STRONGLY DISORDERED SPIN SYSTEMS AND SUPERCONDUCTORS: SOLUTION ON BETHE LATTICE.

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0. General problem of noise/dynamics in strongly disordered quantum systems at low T.
   Novel (?) type of transition between noisy and quiet behaviors.
1. Experimental systems:
   A. strongly disordered superconducting films of InO, TiN, Bi etc ← this talk
   B. Paramagnetic spins in insulators and superconductors (but not in normal metals).
   C. Large Josephson arrays
2. Brief summary of data and theoretical models of SI transition.
3. Conclusion: plausible model for the SI transition in disordered films is the one that also might show noise freezing out at very low T.

1. Bethe lattice model: toy model for SIT transition in disordered superconductors and spin systems.
2. Solution of Bethe lattice model: importance of RSB.
3. Qualitative picture of both the SI and noisy-quiet transition:
   Very strong inhomogeneity, insulator and 'superinsulator'.

Very recent experimental data from Grenoble group (*Sacépé, Dubouchet, Chapelier, Sanquer, Shahar et al*) that confirm theoretical predictions……………………Conclusions.

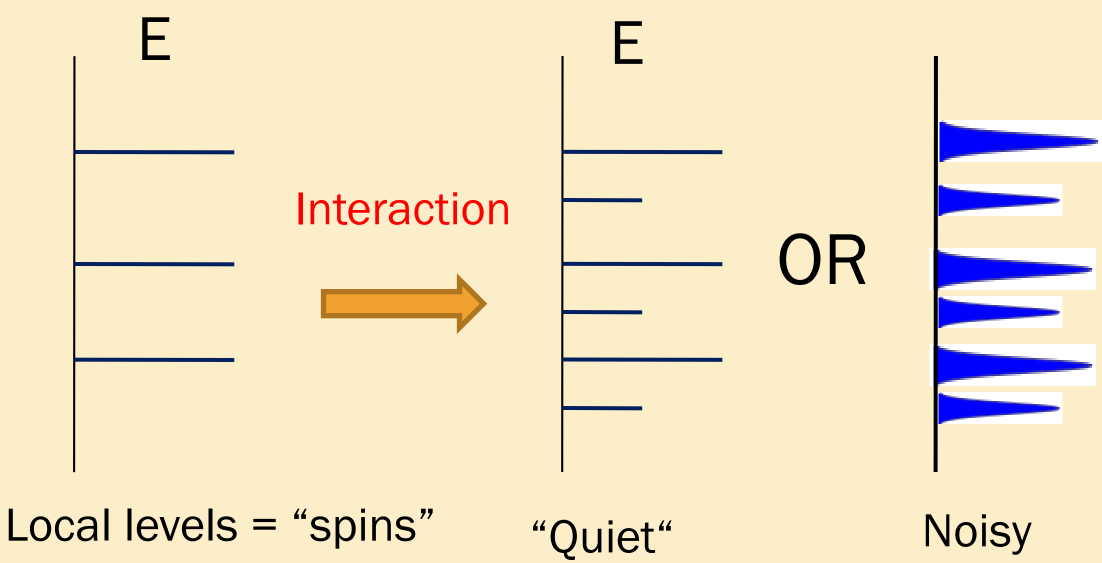
FUNDAMENTAL (PHILOSOPHICAL) QUESTION OF NOISE GENERATION AT VERY LOW T

Localized modes + interaction between them

? ↓ ?

Energy delocalization

Local spectrum



Which model to study?

Ideally it should correspond to a well studied experimental system to test new theoretical methods…

SI transition in films is a good candidate

Paramagnetic spins in insulators and superconductors

Josephson arrays are described by more complicated models due to long range Coulomb interaction.

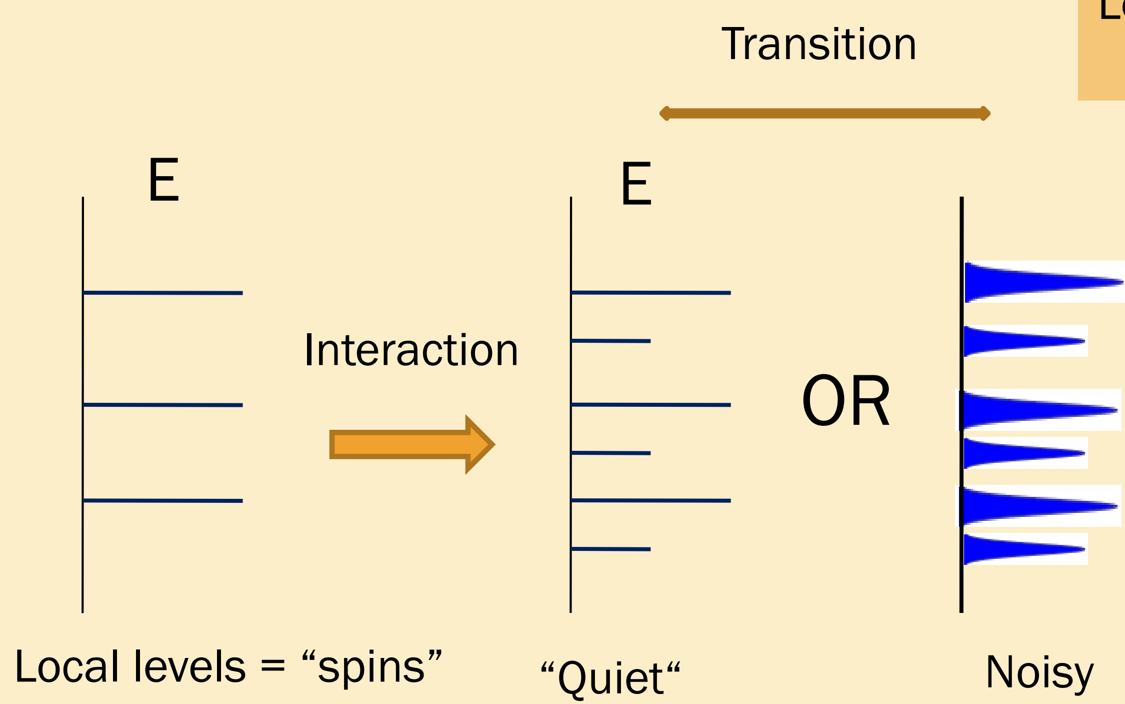
FUNDAMENTAL (PHILOSOPHICAL) QUESTION OF NOISE GENERATION AT VERY LOW T

Two cases (and two transitions):

Levels are sharp only at low energies

Or

Levels are sharp at all energies



Side remark.

