Integer Quantum Hall Edge States

Out of Equilibrium

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Work with Dmitry Kovrizhin, in preparation

and: PRB 81 (2010), PRB 80 (2009)

Earlier collaboration: Y. Gefen and M. Veillette, PRB 76 (2007)

Outline

Experimental motivation

QH Mach-Zehnder interferometers out of equilibrium

Generating & observing evolution of

non-equilibrium electron distribution in QHE edge states

Theoretical Idealisation

Time evolution of electron momentum distribution

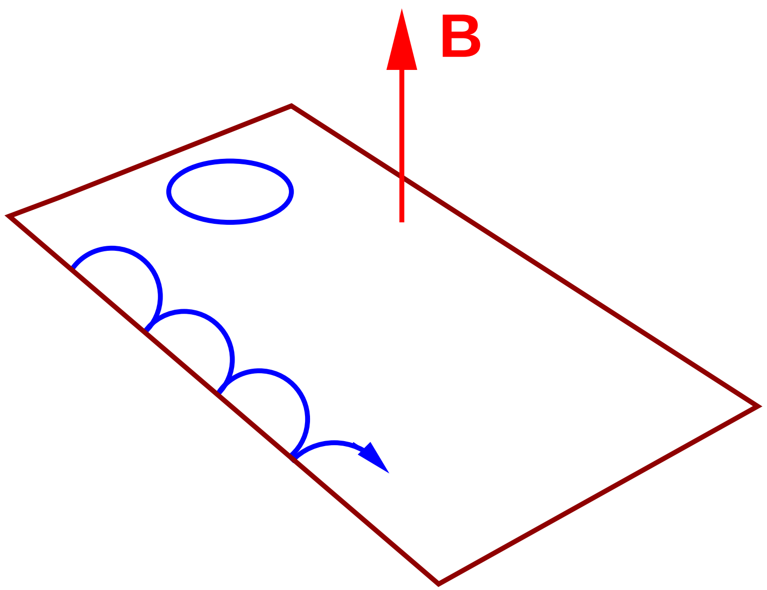
Results

Non-thermal steady state

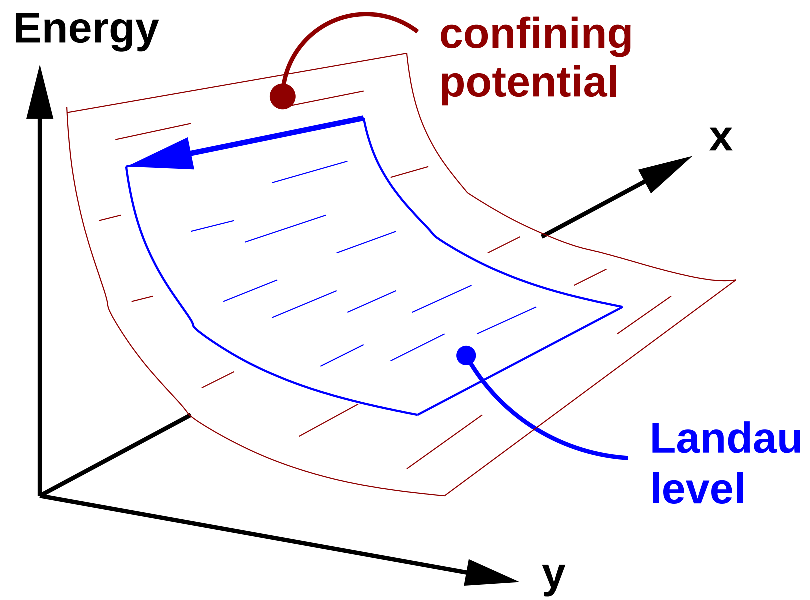
Interaction effects in MZ interferometers

Quantum Hall Edge States

Classical skipping orbits



Quantum edge states

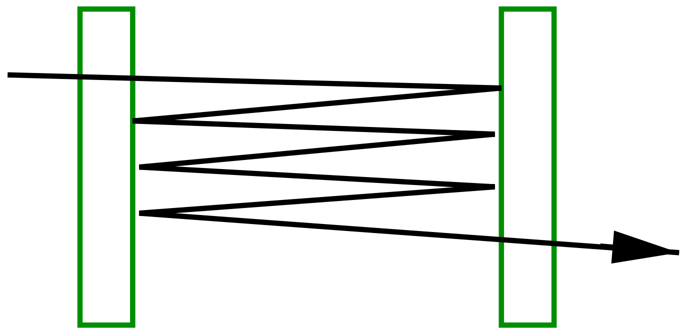


Two-dimensional electron gas in magnetic field

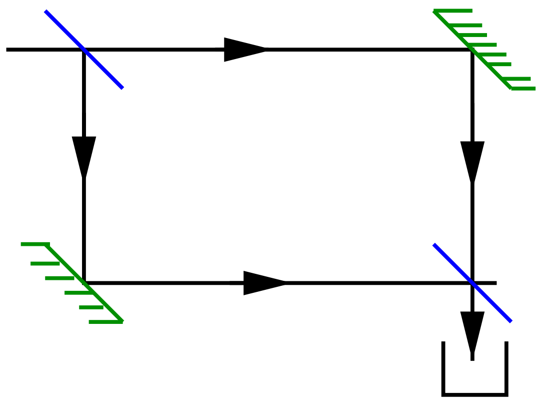
Edge state Hamiltonian: $\mathcal{H} = \int \psi^\dagger(x)(-i\hbar v \partial_x)\psi(x)dx$

