EFT for Black Hole Horizons

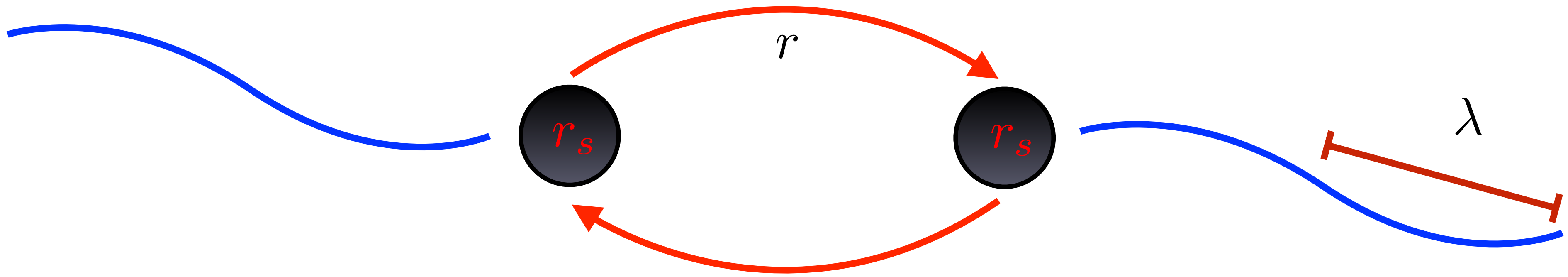
Walter Goldberger
Yale U.

Based on work w/ Rothstein, in progress

Part I: “Classical”

Black hole binary inspirals

Gravitational dynamics of radiating classical BH (or NS) binary systems in the non-relativistic limit is experimentally relevant (LIGO/VIRGO,…)



Even for $v \ll 1$, the non-linear nature of GR makes this a difficult problem, involving a hierarchy of length scales