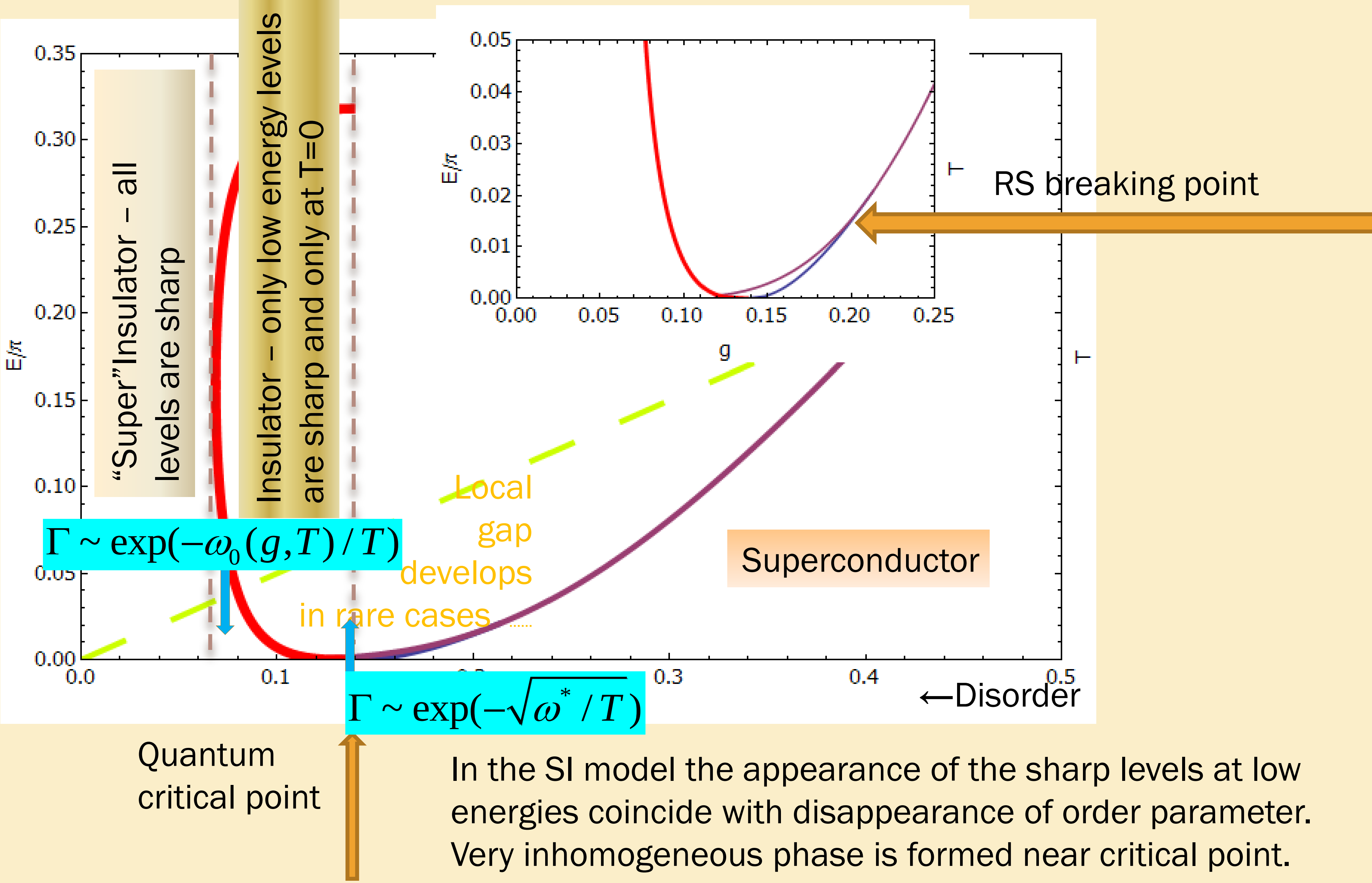
Nothing is really novel…

Symmetry broken by the transition:
Independent time reparametrizations at different space locations
Equivalent to

