

Much Ado About Naturalness

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The Naturalness Strategy

Param	UV sensitivity	Natural if	NP	Scale	Natural?
“ m_e ”	$e^2 \Lambda$	$\Lambda \lesssim 5 \text{ MeV}$	Positron	511 keV	✓
$m_{\pi^\pm}^2 - m_{\pi^0}^2$	$\frac{3\alpha}{4\pi} \Lambda^2$	$\Lambda \lesssim 850 \text{ MeV}$	Rho	770 MeV	✓
$m_{KL} - m_{KS}$	$\frac{s_c^2 f_K^2 m_{K_L^0}}{24\pi^2 v^4} \Lambda^2$	$\Lambda \lesssim 2 \text{ GeV}$	Charm	1.2 GeV	✓
m_H^2	$-\frac{6y_t^2}{16\pi^2} \Lambda^2 + \dots$	$\Lambda \lesssim 500 \text{ GeV}$?	?	?

Implementation is up to us

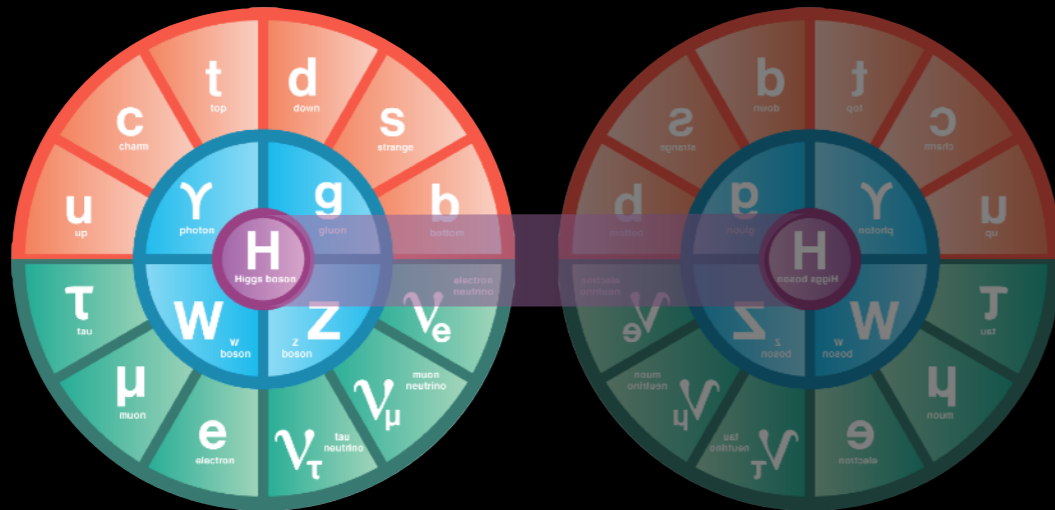
We've refined this strategy using some rules of thumb,
for example...

1. The Standard Model coupled to gravity is a generic EFT.
2. The solutions to the hierarchy problem involve symmetries, low cutoffs, or anthropics.
3. Symmetries imply new particles charged under the SM.

Rules of thumb probably still correct; continuing to test them experimentally is an excellent idea. But it is hard to say much new along these lines, so null results invite exploring other avenues.

1. The Standard Model coupled to gravity is a generic EFT.
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3. ~~Symmetries imply new particles charged under the SM.~~

Discrete symmetries



E.g. “Twin Higgs”
[Chacko, Goh,
Harnik '05, ...]

Higgs is a pNGB of an accidental SU(4),
but spectrum only respects a Z_2

$$\Delta V = -\frac{6y_t^2}{16\pi^2}\Lambda^2 (|H_A|^2 + |H_B|^2) + \dots$$

$$\Delta m_H^2 = -\frac{6y_t^2}{16\pi^2}\Lambda^2 + \frac{6y_t^2}{16\pi^2}\Lambda^2 - 6\frac{y_t^2}{16\pi^2}(m_T^2 - m_t^2) \log \frac{\Lambda^2}{m_T^2}$$

Still a plethora of
new particles, not
interacting via SM
gauge forces but
coupling to Higgs.

Why Not?

Higgs portal maintains equilibrium down to $T \sim \text{GeV}$

$$\Delta N_{\text{eff}} \gg 1$$

Options are

Change the cosmology

*Signals in CMB: N_{eff} , $\sum m_\nu$,
twin BAO...*

- RHN decay
- Saxion decay
- Early ν' decoupling

Change the spectrum

*Copious new physics at $\sim \text{few TeV}$
Higgs signals @ LHC*

- Fraternal Twin Higgs
- Holographic Twin Higgs
- Composite Twin Higgs
- Orbifold Higgs
- ...

[Chacko, NC, Fox, Harnik '16; NC, Koren, Trott '16; Chacko, Curtin, Geller, Tsai '18, ...]

		<i>scalar</i>	<i>fermion</i>
<i>strong direct production</i> {	<i>QCD</i>	SUSY	Composite Higgs/ RS
<i>DY direct production</i> {	<i>EW</i>	folded SUSY	Quirky Little Higgs
<i>Higgs portal direct production</i> {	<i>singlet</i>	Hyperbolic Higgs / Accidental SUSY	Twin Higgs

Mirror Glueballs

Higgs portal observables

Higgs coupling shifts

~ tuning

When all is said and done, scale of new charged states (c.f. usual continuous symmetry solutions)

[NC, Howe '13; Contino et al. '17]