Gravitational radius: $r_g = 2G_N M$

Physical radius: $r_s (= r_g \text{ for BH})$

Orbital scale: r

Radiation wavelength λ

$$r_g \sim r_s \gg r \gg \lambda$$

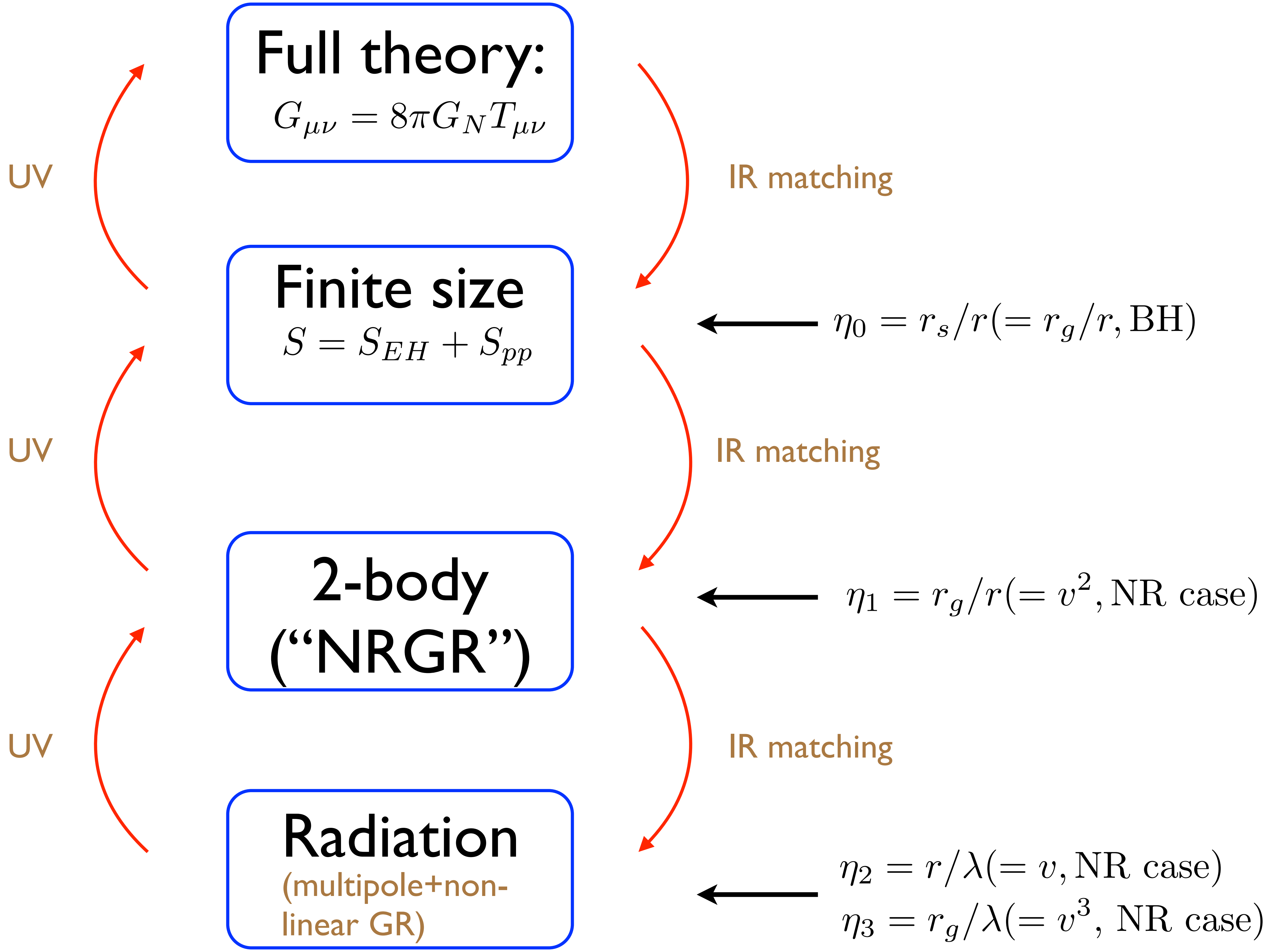
$$r \sim r_g/v^2$$

$$\lambda \sim r/v \sim r_g/v^3$$

"correlated scales"

Experiments will be sensitive to at least v^6 = "3PN" corrections beyond Newtonian gravity (Thorne et al 1994). (5PN considered feasible). Numerical GR results also motivate computing higher order corrections.

Tower of gravity EFTs:



Independent EFTs with distinct expansion parameter coincide in PN limit.
UV divergence in EFT_{i+1} corresponds to IR effect in EFT_i

Finite size effects:

What can we learn about the internal structure of compact objects from binary inspirals at LIGO?

In the inspiral phase, binary constituents can be treated as point-like. Finite size effects encoded in an worldline EFT coupled to gravity

DOFs:

$x^\mu(\lambda) =$ worldline CM coordinate

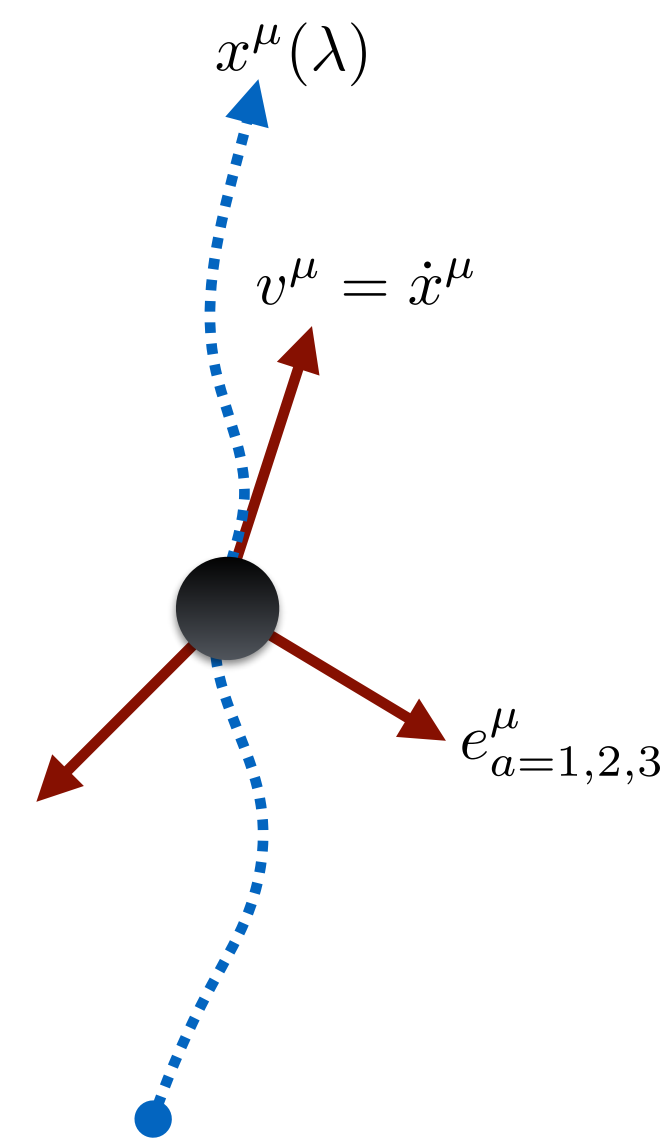
$e^\mu_{a=1,2,3}(\lambda) =$ local frame (SPIN)

Symmetries:

Diff. invariance $x^\mu \mapsto x^\mu + \xi^\mu(x)$

Worldline RPI $\lambda \mapsto \lambda'(\lambda)$

Local $SO(3)$ rotations acting on (for BH only) e^μ_a



EFT for gravity coupled to BH, in the point particle limit:

$(\hbar = c = 1)$

$(m_{Pl}^2 = 1/(32\pi G_N))$

$S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{g} R(x)$

$$S = S_{EH} + S_{pp}$$

The most general (mod. e.o.m's) point particle Lagrangian consistent with symmetries (ignoring spin, assume parity invariance), organized in a derivative expansion:

$$S_{pp} = -m \underbrace{\int d\tau}_{\mathcal{O}(\partial^0 g)} + c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + \underbrace{c_B \int}_{\mathcal{O}(\partial^4 g)} d\tau B_{\mu\nu} B^{\mu\nu} + \cdots$$

w/ $E_{\mu\nu} = R_{\mu\alpha\nu\beta} v^\alpha v^\beta$ = "electric" curvature tensor

$B_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\rho\sigma\lambda} v^\rho v^\alpha R_{\nu\alpha}{}^{\sigma\lambda}$ = "magnetic" curvature tensor

(Note: $\mathcal{O}(\partial^2 g)$ terms, eg $\int d\tau g^{\mu\nu} R_{\mu\nu}$ are redundant due to source free eom $R_{\mu\nu} = 0$).

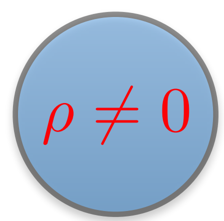
The curvature couplings

$$S_{pp} \supset c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu}$$

describe the $\ell = 2$ linear tidal response of the compact object to external gravitational fields.

E.g, Newtonian spherical self-gravitating star (radius R). No external gravitational field:

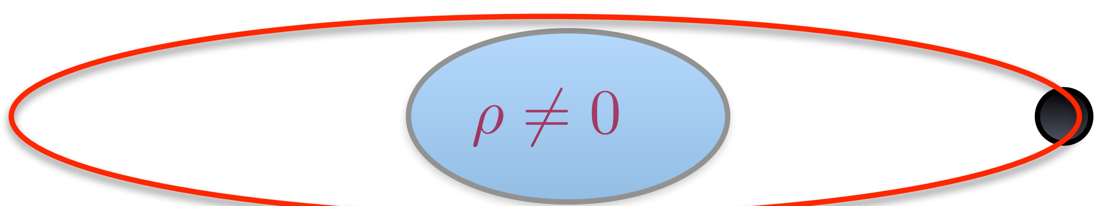
$$\Phi = 0$$



$$Q_{ij} = \int d^3\vec{x} \rho \left(x^i x^j - \mathrm{trace} \right) = 0$$

Turn on weak external perturbation:

$$\Phi \neq 0$$



$$\delta Q_{ij} = -\frac{2}{3} k \, \frac{R^5}{G_N} E_{ij}$$

$$E_{ij} = \partial_i \partial_j \Phi$$

w/ dimensionless (gravitational) "Love number" $k \sim \mathcal{O}(1)$ that depends on fluid eqn. of state.

In the point particle EFT, the induce quadrupole moment is:

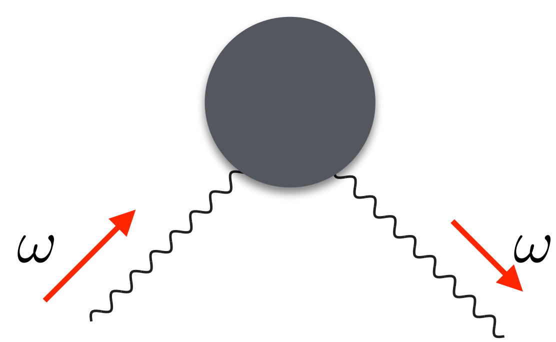
$$\delta Q_{ij} = -\frac{\delta}{\delta E_{ij}} S_{pp} = -2c_E E_{ij} \qquad \text{vs.} \qquad \delta Q_{ij} = -\frac{2}{3}k\,\frac{R^5}{G_N}E_{ij}$$

so we expect on dimensional grounds:

$$c_E \sim R^5/G_N$$

Same scaling also holds for relativistic (compact) objects. Eg elastic graviton+BH scattering amplitude: $(r_s\omega \ll 1)$

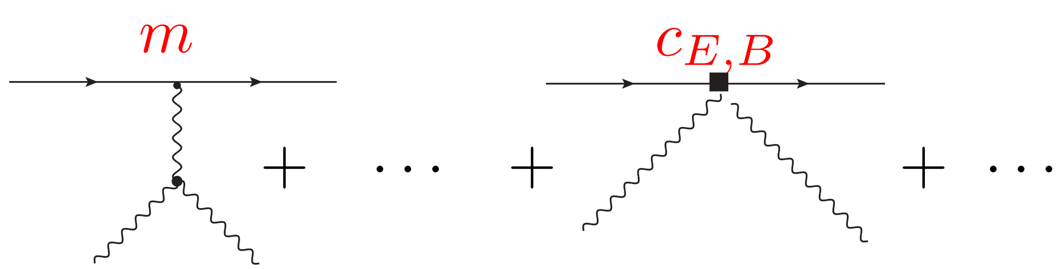
Full theory: $i\mathcal{A} =$



$= r_s f(r_s\omega)$

$\Longrightarrow \quad c_{E,B} \sim r_s^5/G_N$

EFT: $i\mathcal{A} =$



$\sim \frac{c_{E,B}}{m_{Pl}^2}\omega^4$

from matching
(WG, '06)

