Enhancement of superconducting Tc near SIT

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The Hamiltonian

3d lattice tight-binding model with diagonal disorder

Local attraction

\[ H = \epsilon_r \delta_{r',r} + V \delta_{r,r+\hat{a}} + H_{\text{int}} \]

\[ H_{\text{int}} = -\frac{\lambda}{\nu_0} \sum_r \psi_\uparrow^+(r)\psi_\downarrow^+(r)\psi_\downarrow(r)\psi_\uparrow(r) \]

NO Coulomb interaction
NO Coulomb interaction?

Where can it happen?

- Cold fermionic atoms in disordered optical traps: atoms are neutral
- In solids with a certain band structure: large background dielectric constant
Q: What does the strong disorder do to superconductivity?

A1: weak disorder do not do anything; strong disorder kills superconductivity

A2: eventually disorder kills superconductivity but before killing it enhances it
Weak and strong disorder

Anderson transition

Extended states  Critical states  Localized states

$\ell \sim \lambda^F$
FAQ: Why superconductivity is possible even when single-particle states are localized

**Single-particle conductivity:** only states in the energy strip $\sim T$ near Fermi energy contribute.

**Superconductivity:** states in the energy strip $\sim \Delta \sim T_c$ near the Fermi-energy contribute.

Interaction sets in a new scale $\Delta \sim T_c$ which stays constant as $T \rightarrow 0$.

$R(T) = \left( \frac{1}{\nu_0 T} \right)^{1/3}$
Multifractality of critical and off-critical states

\[ \sum_{r} |\Psi_i(r)|^{2n} = \frac{1}{d_{n}(n-1)} \]
Matrix elements

Interaction comes to play via matrix elements

\[ \lambda M_{nm} = \lambda V \int d^d r \; \Psi_n^2(r) \Psi_m^2(r) \]
Ideal metal and insulator

\[ \langle M_{nm} \rangle = \int V d^d r \left( |\Psi_n(r)|^2 |\Psi_m(r)|^2 \right) \]

**Metal:**
\[ V \quad V \quad \frac{1}{V} \quad \frac{1}{V} = 1 \]
Small amplitude 100% overlap

**Insulator:**
\[ V \quad \xi^d \quad \frac{1}{\xi^d} \quad \frac{1}{\xi^d} \times \left( \frac{\xi^d}{V} \right) = 1 \]
Large amplitude but rare overlap
Critical enhancement of correlations

Amplitude higher than in a metal but almost full overlap

States rather remote $\left(\delta \ll |E-E'| < E_0\right)$ in energy are strongly correlated
Simulations on 3D Anderson model

\[ E_0 = \left(v_0 \xi_0^3\right)^{-1} \sim \frac{D}{W_c} \]

\[ \xi = \lambda_F \left| \frac{W_c}{W_c - W} \right|^\nu \]

\[ \delta_\xi = \left(v_0 \xi^d\right)^{-1} \]

\[ \ell_0 \sim \lambda_F W_c^{1/3} \]

Ideal metal: \( \xi < \xi_0 \)

Multifractal metal: \( \xi > \xi_0 \)

Critical power law persists

\[ E_0 = \left(\frac{E - E'}{|E - E'|/W/2}\right)^{1-d_2/d} \]
Multifractal metal

Wave function does not occupy all the available space: enhanced amplitude (normalization)

Regions with enhanced amplitude are strongly correlated for different wave functions as long as

$L_\omega < \xi$

and disorder
\[ \ell_0 \sim \lambda_F W_{c}^{1/3} \]

Is a pixel of fractal pattern

\[ E_0 \sim \frac{E_F}{W_{c}} \sim \frac{E_F}{16} \]
Enhancement of matrix element

\[ \lambda M_{nm} = \lambda V \int d^d r \, \Psi_n^2(r) \Psi_m^2(r) \]
What to do at strong disorder?

\( \Delta(r) \) cannot be averaged independently of the MF kernel \( K(r,r') \): No "Anderson theorem"

Fock space instead of the real space

\[
H_{\text{eff}} = -2 \sum_i \varepsilon_i S_i^z - U \sum_{i \neq j} M_{ij} (S_i^+ S_j^- + S_i^- S_j^+) 
\]

Single-particle states, strong disorder included

<table>
<thead>
<tr>
<th>Superconducting phase</th>
<th>Normal phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle S_i^{x,y} \rangle \neq 0 )</td>
<td>( \langle S_i^{x,y} \rangle = 0 )</td>
</tr>
</tbody>
</table>
Why the Fock–space mean field is better than the real–space one?

