Entanglement Detection by Interfering Quantum Many-body Twins

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HARVARD UNIVERSITY v MIT
CENTER FOR ULTRACOLD ATOMS

Entanglement in Many-body Physics

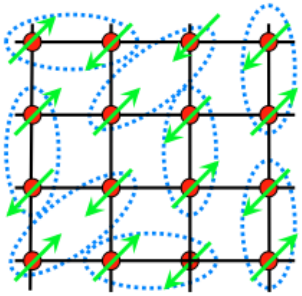
- **Novel states of matter:**

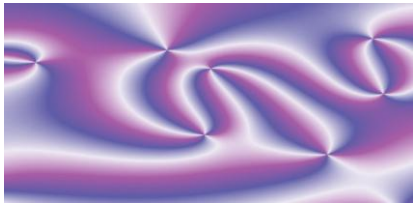
Order beyond simple broken symmetry

Example: Topological order, spin liquid, fractional quantum Hall

Characterized by quantum entanglement !

- **Correlations near quantum critical points –** scaling laws of entanglement

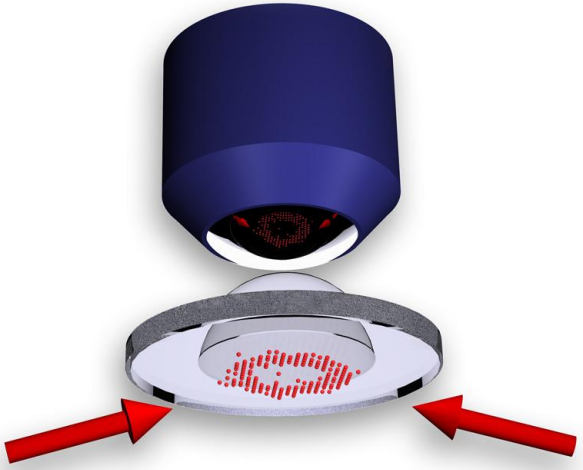




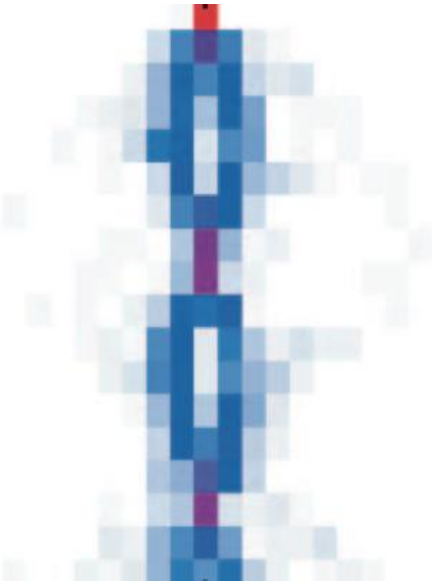
- **Challenge:**

Entanglement not detected in traditional CM experiments

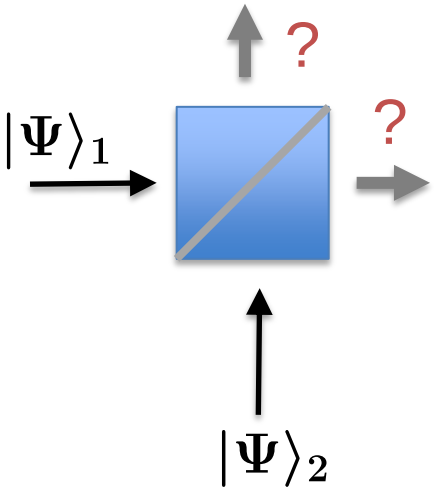
Entanglement in ultracold atom synthetic quantum matter?