Given that $c_E \sim R^5/G_N$, we expect finite size/tidal corrections to potentials and radiation to scale as

$$\text{Tidal effects} \sim \left(\frac{R}{r}\right)^5 = \left(\frac{R}{r_s}\right)^5 \times v^{10}$$

which is formally a 5PN effect. Specifically

Black hole: $R = r_s$ → Tidal effects at 5PN

Neutron start: $R \sim \mathcal{O}(10) \times r_s$ → Enhancement by a factor of $\sim 10^5$

(Flanagan+Hinderer, 2007)

(in fact the NS/NS inspiral event GW17087 at LIGO has already placed very crude constraints on the neutron star tidal coefficients…)

For neutron stars, c_E depends on the EOS and has been calculated numerically in (Flanagan+Hinderer, 2007)

For the case of Schwarzschild black holes in $d = 4$, the tidal response in the full theory $R_{\mu\nu} = 0$ has been computed analytically by

Damour+Nagar, 2009

Binnington+Poisson, 2009

Kol+Smolkin, 2011

Steinhoff et al 2013

while the EFT side corrections were shown to vanish in Kol+Smolkin, 2011. The result for BH's in $d = 4$

$$c_E^{BH} = c_B^{BH} = 0$$

so no (static) tidal response at $\ell = 2$ (and likely also for $\ell > 2$ ...)

BH absorption and Compact Binaries

The results on tidal coefficient suggests that finite size effects are absent for black holes.

But formalism outlined so far neglects dissipation, ie absorption of energy and angular momentum by the compact objects themselves.

On general grounds, dissipation implies the existence of low frequency modes with $\omega \sim \omega_{GW}$ (eg NS: hydro modes,.... BH: horizon absorption) not captured by the point particle EFT

$$S_{pp} = -m\int d\tau + c_E \int d\tau E_{\mu\nu}E^{\mu\nu} + c_B \int d\tau B_{\mu\nu}B^{\mu\nu} + \cdots$$

eg, for a Schwarschild black hole, the spectrum contains an infinite tower of modes labeled by $SO(3)$. In this case there are some zero modes:

Mode	Freq.	J^P
$m(\lambda)$	0	0
$x^\mu(\lambda)$	0	1^+
$\omega_{ij}(\lambda)$ (spin)	0	1^-

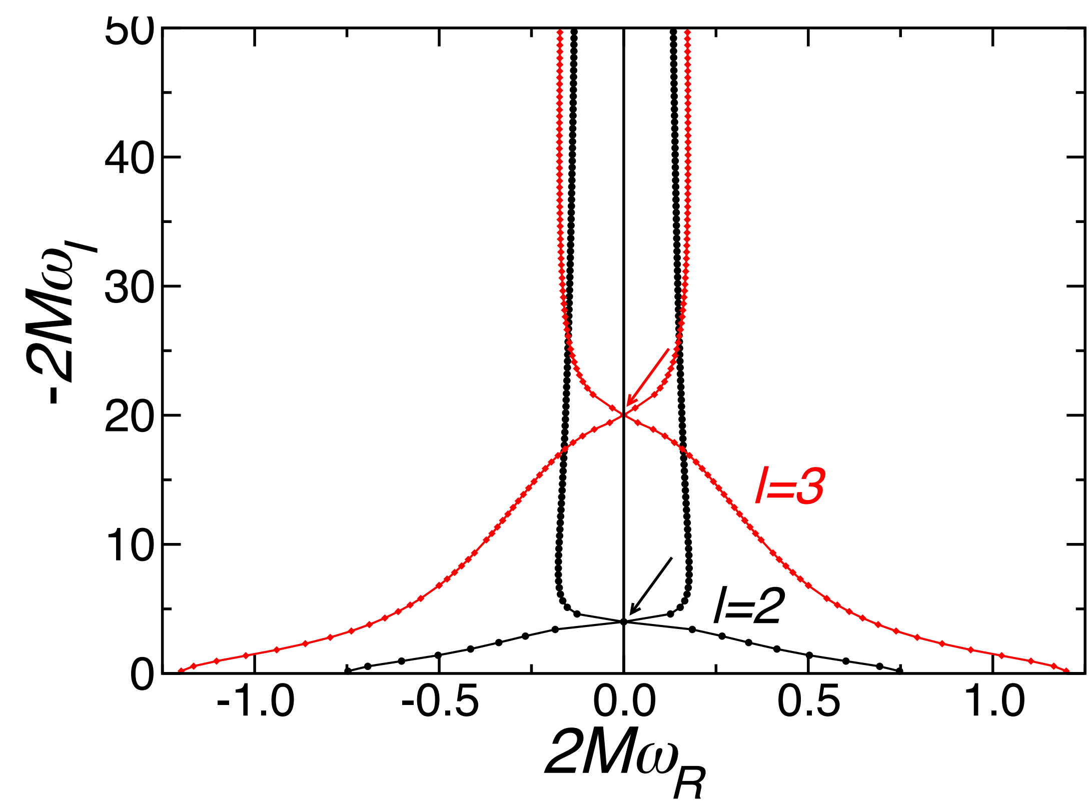
there are also an infinite tower of "quasinormal modes"…