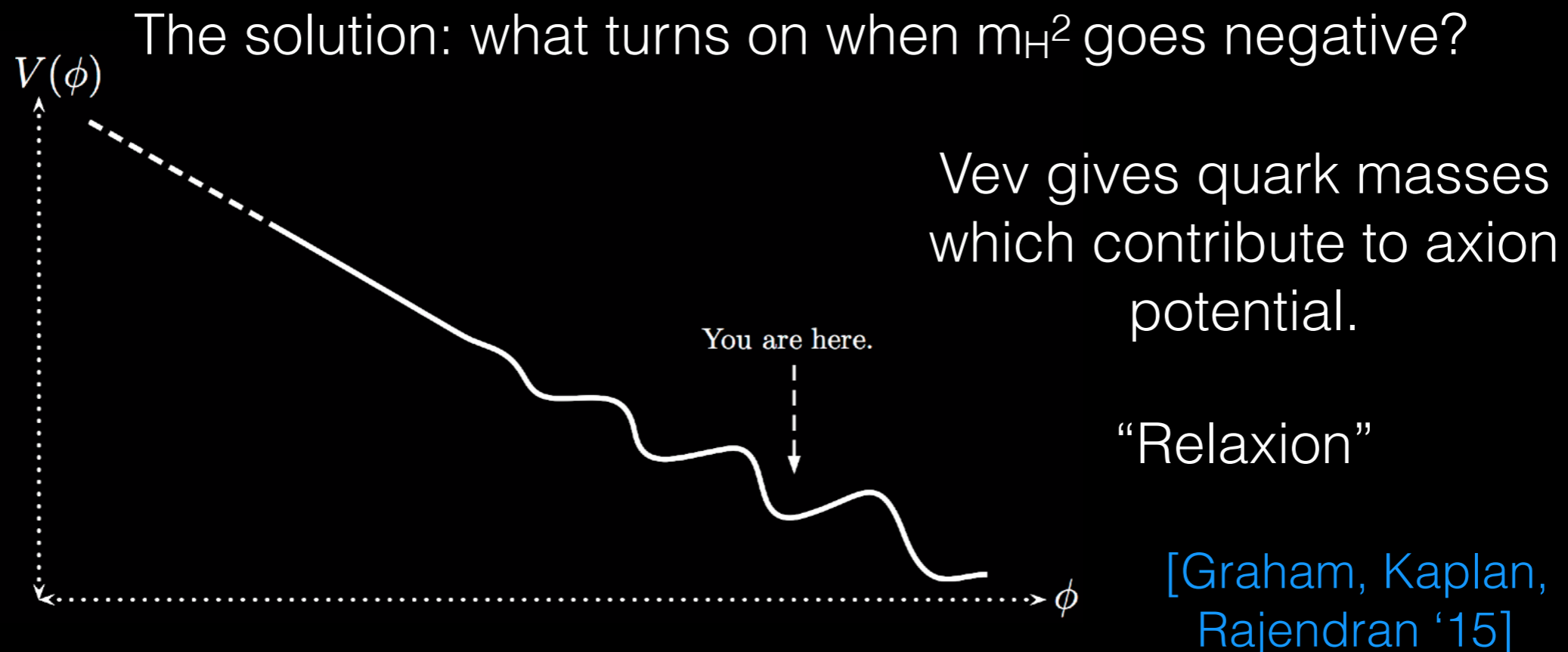
$$m_*^2 \sim m_{*,\text{cts}}^2 \times \left(\frac{g_*}{g_{SM}} \right)^2$$

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Relaxion

What if the weak scale is selected by scanning?

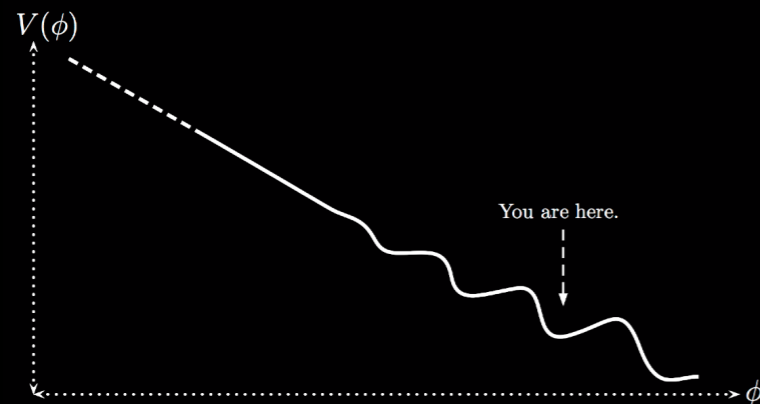
The idea: couple Higgs to field whose minimum sets $m_H=0$
The problem: How to make $m_H=0$ a special point of potential?



But: immense energy stored in evolving field, need dissipation.

Relaxion

Simplest version: an axion coupled to QCD during inflation.



$$\Lambda^4(H) \cos(\phi/f) + F(g\phi) + (-M^2 + g\phi) |H|^2$$

Viable for Higgs + non-compact axion + inflation w/

- Very low Hubble scale ($\ll \Lambda_{\text{QCD}}$)
- 10 Giga-years of inflation

Why not? Various other subtleties regarding technical naturalness, trans-Planckian field excursions, CC, fine-tuning to inflationary sector; need to solve strong CP problem. *New UV considerations.*

Extensive development, e.g. [Espinosa et al. '15; Hardy '15; Gupta et al '15; Batell, Giudice, McCullough '15; Choi, Im '15; Kaplan, Rattazzi '15; Di Chiara et al. '15; Ibanez et al. '15; Hook, Marques-Tavares '16; Nelson, Prescod-Weinstein '17; ...]