Quantum gas microscopy

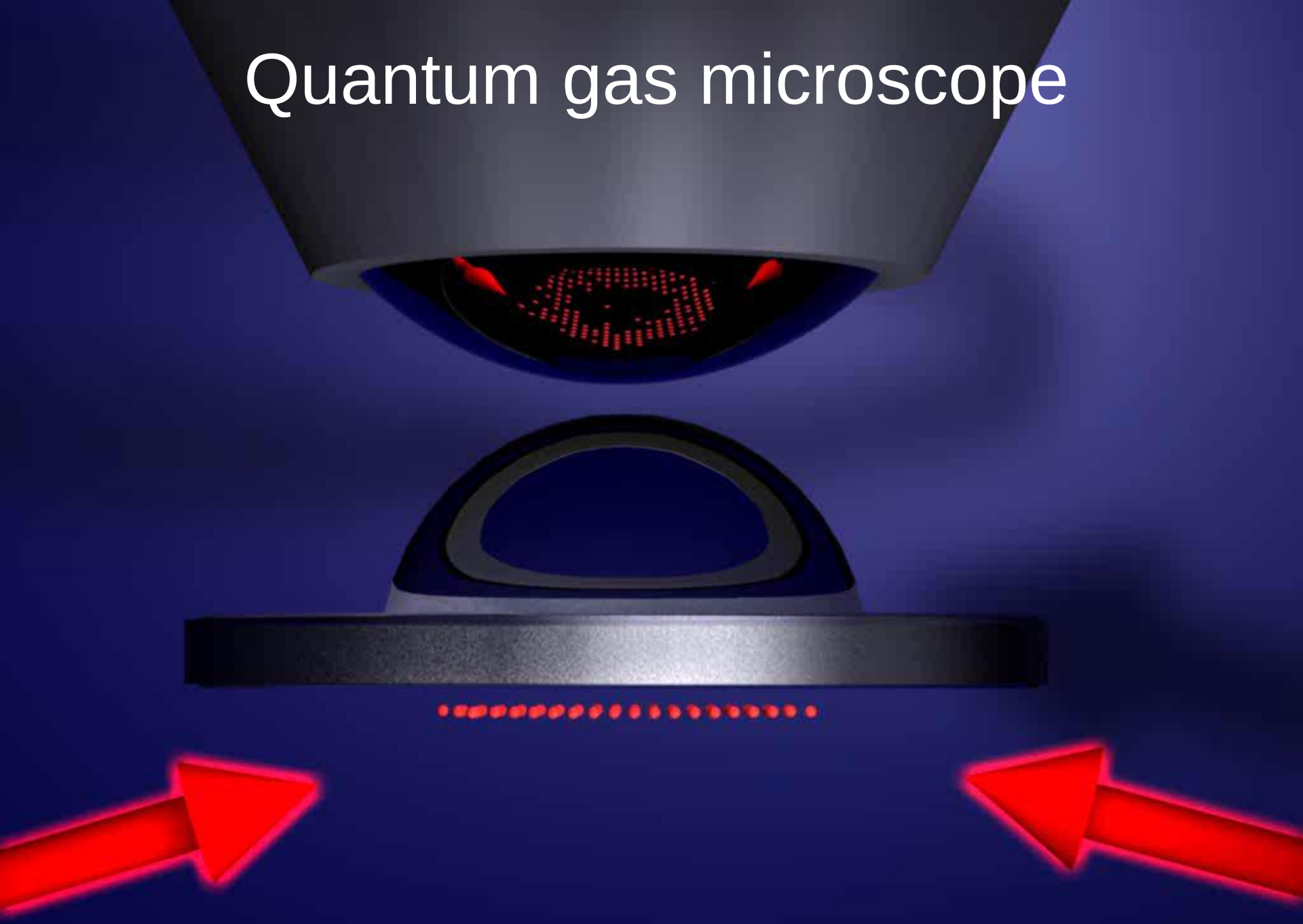


Coherent control



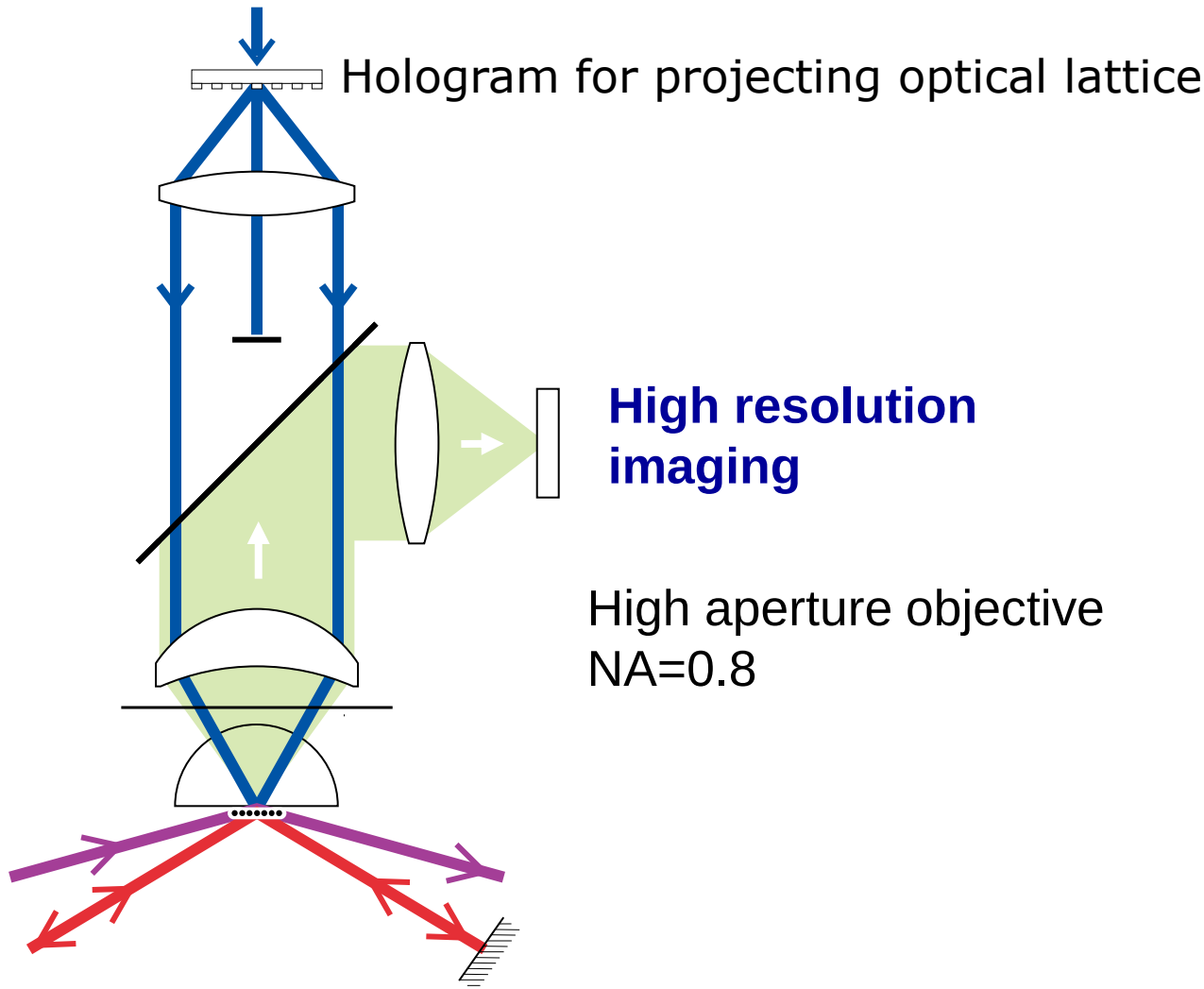
Renyi entropy

Quantum gas microscope

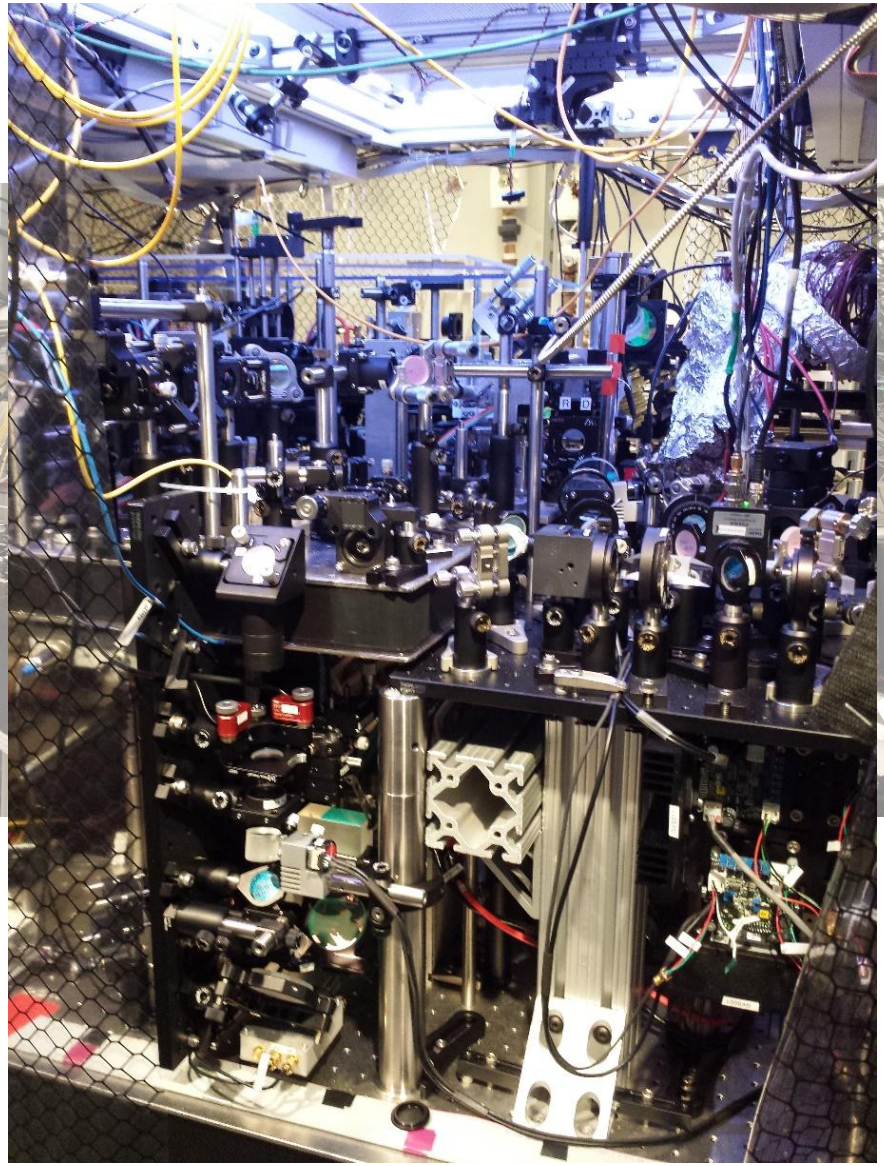


Bakr *et al*., Nature 462, 74 (2009), Bakr *et al*., Science.1192368 (June 2010)

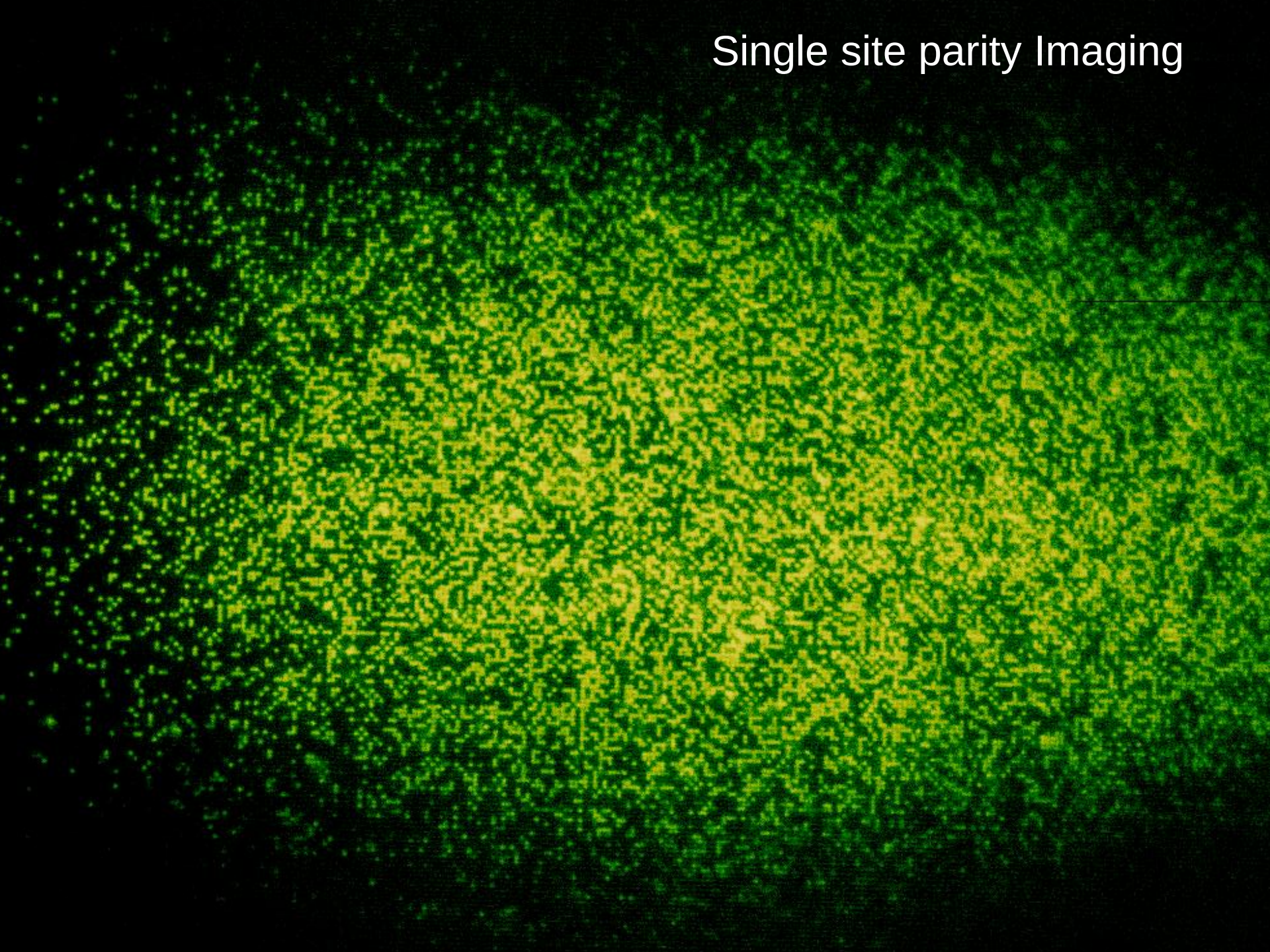
Quantum gas microscope



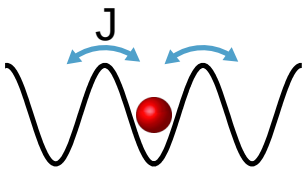
… and the whole apparatus



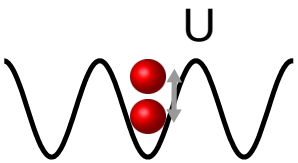
Single site parity Imaging



tunneling J

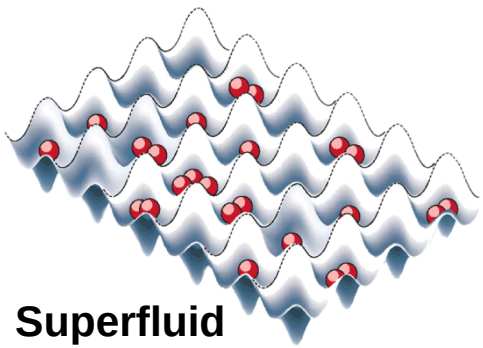


interaction U

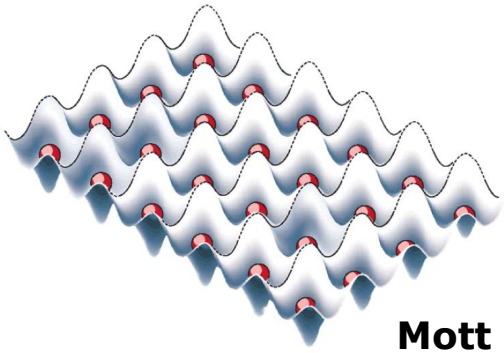


$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i (n_i - 1)$$

Bose Hubbard Model



Superfluid

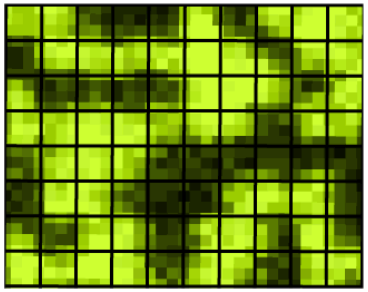


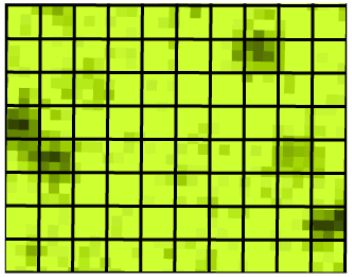
Mott insulator

$U \ll J$

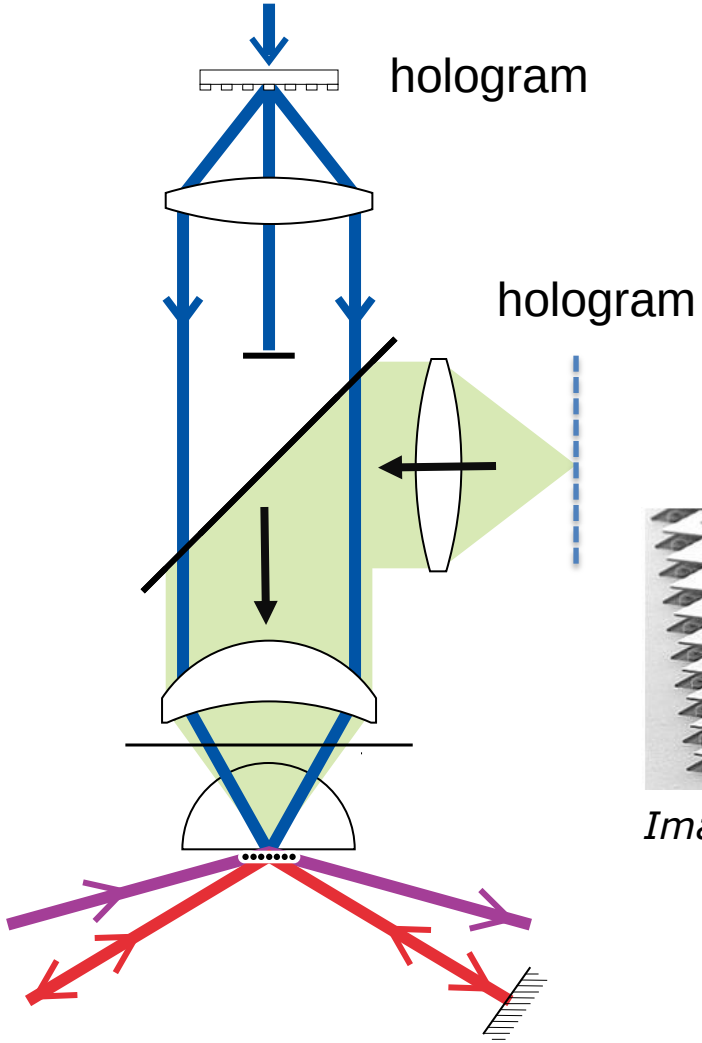
U/J

$J \ll U$





Addressing individual atoms



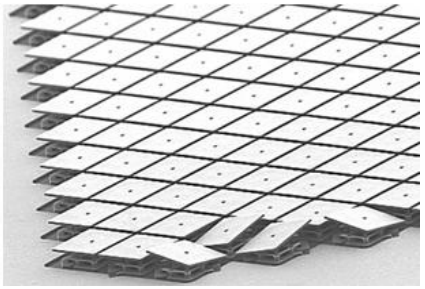


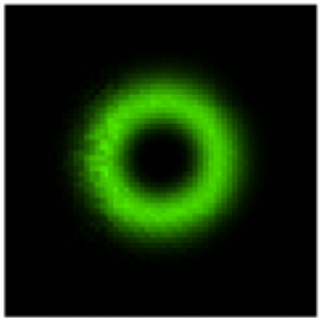
Image: EKB Technologies

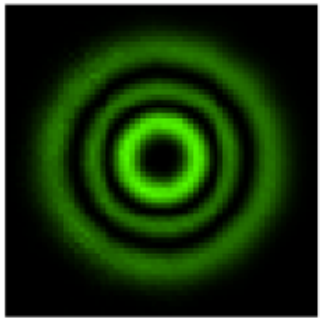
2D quantum gas of Rb-87
in optical lattice