n	$\ell = 2$		$\ell = 3$		$\ell = 4$	
0	0.37367	-0.08896 i	0.59944	-0.09270 i	0.80918	-0.09416 i
1	0.34671	-0.27391 i	0.58264	-0.28130 i	0.79663	-0.28443 i
2	0.30105	-0.47828 i	0.55168	-0.47909 i	0.77271	-0.47991 i
3	0.25150	-0.70514 i	0.51196	-0.69034 i	0.73984	-0.68392 i

Table 1: *The first four QNM frequencies* (ωM) *of the Schwarzschild black hole for* $\ell = 2, 3,$ *and* 4 *[135]. The frequencies are given in geometrical units and for conversion into kHz one should multiply by* $2\pi(5142Hz) \times (M_\odot/M)$.

(from Kokkotas and Schmidt, gr-qc/9909058).

which are increasingly "broad resonances," rather than "quasiparticles":



Schwarzschild QNMs for $\ell = 2, 3$

(Berti et al CQG (2009)):

Even though the form of the internal spectrum depends on the details of the internal structure, can incorporate the effects of dissipation in a model independent w/o the need to explicitly track the light DOFs

The idea is to treat the compact object as $R \to 0$ as an "atom", i.e a worldline with local operators coupled to gravitons. For a spherical symmetric object, the leading interactions with gravitons take the form

$$S_{int} = -\int d\tau(\lambda) Q^E_{ab}(\lambda) E^{ab}(x) - \int d\tau(\lambda) Q^B_{ab}(\lambda) B^{ab}(x).$$

With operators $Q^{ab}_E(\lambda),\ Q^{ab}_B(\lambda) \quad \ldots$ acting on the Hilbert space of internal states. These are gravitational analogs of the EM dipole interaction

$$H_{em} = -\hat{\vec{p}} \cdot \vec{E}$$

Microscopic properties are then encoded in the correlation functions

$$\langle Q^{E,B} \cdots Q^{E,B} \rangle$$

which can be related to observable quantities of the compact object.

Example: Graviton absorption and power dissipation

Consider an compact object of mass M . Graviton absorption amplitude in the object's rest frame:

$$i\mathcal{A}(g_h(k) + M \to X) = \langle X|Te^{-i\int dt H_{int}}|k,h;M\rangle$$

$$\approx -\int dt \langle X|Q^E_{ij}(t)|M\rangle\langle 0|E_{ij}(t,0)|k,h\rangle + (E \leftrightarrow B)$$

absorption cross section is

$$\sigma_{abs}(\omega) = \lim_{T\to\infty}\frac{1}{T}\cdot\frac{1}{2\omega}\sum_X |\mathcal{A}(g(k)+M\to X)|^2$$

then, assuming unitarity (even for BHs!):

$$\sum_X |X\rangle\langle X| = \mathbb{I}$$

$$\Rightarrow$$

$$\sigma_{abs}(\omega) = \frac{\omega^3}{8m_{Pl}^2}\int dt e^{-i\omega t}\epsilon_{ij}(k)\epsilon^*_{rs}(k)\left[\langle M|Q^E_{rs}(0)Q^E_{ij}(t)|M\rangle + \langle M|Q^B_{rs}(0)Q^B_{ij}(t)|M\rangle\right],$$

where the 2-pt. correlators are in the initial state of the compact object

$$\langle Q^E(0)Q^E(x^0)\rangle = \langle M|Q^E(0)Q^E(x^0)|M\rangle$$

(alternatively, initial state could be mixed/thermal)