Edge State Interferometer Design

Fabry-Perot



Mach-Zehnder



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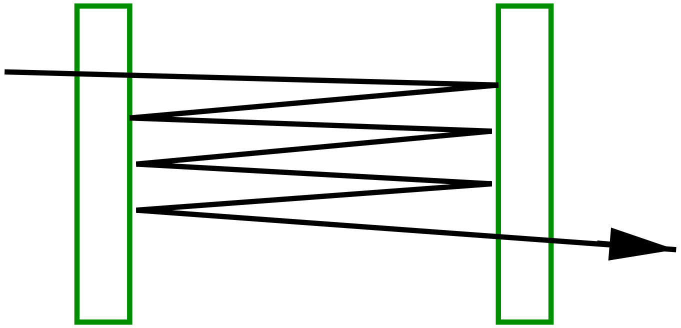
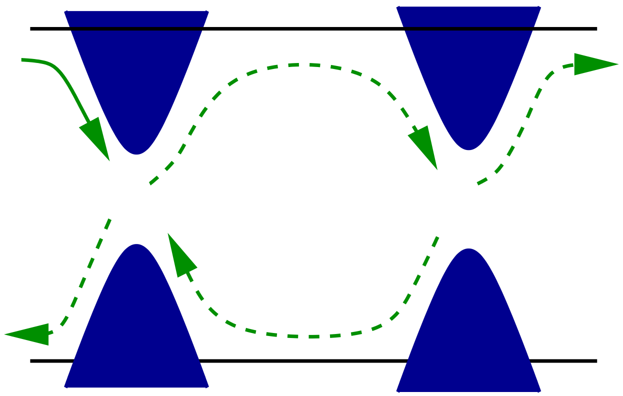
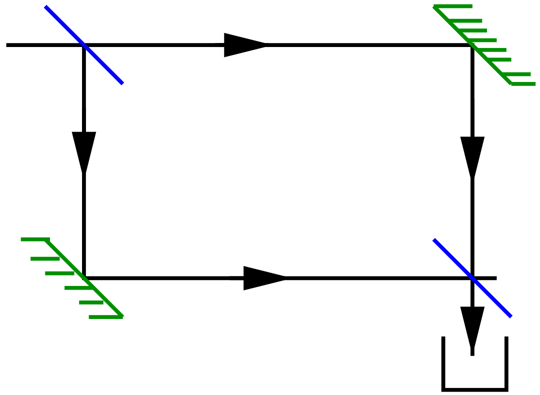




Mach-Zehnder



Edge State Interferometer Design

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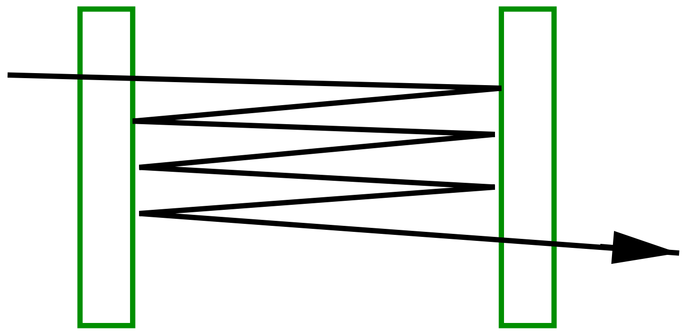