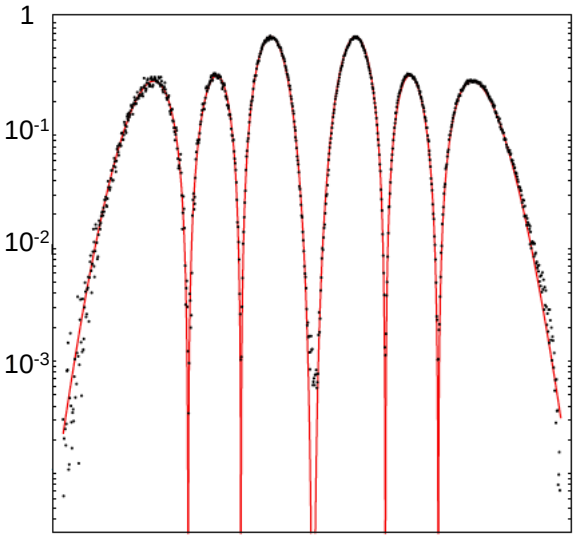
Arbitrary beam shaping

- Weitenberg et al., **Nature** 471, 319-324 (2011)
  Zupancic, P., Master's Thesis, LMU Munich/Harvard 2013
- Cizmar, T *et al.,* **Nature Photonics** 4, 6 (2010)

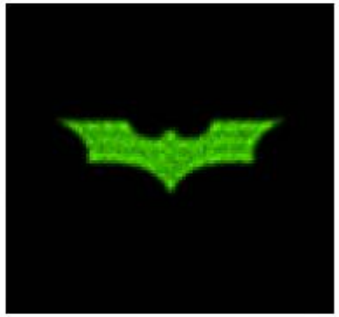
High-order Laguerre Modes

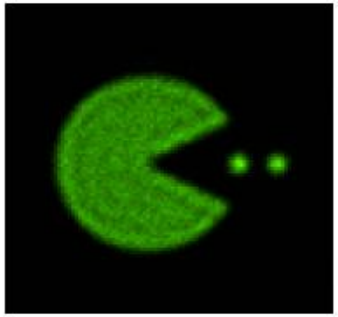


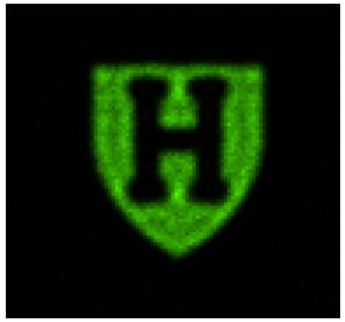




Laguerre-Gauss profile



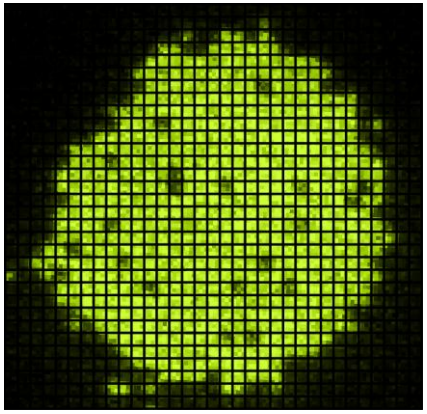




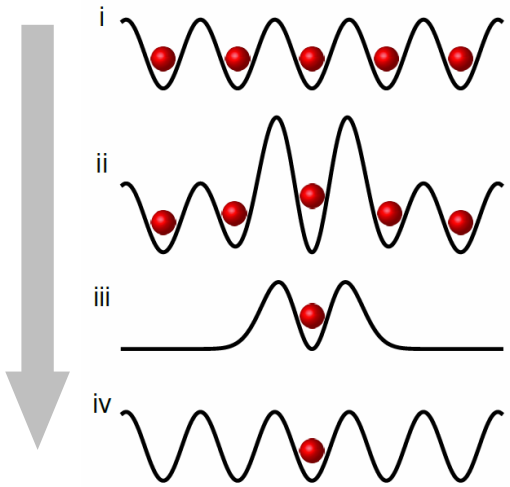
Quantum Gas Microscope

Versatile number state preparation

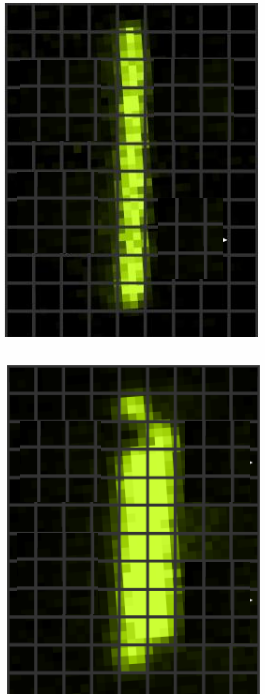
Low entropy
Mott Insulator



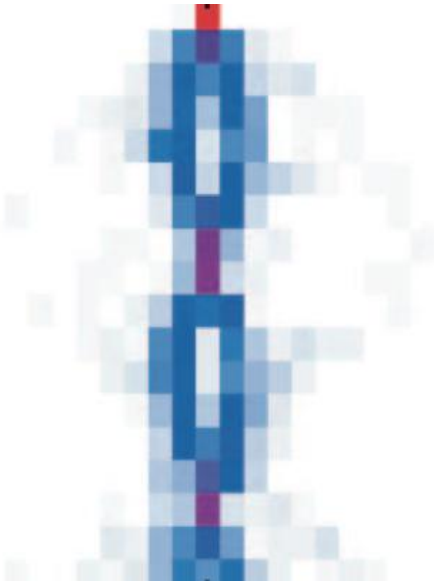
Hermite-Gauss
repulsive potentials



High fidelity
state preparation



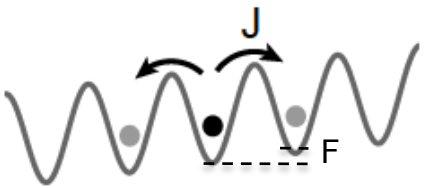
Quantum gas microscopy

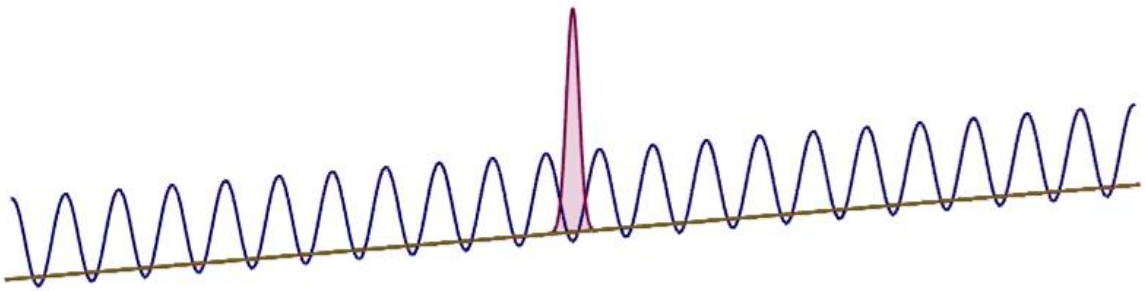


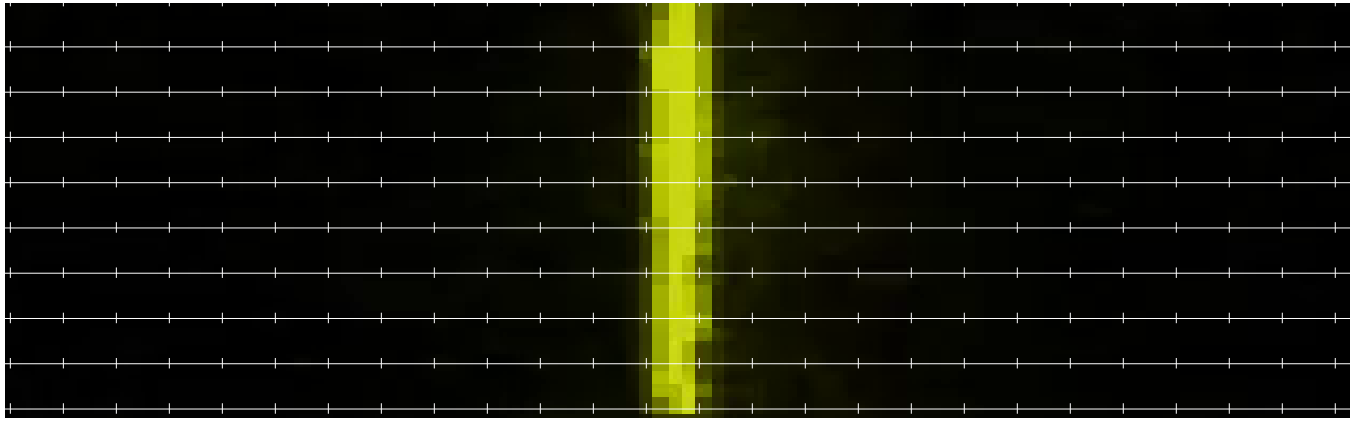
Coherent control

Renyi entropy

Single-Particle Bloch oscillations

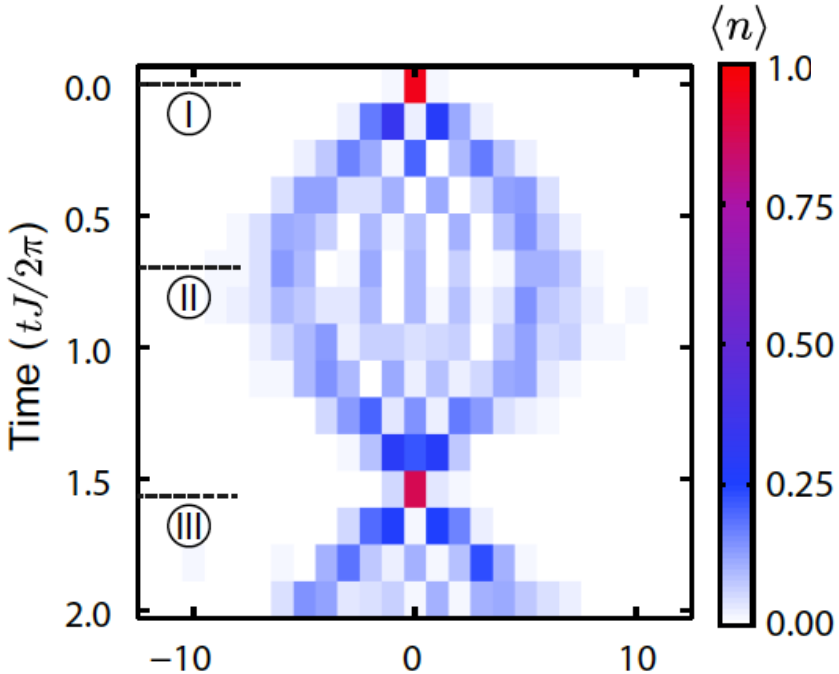


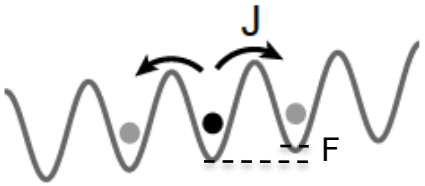




P. M. Preiss et al, **Science 347, 1229 (2015)**

Single-Particle Bloch oscillations





- Temporal period $T_B = \frac{2\pi}{F}$, spatial width $L_B = \frac{4J}{F}$
- Delocalized over ~14 sites = 10μm.
- Revival probability 96(3)%

