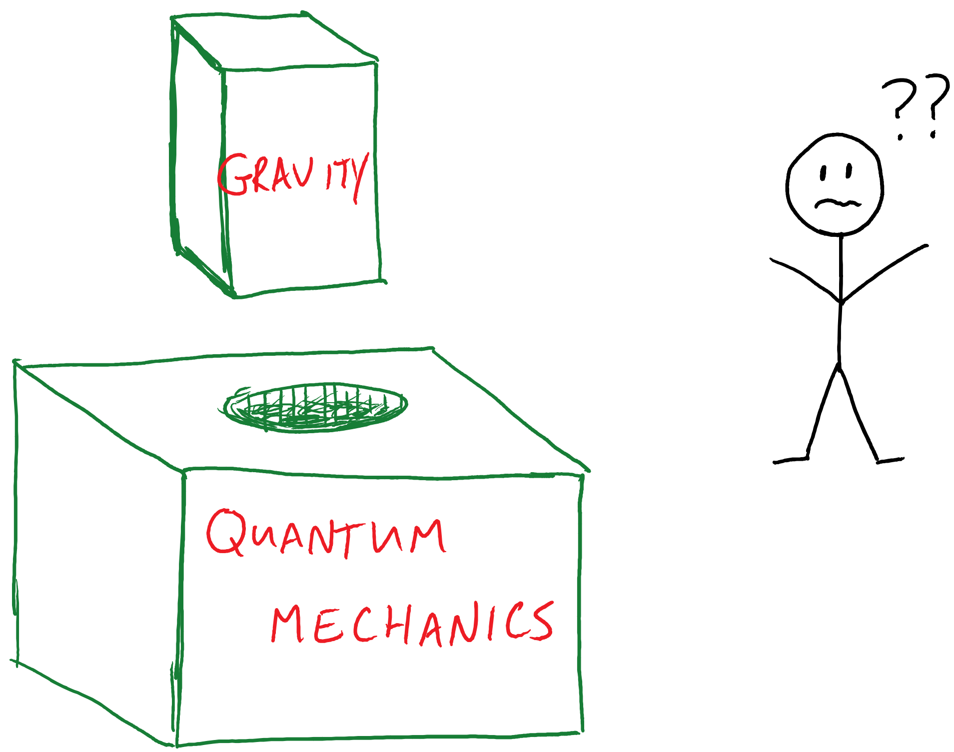
KITP, April 2015

One of the greatest challenges for theoretical physics:

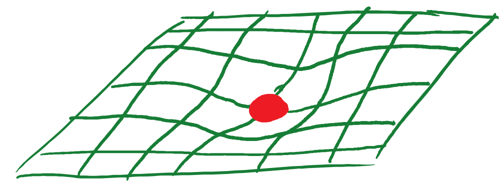


GRAVTY describes the dynamics of spacetime geometry and its interaction with matter.

Central result:

EINSTEIN'S EQUATION:

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$



measures intrinsic curvature of spacetime

measures energy + momentum density + flow

Fitting this into the framework of quantum mechanics has been very difficult!

1997 : Remarkable progress via AdS/CFT correspondence in string theory (Maldacena)

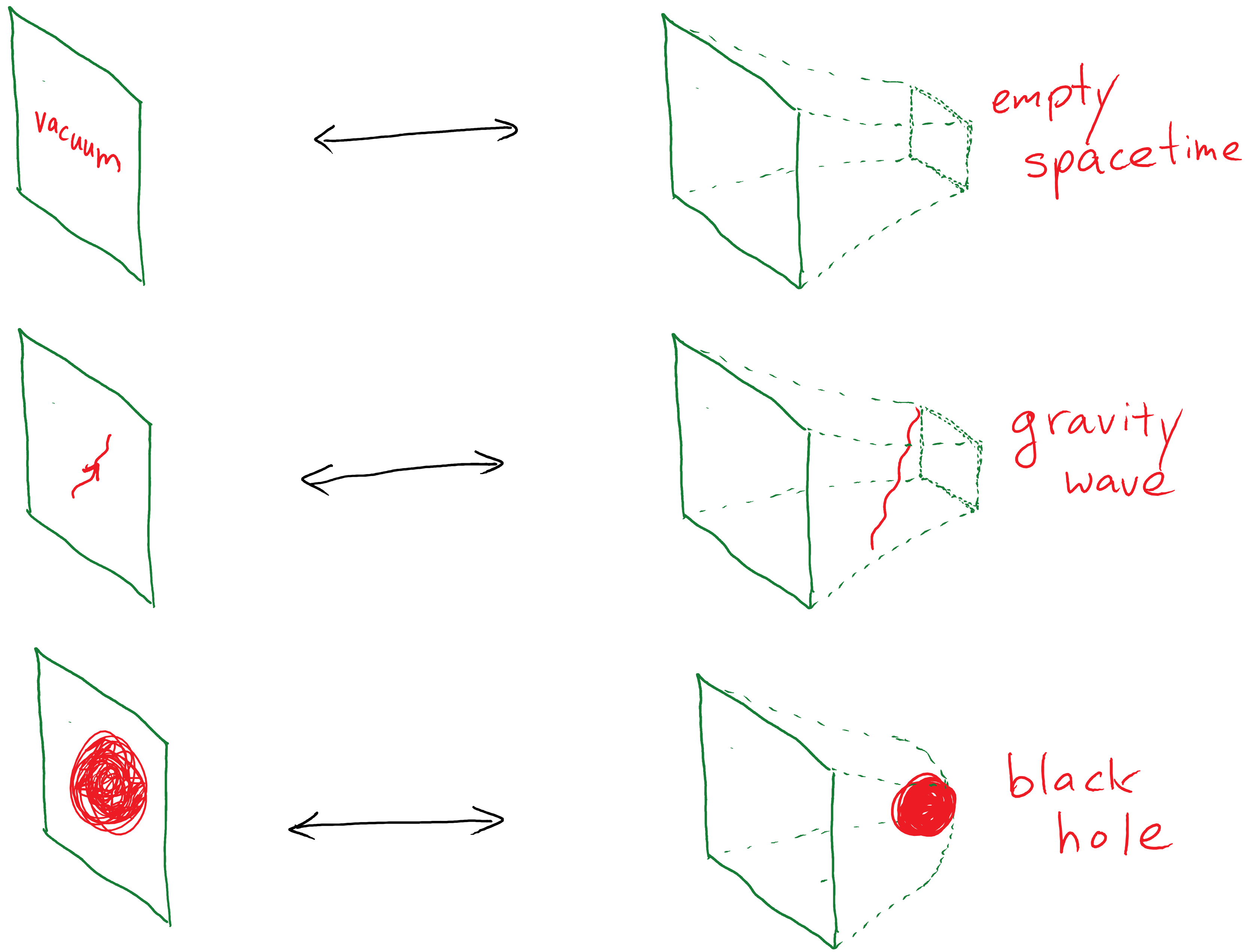
QUANTUM GRAVITY (certain examples)

=

exactly equivalent

ORDINARY QUANTUM SYSTEM (e.g Quantum Field Th. on fixed spacetime)

Different QFT states ⟷ Different spacetimes



Big Question:

How/why do spacetime/gravity emerge from QFT physics?