$$g_{oi}(x) \longrightarrow g_{oi}(x)+\varepsilon_{oi}(x)$$

for metric tensor
Discussed in the context of quantum gravity

PHASE DIAGRAM OF BETHE LATTICE MODEL

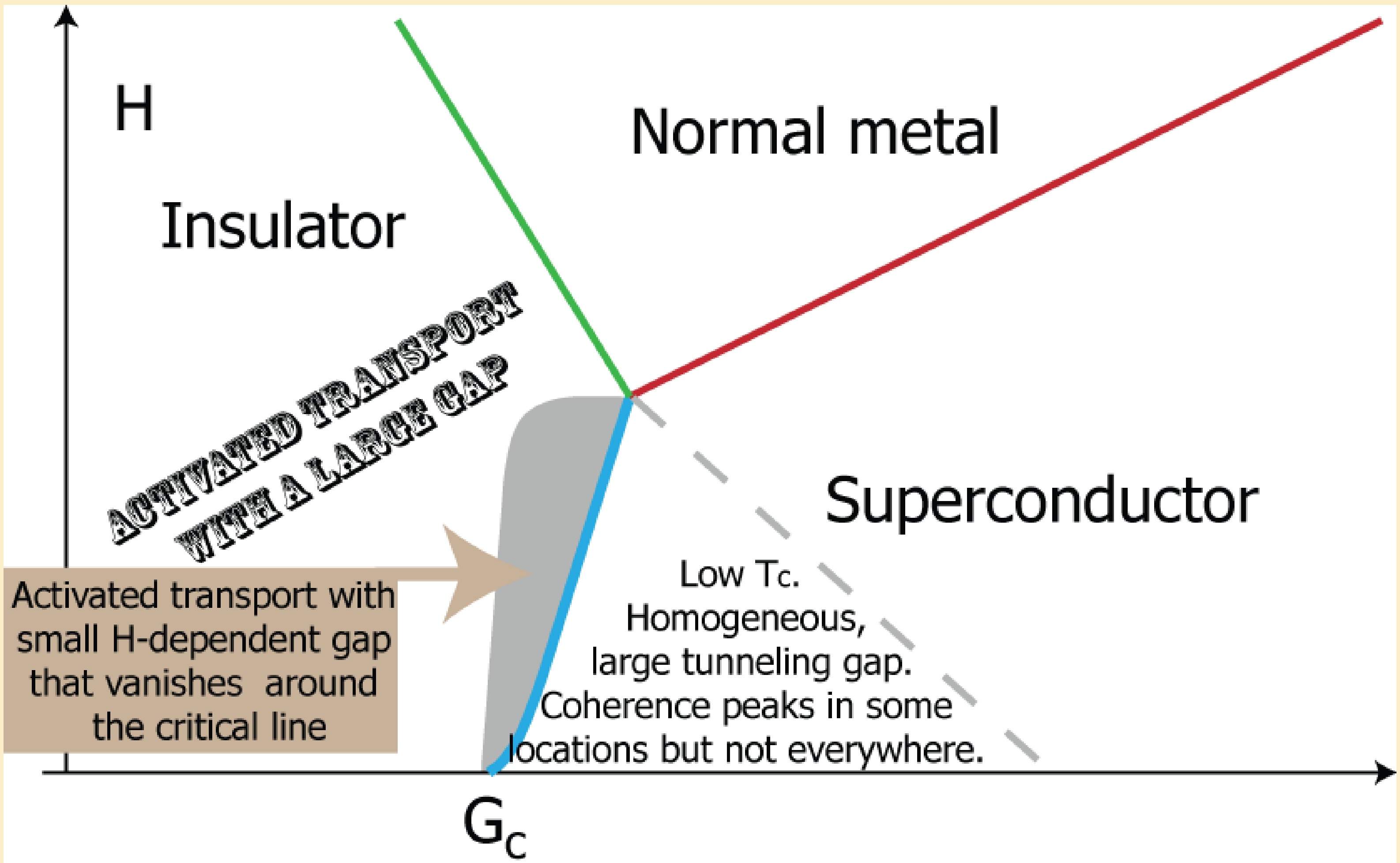


Quantum critical point

In the SI model the appearance of the sharp levels at low energies coincide with disappearance of order parameter. Very inhomogeneous phase is formed near critical point.

SI - EXPERIMENTAL PHASE DIAGRAM

InO, TiN, thin Be, some Bi…



What are the properties of Superconductor – Insulator transition at very low T?

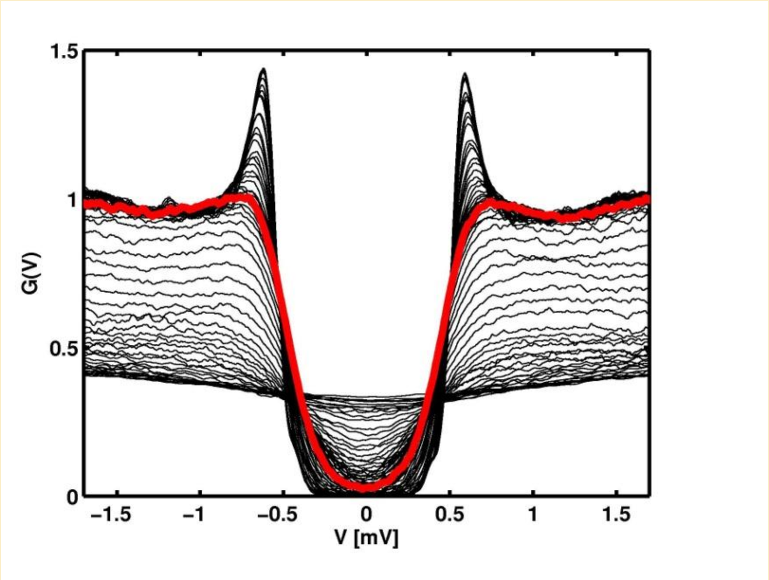
ALTERNATIVE SCENARIOS OF SUPERCONDUCTOR-INSULATOR TRANSITIONS

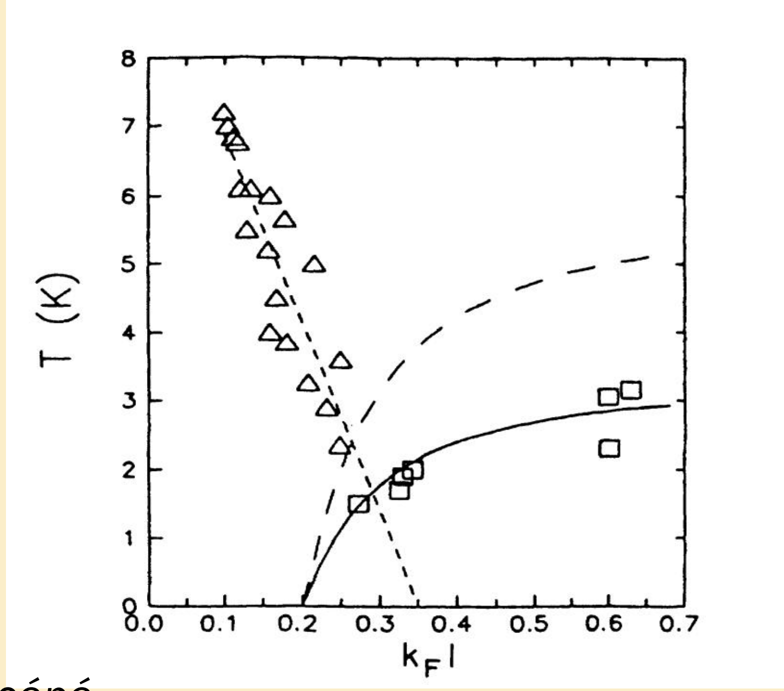
- Fermi model (suppression of fermion pairing by Coulomb interaction.
- Bose model (preformed Cooper pairs)

- Competition between Coulomb repulsion and Cooper pair hopping (large scale physics) – similar to transition in Josephson junction arrays.

- Competition between disorder and Cooper pair hopping

SUPERCONDUCTOR-INSULATOR: EXPERIMENTAL EVIDENCE





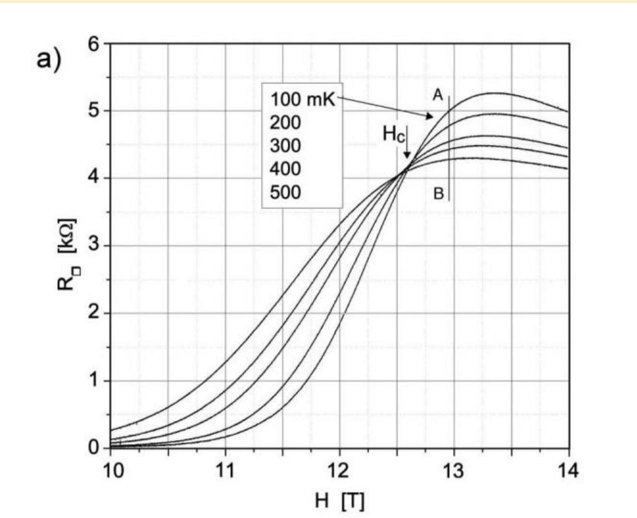
Direct evidence for the gap above the transition (*Sacépé, Dubouchet, Chapelier, Sanquer, Shahar et al*).
Activation behavior does not show gap suppression at the critical point as a function of the disorder (*Sahar, Ovaduyahu, 1992*).

Conclusion: Gap persistence rules out fermion mechanisms
Indicates preformed Cooper pairs.