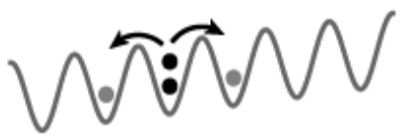
P. M. Preiss et al, **Science 347, 1229 (2015)**

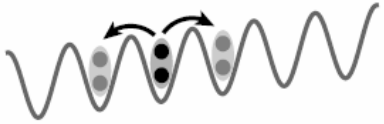
Bloch oscillations of two atoms

Two atoms starting on same site

Weak interaction

(I) u = 0.3





- Independent oscillations
- Clean revival

- Complex dynamics
- Asymmetry

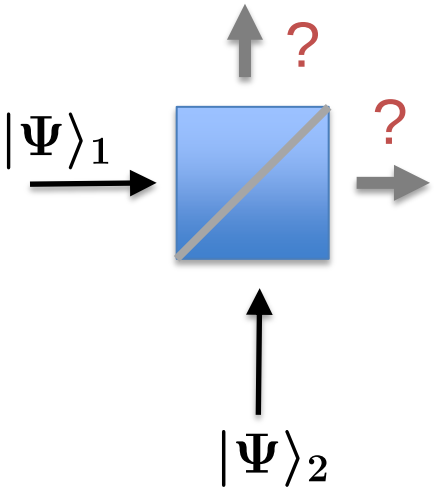
- Bloch oscillations of pairs
- Frequency-doubled BO

P. M. Preiss et al, **Science 347, 1229 (2015)**

Also see : R. Khomeriki *et al.,* PRA **81** (2010), G. Corrielli *et al*., Nature Comm. **4** (2013); F. Meinert *et al.*, Science **344**, 1259 (2014)

Quantum gas microscopy

Coherent control



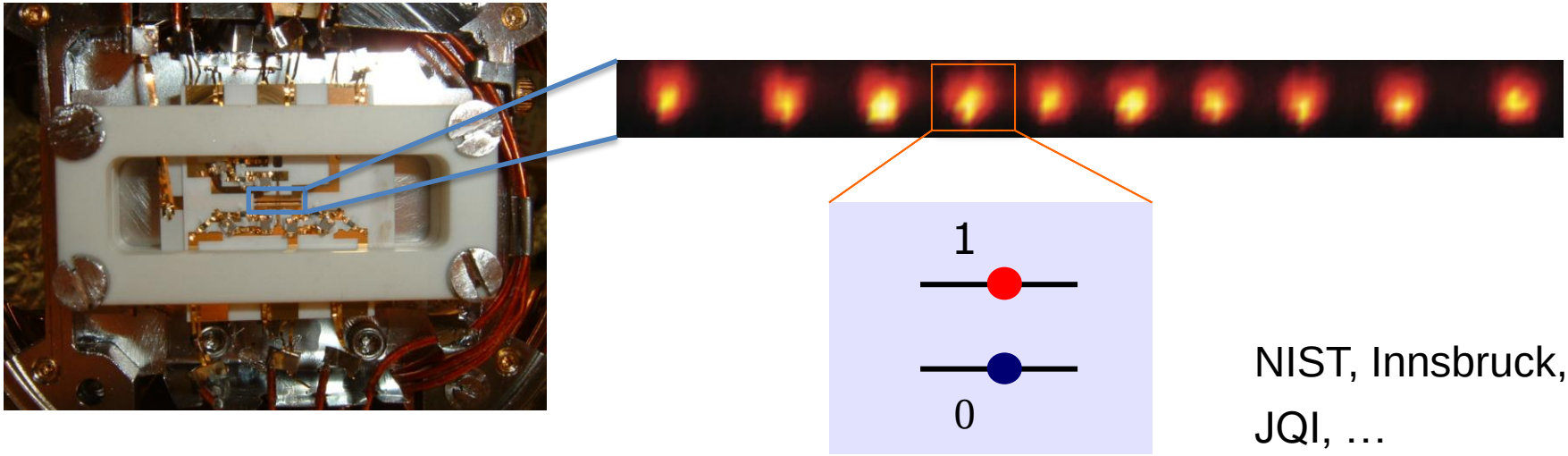
Renyi entropy

How to detect entanglement in experiments?

- Not accessible in conventional solid state materials – fast time scales, limited control at the single particle level
- Synthetic ‘engineered’ matter with cold atoms!

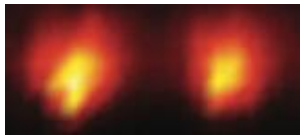
✓ Controllable Hamiltonian
✓ Long decoherence times
✓ Accessible time scales
✓ Detection at the single atom level

- **Entanglement in Spin systems**



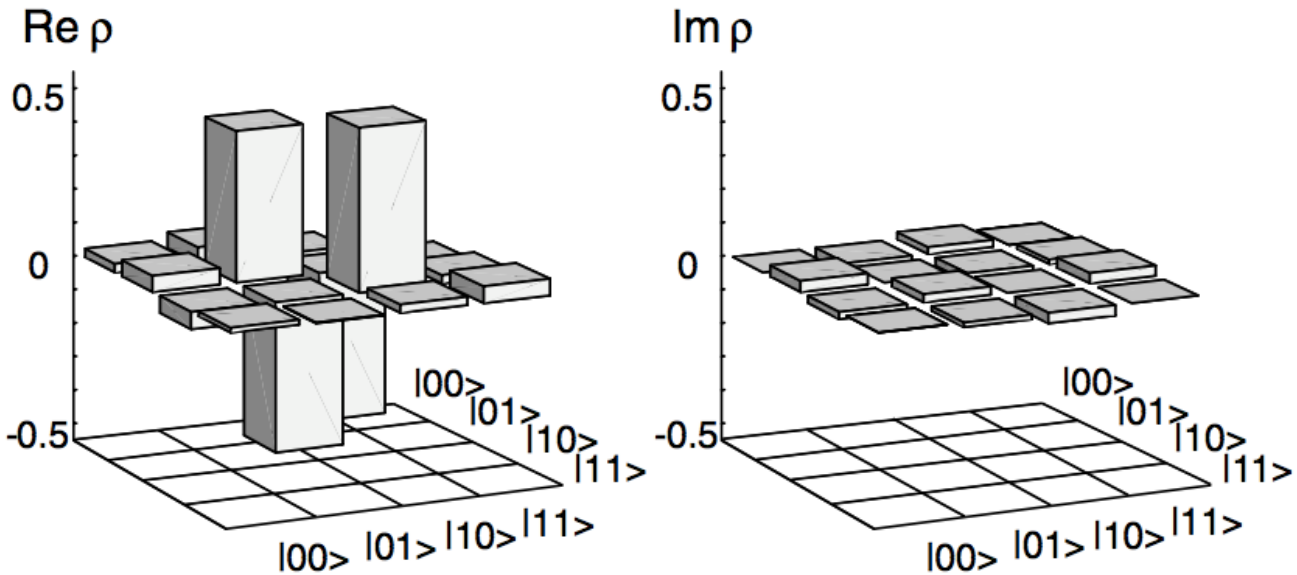
NIST, Innsbruck, JQI, …

How to detect entanglement in experiments?



$$|\psi\rangle = |10\rangle - |01\rangle$$

State Tomography



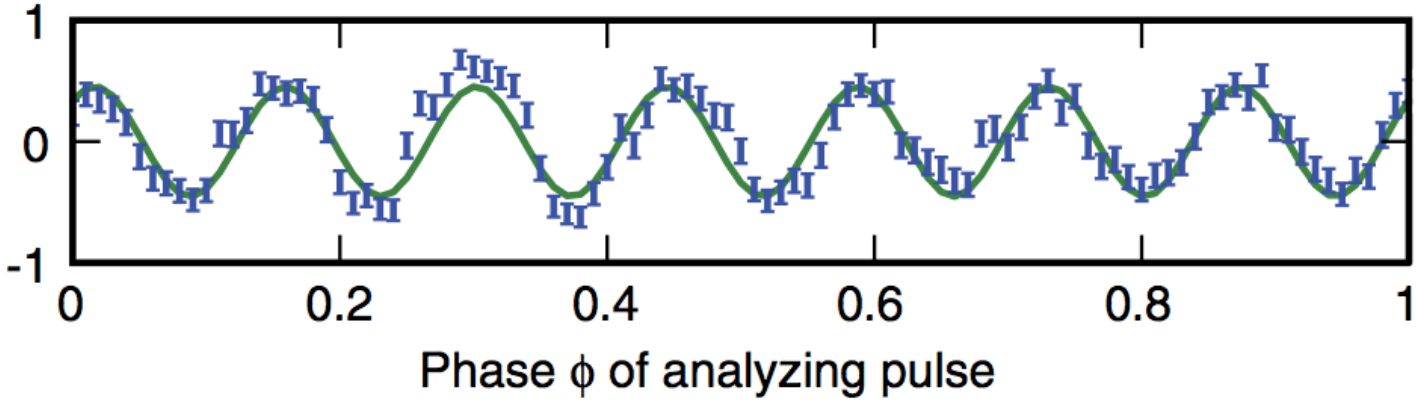
C. F. Roos et al, PRL 92, 220402 (2004)

- Exponentially hard
- No general scheme for other systems

How to detect entanglement in experiments?

Entanglement Witness

$$|\psi_{GHZ}\rangle = |000...0\rangle + |111...1\rangle$$

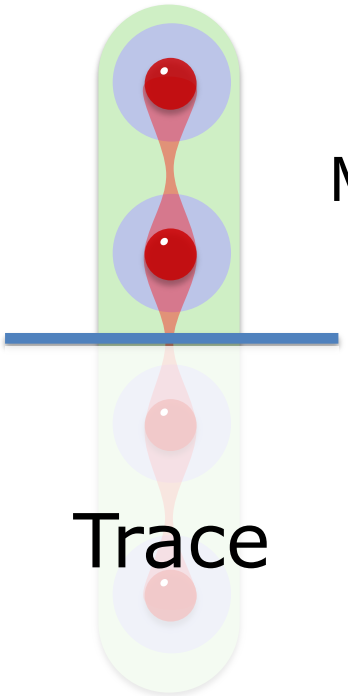


14 ions

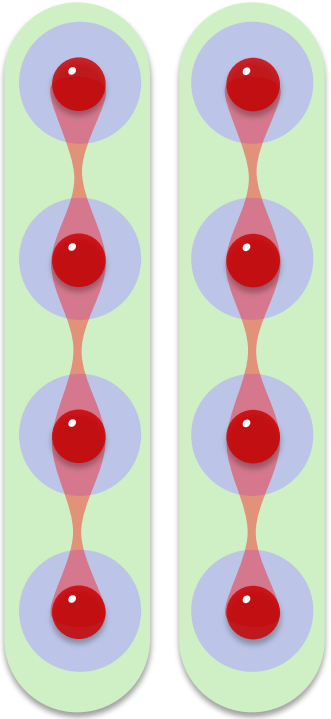
T. Monz et al, PRL 106, 130506 (2011)

Also see : Entanglement generated from spin squeezing , Vuletic group (Nature 2015)