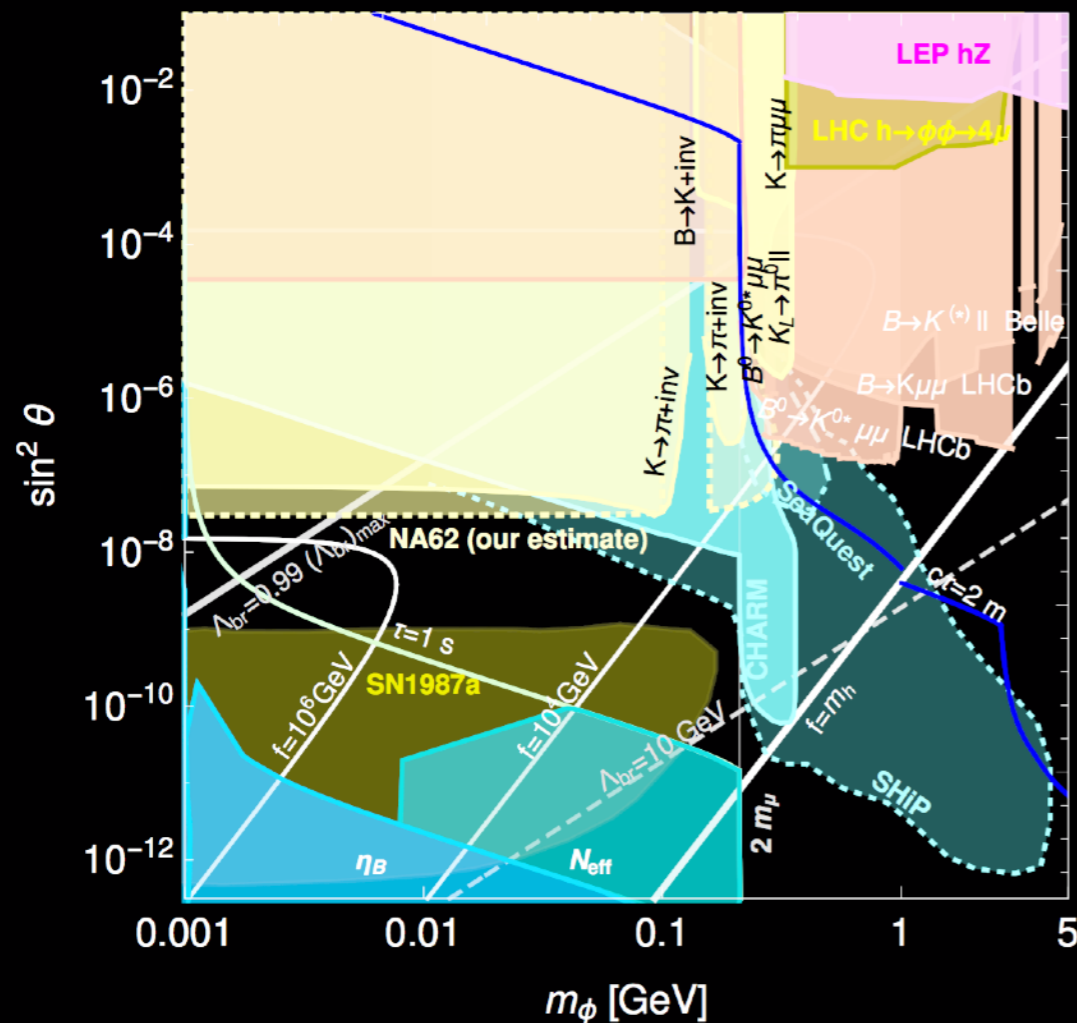
See also: NNaturalness [Arkani-Hamed et al. '16]

New Signals

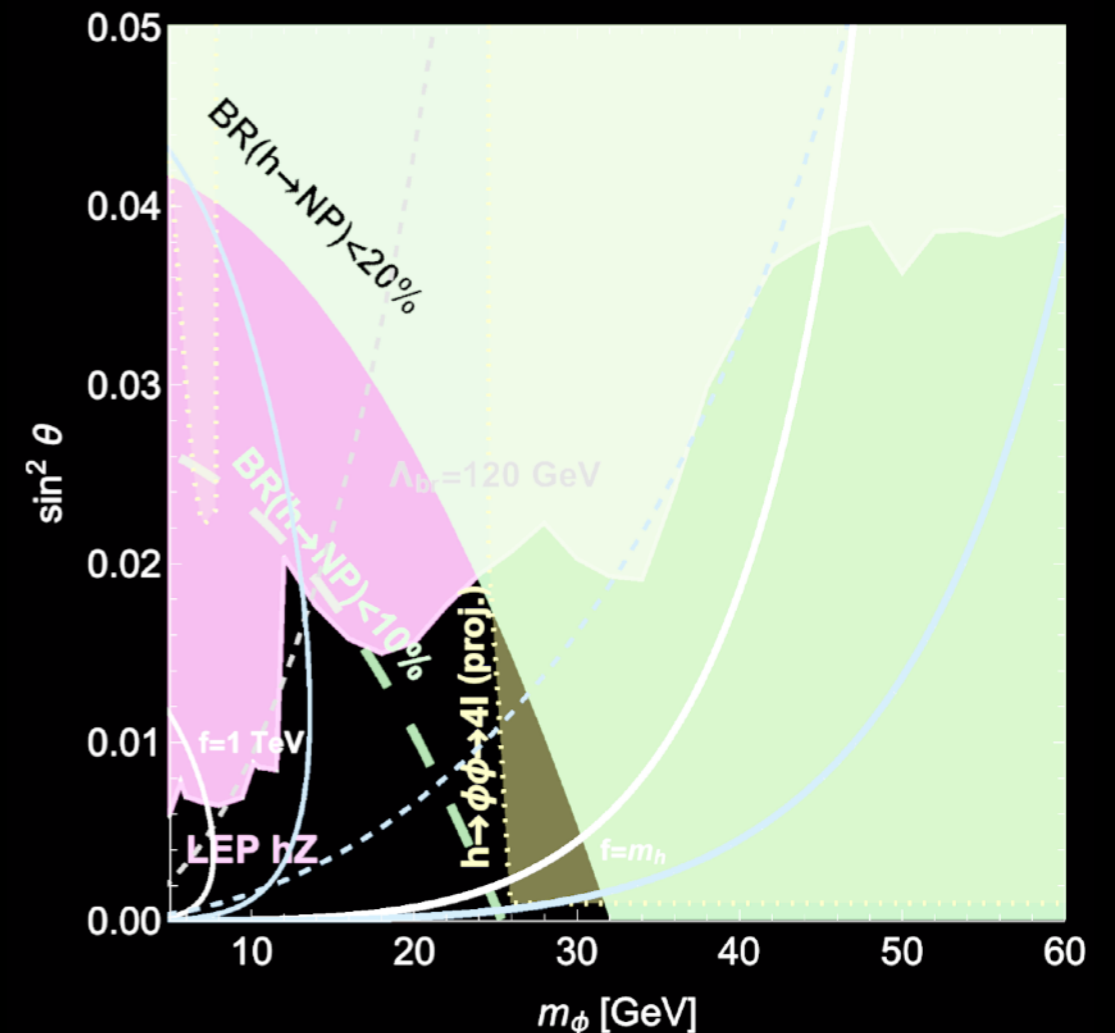
Higgs portals $g\phi|H|^2$ $\Lambda^4(H) \cos(\phi/f)$

$\Lambda^4(H) \cos(\phi/f)$ gives ϕ - H mixing* w/ $\sin\theta \approx \frac{\Lambda^4}{vf m_h^2}$

[Flacke, Friqule, Fuchs, Gupta, Perez '16]



[Flacke, Friqule, Fuchs, Gupta, Perez '16]



+5th force for $m_\phi < eV$ & cosmology for $eV < m_\phi < MeV$

*assuming $\langle \phi \rangle$ breaks CP

Particle production relaxion

Alternative possibility: keep bumps across entire potential, turn on dissipation at a special point of potential.