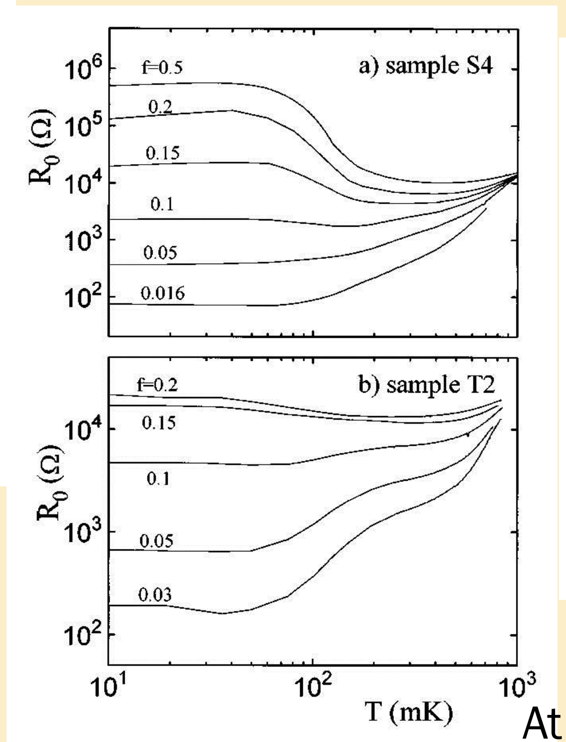
SUPERCONDUCTOR-INSULATOR: EXPERIMENTAL EVIDENCE

If Josephson/Coulomb model is correct, the same behavior should be observed in Josephson arrays…

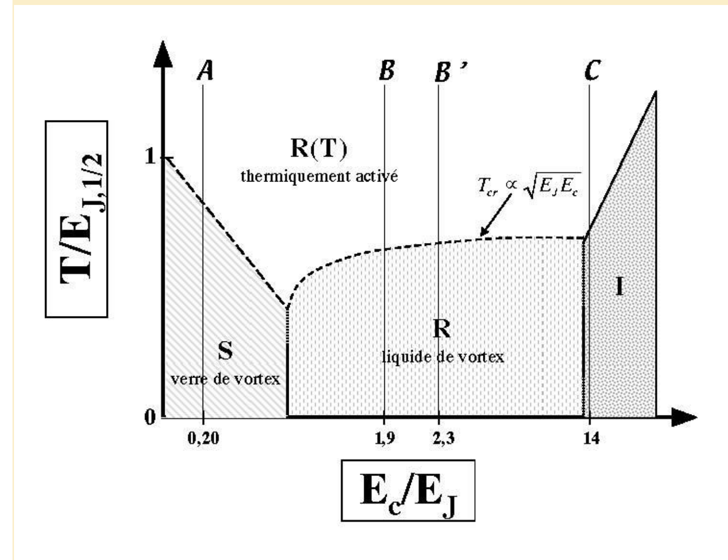
BUT IT IS NOT



Disordered films (*Kapitulnik*)



Zant and Mooji, 1996



At non-zero field Josephson arrays of more complex (dice) geometry show temperature independent resistance in a wide range of E_J/E_c. (*Pannetier and Serret 2002*)

BOSE MODEL (PREFORMED COOPER PAIRS)

- Competition between Cooper pairing and disorder, i.e. no Coulomb interaction. *(Ma and Lee, 1985, Kapitulnik and Kotliar 1985)*

  Potential disorder does not affect the superconductivity provided that

  $T_c \gg \delta_L = 1/\nu_o \xi^D$ – level spacing in the volume of localization.

  For $T_c \ll \delta_L \ll \omega_D$ *local* pairing is still possible leading to parity gap: all low lying excitations are Cooper pairs localized in fractal eigenstates of localization problem (Feigelman, Kravtsov and others).

  Superconductor-insulator transition happens when boson hopping M_{ij} between these states is comparable to the spread of the individual energies. Model Hamiltonian:

$$H = \sum_j \xi_j c_j{}^\dagger c_j - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} c_j{}^\dagger c_j{}^\dagger c_i c_i \quad \text{with } M_{ii} \gg Z M_{ij}$$

In the insulating phase the transport is via Cooper pair hopping.
Why the gap?

TOY MODEL OF SIT DRIVEN BY DISORDER WITH PURELY ATTRACTIVE INTERACTION AND PREFORMED PAIRING.

- Basis of exact single particle states. Close to insulator-metal transition localized single particle states are large and have many overlaps.

$$H = \sum_j \xi_j c_j^{\dagger} c_j - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} c_j^{\dagger} c_j^{\dagger} c_i c_i \quad \text{with } M_{ii} \gg Z M_{ij},\ Z \gg 1$$

- Leave out single particle states (spin representation, confirmed by Grenoble data):

$$H = -\sum_j 2\xi_j \sigma_j^{z} - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} (\sigma_i^{+} \sigma_j^{-} + \sigma_i^{-} \sigma_j^{+}) \quad \mathrm{Z} \gg 1$$

- What are general properties of the quantum transition in the models in random field? Applies also to strongly disordered magnets (paramagnet-ferromagnet transition).

$$H = -\sum_j 2\xi_j \sigma_j^{z} - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} \sigma_i^{x} \sigma_j^{x} \quad \text{with } Z \gg 1$$

TOY MODEL OF SIT DRIVEN BY DISORDER WITH PURELY ATTRACTIVE INTERACTION AND PREFORMED PAIRING.

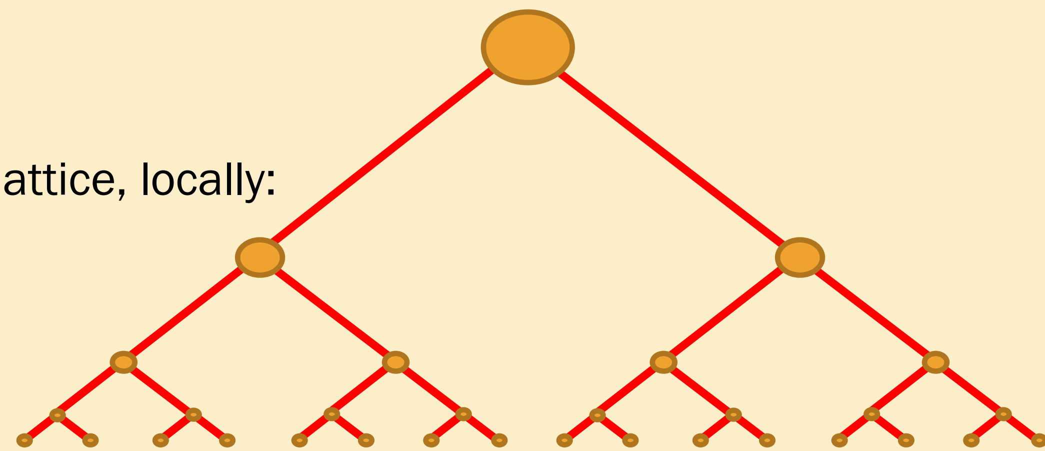
- Because number of neighbors is large the loops can be neglected. The model on Bethe lattice is believed to reproduce the main features of the transition and phases on both sides (formally we ignore small 1/Z effects but keep 1/Log(Z):

$$H = -\sum_j \xi_j \sigma_j^{\ z} - \frac{g}{K}\sum_{i,j} \sigma_{i,k+1}^{\ \ x}\sigma_{j,k}^{\ \ x} \quad \xi_j > 0 \ \text{ with } K \gg 1$$