Entanglement in the motional degree of freedom

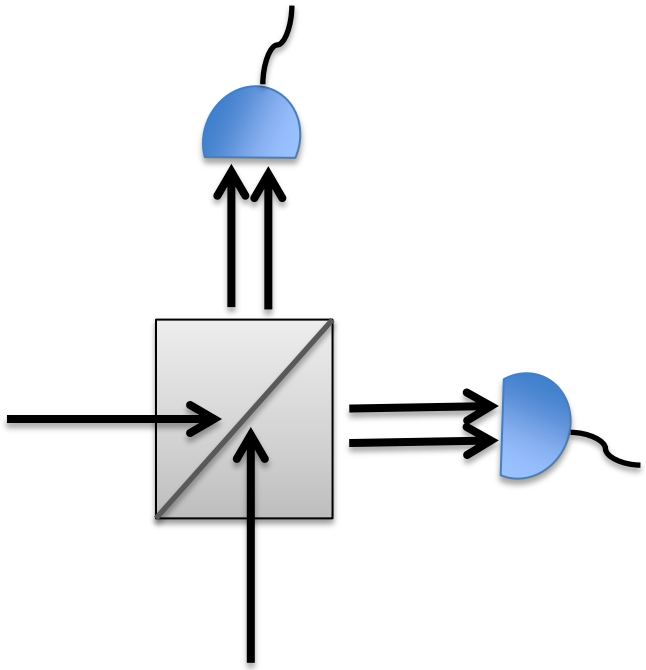


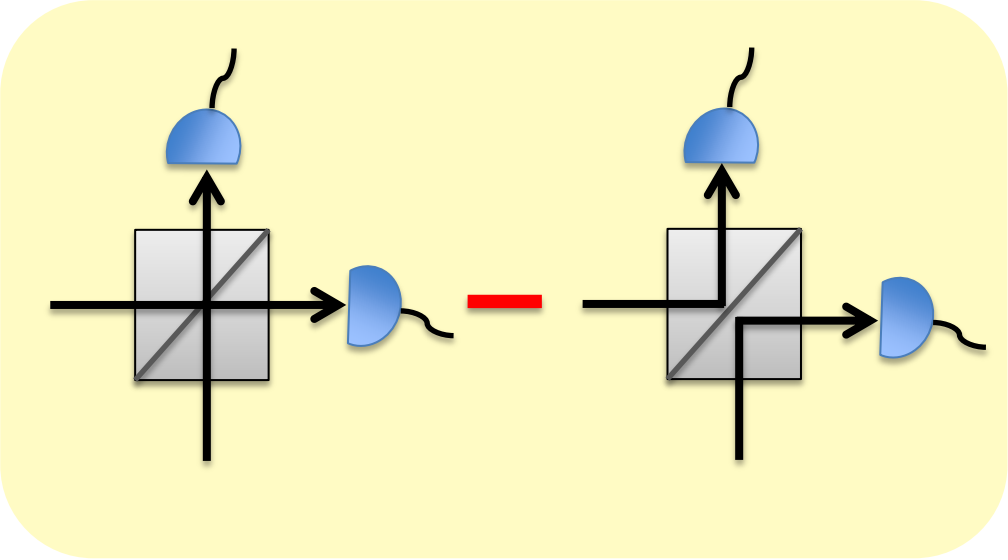
Mixed state



‘Quantum compare’ two copies

Hong-Ou-Mandel interference

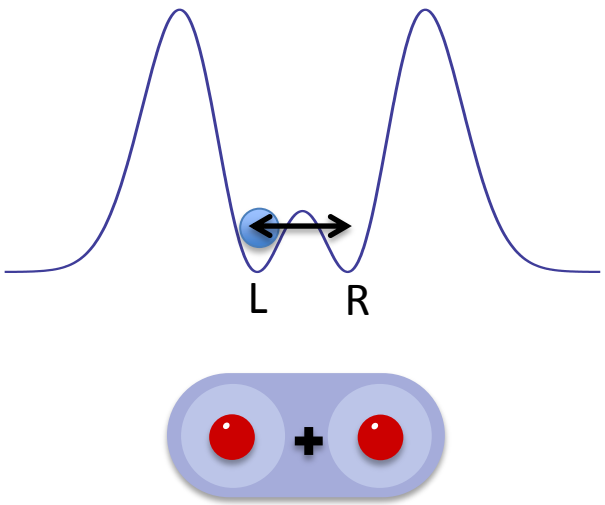


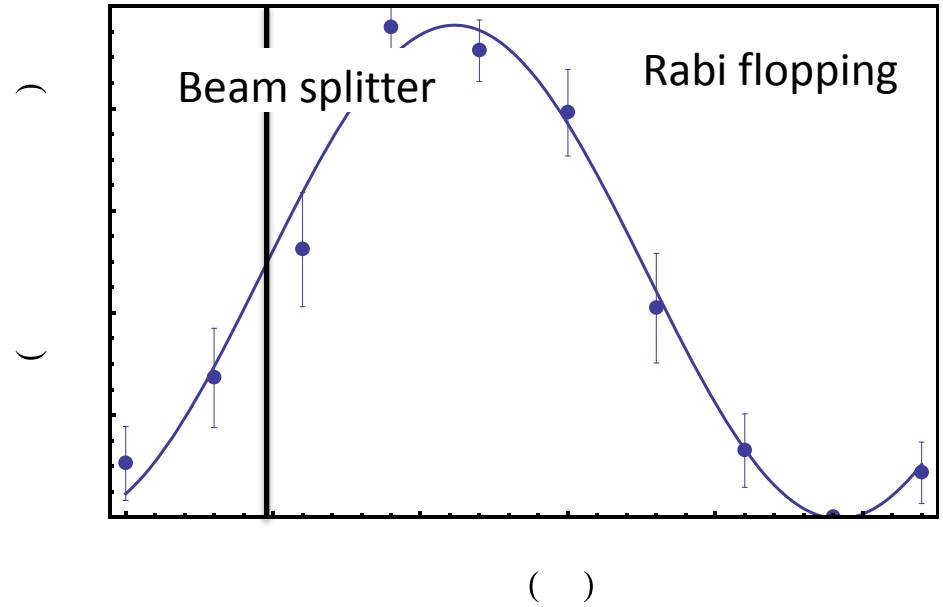


No coincidence detection
for **identical** photons

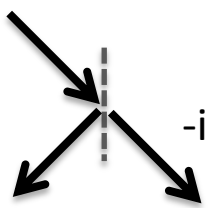
Hong C. K., Ou Z. Y., and Mandel L. Phys. Rev. Lett. 59 2044 (1987)

Beam splitter operation: Rabi flopping in a double well

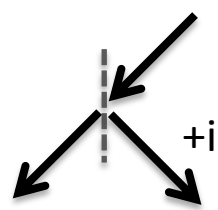




$$a_L^\dagger \to a_L^\dagger - i a_R^\dagger$$



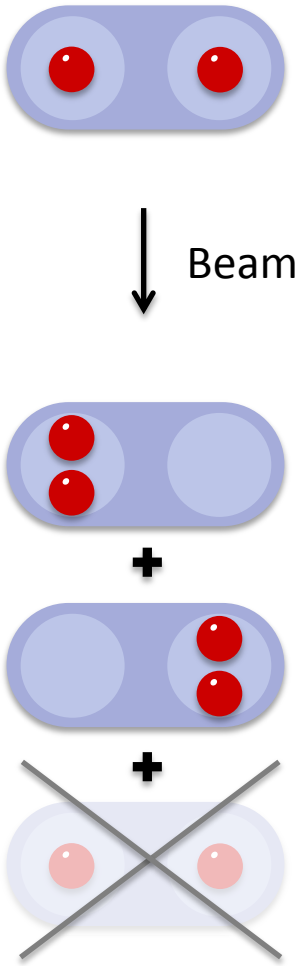
$$a_R^\dagger \to a_L^\dagger + i a_R^\dagger$$



Also see: Kaufman A M *et al.*, **Science 345, 306 (2014)**

Two bosons on a beam splitter

Hong-Ou-Mandel interference



$$a_L^\dagger a_R^\dagger$$

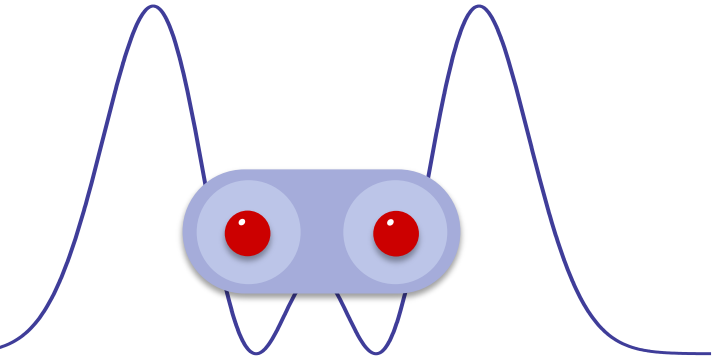
$$a_L^\dagger \to a_L^\dagger - i a_R^\dagger$$

$$a_R^\dagger \to a_L^\dagger + i a_R^\dagger$$

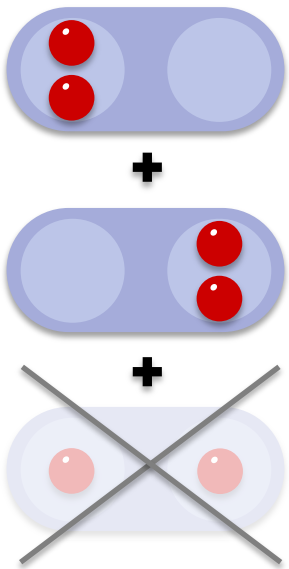
$$a_L^\dagger a_L^\dagger$$

$$+$$

$$a_R^\dagger a_R^\dagger$$



Beam splitter

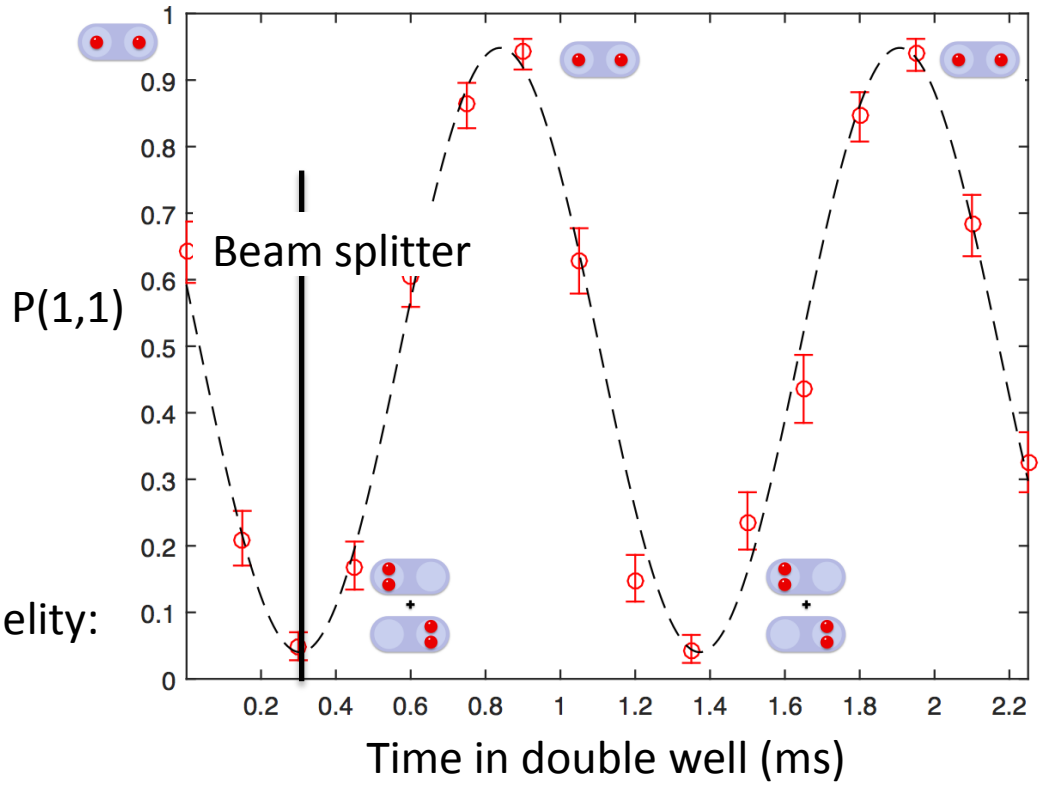


measured fidelity:

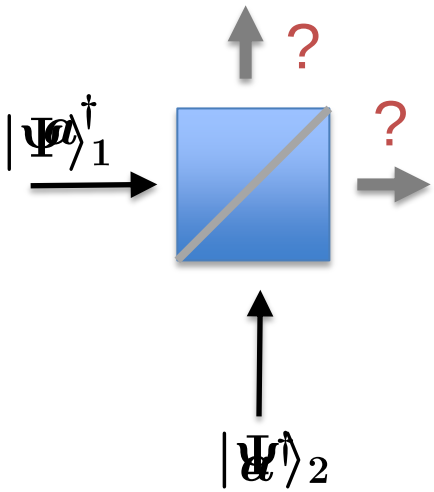
96(4)%

4(4)%

limited by interaction



HOM-Interference of Many-Body States



How "identical" are the **particles**?