Which states $|\psi\rangle$ correspond to classical geometry?

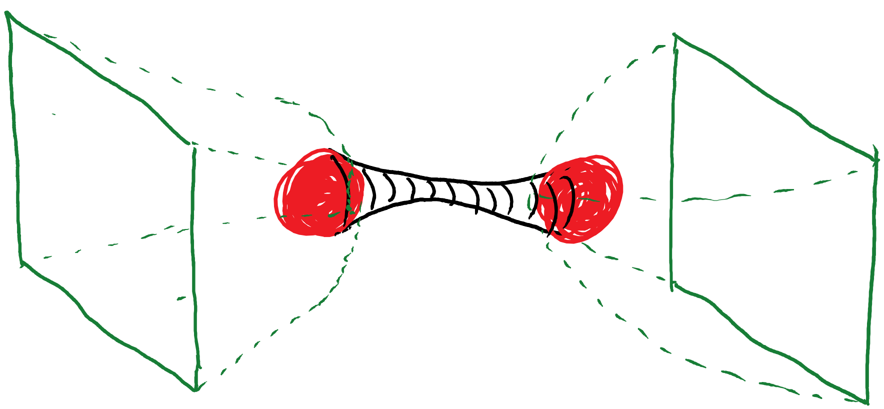
What can we learn about gravity?

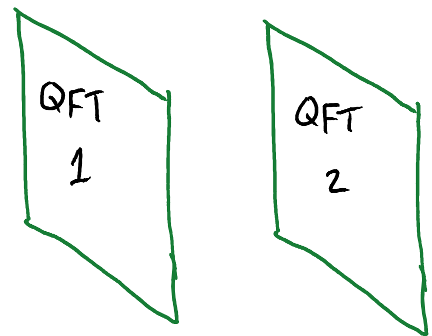
Recent work: physics of entanglement + quantum information is crucial!

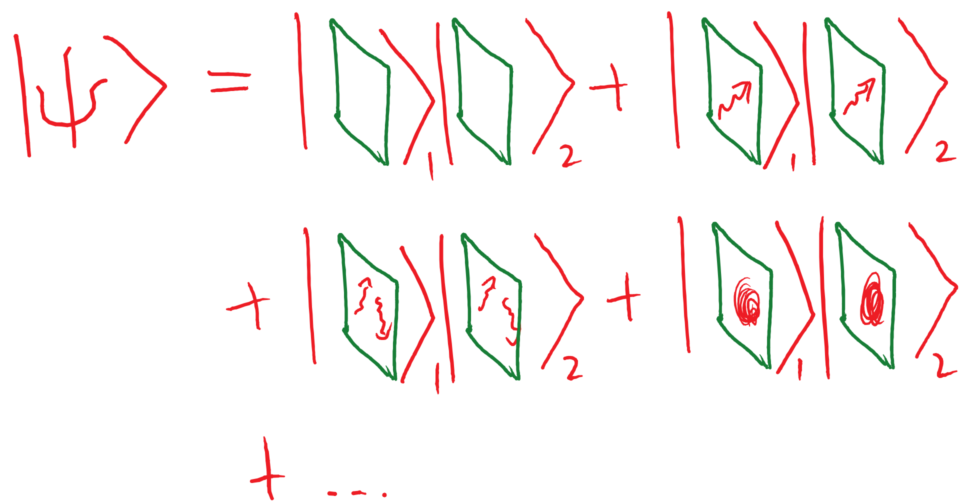
Maldacena 2001:

2 separate spacetimes connected by wormhole

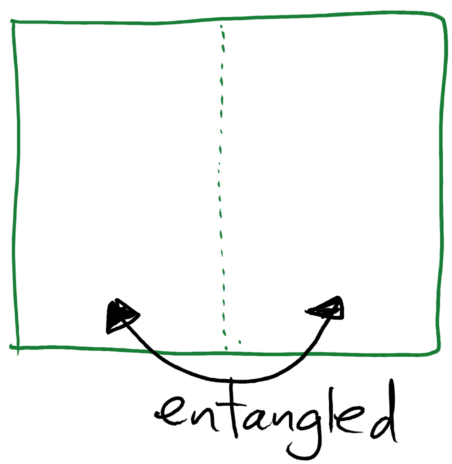
2 separate QFTs entangled with one another