Novel source of dissipation: particle production

Consider axion-like couplings to massive gauge field:

$$\mathcal{L} \supset -\frac{\phi}{4f} F \tilde{F}$$

E.O.M. for transverse polarizations:

$$\ddot{A}_{\pm} + \left(k^2 + m_A^2 \pm \frac{k\dot{\phi}}{f} \right) A_{\pm} = 0$$

For $\dot{\phi} \approx \text{constant}$ $A_{\pm}(k) \propto e^{i\omega_{\pm}t}$ $\omega_{\pm}^2 = k^2 + m_A^2 \pm \frac{k\dot{\phi}}{f}$

Exponentially growing solution for $\omega_{\pm}^2 < 0 \Rightarrow |\dot{\phi}| \gtrsim 2fm_A$

Growing mode drains energy from $\dot{\phi}$

Particle production relaxion

Apply to relaxion: use electroweak gauge fields

Instead of

$$\frac{\phi}{f} G \tilde{G}$$

+ inflation



Use coupling to EWK gauge bosons:

$$\frac{\phi}{f} \left(g^2 W \tilde{W} - g'^2 B \tilde{B} \right) + \Lambda_c^4 \cos \frac{\phi}{f'}$$

Exponential production of EWK gauge bosons around $h \sim v$ slows evolution

Important subtlety: can't couple to pairs of photons!

(Not a tuning, can be made natural with symmetries, e.g., $SU(2)_L \times SU(2)_R$)

Requiring sub-Planckian field excursions
& avoiding overshoot bounds cutoff

Corresponding decay constant

$$\Lambda \lesssim (M_{\text{Pl}} v^5)^{1/6} \sim 50 \text{ TeV}$$

$$f \sim \frac{\dot{\phi}}{v} \sim \frac{\Lambda^2}{v} \lesssim 10^4 \text{ TeV}$$

New Signals

Even if tree-level relaxion couplings to SM states are engineered to be

$$\frac{\phi}{f} \left(g^2 W \widetilde{W} - g'^2 B \widetilde{B} \right) \quad \text{in the UV...}$$

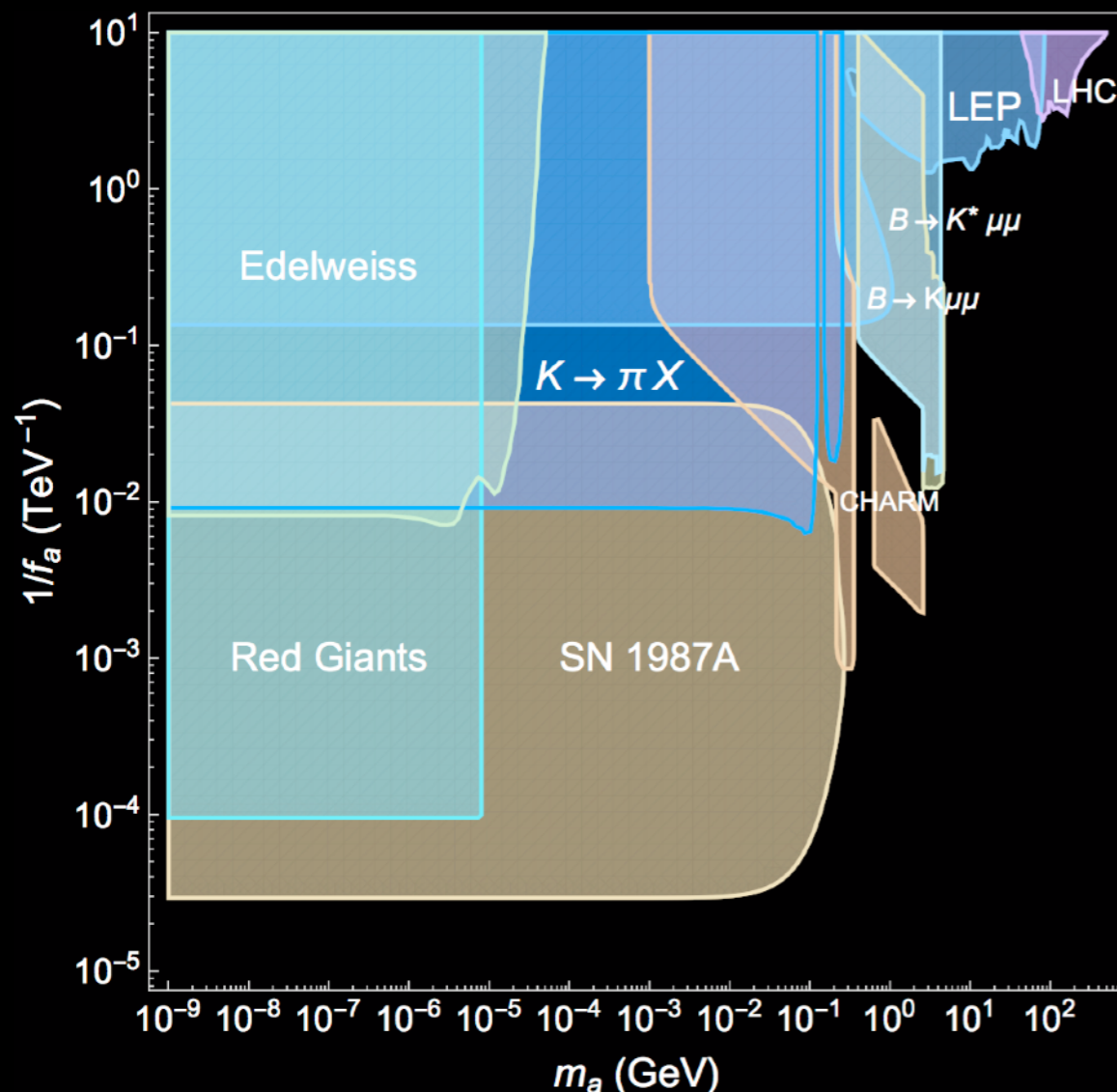
...radiative couplings to fermions induced at one loop, photon pairs at one & two loops [Bauer, Neubert, Thamm '17; NC, Hook, Kasko '18]