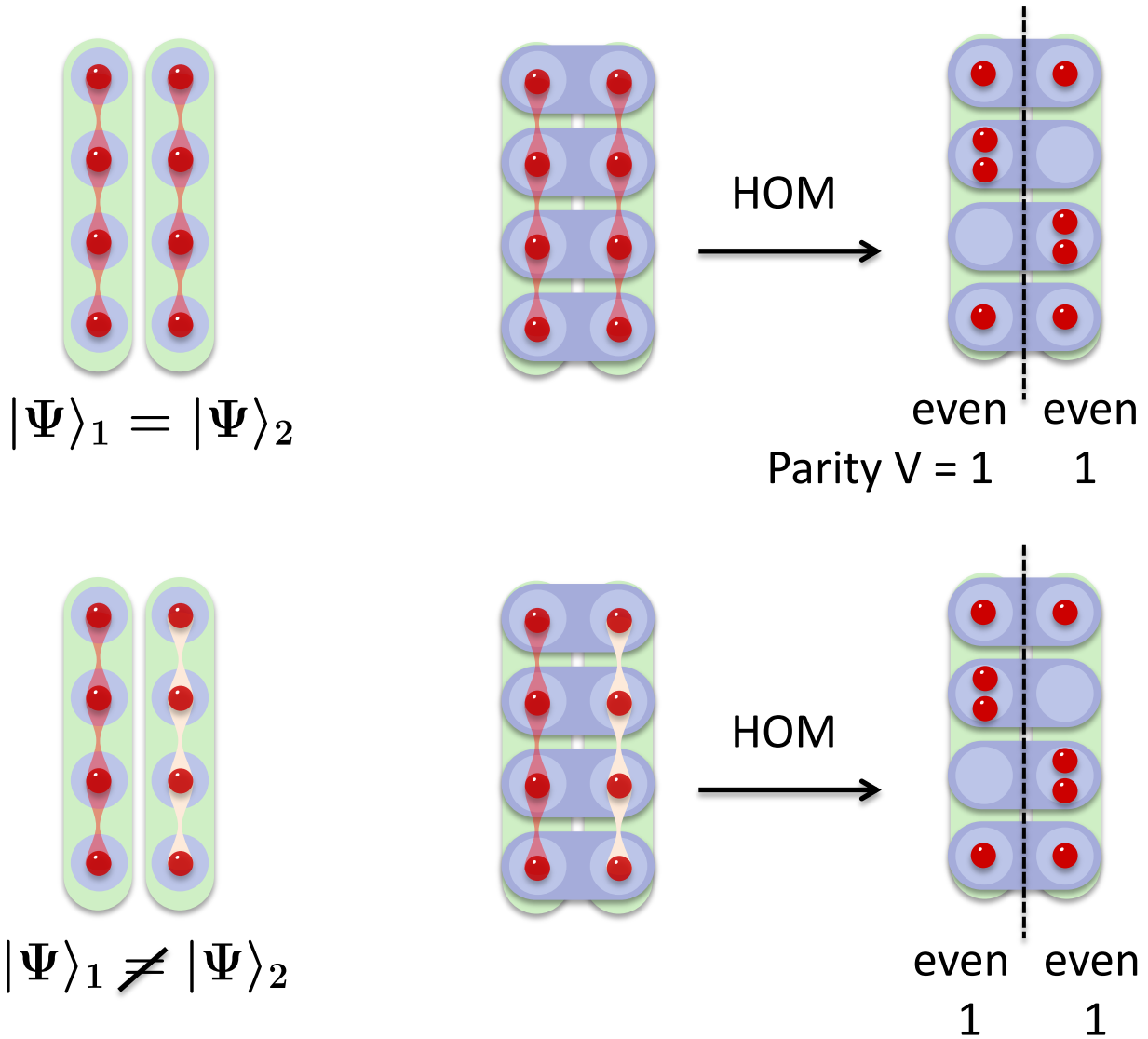
vs.

How "identical" are the **states**?

If $|\Psi\rangle_1 = |\Psi\rangle_2$, **deterministic number parity** after beam splitter

Alves and Jaksch, PRL **93** (2004)
Daley et al., PRL **109** (2012)

HOM-Interference of Many-Body States

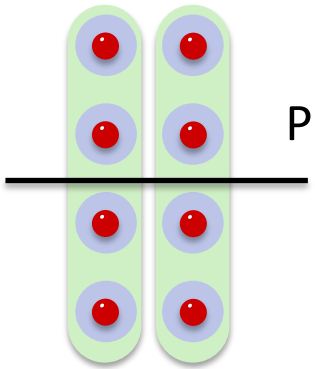


$$\langle V \rangle = \mathrm{Tr}\rho_1\rho_2$$
$$= \mathrm{Tr}\rho^2$$
$$\rho_1 = \rho_2 = \rho$$

Alves C M, Jaksch D, PRL 93, 110501 (2004), Daley A J et al, PRL 109, 020505 (2012)

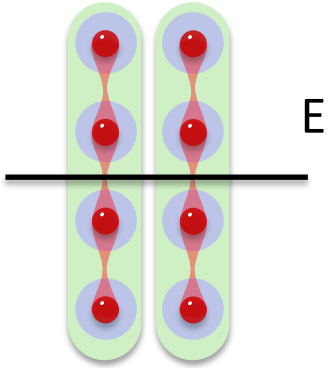
Measuring many-body entanglement

Mott Insulator



Product State

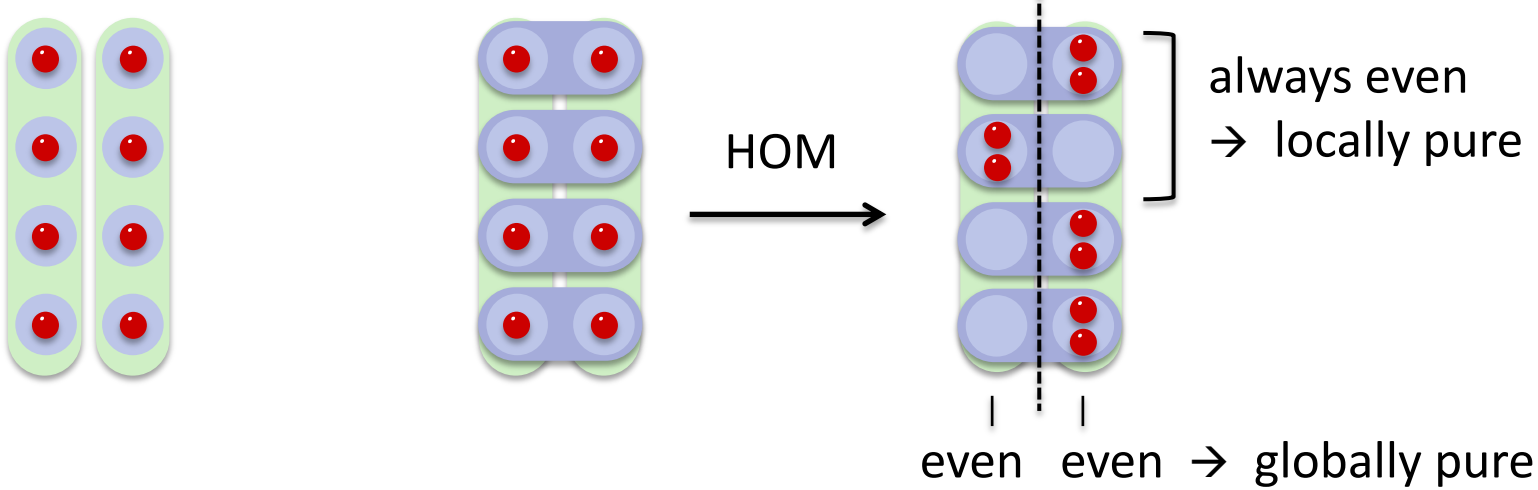
Superfluid



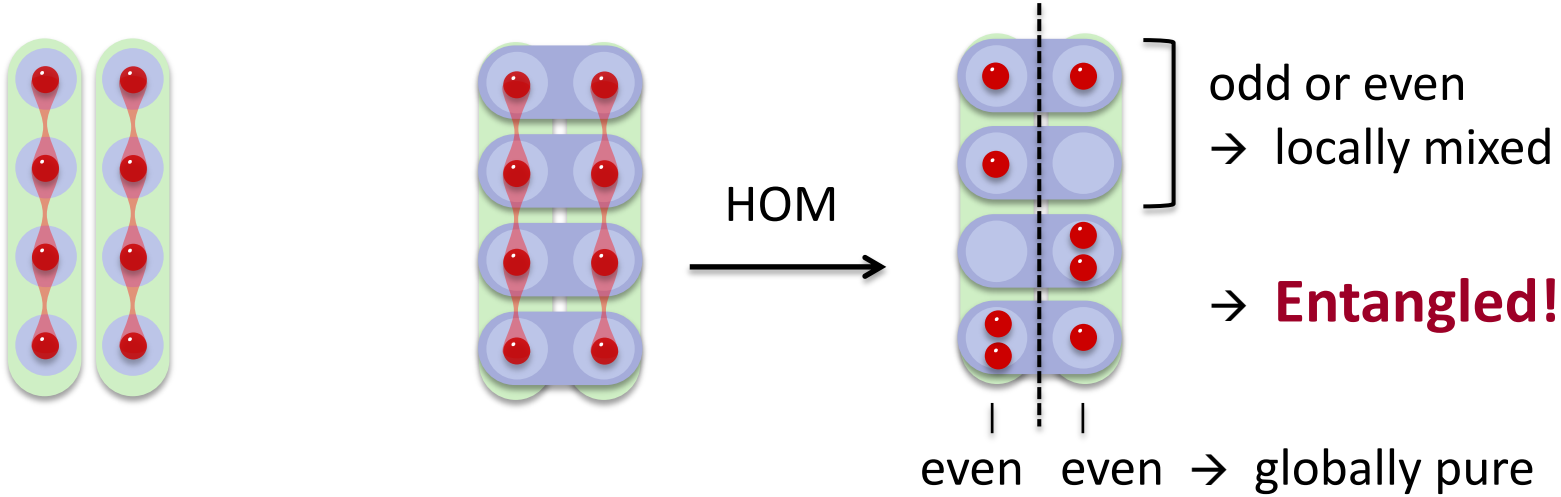
Entangled

Measuring many-body entanglement

Mott Insulator



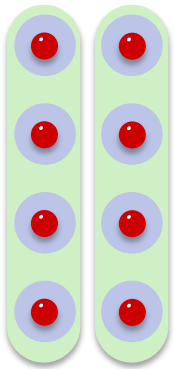
Superfluid

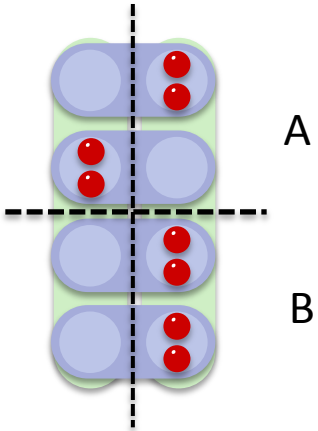


Ref: Alves C M, Jaksch D, PRL 93, 110501 (2004), Daley A J et al, PRL 109, 020505 (2012)