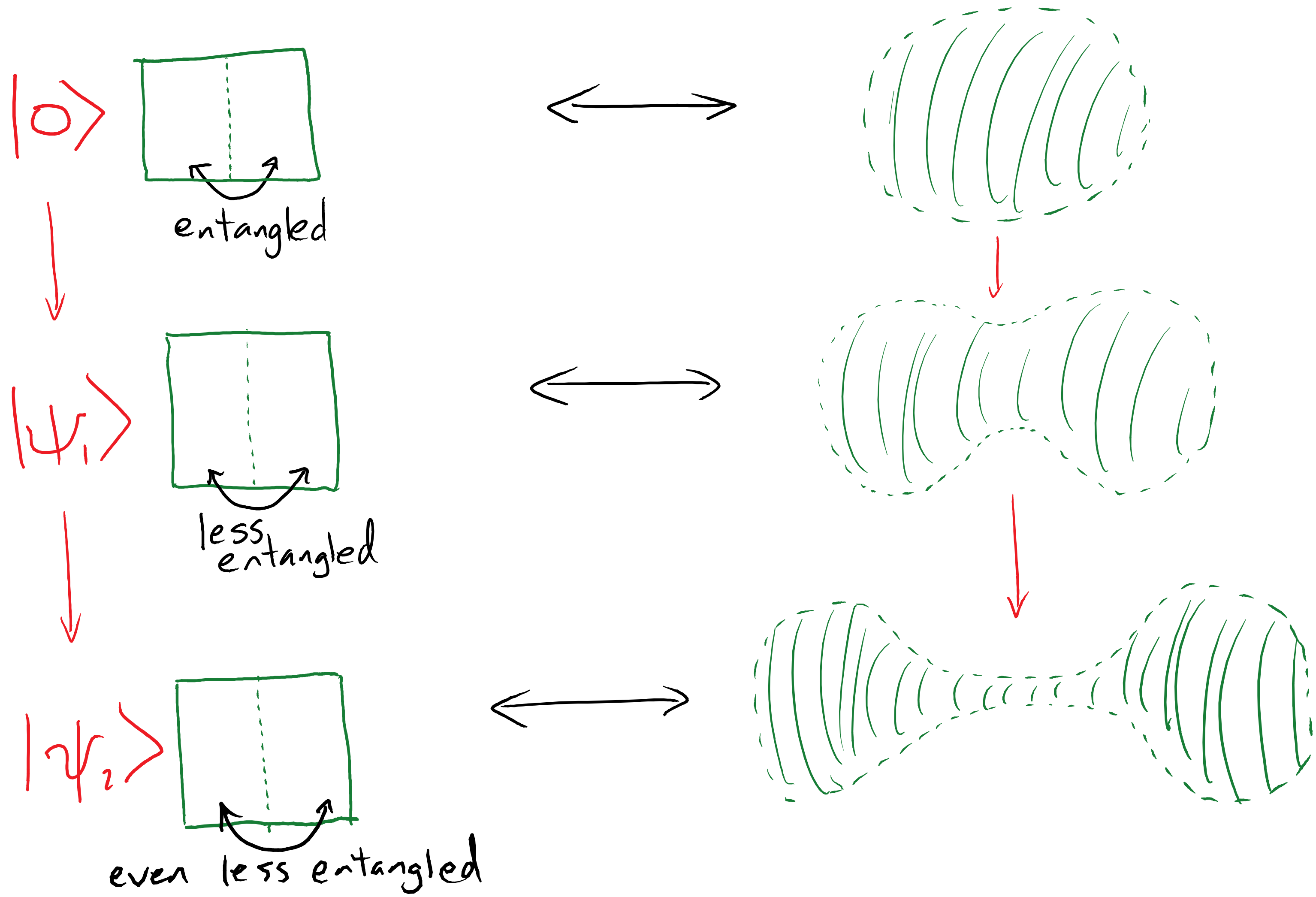


But: lots of entanglement already in QFT ground state dual to empty spacetime.



QFT fields in any region entangled with fields outside

What happens if we remove this entanglement?



2 regions of space pinch off from each other!

MvR; MvR, Czech, Nogueira, Karczmarek

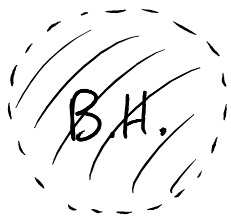
Suggests that classical spacetime geometry emerges via entanglement of degrees of freedom in dual QFT!

MvR, Swingle 2009

No classical spacetime without quantum entanglement.

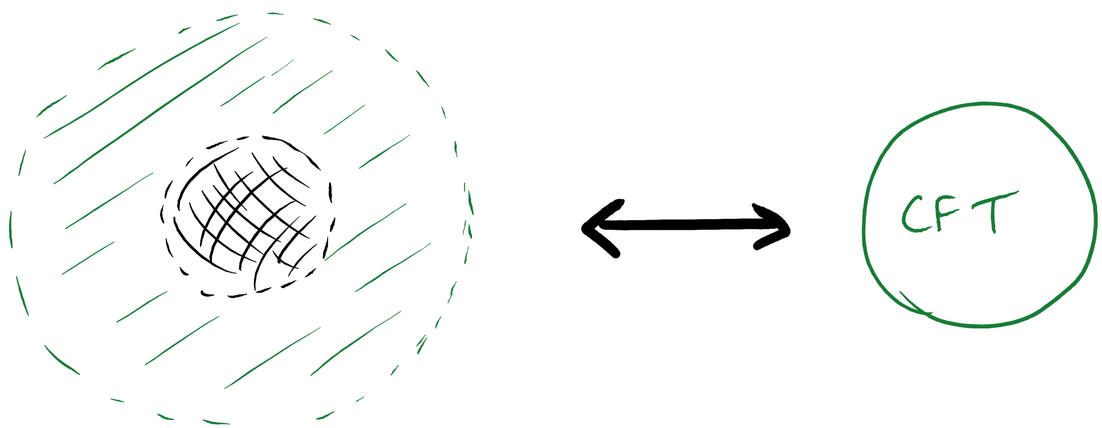
Can we be more quantitative?

First hints: black holes + thermodynamics 1970s



$\frac{\text{Area}}{4G_N}$ behaves like entropy.

AdS/CFT:



in thermal ensemble

$$\left\{ |E_i\rangle, \; p_i = \frac{1}{Z} e^{-\beta E_i} \right\}$$

$$\frac{\text{Area}}{4G_N} = S_{CFT} = -\sum_i p_i \log p_i$$

Key point (Ryu + Takayanagi)

Entropy of subsystems also has a geometric interpretation.

Entropy of subsystems (= Entanglement Entropy)

Consider any quantum system w. subsystem A

$$\uparrow\ \downarrow\ \uparrow\ \boxed{\downarrow\ \downarrow\ \uparrow\ \downarrow}\ \uparrow\ \uparrow\ \uparrow \qquad \text{state } |\psi\rangle$$