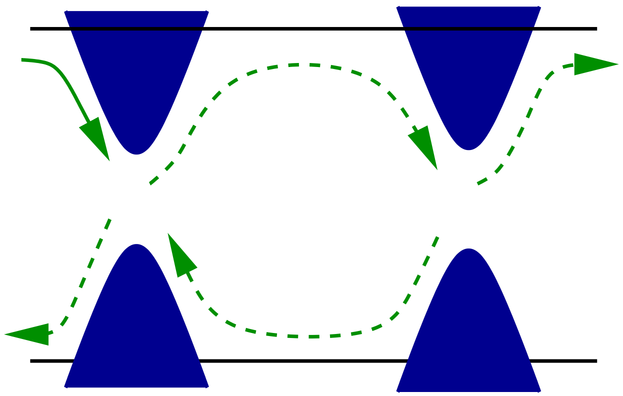

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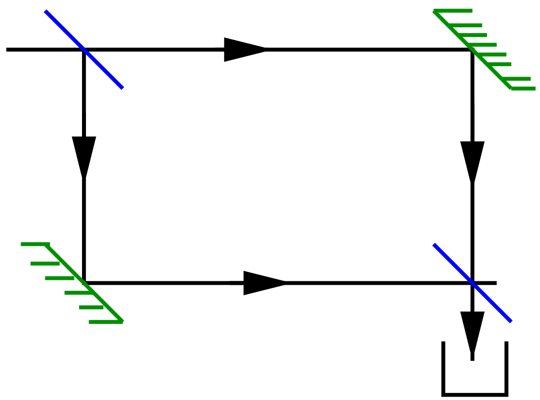
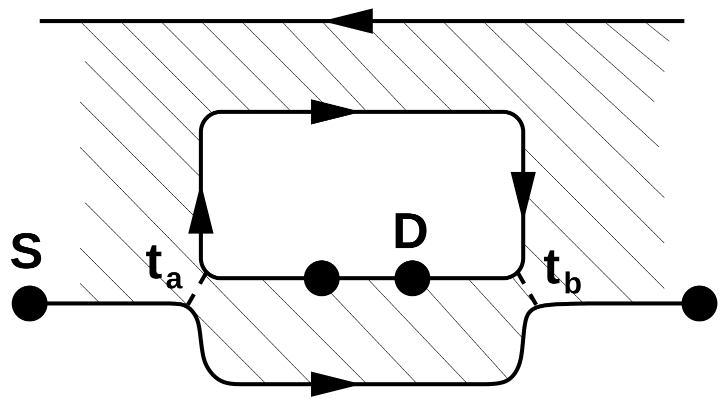
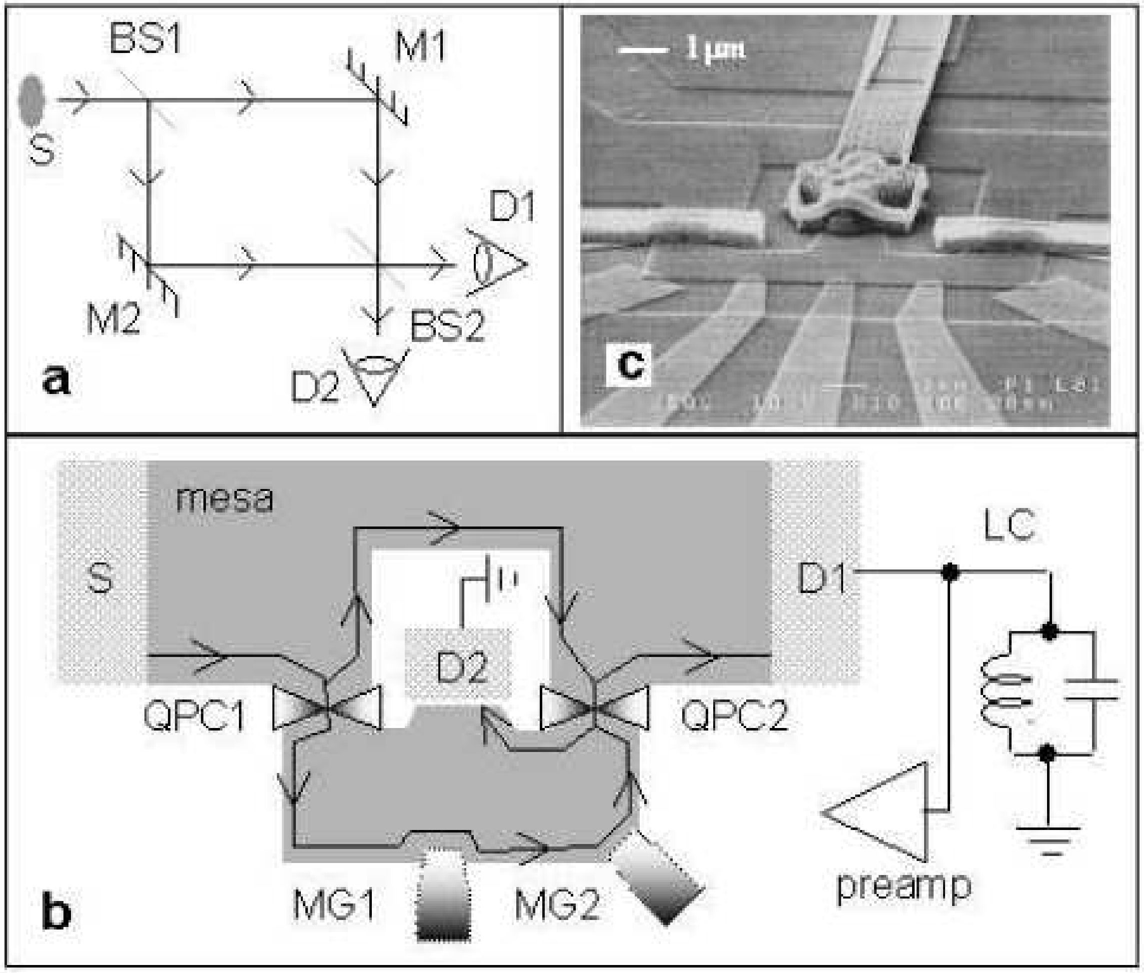