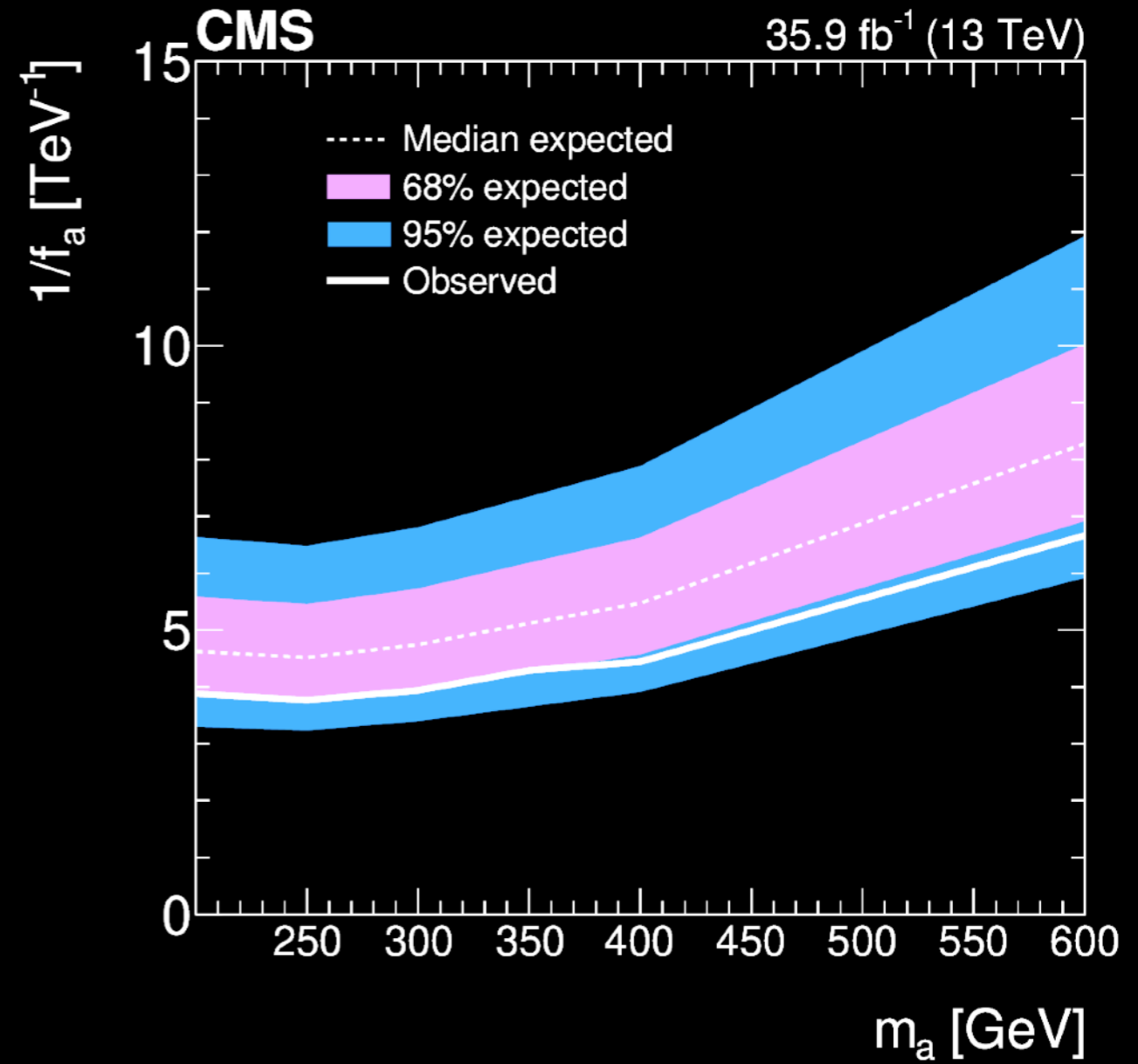
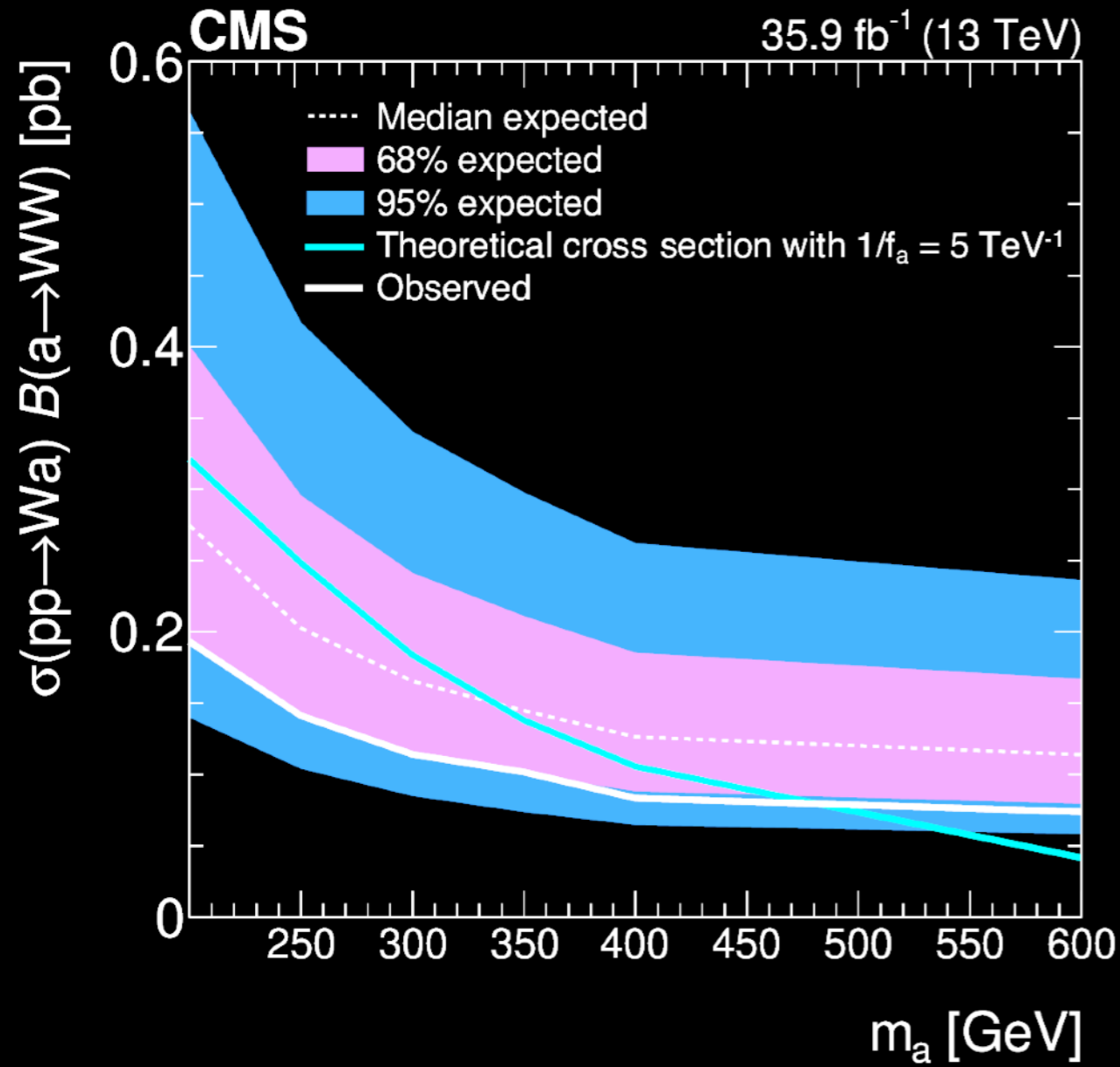
$$f_\gamma \sim 16\pi^2 \frac{m_W^2}{m_a^2} f_a + (16\pi^2)^2 \frac{m_f^2}{m_a^2} f_a$$