subsystem A → described by ENSEMBLE $\{p_i, |\psi_i^A\rangle\}$

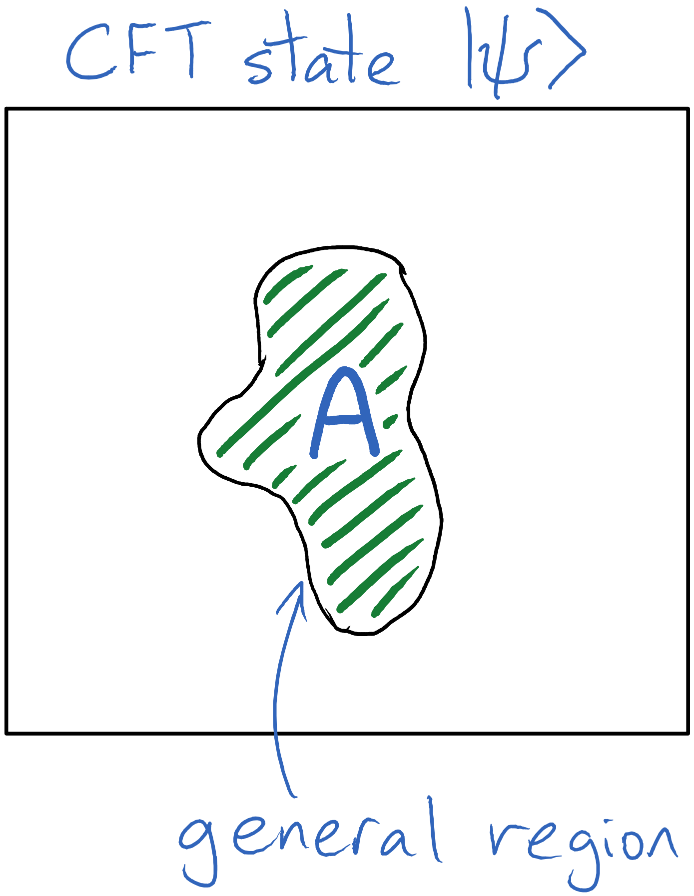
$\{p_i\} \neq \{1\}$ → A is ENTANGLED w. rest

→ classical uncertainty about state of A

Quantify via entropy:

$$S(A) = -\sum_i p_i \log p_i$$

QFT entanglement entropy:



$S(A)$ divergent but can define sensible quantities:

$$S_{|\psi\rangle}(A) - S_{|vac\rangle}(A)$$

$$S(A) + S(B) - S(A \cup B) \equiv I(A,B)$$

mutual information

Example:

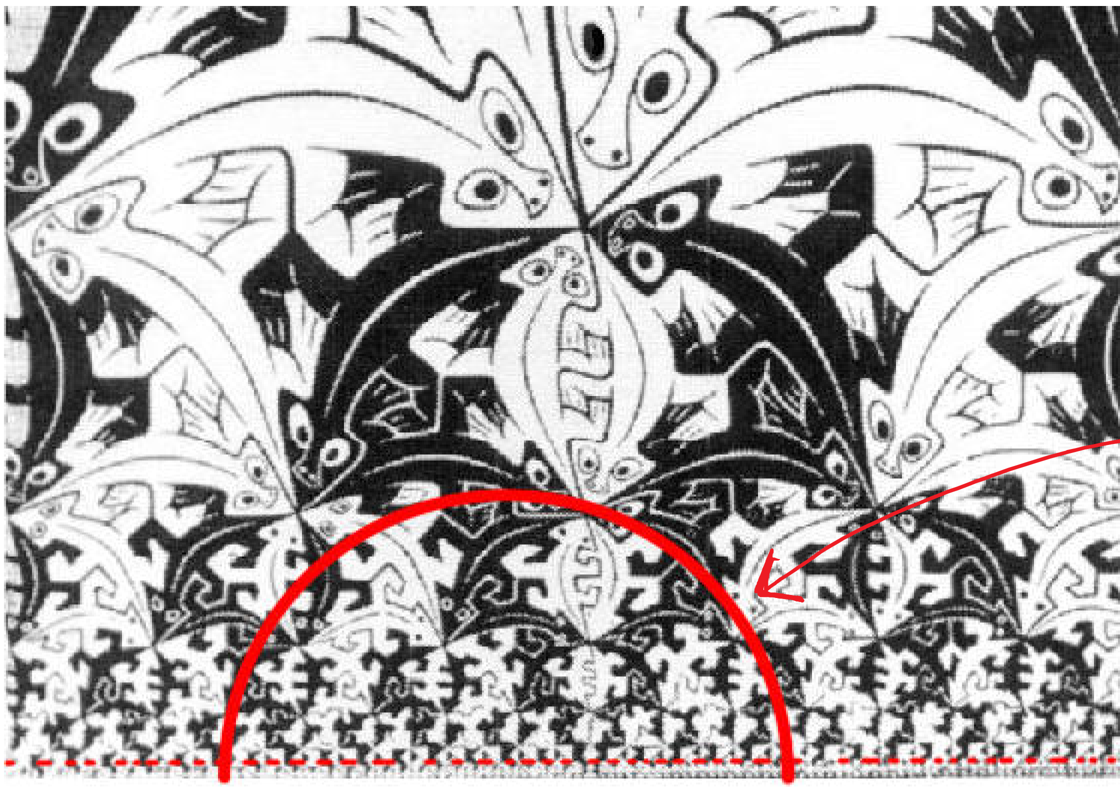
"central charge" = measure of number of d.o.f.

$$S = \frac{c}{3} \log\left(\frac{L}{\varepsilon}\right)$$

for ANY 1+1D conformal field theory.

Ryu + Takayanagi: this result can be represented geometrically.

spatial slice of anti-de Sitter space



$\longleftarrow L \longrightarrow$

BOUNDARY

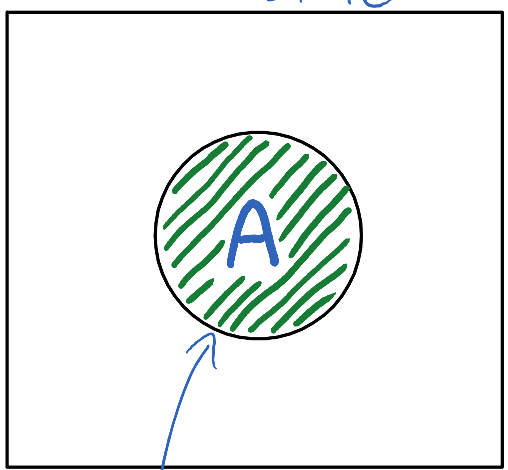
shortest length curve that has same boundary as interval L

$$\text{length} = \frac{c}{3} \ln\left(\frac{L}{\varepsilon}\right)$$

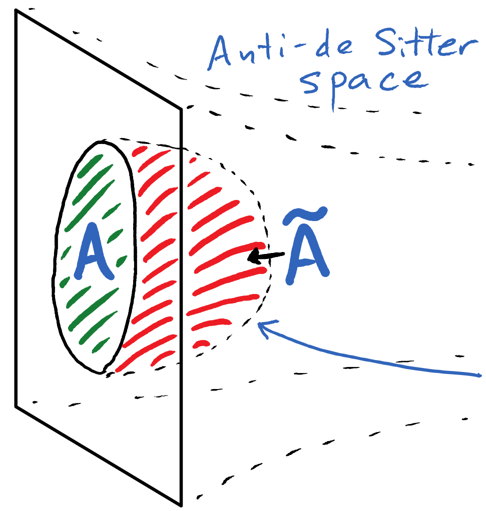
Higher dimensions:

for any CFT:

vacuum state



ball shaped region



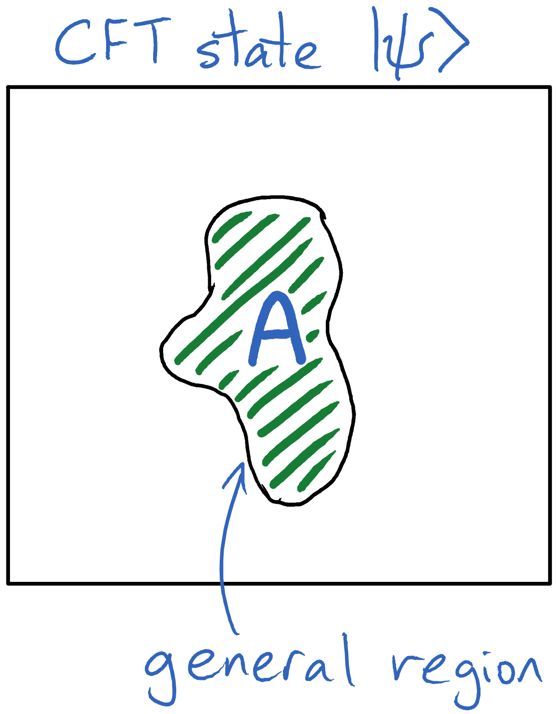
minimal area surface with same boundary as A

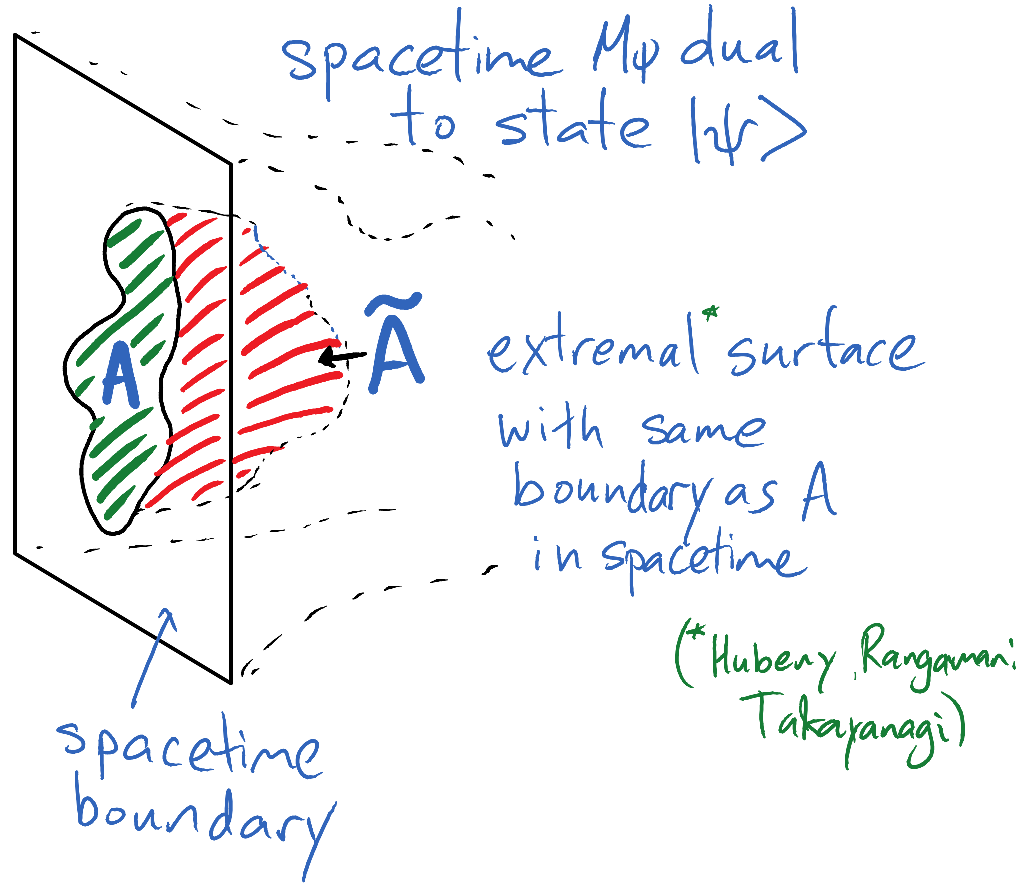
$$S(A) \propto \text{Area}(\tilde{A})$$

c.f. $S = \frac{\text{Area}}{4G_N}$ for black holes!

Ryu-Takayanagi: Conjectured this to hold for general regions A + general states $|\psi\rangle$ in "holographic" CFTs

Geometry From Entanglement





$|\psi\rangle \rightarrow$ calculate S_A for many A $\rightarrow$ find M_ψ s.t. $\mathrm{Area}(\tilde{A}) = S_A$

can (plausibly) reconstruct geometry from entanglement!

Question: Couldn't I do this for any state in any field theory?