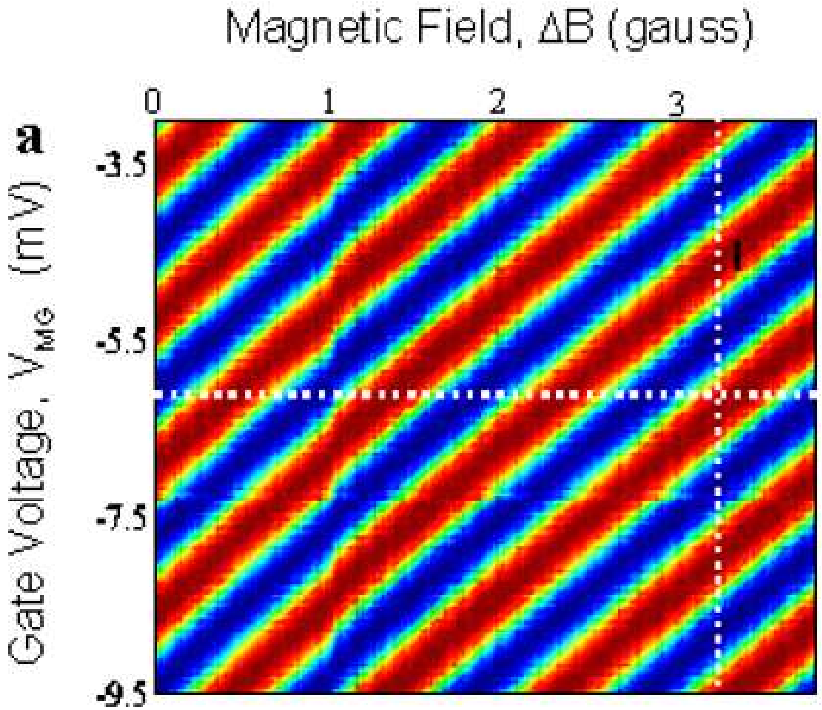
Experimental system

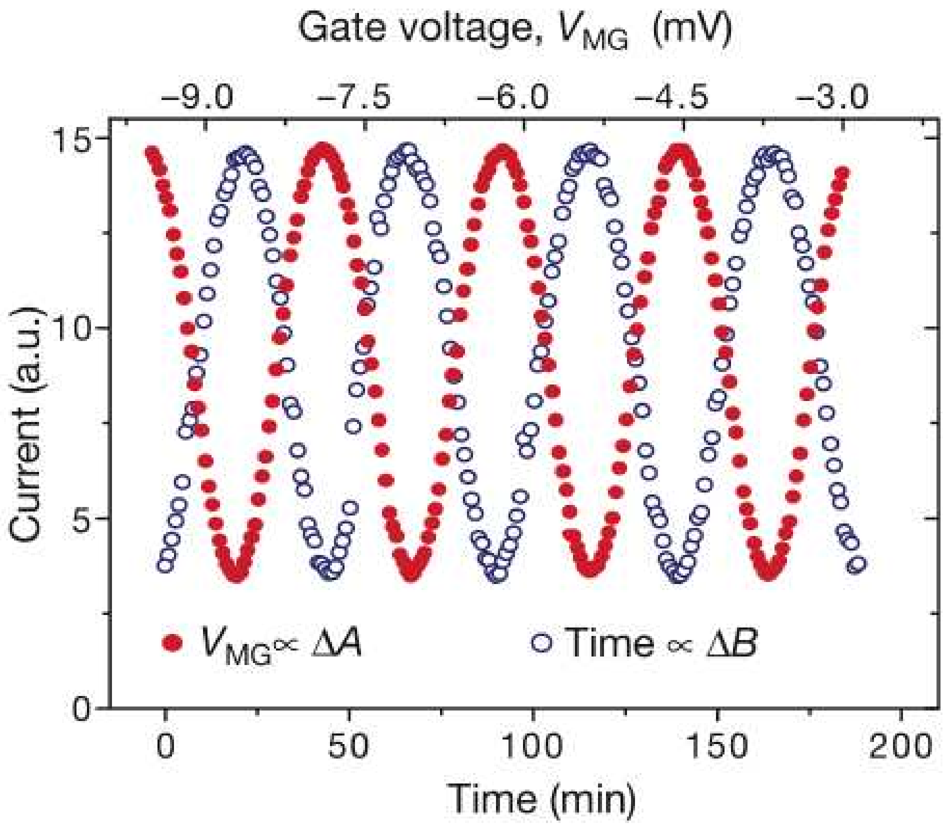


Heiblum Group, Weizmann Institute

Ji *et al.*, Nature (2003)