Astrophysical and collider signatures abound; still viable parameter space [Fonseca, Morgante, Servant '18]

[NC, Hook, Kasko '18]



1905.04246



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Indirect UV/IR: WGC

(Electric) weak gravity conjecture: an abelian gauge theory must contain a state of charge q and mass m satisfying

$$q > \frac{m}{M_{Pl}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa '07]

[Cheung, Remmen '14]: If mass of WGC particle is UV sensitive, then for fixed UV-insensitive parameters, satisfying the WGC would mandate fine-tuning. (Or: would orchestrate correlations among UV contributions)

Application to SM: charge SM fermions under weakly gauged (unbroken) $U(1)_{B-L}$ (bounds currently $q \lesssim 10^{-24}$). Cancel anomalies with RHN ν_R

Neutrino mass from EWSB
 $y_\nu H \bar{L} \nu_R \rightarrow m_\nu \sim y_\nu v$

If lightest neutrino is WGC particle,
 $m_\nu \sim 0.1 \text{ eV}, q \gtrsim 10^{-29}$

For fixed y, q , satisfying WGC places an upper bound on v

Indirect UV/IR: WGC

Things that could go wrong:

- WGC could be satisfied by states outside EFT
- Satisfying WGC could compel the appearance of a new light state that enforces apparent UV correlations (e.g. relaxion)
- Apparently UV-sensitive parameters might control apparently UV-insensitive ones (e.g. emergent gauge fields)

Thing that certainly goes wrong:

- Magnetic WGC implies cutoff of U(1) at $\Lambda \lesssim gM_{Pl}$

Indirect UV/IR: WGC

First order of business: can m, Λ be raised to the weak scale?

New $U(1)_X$ plus matter acquiring some mass from the Higgs. E.g...

[NC, Garcia Garcia, Koren '19]

	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
L	\square	$+1/2$	$+1$
L^c	\square	$-1/2$	-1
N	$-$	0	$+1$
N^c	$-$	0	-1

$$-\mathcal{L} \supset \{m_L L L^c + m_N N N^c + y H^\dagger L N^c + \tilde{y} H L^c N\} + \text{h.c.}$$

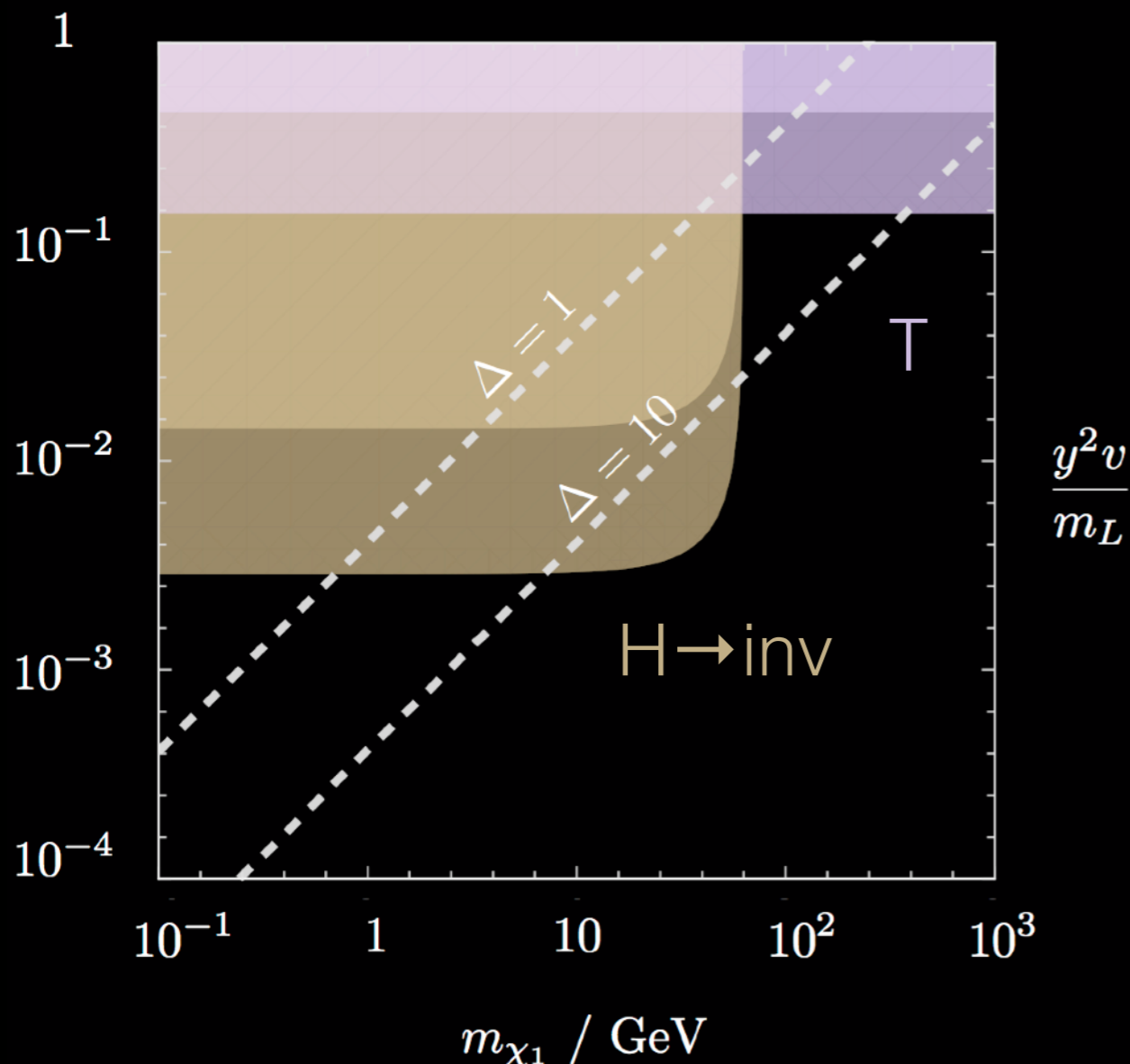
Best option: $m_N < m_L$, lightest mass eigenstate χ_1 is WGC particle