Measuring many-body entanglement

Mott Insulator



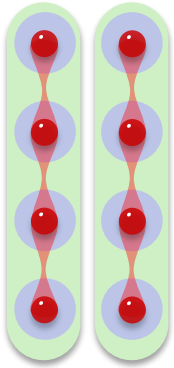


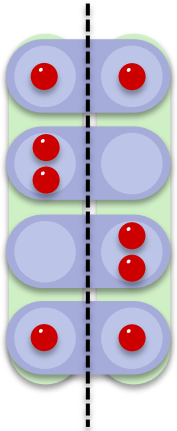
$$\rho_A = \mathrm{tr}_B\{\rho\} = |\Psi_A\rangle \otimes \langle\Psi_A|$$

$$S_n(\rho_\alpha) = \frac{1}{1-n}\log \mathrm{Tr}\{\rho_\alpha^n\}$$

$$S(\rho) = -\log \mathrm{Tr}\rho^2$$

Superfluid





$$\mathrm{Tr}\rho_A^2 < \mathrm{Tr}\rho_{AB}^2$$

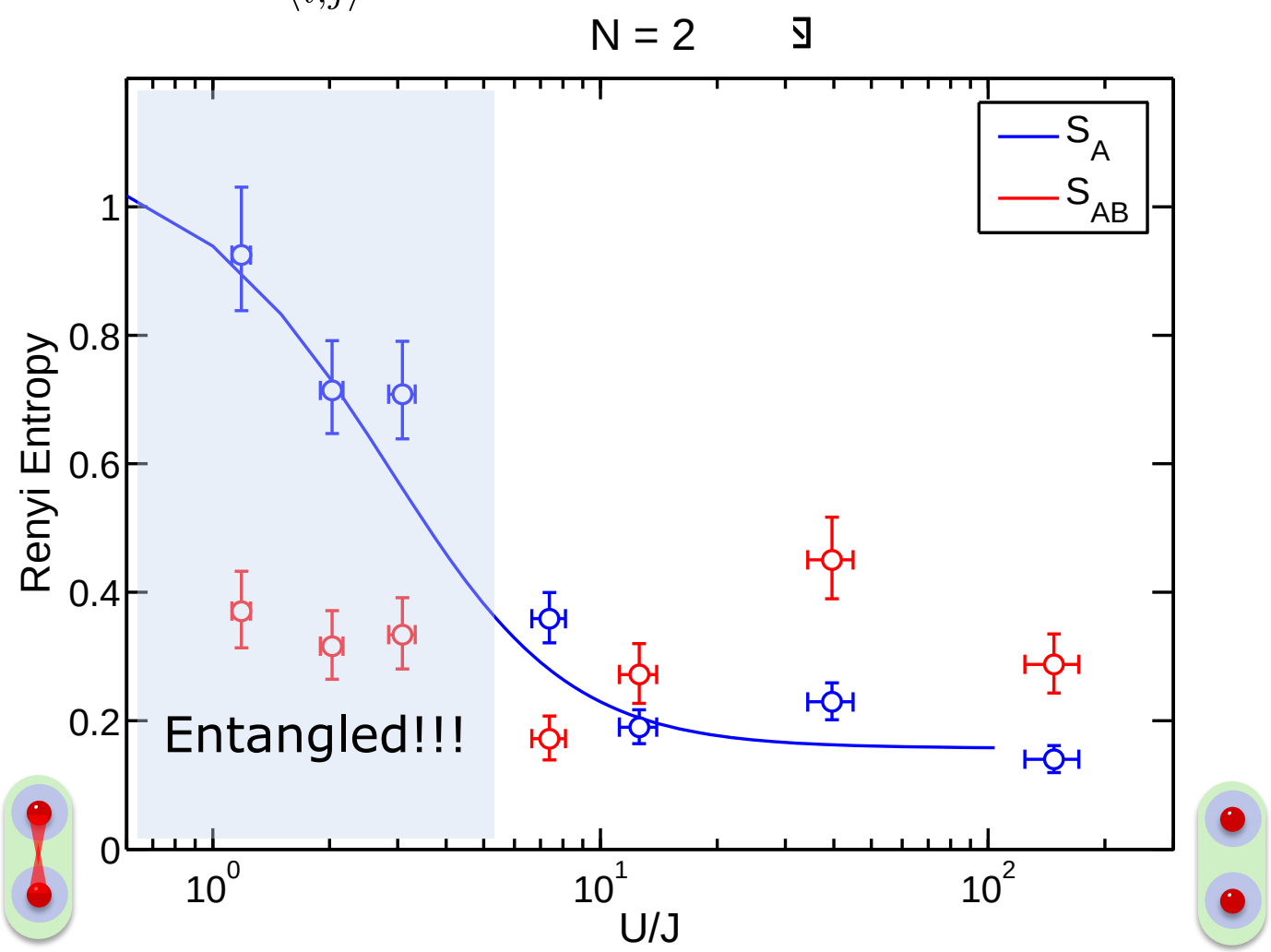
$$S_A > S_{AB}$$

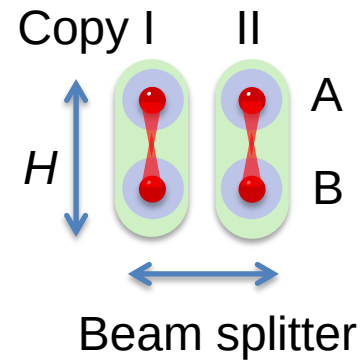
Subsystems are entangled

Ref: Alves C M, Jaksch D, PRL 93, 110501 (2004), Daley A J et al, PRL 109, 020505 (2012)