Fringes in Edge State Interferometer

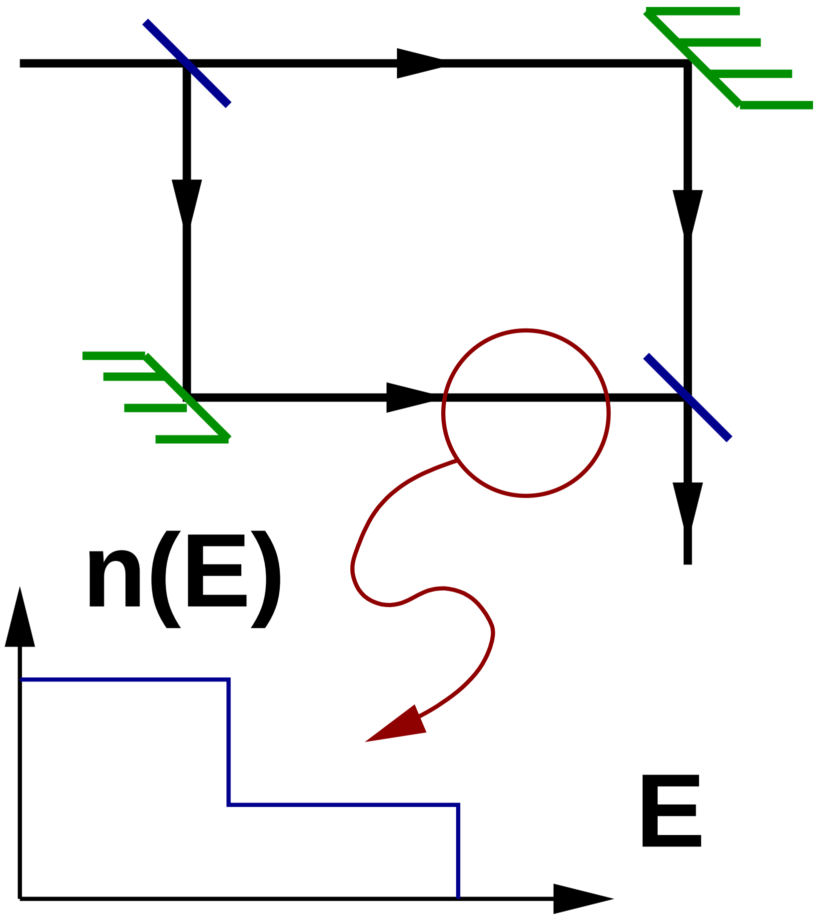




G_{SD} **vs** $Flux\ density$ **and** $Area$

Interferometer out of equilibrium

Decoherence from inelastic scattering