Equivalent to the 'superconducting' model:

$$H = -\sum_j \xi_j \sigma_j^{\ z} - \frac{g}{K}\sum_{i,j} (\sigma_{i,k+1}^{\ \ x}\sigma_{j,k}^{\ \ x} + \sigma_{i,k+1}^{\ \ y}\sigma_{j,k}^{\ \ y}) \text{ with } K \gg 1,\ \xi_j \subset (-W, W)$$

Bethe lattice, locally:



MODEL SOLUTION 1: CAVITY EQUATIONS.

Main idea: cavity equations.
Introduce effective field that simulates the effect of spins at higher levels:

$$H_{eff} = -\xi_0\sigma_0^z - h_0\sigma_0^x \qquad \left\langle \sigma_0^x \right\rangle_0 = \frac{h_0}{\sqrt{h_0^{\,2}+\xi_0^{\,2}}}\,\mathrm{Tanh}\left[\frac{\sqrt{h_0^{\,2}+\xi_0^{\,2}}}{T}\right]$$

$$H_{sn} = -\xi_0\sigma_0^z - \sum_j (\xi_j\sigma_j^z + \sigma_0^x\sigma_j^x + h_j\sigma_j^x) \qquad \text{Choose } h_0 \text{ so that } \left\langle \sigma_0^x \right\rangle_H = \left\langle \sigma_0^x \right\rangle_0$$

Roughly - this approximation is sufficient to get the transition temperature to O(1/K):

$$h_{k+1} = \frac{g}{K}\sum_j \frac{h_{k,j}}{\sqrt{\xi_{k,j}^{\;2}+h_{k,j}^{\;2}}}\,\mathrm{Tanh}\,\frac{\sqrt{\xi_{k,j}^{\;2}+h_{k,j}^{\;2}}}{T}$$

Can be further improved (leading order in 1/K):

1. Diagonalize H analytically/numerically
2. Find <σ>

The difference at large K is not significant.

If averaged over uniform distribution of ξ we get usual BCS-like equation:

$$h = g\int_0^\infty \frac{d\xi h}{\sqrt{\xi^2+h^2}}\mathrm{Tanh}\left[\frac{\sqrt{\xi^2+h^2}}{T}\right]$$

that tells us that $T_c > 0$ for any $g > 0$.

MODEL SOLUTION 2: EQUATION FOR T_C.

To find T_c we need to find when infinitely small field applied at the boundary leads to large field in the center:

$$h_0 = Z h_N \qquad Z = \sum_{\{i[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i[k]}/T]}{\xi_{k,i[k]}}$$

That is whether $Z=exp(fN)$ with $f>0$ ("magnet" or "superconductor") or $f<0$ (paramagnet)?

Non-trivial physics is due to the fact that Z is not necessarily self-averaging quantity! Consider higher moments:

$$K \left\langle \left[\frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i}/T]}{\xi_{k,i}} \right]^n \right\rangle = \sqrt{\frac{3\pi}{4K}} K^{1-n} g^n / T^{n-1}$$

The moments diverge at $T=g/K$ which becomes higher than 'average' $T_c=exp(-1/g)$.

MODEL SOLUTION 3: EQUATION FOR T_C.

$$h_0 = Zh_N \qquad Z = \sum_{\{i[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i[k]}/T]}{\xi_{k,i[k]}}$$

Z=exp(fN) with *f>0* ("magnet" or "superconductor") or *f<0* (paramagnet, non-supercond)?

Reminder: Non-trivial physics is due to the fact that *Z* is not necessarily self-averaging quantity!
For T<g/K Z is not self-averaging and typical Z_{typ}=expN<f> might be different from <Z>.
Typical lattice shows the transition when <f> > 0.
To find average <f> use replica trick:

$$Z^n = \sum_{\{i_a[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i_a[k]}/T]}{\xi_{k,i_a[k]}} \qquad a = 1..n$$

Solve the problem for n replicas and continue to n=0. Similar problems were solved in the context of directed polymer physics (*Derrida and Spohn*).
Replica symmetric solution (i.e. all replicas are independent) gives the BCS-like result.
However at low T(g) replica symmetry breaks down.