\[ H_{\text{eff}} = -2 \sum_i \varepsilon_i S_i^z - \lambda \sum_{i \neq j} M_{ij} (S_i^x S_j^x + S_i^y S_j^y) \]

\[ M_{ij} = \int dr \; \Psi_i^2(r) \Psi_j^2(r) \]

Infinite or large coordination number for extended and weakly localized states: supports mean field approximation.

\[ \Delta_i = \lambda \sum_j \Delta_j \frac{\tanh(E_j / 2T)}{E_j} \]

\[ M_{ij} \]

\[ \text{M}_{ij} \text{ is energy dependent} \]
MF critical temperature close to critical disorder

At a small $\lambda$
parametrically large enhancement of $T_c$

\[
\Delta(E) = \lambda \int dE' \Delta(E') \frac{\tanh(E'/2T)}{E'} M(E-E')
\]

\[
M(E-E') = \left( \frac{E_0}{|E-E'|} \right)^{1-d_2/d}
\]

Critical enhancement of correlations

\[
T_c \sim E_0 \lambda^{1/(1-d_2/d)} \sim E_0 \lambda^{1.78} \]

\[
\omega_D \exp[-1/\lambda]
\]

M.V. Feigelman, L.B. Ioffe, V.E.K. and E. Yuzbashyan,
Phys.Rev.Lett. v.98, 027001 (2007);
Isotop effect

\[ T_c \sim E_0 \lambda^{1.78} \]

\[ E_0 \sim \frac{E_F}{W_c} \sim \frac{E_F}{16} \]

No isotop effect
The phase diagram

\[ \lambda_{BCS} = 0.2 \]

Extended states

Localized states

Mobility edge
Relevant experiment?
Li-doped ZrNCl

Without Li, ZrNCl is a band insulator with a gap of 1.6 eV.

Intercalated Li atoms, provide electrons to ZrN layer, source of disorder?
Why “natural” in this compound?

Initial motivation for our theory: direct SIT in amorphous films of InO, TiN

Superconductivity survives until disorder is strong enough to cause Anderson localization. Why Coulomb interaction does not destroy it well before?

In Mendeleev’s periodic Table

<table>
<thead>
<tr>
<th>Element</th>
<th>(Z)</th>
<th>(Z)-number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Zr</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Hf</td>
<td>72</td>
<td>72</td>
</tr>
</tbody>
</table>

Li-doped HfNCl is a similar SC with higher \(T_c\sim25K\)
Electron-phonon attraction constant $\sim 0.5$
follows from band-structure calculations.
Does the band structure support large background dielectric constant?
\[ \varepsilon \sim \frac{\sigma \nu_i \nu_f}{\omega_0 - \omega} \propto \frac{\nu_i \nu_f}{\omega_0} \]

High peaks in DoS close to low DoS conduction band

\[ \nu_i \sim 0.4 \text{ states/eV} \sim 0.2 \nu_{\text{Gold}} \]

\[ \nu_f \sim 120 \text{ states/Ry} \sim 3 \nu_{\text{Gold}} \]

\[ \omega_0 \sim 0.5 \text{ eV} \sim 0.1 \varepsilon_{F_{\text{Gold}}} \]

DoS in good metals (gold)
\[
\sim 2 \text{ states/eV (per atom)}
\sim 30 \text{ states/Ry}
\]

Fermi surface occupies a very small part of the Brillouin zone

\( E_{F} \ll a \):
clear separation between low-energy and "high-energy" physics

\( R_y = 13.6 \text{ eV} \)
Al N

No high peaks in the DoS close to conduction band

\[ \varepsilon_0 \sim 6 \]

\[ \omega_0 \sim 6 \text{ eV} \]
$\varepsilon_0 \sim 50$

For large-distance screening

Does not seem to be unreasonable as long as the high DoS peak at ~0.6 eV persists

Small-distance (large $k \sim 1/a$) screening may be much weaker because of the effect of dispersion

$$\omega_0(k) = \omega_0 + k^2 / 2m$$
Conclusions

- Enhancement of superconductive $T_c$ by near-critical disorder: combined action of “holes” in the single-particle wavefunction amplitude and correlation in positions of peaks for different wavefunctions.

- Power-law dependence of $T_c$ on the attractive interaction constant $\lambda$ at near-critical disorder. The exponent depends on the critical exponent: the fractal dimension $d_2$.