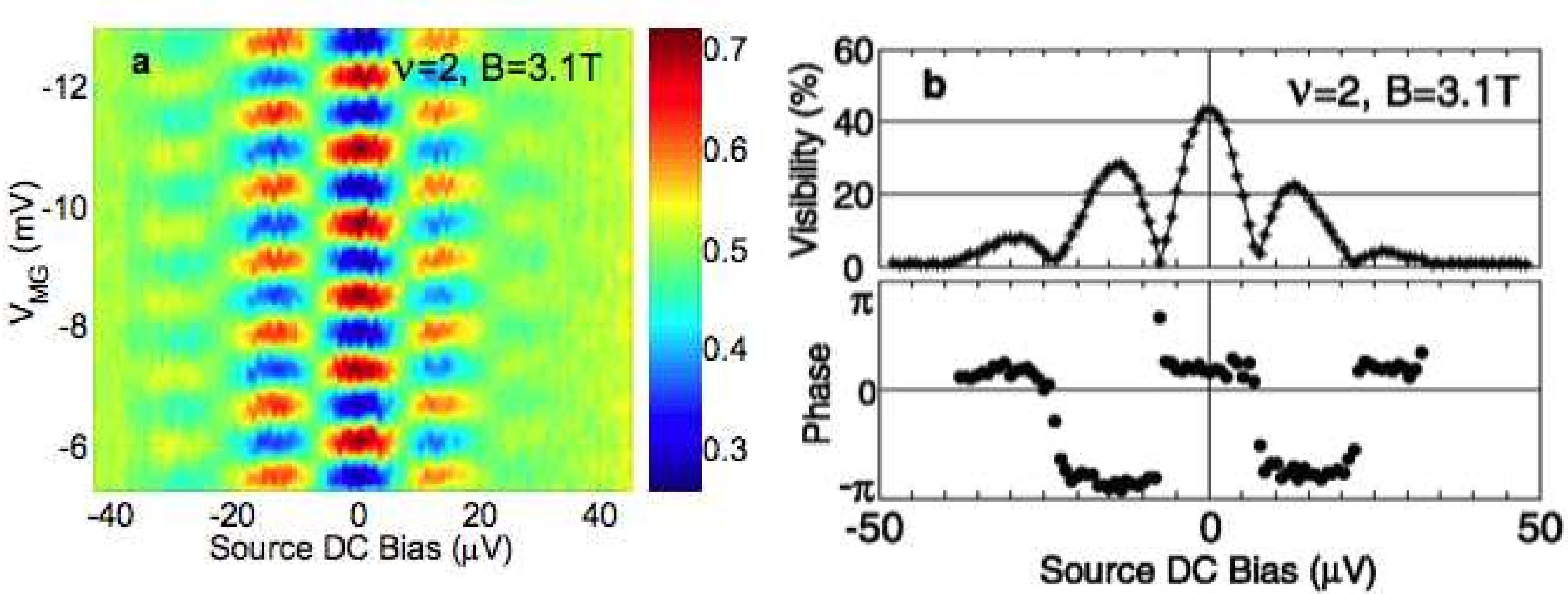


Surprises from experiment

Oscillatory dependence of visibility on bias

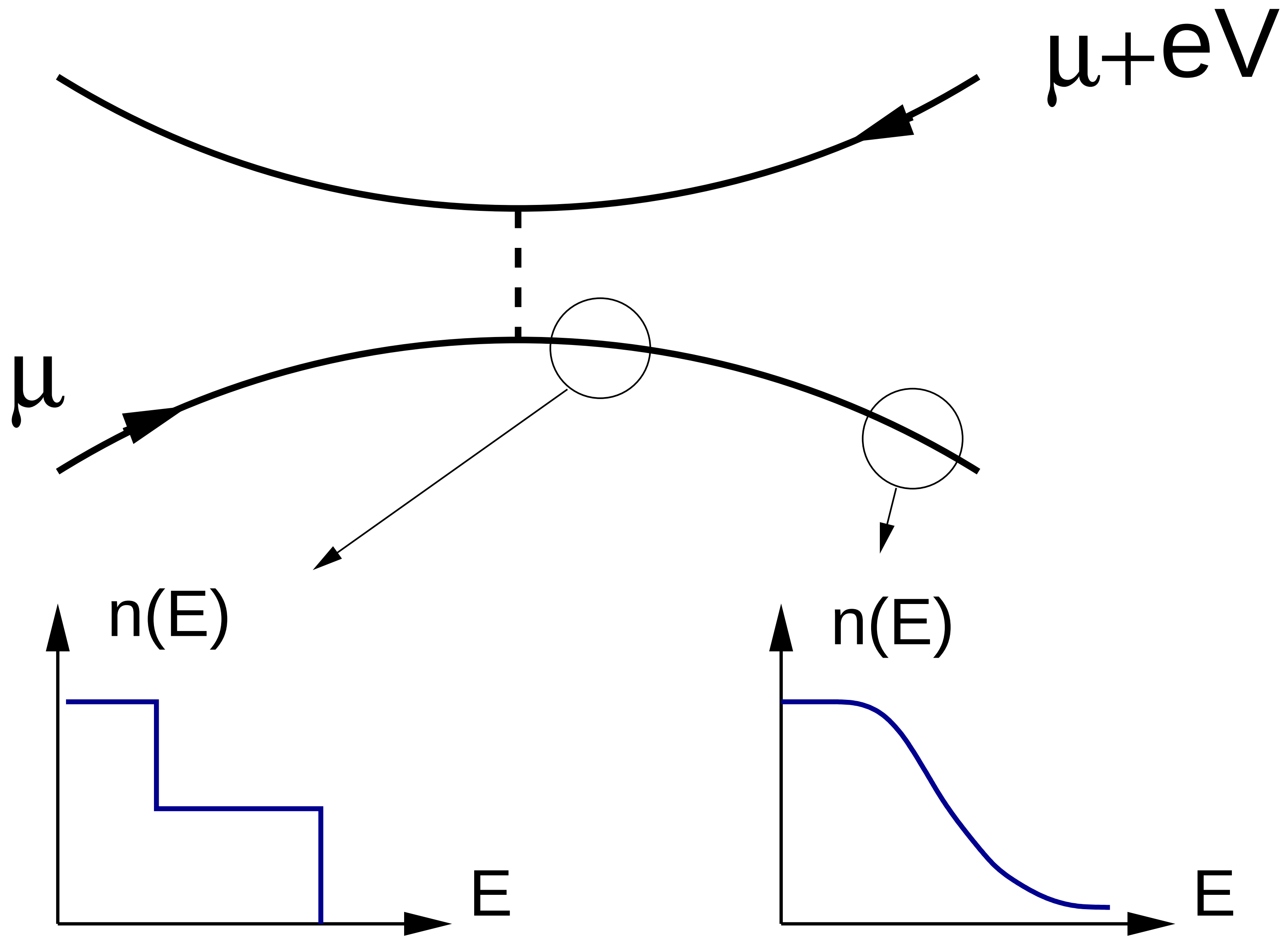
Differential conductance $G(\Phi_{AB}) = G_0 + G_1 \cos(\Phi_{AB})$