Then for fixed
(technically natural)
 $g, m_L, m_N, y,$

$$v^2 \lesssim \frac{2}{y^2} \left(m_{\chi_1}^2 + m_{\chi_1} (m_L - m_N) - m_L m_N \right)$$

Indirect UV/IR: WGC

Still have a notion of sensitivity of the weak scale to parameters involved in the bound



Quantify with e.g. $\Delta_x \equiv \left| \frac{\partial \log v^2}{\partial \log x} \right|$

Here $\Delta_{\text{max}} \sim \frac{m_N m_L}{y^2 v^2}$

Not surprising: WGC particle should get “most of” its mass from EWSB.

Surprisingly predictive: look for new singlet fermions coupled to the Higgs at/below the weak scale.

DM story interesting...

Indirect UV/IR: WGC

Second order of business: can the magnetic WGC scale be something less severe than the SM cutoff? Only confident that Λ ~ scale associated w/ structure of magnetic monopoles

E.g. t' Hooft-Polyakov monopoles $SU(2)_X \xrightarrow{\langle \text{Adj} \rangle} U(1)_X$

$$“\Lambda” = m_W = g_2 f = 2g f \lesssim 2g M_{\text{Pl}}$$

W's would trivialize bound from vanilla electric WGC, but not e.g. unit charge version (charge ± 2 under $U(1)_X$)

Alternately: WGC w/ scalar fields
[Palti '17, Palti & Lüst '17]

$$m^2 \lesssim (g^2 - \mu^2) M_{\text{Pl}}^2$$

Worth refining conjectures & exploring further even out of skepticism: could point to WGC conjectures ripe for counterexamples.

Direct UV/IR Mixing

Take the bull by the horns...

Study field theories with UV/IR mixing

Canonical example:

QFT on non-commutative spacetime $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$

UV/IR mixing from “uncertainty principle” $\Delta\hat{x}^\mu \Delta\hat{x}^\nu \geq \frac{1}{2} |\theta^{\mu\nu}|$

Caveats: Lorentz violating; Minkowski NCQFT unitary only for space-space non-commutativity (i.e. $\theta^{0i}=0$).

Not the theory of our universe, but a useful toy model. (See e.g. [Heckman & Verlinde '14])



NCQFT

Two common approaches:

1. QFT on commutative coordinates w/ star product:

$$f(x) \star g(x) = \exp \left(\frac{i}{2} \theta_{\mu\nu} \partial_y^\mu \partial_z^\nu \right) f(y) g(z) \Big|_{y=z=x}$$

2. Seiberg-Witten map [Seiberg, Witten '99]:

$$\text{i.e., } f \star g = f \cdot g + \frac{i}{2} \theta^{\mu\nu} \partial_\mu \cdot \partial_\nu g + \mathcal{O}(\theta^2) \quad \text{and e.g.}$$

$$\hat{A}_\mu[A] = A_\mu + \frac{1}{4} \theta^{\rho\sigma} \{A_\sigma, \partial_\rho A_\mu\} + \frac{1}{4} \theta^{\rho\sigma} \{F_{\rho\mu}, A_\sigma\} + \mathcal{O}(\theta^2)$$

Equivalent to any finite order in θ (i.e., option (2) defines a low-energy effective action), but UV/IR mixing only apparent in (1).

NCQFT: ϕ^4

Consider just ϕ^4 in Euclidean $d=4$: $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} g^2 \phi \star \phi \star \phi \star \phi$

Quadratic terms identical to commutative theory


Interactions associated w/ additional phases: $V(k_1, k_2, k_3, k_4) = e^{-\frac{i}{2} \sum_{i < j} k_{i\mu} \theta^{\mu\nu} k_{j\nu}}$

Not invariant under arbitrary permutations of k

Planar graphs: reduces to an overall phase involving external momenta

Nonplanar graphs: additional phases from crossing lines

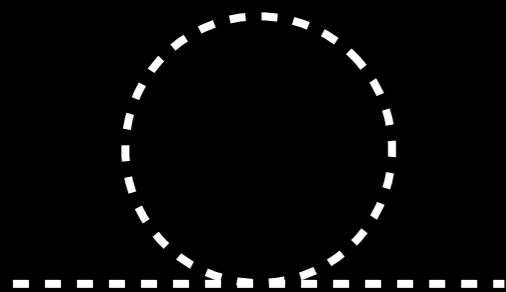
Feynman rules as usual modulo phases in nonplanar diagrams:



$$\sim e^{ip_\mu \theta^{\mu\nu} k_\nu}$$

NCQFT: ϕ^4

Compute one-loop radiative corrections to scalar 2-pt function.
Both “planar” and “non-planar” diagrams at one loop:

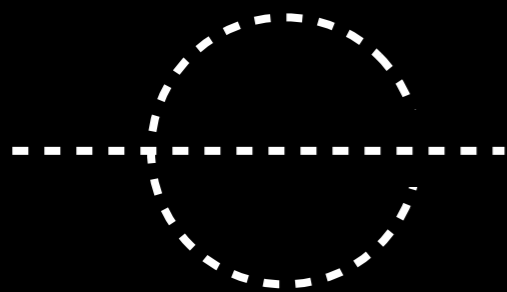


$$\sim \int \frac{d^4 k}{k^2}$$

UV divergent

$$\Gamma_1^{2,p} = \frac{g^2}{48\pi^2} \left(\Lambda^2 - m^2 \log \left(\frac{\Lambda^2}{m^2} \right) + \dots \right)$$

Akin to commutative theory



$$\sim \int \frac{d^4 k}{k^2} e^{ip\theta k} \sim \frac{1}{p\theta^2 p}$$

IR divergence!

$$\Gamma_1^{2,np} = \frac{g^2}{96\pi^2} \left(\Lambda_{\text{eff}}^2 - m^2 \log \left(\frac{\Lambda_{\text{eff}}^2}{m^2} \right) + \dots \right)$$

Likewise, but with $\Lambda \rightarrow \Lambda_{\text{eff}}$

Appearance of
a new “scale”:

$$\Lambda_{\text{eff}}^2 = \frac{1}{1/\Lambda^2 + p \circ p}$$

where

$$p \circ k = |p^\mu \theta_{\mu\nu}^2 k^\nu|$$

$$\Lambda_{\text{eff}}^2 \rightarrow \begin{cases} \Lambda^2 & p \circ p \rightarrow 0 \\ \frac{1}{p \circ p} & \Lambda \rightarrow \infty \end{cases}$$

New “poles”

1-loop 1PI quadratic effective action: $\frac{1}{2} \left(p^2 + M^2 + \frac{g^2}{96\pi^2 (p \circ p + 1/\Lambda^2)} + \dots \right) \phi(p)\phi(-p)$

w/ renormalized mass M: $M^2 = m^2 + \frac{g^2 \Lambda^2}{48\pi^2} - \frac{g^2 m^2}{48\pi^2} \ln \frac{\Lambda^2}{m^2}$

Action @ infinite cutoff: $\frac{1}{2} \left(p^2 + M^2 + \frac{g^2}{96\pi^2 (p \circ p + 1/\Lambda^2)} + \dots \right) \phi(p)\phi(-p)$

Two poles in $\Lambda \rightarrow \infty$ action:

1. Usual one (ϕ quanta) at $p^2 + m^2 = \mathcal{O}(g^2)$

2. New one at $p \circ p = -\frac{g^2}{96\pi^2} \frac{1}{p_c^2 + m^2} + \dots$

Second pole signals existence of new light “particle” arising from high-momentum modes of ϕ

Wilsonian interpretation

Normally require renormalizable Wilsonian action to satisfy

1. Correlation functions well-defined as $\Lambda \rightarrow \infty$
2. Correlation functions at finite Λ differ from limiting value by $O(1/\Lambda)$ at all momenta

Badly violated here *at small p*.

$$\frac{1}{2} \left(p^2 + M^2 + \frac{g^2}{96\pi^2(p \circ p + 1/\Lambda^2)} + \dots \right) \phi(p)\phi(-p)$$

Restore Wilsonian interpretation w/ new particle χ :

$$\delta\mathcal{L} = \frac{1}{2}\partial\chi \circ \partial\chi + \frac{1}{2}\Lambda^2(\partial \circ \partial\chi)^2 + i\frac{1}{\sqrt{96\pi^2}}g\chi\phi$$

Quadratic, so integrate out:

$$+ \frac{1}{2} \frac{1}{96\pi^2} \left(\frac{g^2}{p \circ p} - \frac{g^2}{p \circ p + 1/\Lambda^2} \right) \phi(p)\phi(-p)$$

What have we learned?

High-momentum modes of massive fields in a non-commutative scalar theory are “dual” to additional (peculiar) light fields

4d fields in case of quadratic divergences, 5d for linear divergences, 6d for logarithmic divergences

In a fantasy application to the hierarchy problem, apparently light scalars are the χ fields, not the ϕ fields

$$\delta\mathcal{L} = \frac{1}{2}\partial\chi \circ \partial\chi + \frac{1}{2}\Lambda^2(\partial \circ \partial\chi)^2 + i\frac{1}{\sqrt{96\pi^2}}g\chi\phi$$

Just a fantasy in this setting, but worth understanding basic features & trying to extract lessons [NC, Koren, *to appear*]
Other controlled QFTs with similar features?



Conclusions



1. The Standard Model coupled to gravity is a generic EFT.
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Conclusions

The background features several Feynman diagrams. On the left, a vertical blue line with an upward arrow is accompanied by a red wavy line. In the center, a blue oval loop contains a red wavy line. To the right, another vertical blue line with an upward arrow is also accompanied by a red wavy line. At the bottom left, there is a diagram with a blue circle containing a purple circle, with red wavy lines and arrows. At the bottom right, a red scalloped shape is connected to a blue line that extends from the right edge of the slide.

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Relaxing these rules of thumb is constructive and leads to new signatures associated with naturalness. Only beginning to explore the possibilities....

Conclusions

Hard not to notice patterns among the three naturalness problems...

	Hierarchy Problem	CC Problem	Strong CP
Continuous symm.	SUSY, global	SUSY	$U(1)_{PQ}$
Discrete symm.	Z_2	$E \rightarrow -E$	P/CP
Dynamical field	Relaxion	Abbott	$U(1)_{PQ}$
Anthropics	Atomic principle	Structure formation	?
UV/IR mixing	WGC/NCQFT/...	Holography	?

Thank you!