Matching to the full theory

For the case of black holes, the low frequency $\sigma_{abs}(\omega)$ can be calculated analytically, by finding the graviton wavefunctions in the BH background:

$$\Box_{BH} h_{\mu\nu} = 0$$

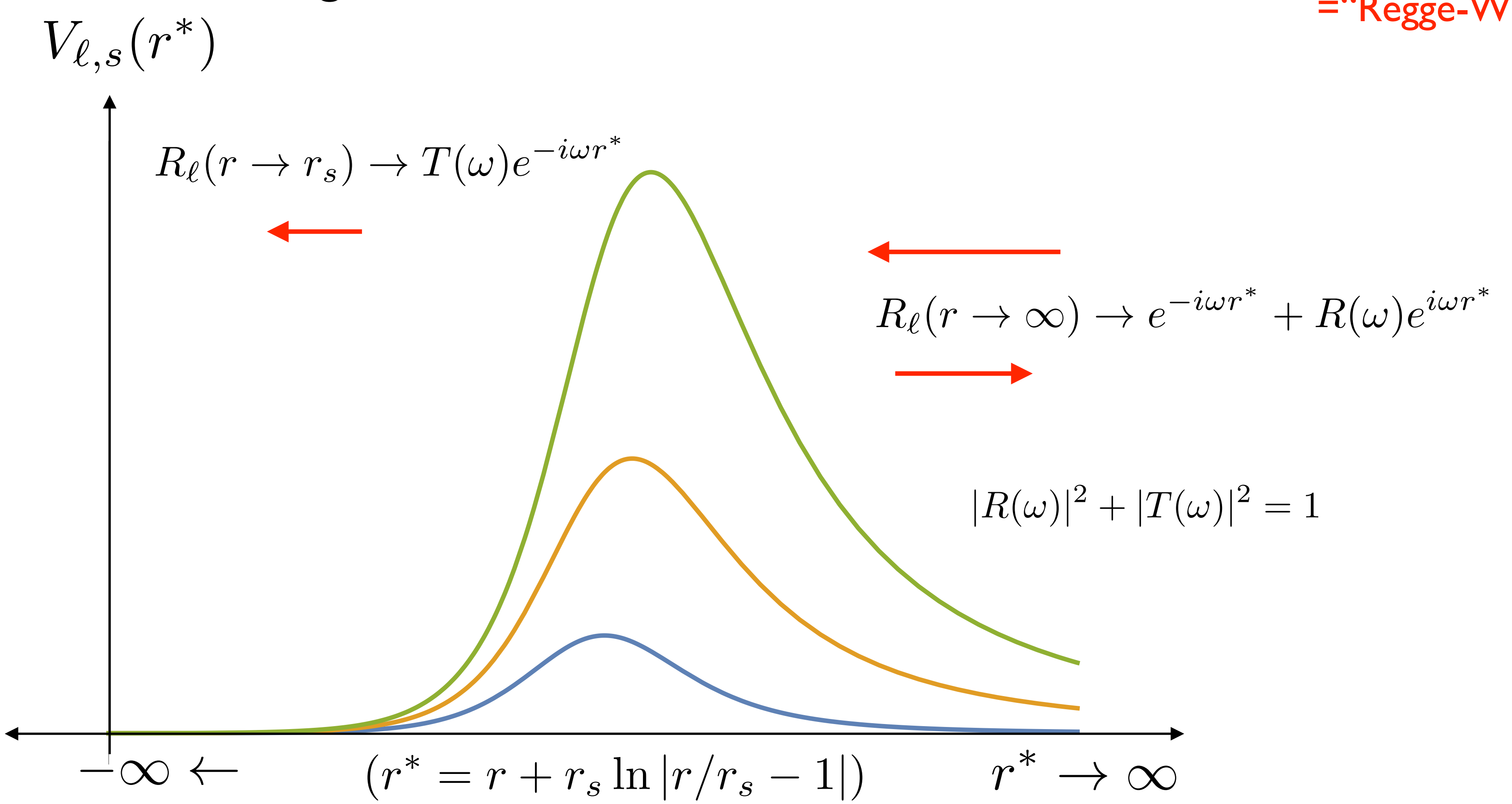
$$h_{\mu\nu}(x) = e^{-i\omega t}\frac{R_\ell(r)}{r}Y^{\ell m}_{\mu\nu}(\Omega)$$

$$\left(-\frac{d^2}{dr^{*2}} + V_\ell(r)\right)R_\ell(r) = \omega^2 R_\ell(r)$$

$$V_\ell(r) = \left(1-\frac{r_s}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{3r_s}{r^3}\right)$$

Schrodinger eqn for radial modes ="Regge-Wheeler" eqn.