Fringe visibility $\mathcal{V} = |G_1|/G_0$



Neder *et al.*, PRL (2006) **Also Regensburg, Basel and Saclay groups**

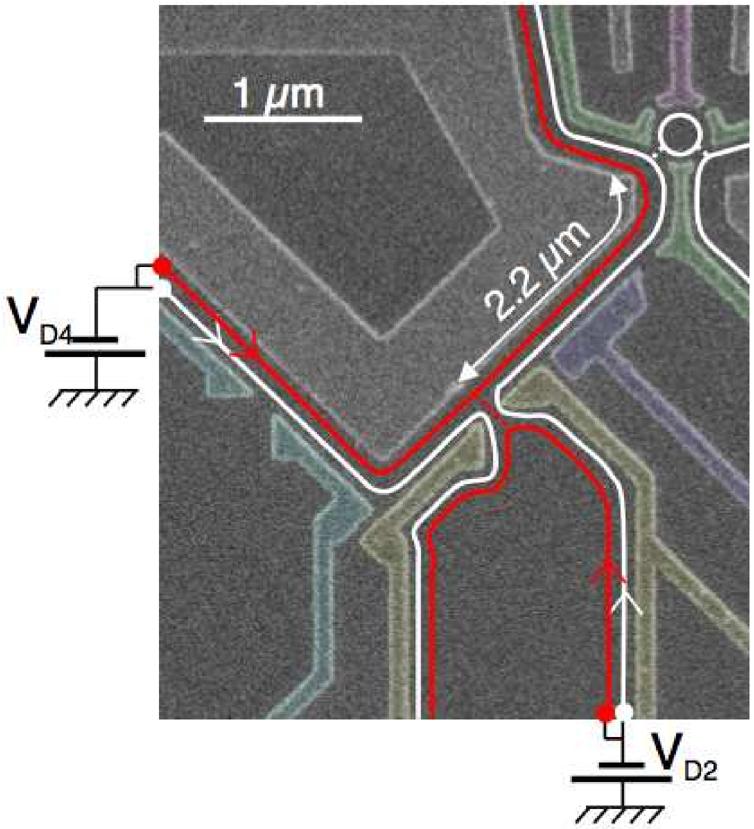
Focussing on non-equilibrium aspects



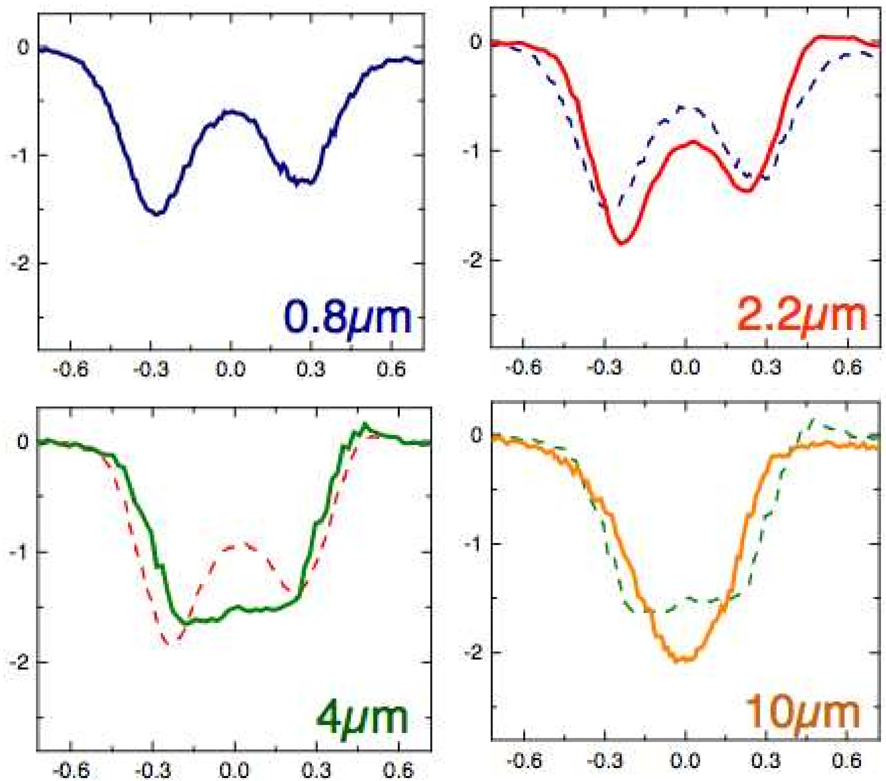
Experiment – Actual

le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

Sample Design



Evolution of Distribution



$\partial n(E)/\partial E$ **vs.** E

Theoretical Idealisation

Evade treatment of point contact - treat quantum quench

Study time evolution in translationally-invariant edge

For approx theory with QPC see: Lunde *et al*, (2010) & Degiovanni *et al* (2010)

Standard model of edge states at $\nu = 1$ & initial state

As electrons

$$\mathcal{H} = -i\hbar v \int \mathrm{d}x \psi^\dagger(x)\partial_x\psi(x) + \int \mathrm{d}x \int \mathrm{d}x'\; U(x-x')\rho(x)\rho(x')$$