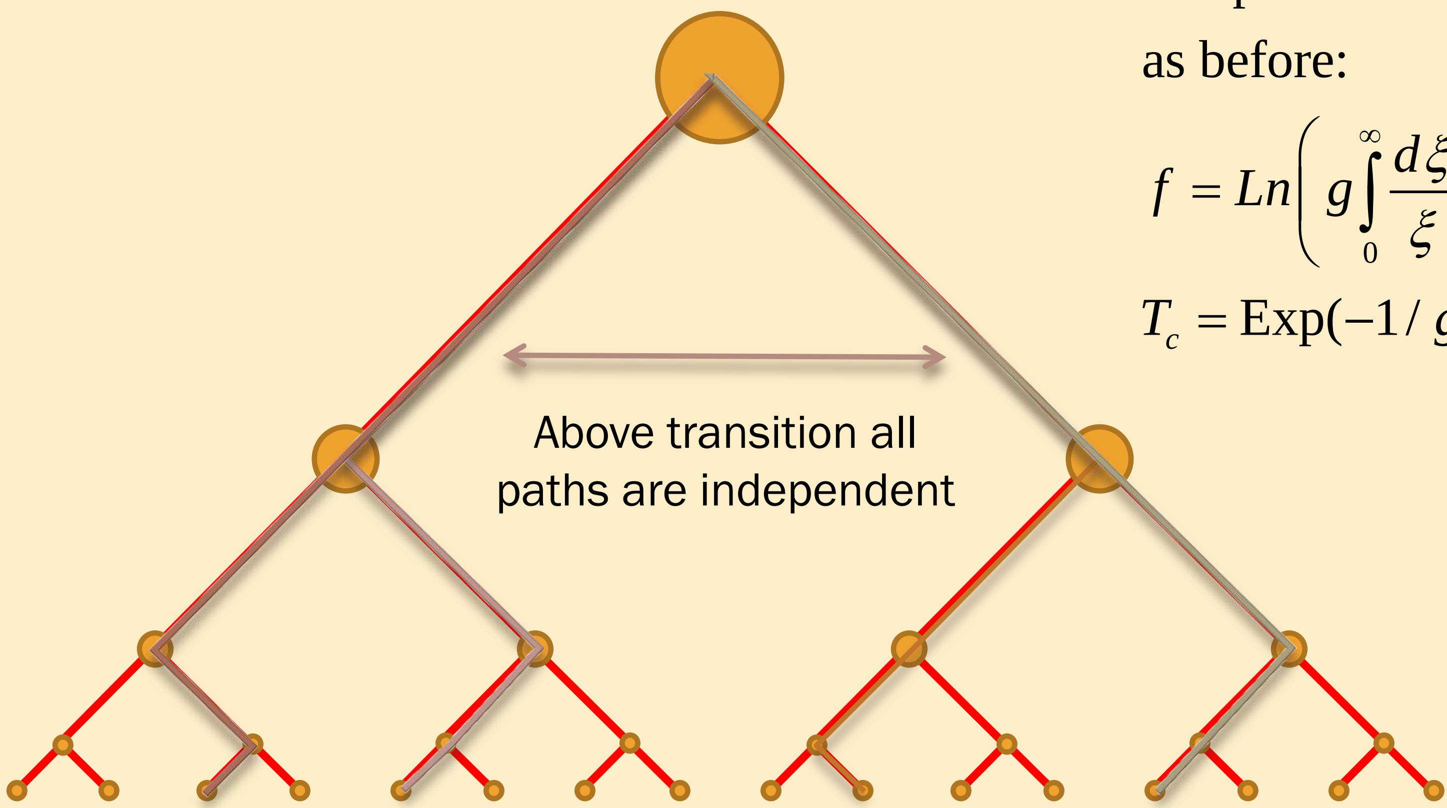
EQUATION FOR T_c IN ONE STEP RSB.

$$Z^n = \sum_{\{i_a[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i_a[k]}/T]}{\xi_{k,i_a[k]}} \quad a = 1..n$$

Assumption that all paths are independent leads the same result as before:

$$f = Ln\left(g \int_0^\infty \frac{d\xi}{\xi} \mathrm{Tanh}\left[\frac{\xi}{T}\right]\right)$$

$$T_c = \mathrm{Exp}(-1/g)$$

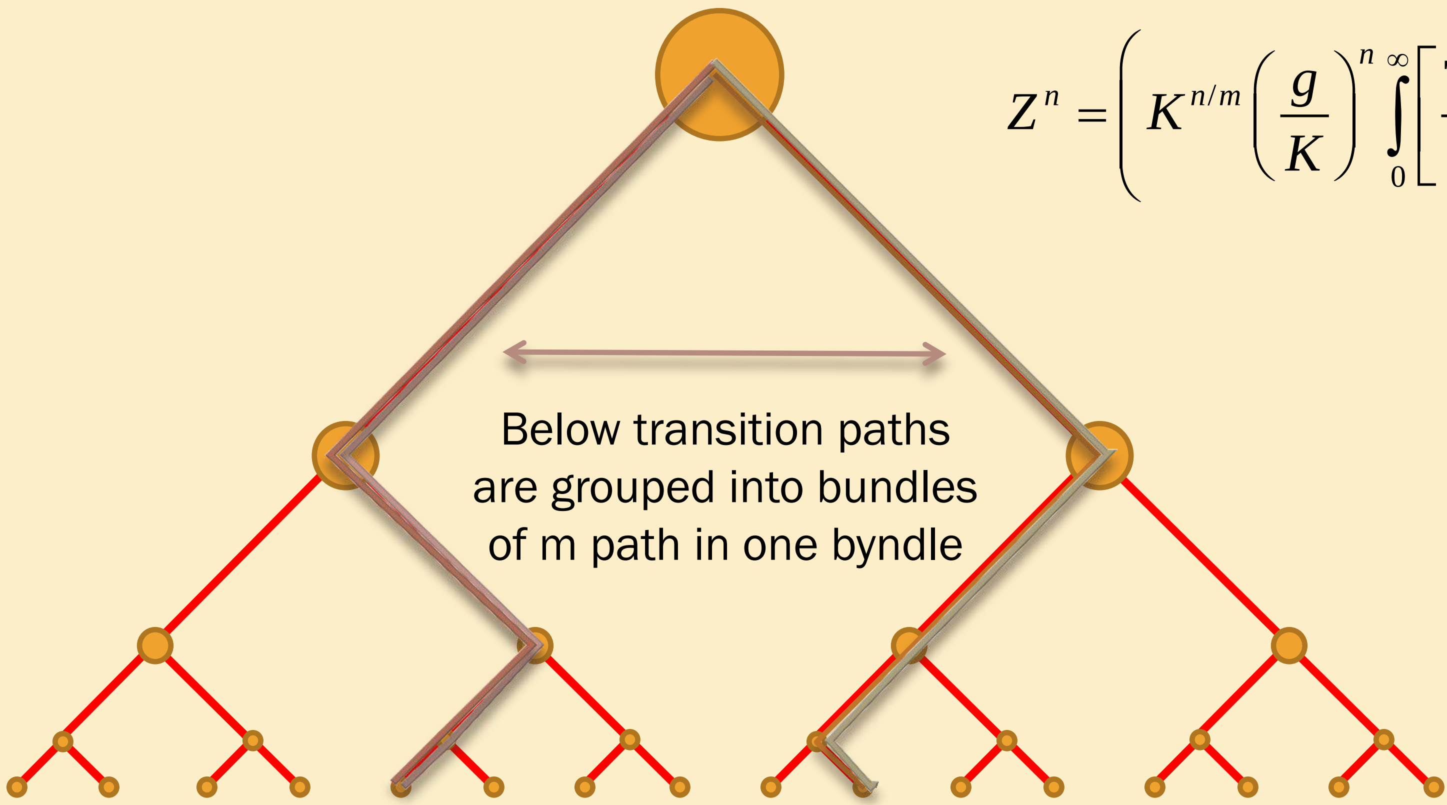


EQUATION FOR T_C IN ONE STEP RSB.

$$Z^n = \sum_{\{i_a[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i_a[k]}/T]}{\xi_{k,i_a[k]}} \qquad a = 1..n$$

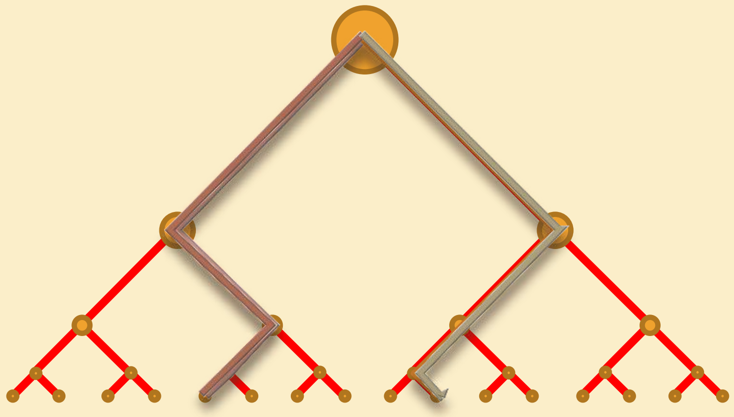
Bundled average (n/m bundles):

$$Z^n = \left(K^{n/m} \left(\frac{g}{K}\right)^n \int_0^\infty \left[\frac{\mathrm{Tanh}(\xi/T)}{\xi} \right]^m d\xi \right)^N$$



EQUATION FOR T_c IN ONE STEP RSB.