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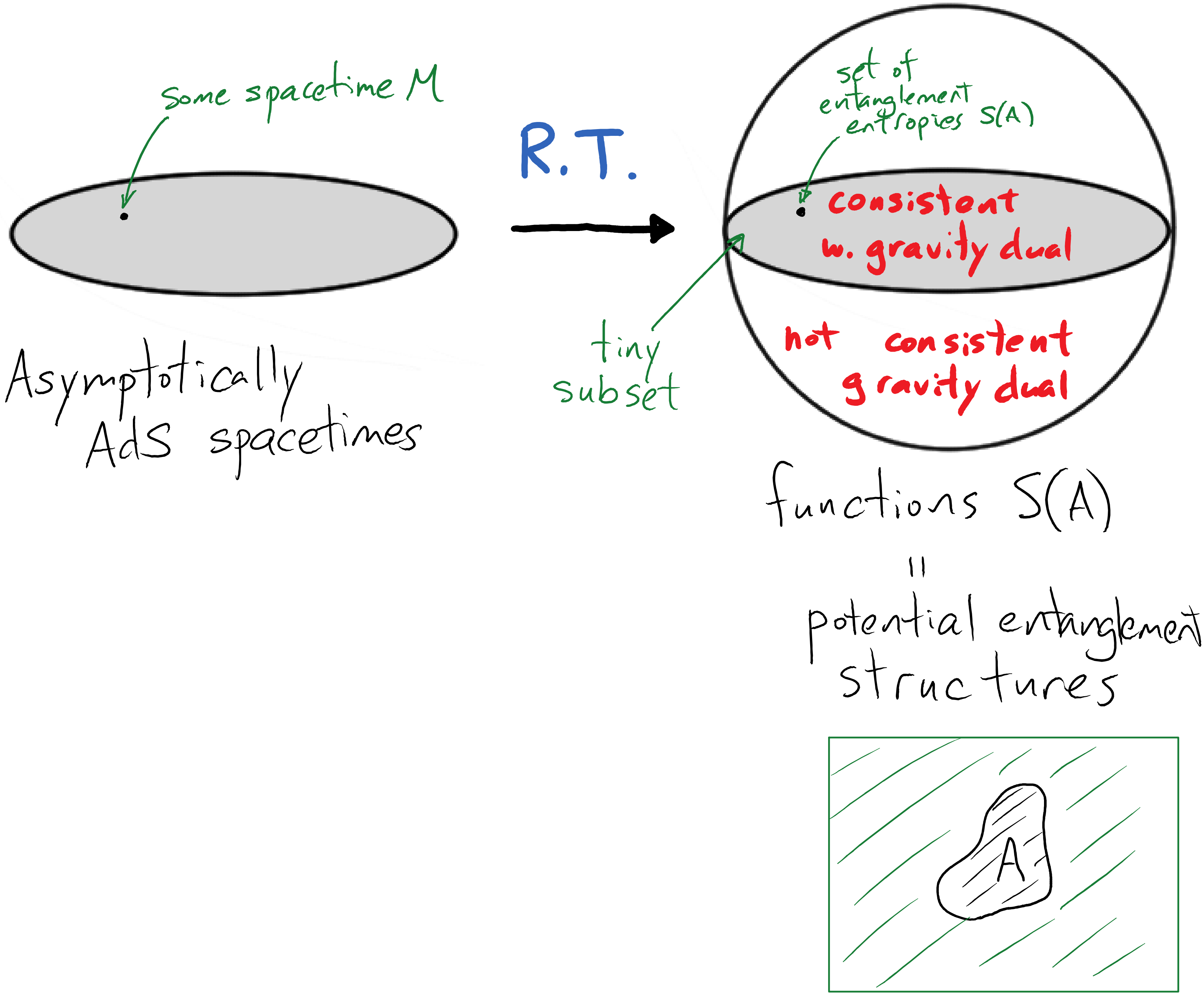
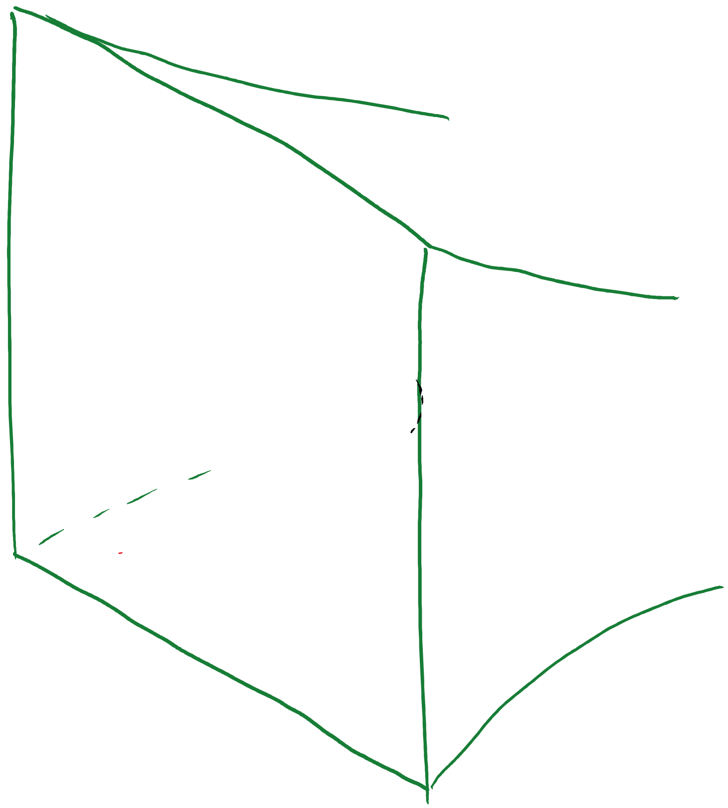
A: No! Usually no M will reproduce entanglement structure $S(A)$

Entanglement structure of states w. geometrical dual is highly constrained


Some spacetime M
Asymptotically AdS spacetimes
R.T.
set of entanglement entropies S(A)
consistent w. gravity dual
tiny subset
not consistent gravity dual
functions S(A)
=
potential entanglement structures
A

Example: states dual to vacuum solutions of Einstein's equations



$S(A)$ for infinitesimal regions (equivalent to $\langle T_{\mu\nu} \rangle$)

$\big\downarrow$ R.T.

asymptotic metric

$\big\downarrow$ Einst. Eqn.

interior metric (to some distance)

$\big\downarrow$ R.T.

$S(A)$ for any region A (not too large)

Entanglement structure determined by local data!

General Questions:

* Which entanglement structures are consistent w. geometrical desciption? *

* Why do holographic Hamiltonians prefer states w. this structure? *

Can we learn anything about gravity?