BCs for scattering:



$$\sigma_{abs}(\omega) \sim |T(\omega)|^2$$

(QNMs: Same eqn. but purely outgoing bc's at the horizon and infinity)

These absorption coefficients were computed by Page (1975) for massless particles of arbitrary spin in the case of Kerr black holes:

$$\sigma_s(\omega) = \pi\omega^{-2}\sum_{l,m}\Gamma_{s\omega l m p} \underset{\omega\to 0}{\sim} \begin{cases} A, & s=0 \\ 2\pi M^2, & s=\frac{1}{2} \\ \frac{4}{9}A(3M^2-a^2)\omega^2, & s=1 \\ \frac{16}{225}A(5M^2+\frac{5}{2}M^2a^2+a^4)\omega^4, & s=2. \end{cases}$$

Using his result we can match the two-point functions in the case $s=2$

$$\int dt e^{i\omega t}\langle M|Q_{ij}^{E,B}(t)Q_{kl}^{E,B}(0)|M\rangle = \frac{1}{2}A_+(\omega)\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}\right),$$

and

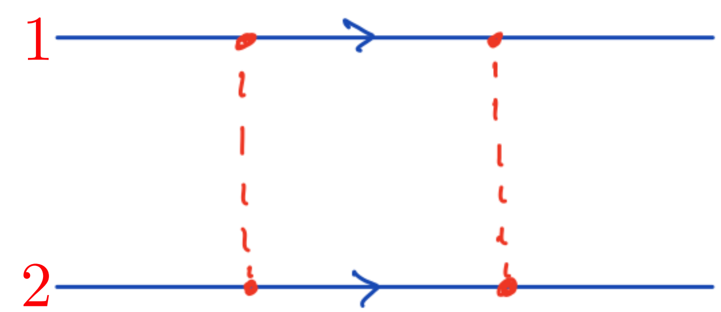
$$\sigma_{abs}^{\ell=2}(\omega) = \frac{\omega^3}{4m_{Pl}^2}A_+(\omega) \approx 4\pi r_s^2\left[\frac{(r_s\omega)^4}{45}+\mathcal{O}(r_s\omega)^6\right]$$

$$A_+(\omega>0) = \frac{1}{2G_N}\frac{r_s^6\omega}{45}+\mathcal{O}(r_s^8\omega^3).$$

$$A_+(\omega<0)=0$$

IF NO EMISSION FROM BH

The same two-pt. function that controls BH absorption also shows up in BH-BH binary dynamics. Using the NRGR formalism, this is given by the box diagram with potential exchange

$$i\Gamma_{eff}[x_1,x_2] =$$ 

$$+(1\leftrightarrow 2)+\cdots$$

$$= -\frac{1}{2}(G_N m_2)^2\int dt dt' q_{ij}[\mathbf{x}_{12}(t)]\langle Q_{ij}(t)Q_{kl}(t')\rangle q_{kl}[\mathbf{x}_{12}(t')] + (1\leftrightarrow 2),$$

$$(q_{ij}[\mathbf{x}]\equiv \partial_i\partial_j|\mathbf{x}|^{-1})$$

Absorption power spectrum:

$$\frac{2}{T}\mathrm{Im}\Gamma_{eff} = \int\frac{d\omega}{\omega}\frac{dP}{d\omega} \quad\Longrightarrow\quad \frac{dP_{abs}}{d\omega} = -\frac{1}{T}\frac{G_N}{64\pi^2}\sum_{a\neq b}\frac{\sigma^{(b)}_{abs}(\omega)}{\omega^2}m_a^2|q^{(a)}_{ij}(\omega)|^2,$$

NOTE: Valid also for NS. Contains info about EoS

$$P_{abs} = \frac{16}{45}G_N^7 m_1^2 m_2^2\left\langle\frac{1}{2}\sum_{a\neq b}m_a^4\dot{q}^{(b)}_{ij}\dot{q}^{(b)}_{ij}\right\rangle = \frac{32}{5}G_N^7 m^6\mu^2\left\langle\frac{\mathbf{v}^2}{|\mathbf{x}|^8}+32\frac{(\mathbf{x}\cdot\mathbf{v})^2}{|\mathbf{x}|^{10}}\right\rangle.$$

(in agreement with Poisson (1995) in the extreme mass limit $m_1\gg m_2$)

Note: $P_{abs}/P_{quad}\sim v^8$ is a (small) 4PN effect. Absorption enhanced to v^5 for rotating black holes (see Porto; Endlich+Pencco for EFT description)

Real part of effective action/box diagram: Leads to finite size corrections to two-body eqns of motion. To ensure causal result must compute using IN-IN formalism. The appropriate response function is then the Retarded Green's fn.

$$G^R_{ij,rs}(t) = -i\theta(t)\langle M|[Q^E_{ij}(t), Q^E_{rs}(0)]|M\rangle \equiv \frac{1}{2}G^R(t)\left(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr} - \frac{2}{3}\delta_{ij}\delta_r\right).$$