- Possible experimental system with weak long-range Coulomb interaction
2d weak multifractality: the Ovchinnikov-Mayekawa-Fukuyama-Finkelstein effect

The diffuson diagrams

\[
\frac{\Delta T_c}{T_c} = \frac{\lambda}{g} \ln^3 \left( \frac{1}{T_c \tau} \right)
\]

For long-range Coulomb interaction \( \lambda = -1 \)

\[ V^{-1} = V_0^{-1}(q) - \Pi \approx -\nu_0 \]

For short-range attraction one has effectively \( \ln^2 \) as \( \lambda = 1/\ln \)
Cold atoms trapped in an optical lattice

Fermionic atoms trapped in an optical lattice

Disorder is produced by:

One, two and three-dimensional tight-binding systems with disorder

speckles

Other trapped atoms (impurities)
One-dimensional Anderson localization in systems of cold atoms

Alain Aspect group

J.Billy et al., Nature \textbf{453}, 891 (2008);

Massimo Inguscio group


Work on two- and three-dimensional localization is in progress
BUT...
Sweet life is only possible without Coulomb interaction
Mean-field approximation and the Anderson theorem

\[ \Delta(r) = \int dr' \langle \Delta(r') \rangle \langle K(r, r'; T) \rangle \]

\[ K(r, r'; T) = U \sum_{ij} \eta_{ij}(T) \frac{1}{V} \delta_{ij} \]

\[ \eta_{ij}(T) = \frac{\tanh(E_i / 2T) + \tanh(E_j / 2T)}{E_i + E_j} \]

Wavefunctions drop out of the equation

**Anderson theorem:** $T_c$ does not depend on properties of wavefunctions
Tc at the Anderson transition: MF vs virial expansion

M.V. Feigelman, L.B. Ioffe, V.E.K. and E. Yuzbashyan, Phys.Rev.Lett. v. 98, 027001 (2007);

\[ T_{\text{crit}} = c_\gamma E_0 \lambda^{1/\gamma}, \quad \gamma = 1 - d_2 / d \]

**MF result**

**Virial expansion**

\[ T^0_{\text{crit}}(\lambda, \gamma) = 2.1 \lambda^{1.79 \pm 0.05} \]

MFA: \[ T^0_{\text{crit}}(\lambda, \gamma) = 2.46 \lambda^{1.78} \]
Superconducting transition temperature

Virial expansion on the 3d Anderson model

$3D\ AM, W=4.0, L=20, \omega_0 = 0.5$

$\lambda = 0.08$

$\nu T_c, T_{c,typ}$

$E_c$

Anderson localization transition

$E$ (disorder)
Conclusion:

Enhancement of $T_c$ by disorder

Maximum of $T_c$ in the insulator

Direct superconductor to insulator transition

BUT

Fragile superconductivity:

Small fraction of superconducting phase

Critical current decreasing with disorder
Two-eigenfunction correlation in 3D Anderson model (insulator)

\[
NC(\omega) \propto \ln^{d-1} \left( \frac{\xi}{\omega} \right)
\]

Ideal insulator limit only in one-dimension

\[
|E - E'|^{-1 + d_s/d}
\]

Mott's resonance physics

critical, multifractal physics
Superconductor-Insulator transition: percolation without granulation

Coordination number $K >> 1$

Only states in the strip $\sim T_c$ near the Fermi level take part in superconductivity

Coordination number $K = 0$
First order transition?

\[ f(x) \rightarrow 0 \text{ at } x \ll 1 \]
Conclusion

- Fractal texture of eigenfunctions persists in metal and insulator (multifractal metal and insulator).
- Critical power-law enhancement of eigenfunction correlations persists in a multifractal metal and insulator.
- Enhancement of superconducting transition temperature due to critical wavefunction correlations.
disorder
Anderson
localization
transition
BCS
limit
SC
M
IN
SC or IN

T

M

BCS
limit

SC

IN

disorder

Anderson localization transition

KMU
Corrections due to off-diagonal terms

Average value of the correction term increases $T_c$

Average correction is small when
Melting of phase by disorder

\[ \Delta(r) = \sum_{ij} \Delta_{ij} \eta_{ij} \Psi_i(r) \Psi_j(r) \]

In the diagonal approximation

The sign correlation \( <\Delta(r)\Delta(r')> \) is perfect: solutions \( \Delta_i > 0 \) do not lead to a global phase destruction

Beyond the diagonal approximation:

stochastic term destroys phase correlation
How large is the stochastic term?

Stochastic term:

\[ \sum_{m k k}^{≠} m k M U Q \]

\[ d_2 < d / 2 \]

\[ d_2 > d / 2 \] weak oscillations

\[ d_2 < d / 2 \] strong oscillations but still too small to support the glassy solution
More research is needed
Conclusions

- Mean-field theory beyond the Anderson theorem: going into the Fock space
- Diagonal and off-diagonal matrix elements
- Diagonal approximation: enhancement of $T_c$ by disorder.
- Enhancement is due to sparse single-particle wavefunctions and their strong correlation for different energies
- Off-diagonal matrix elements and stochastic term in the MF equation
- The problem of “cold melting” of phase for $d_2 < d/2$