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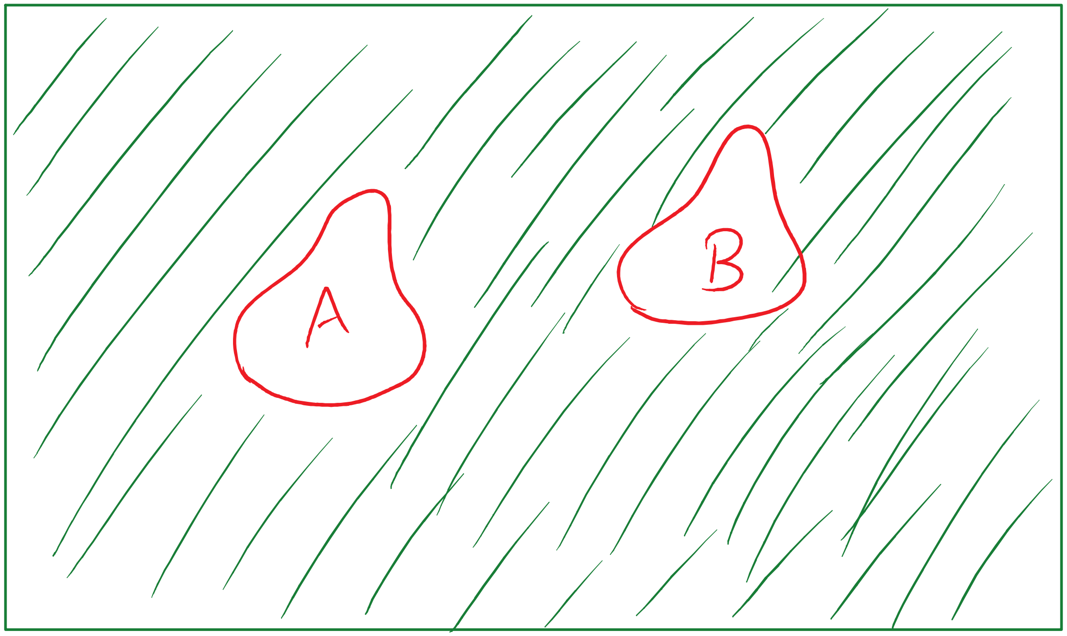
Yes! Constraints on entanglement structure restrict possible spacetimes.

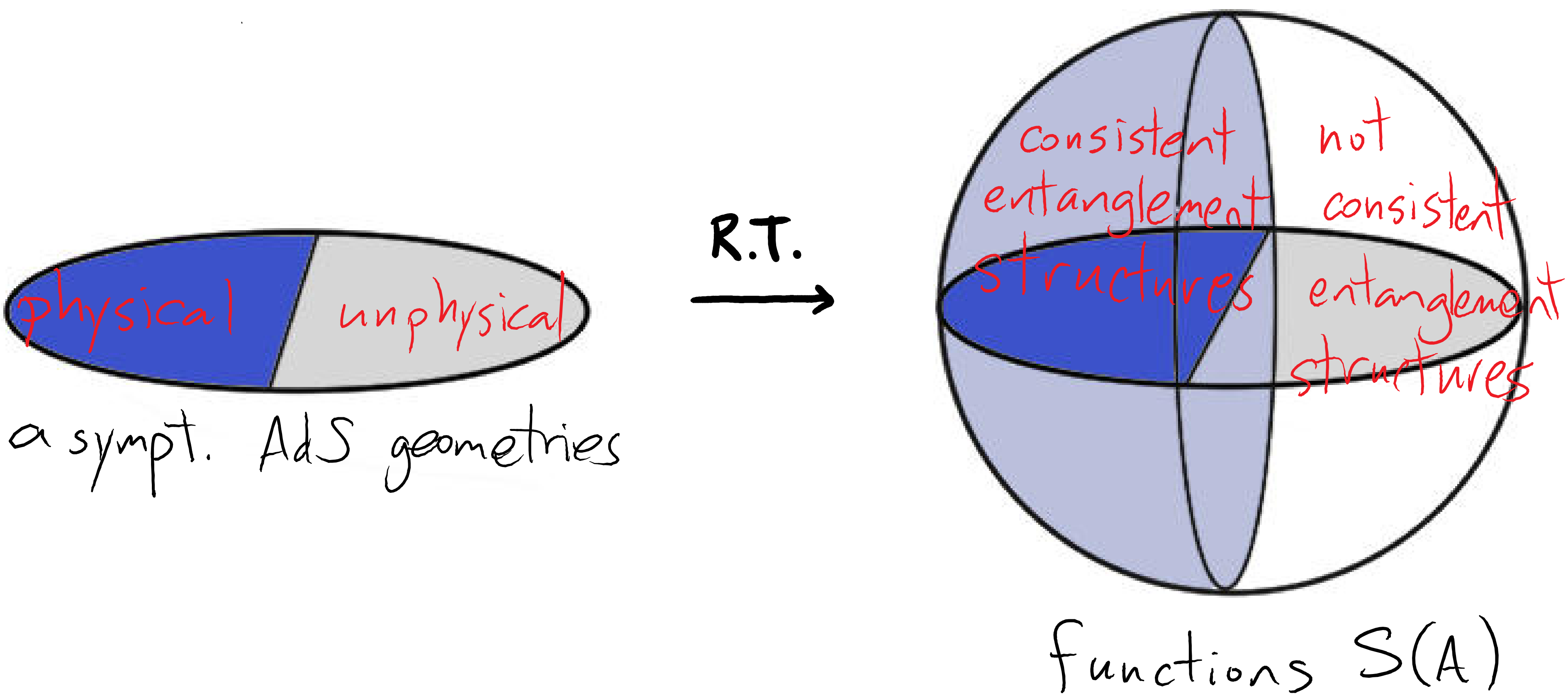
★ Not all functions $S(A)$ represent consistent entanglement structures ★



e.g. must have:

$$S(A) + S(B) - S(A \cup B) \geq 0$$

Question: which geometries give rise to consistent entanglement structures?



ENTANGLEMENT INEQUALITIES

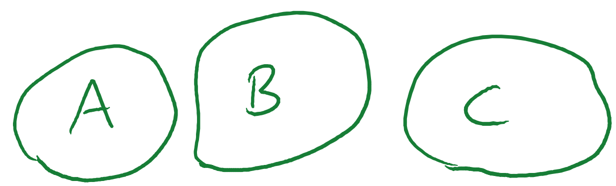
SUBADDITIVITY:

$$I(A,B) \equiv S(A)+S(B)-S(A\cup B) \geq 0$$

MUTUAL INFORMATION: measure of entanglement/correlations between A & B

STRONG SUBADDITIVITY:

$$I(AB,C) \geq I(B,C)$$



$$S(AB)+S(BC) \geq S(ABC)+S(B)$$

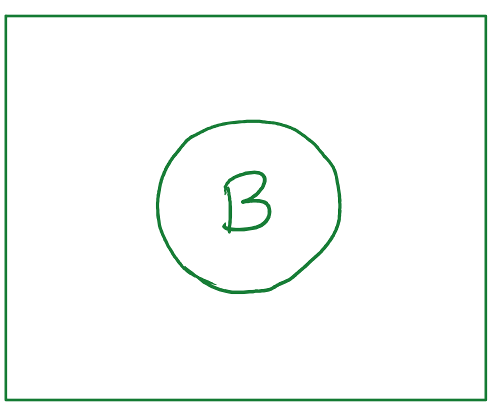
Positivity + Monotonicity of Relative Entropy.

$$S(\rho\|\sigma) = \mathrm{tr}(\rho\log\rho) - \mathrm{tr}(\rho\log\sigma)$$

- measure of distinguishability of ρ, σ
- positive, larger for ρ, σ corresponding to larger subsystems.