Entanglement formation in the ground state

$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i (n_i - 1)$$

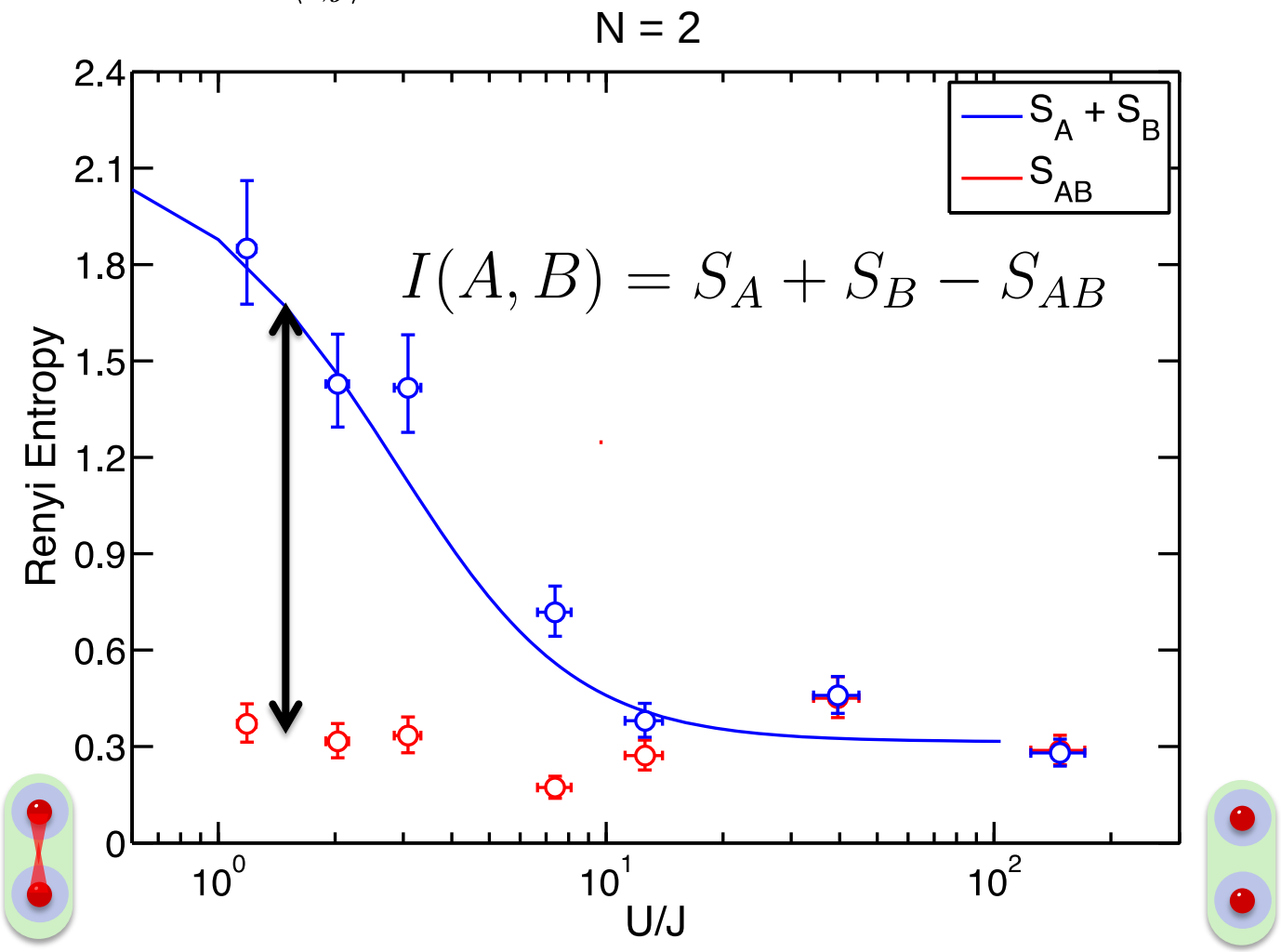


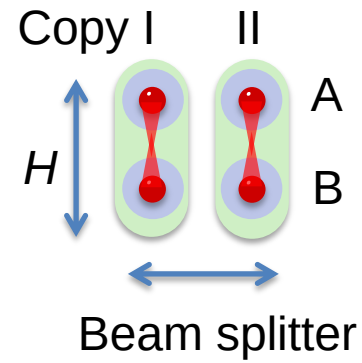


R. Islam et al
(in preparation)

Mutual Information

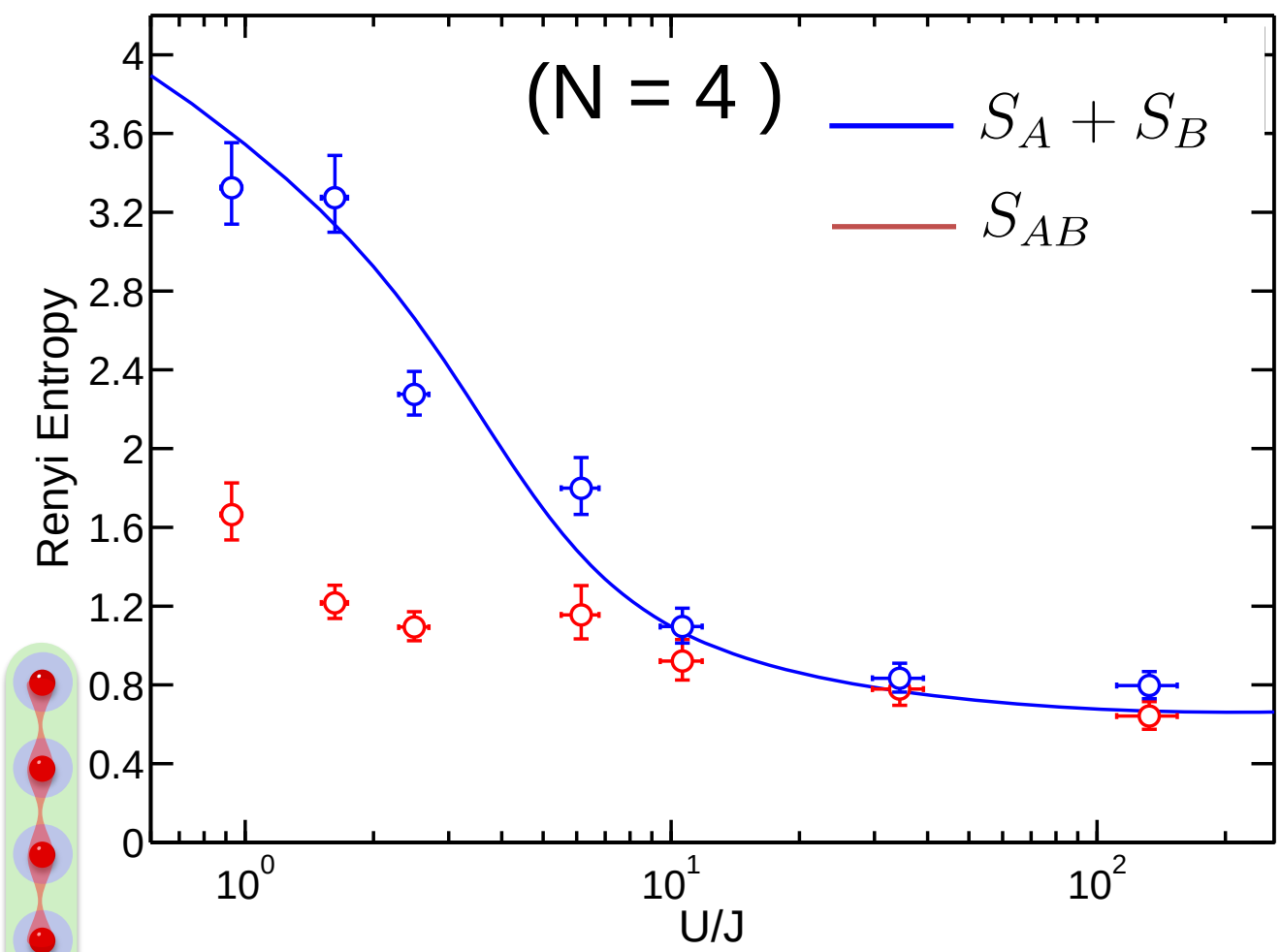
$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i(n_i - 1)$$

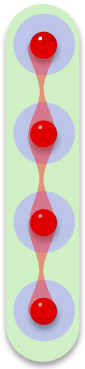


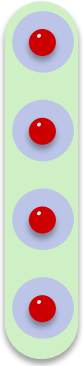


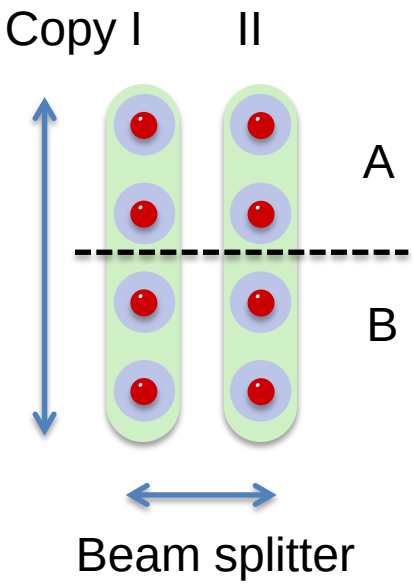
Mutual information

$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i (n_i - 1)$$

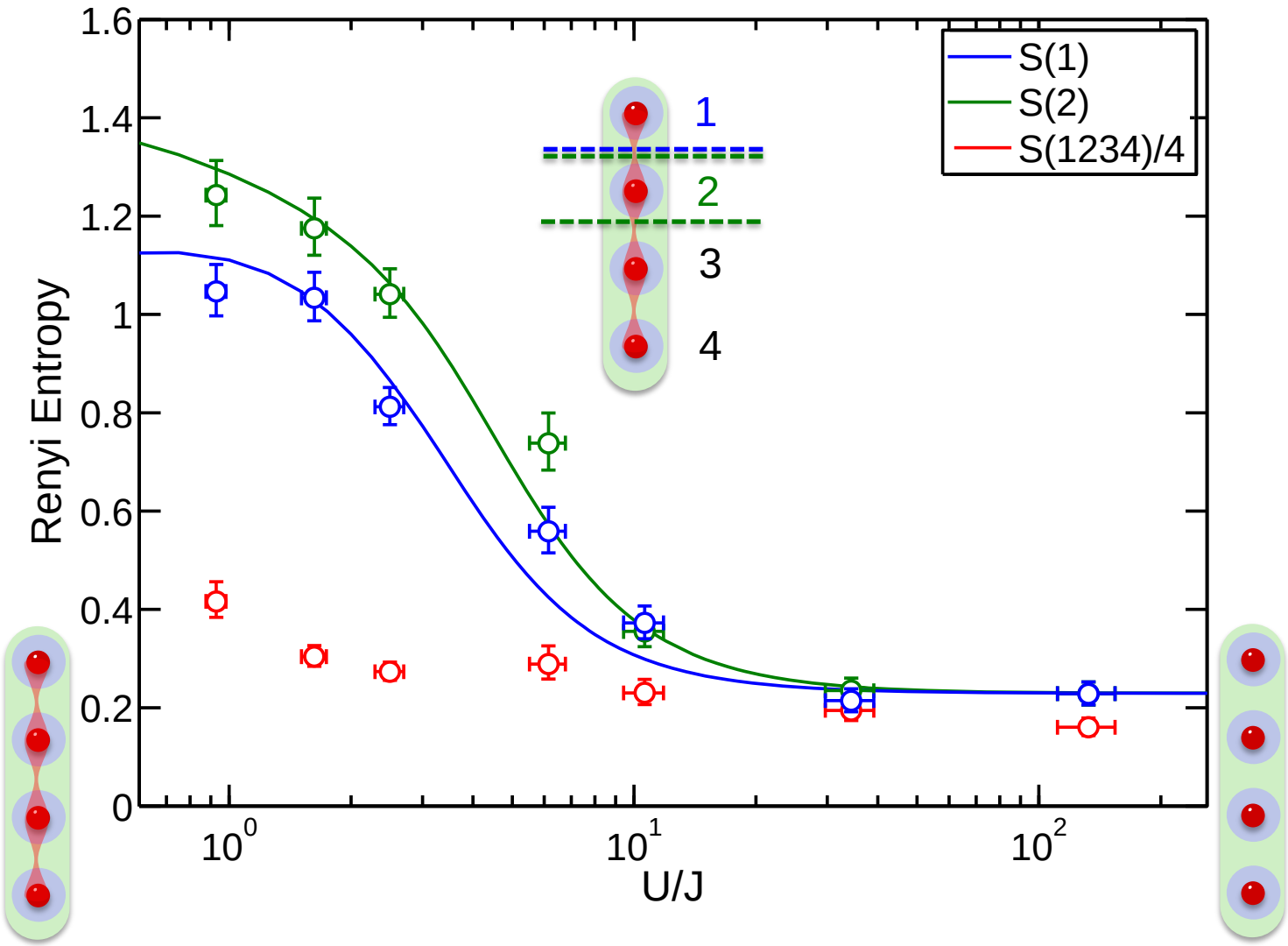




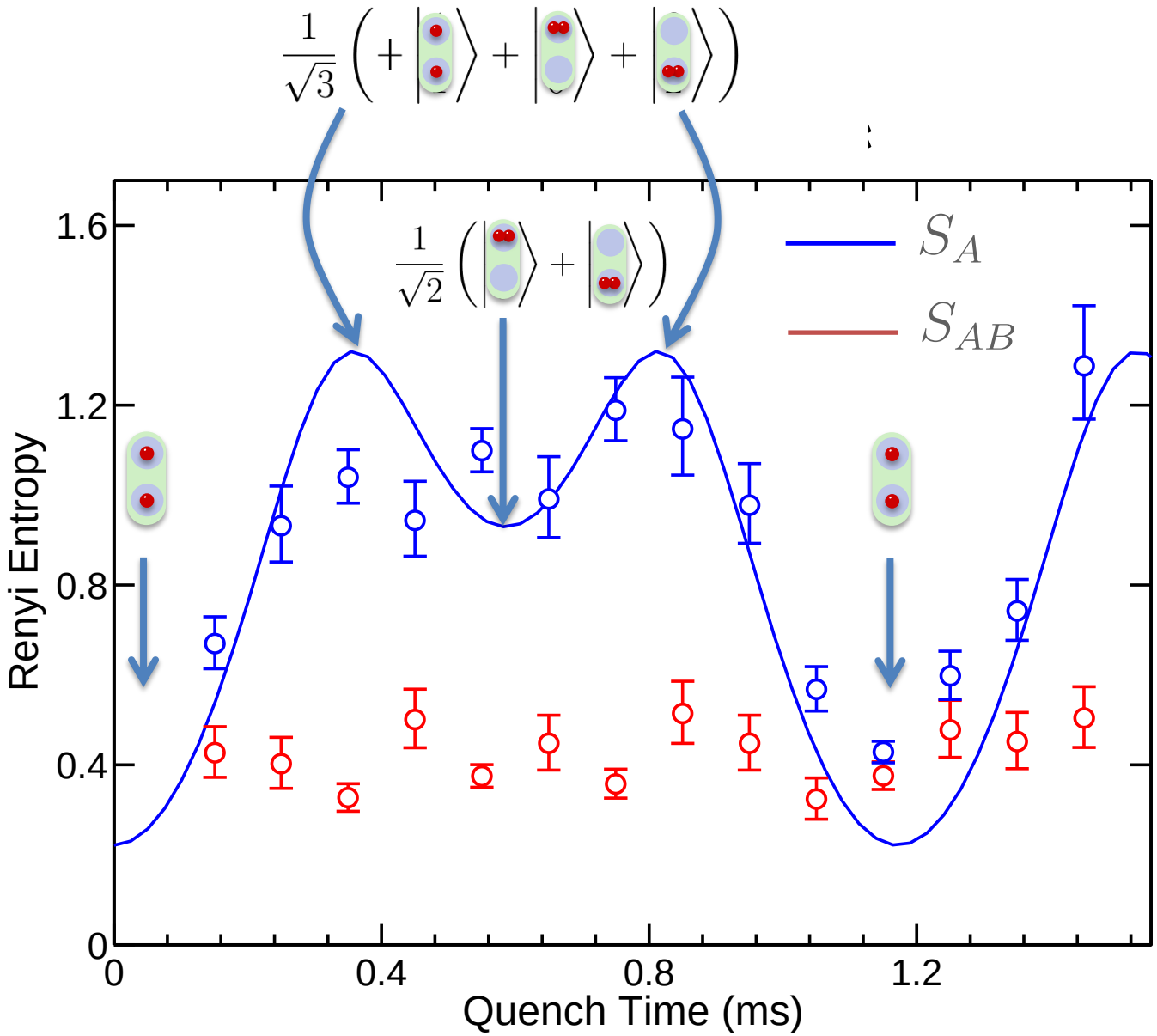


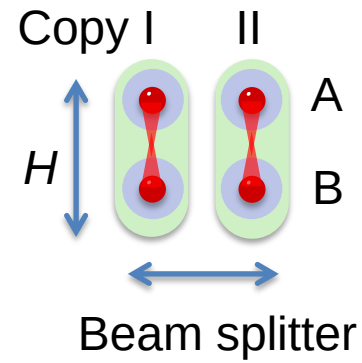


Boundary effect



Non equilibrium: Quench dynamics





Outlook

- **Scaling** of entanglement entropy and mutual information – probe critical points, violation of area law etc.
- Dynamical phenomena with entanglement – MBL phase.

- Overlap of two wave functions $\langle V \rangle = \mathrm{Tr}\rho_1\rho_2$

Sensitivity to perturbation signaling quantum phase transitions.

- Higher order Renyi entropies by interfering more than two copies.

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Aaron Krahn
Susannah Dickerson

Thank you!