Bundled average (n/m bundles):



$$Z^n = \left(K^{n/m} \left(\frac{g}{K} \right)^n \int_0^\infty \left[\frac{\mathrm{Tanh}(\xi / T)}{\xi} \right]^m d\xi \right)^N$$

$$f = \ln \frac{g}{K} + \frac{1}{m} \left[\ln K + \ln \int_0^\infty \left[\frac{\mathrm{Tanh}(\xi / T)}{\xi} \right]^m d\xi \right]$$

Before continuation to $n \to 0$ $n > m > 1$, after $n < m < 1$

RSB occurs when m^* minimizing $f(m)$ becomes m<1.

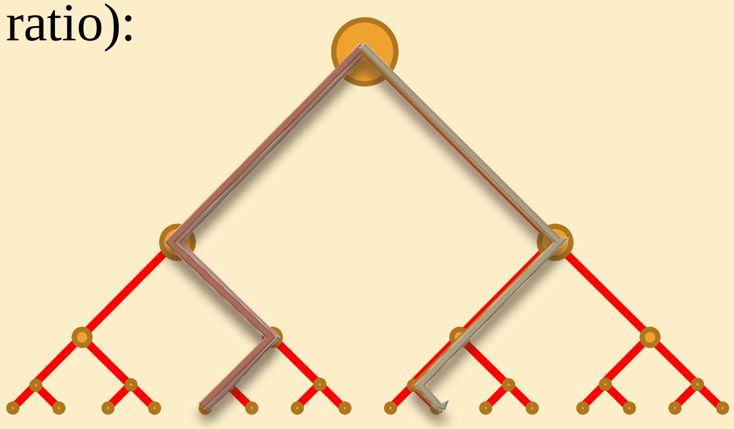
$$\ln \frac{K}{g} = \mathrm{g} \int_0^\infty \left[\frac{\mathrm{Tanh}(\xi / T_{RSB})}{\xi} \right] \ln \left[\frac{\mathrm{Tanh}(\xi / T_{RSB})}{\xi} \right] d\xi$$

Quantum critical point: $g(T = 0) = K \exp(-\frac{1}{m^*} \ln \frac{K}{1 - m^*})$

where m^*: $\quad \ln \frac{K}{1 - m^*} = \frac{m^*}{1 - m^*}$

EFFECTIVE NUMBER OF PATHS AT THE TRANSITION.

Effective number of paths (analogue of participation ratio):



$$\chi = \frac{\sum_{\{i[k]\}} \prod_k \left[\frac{\mathrm{Tanh}[\xi_{k,i[k]} / T]}{\xi_{k,i[k]}} \right]^2}{\left[\sum_{\{i[k]\}} \prod_k \frac{\mathrm{Tanh}[\xi_{k,i[k]} / T]}{\xi_{k,i[k]}} \right]^2}$$

If M paths contribute to Z roughly equally, Z/M $\rightarrow \chi = \dfrac{1}{M}$

Compute $\langle \log(\chi) \rangle$ by replica symmetry breaking in numerator and denominator separately:

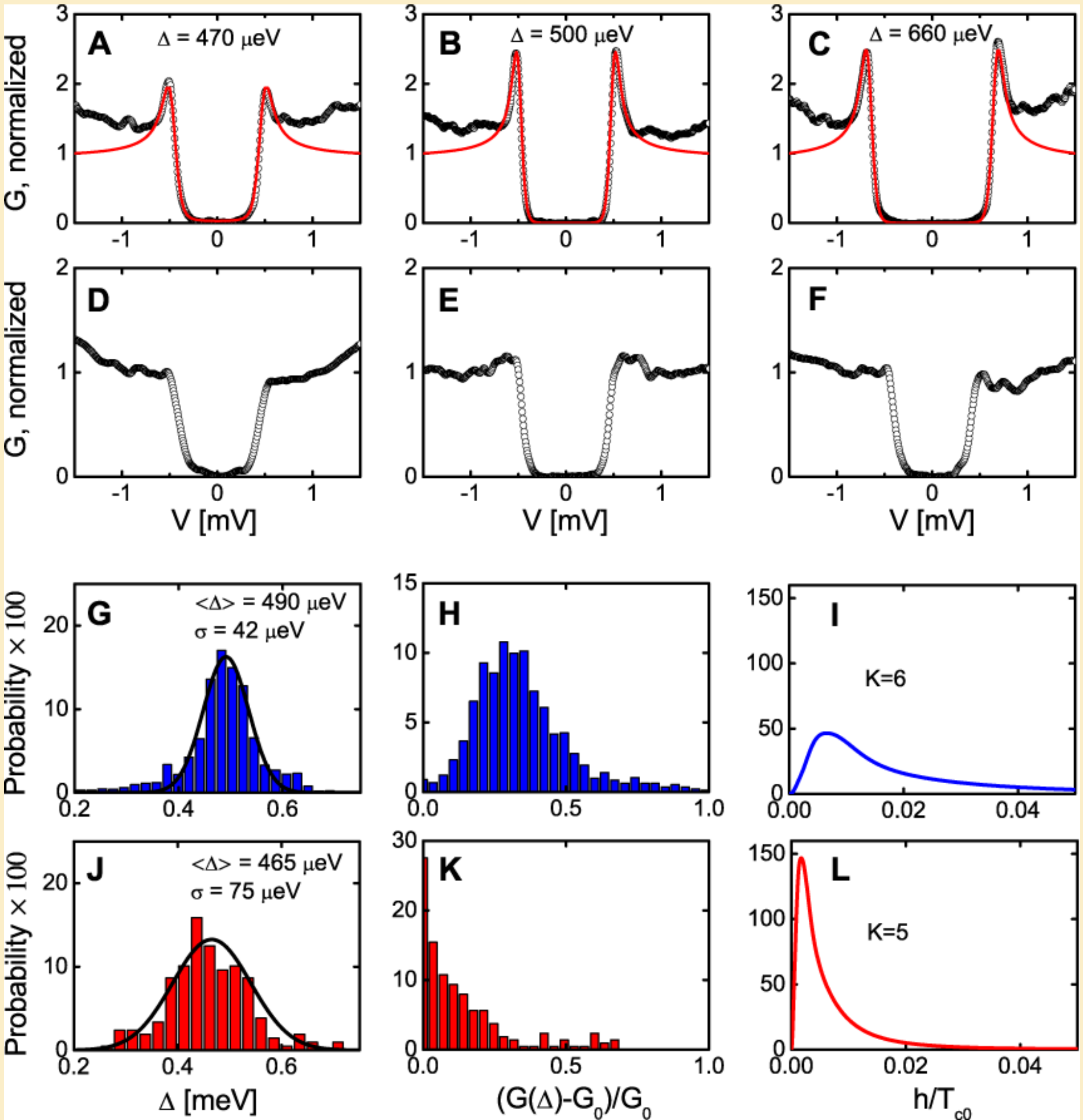
$$f = \frac{1}{m}\left[\ln K + \ln \int_0^\infty \left[\frac{\mathrm{Tanh}(\xi / T)}{\xi} \right]^m d\xi \right] \qquad f_2 = \frac{1}{m}\left[\ln K + \ln \int_0^\infty \left[\frac{\mathrm{Tanh}(\xi / T)}{\xi} \right]^{2m} d\xi \right]$$