Eg, tidal response to applied E-field:

$$\langle Q_{ij}(t)\rangle = -2c_E\bar{E}_{ij}(t,0) + \int_{-\infty}^{\infty} dt' G^R_{ij,rs}(t-t')\bar{E}_{rs}(t',0),$$

so in frequency space,

$$\langle Q_{ij}(\omega)\rangle = -2c_E(\omega)\bar{E}_{ij}(\omega,0),$$

$$c_E(\omega) = c_E - \frac{1}{2}G^R(\omega) = c_E - \frac{1}{2}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{A_+(\omega') - A_+(-\omega')}{\omega - \omega' + i\epsilon}.$$

"dynamical Love number"

In the classical case (no emission) $A_+(-|\omega|) = 0$ we reproduce the full theory result on the vanishing of the BH Love number by fine tuning the counterterm c_E

$$\mathrm{Re}\; c_E(\omega \to 0)|_{classical} \equiv 0 \Rightarrow c_E = -\int_0^\infty \frac{d\omega}{2\pi}\frac{A_+(\omega)}{\omega}. \quad = -\frac{4}{G_N}\int_0^\infty d\omega \frac{\sigma_{abs}(\omega)}{\omega^4}$$

After tuning away the static part, the classical response function is then

$$c_E(\omega)|_{classical} = \frac{i r_s^6 \omega}{360 G_N} + \mathcal{O}(r_s^8 \omega^2).$$

The fact that $\mathrm{Im} c_E(\omega) \neq 0$ is consistent with the dissipative effects discussed earlier.

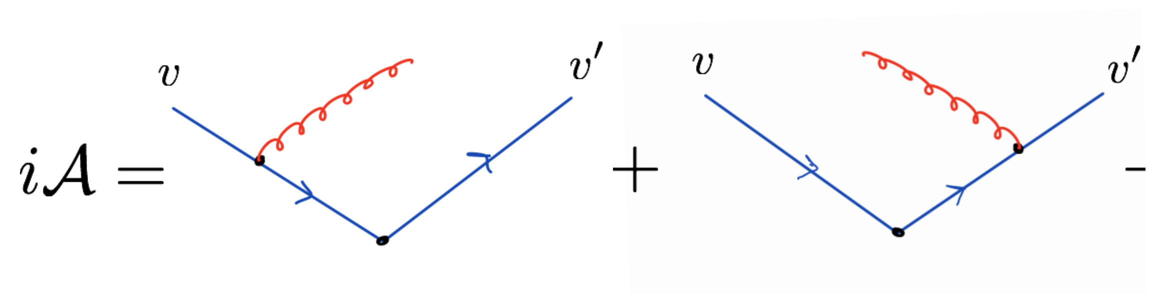
Open questions:

How to use this formalism to go beyond LO in absorption (relevant for LIGO templates; see Poisson et al)?

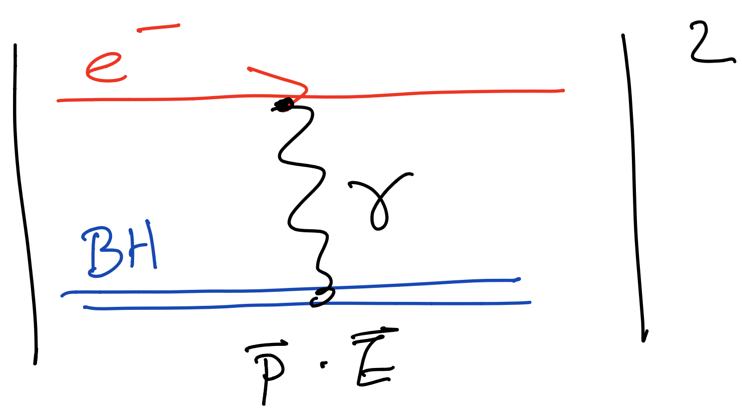
Role of higher-order correlation fns? $\langle Q^{E,B} \cdots Q^{E,B} \rangle$

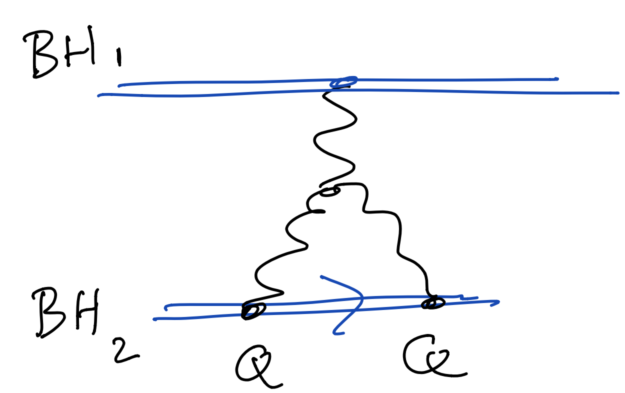
Incorporating quantum effects, e.g Hawking radiation? What does EFT say about interactions of BH with soft radiation, eg.

Soft photon theorem for electrically neutral BHs



Quantum BH scattering (in soft/eikonal limits).





Part II: Quantum