ρ: density matrix for ball B in state $|\psi\rangle$

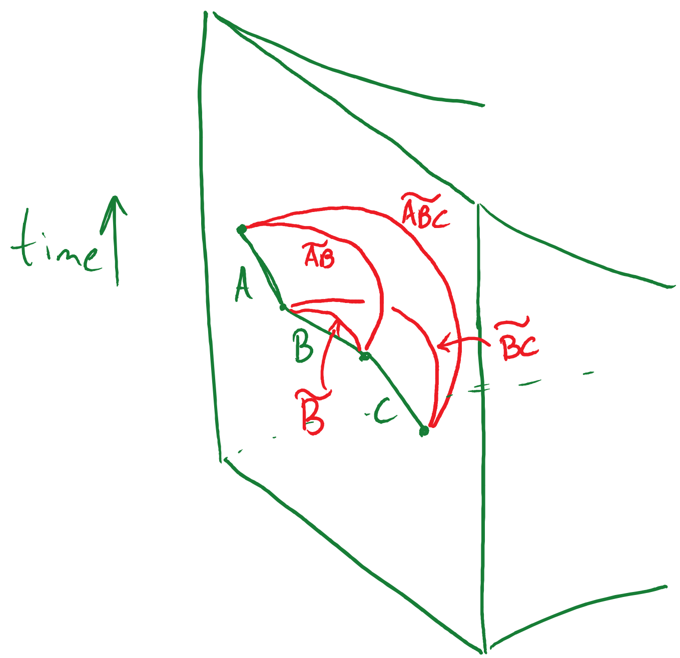
σ: density matrix for ball B in vacuum



$$S(\rho\|\sigma) = \Delta\langle H_\sigma\rangle - \Delta S$$

$$H_\sigma = -\ln\sigma = 2\pi\int \frac{R^2-r^2}{2R}\cdot T_{00}$$

Using R.T., translate these to statements about geometry



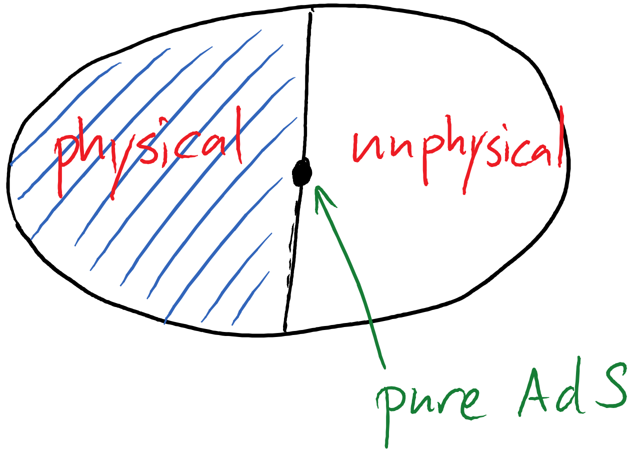
e.g.

Strong subadditivity

$$\Downarrow$$

$$\mathrm{Area}(\widetilde{AB}) + \mathrm{Area}(\widetilde{BC}) \geq \mathrm{Area}(\widetilde{ABC}) + \mathrm{Area}(\widetilde{B})$$

Not true for all geometries

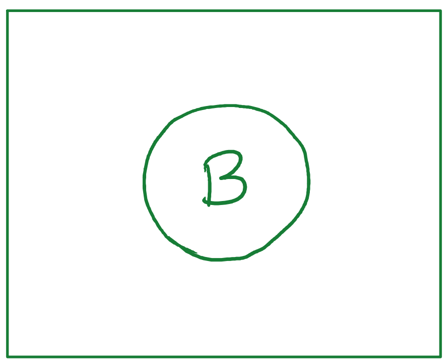


Start w. geometries

M near pure AdS.

$(|\psi\rangle = |vac\rangle + \delta|\psi\rangle)$

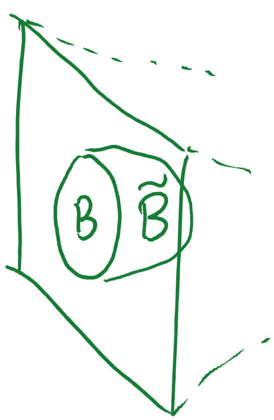
Positivity of relative entropy



$$\Rightarrow \quad \delta S_B = 2\pi \int_B d^dx \, \frac{R^2 - r^2}{2R} \, \delta \langle T_{00} \rangle$$

Blanco
Casini
Hung
Myers

True for all balls in all Lorentz frames.



Translates to constraint on metric perturbation.

Result:

* Physical perturbations to pure AdS must satisfy Einstein's Equations to linear order *

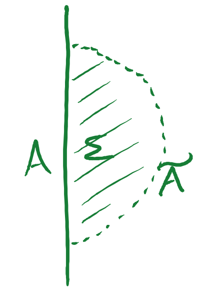
- Lashkari, McDermott, MVR
- Faulkner, Guica, Hartman, Myers, MVR
- Swingle, MVR

More recently:

w Swingle: Refined Ryu-Takayanagi formula: $S_{CFT} = \frac{\text{Area}}{4G_N} + S_\Sigma$

$\Downarrow$ $\delta S = \delta E$

$\delta\langle T_{\mu\nu}\rangle$ is source for linearized Einst. Eqns.



w. Lashkari
Rabideau
Sabella-Garnier

Constraints at non-linear level
(strong subadditivity + positivity/monotonicity of relative entropy)
give rise to energy conditions!

also: Rangamani, Takayanagi
Hubeny, Battacharya
Ooguri, Lin, Marcoli, Stoica

(GRAVITY PROGRAM TALK NEXT WEEK)

Summary

structure of entanglement in QFT ⟷ spacetime geometry

constraints on QFT entanglement ⟷ Einstein Equations (so far: linearized) + Energy Conditions