$$m_2^* = m^* / 2 \rightarrow f_2(m_2^*) = 2 f(m^*)$$

$$\log(\chi) = 0 + O(1)$$

Conclusion: only a small number of path contribute
Exactly at the critical point.

DISTRIBUTION FUNCTION OF THE LOCAL FIELDS



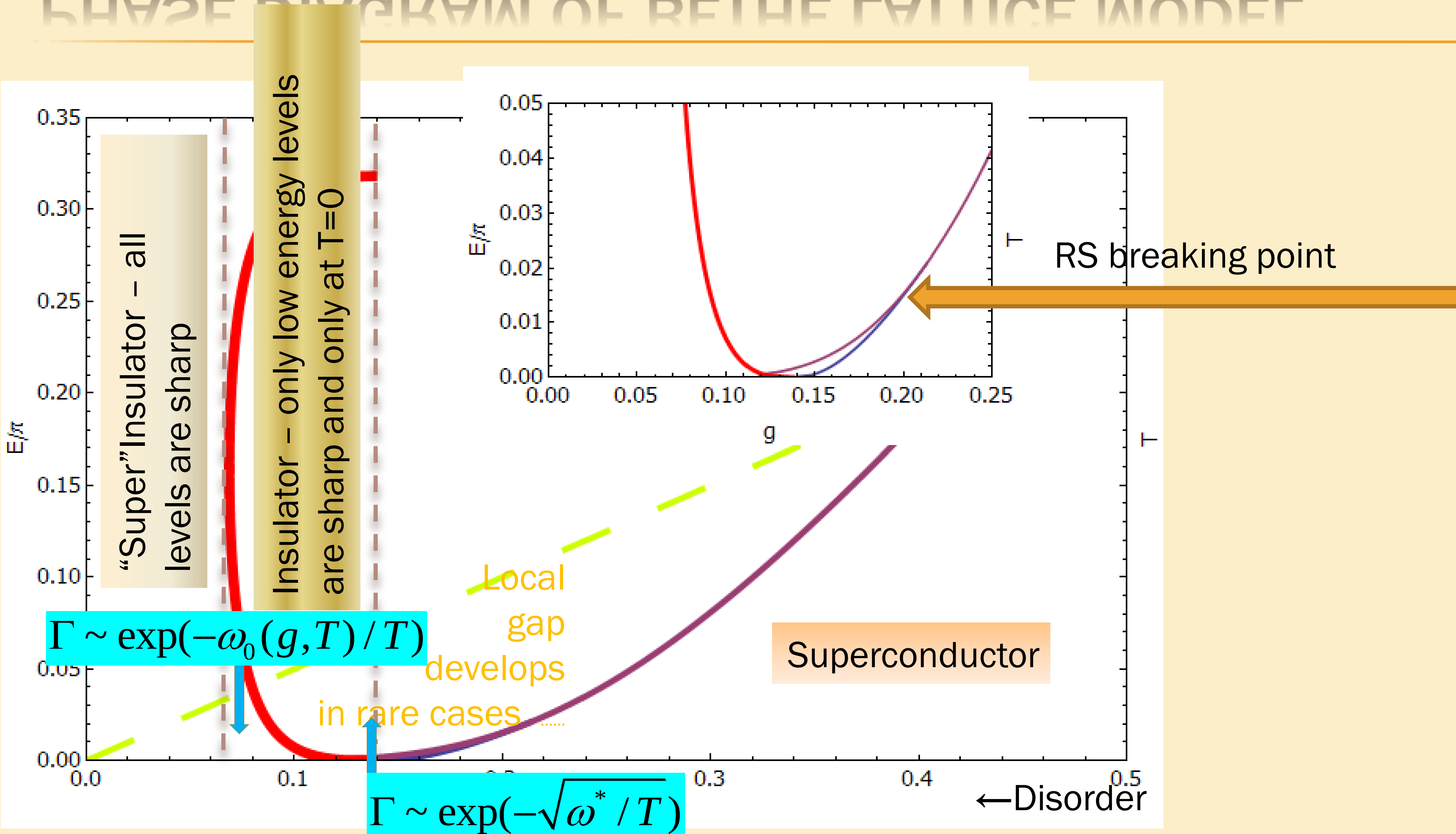
Experimental results
Sacépé, Dubouchet, Chapelier, Sanquer, et al
and theoretical expectations

1. Gap is always large
2. Height of coherence peak measures local order parameter

Sample slightly away from SI transition

Sample very close to SI transition

PHASE DIAGRAM OF BETHE LATTICE MODEL



Quantum critical point

In the SI model the appearance of the sharp levels at low energies coincide with disappearance of order parameter. Very inhomogeneous phase is formed near critical point.

CONCLUSIONS

1. Properties of the disordered superconducting films exhibiting SI transition ask for a different model than Josephson arrays.
2. Good candidate is the model with no Coulomb repulsion (equivalent to magnet in random field)
3. Solution of magnet in random field on Bethe lattice shows formation of a very inhomogeneous (non-self averaging) phase at low T close to quantum critical point.
4. Insulating phase is characterized by zero level width at $T=0$ for sufficiently small $g<g^*$. In the intermediate regime (close to ‘quantum critical point’ g_c) only low energy states $E<E(g-g_c)$ have zero width.