As collective modes

$$\mathcal{H} = \sum_q \hbar\omega(q) b_q^\dagger b_q \qquad \omega(q) = [v + u(q)]\, q$$

$$u(q) = (2\pi\hbar)^{-1} \int \mathrm{d}x\; e^{iqx}\, U(x)$$

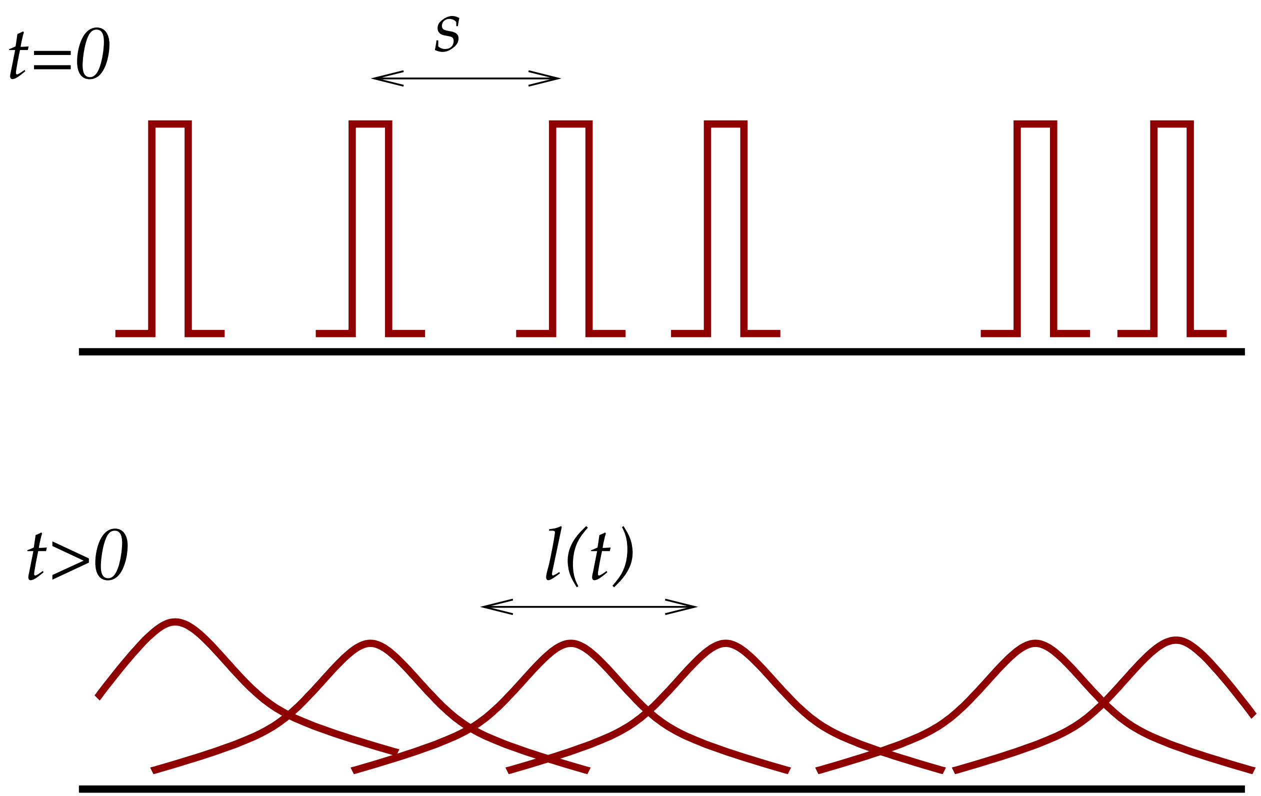
For $\nu > 2 \qquad \psi(x) \to \psi_n(x) \qquad n = 1, \ldots \nu$

Physical picture of equilibration

Edge magnetosplasmon Hamiltonian $\mathcal{H} = \sum_{nq} \hbar\omega_n(q) b^\dagger_{nq} b_{nq}$

Plasmon dispersion $\longrightarrow$ electron equilibration?

Initial quasi-particle separation $s = \hbar v/eV$



Equilibration when wavepacket spread $l(t) \gtrsim s$

Equilibration from two mode velocities

Contact interactions at $\nu = 2$

Edge Hamiltonian $\mathcal{H} = \sum_{nq} \hbar\omega_n(q) b^\dagger_{nq} b_{nq}$

Two linearly dispersing modes $\omega_1(q) = v^{(+)}q$ **&** $\omega_2(q) = v^{(-)}q$

Initial quasi-particle separation $s = \hbar v/eV$

Equilibration when wavepacket spread $l(t) \gtrsim s$

Spread $l(t) = [v^+ - v^-]t$

Equilibration time: $t_{\text{eq}} \sim \frac{\hbar}{eV} \cdot \frac{v^+ + v^-}{v^+ - v^-}$

Equilibration from single mode dispersion

Finite range interactions at $\nu = 1$

Edge Hamiltonian $\mathcal{H} = \sum_q \hbar\omega(q) b_q^\dagger b_q$

Dispersion $\omega(q) = [v + u(q)]\, q \simeq vq - \frac{v}{b}(bq)^3 \ldots$

Wavepacket spread $l(t) \sim b(vt/b)^{1/3}$

Equilibration time $t_{\mathrm{eq}} \sim (\hbar/eV)^3 \cdot (v/b)^2$

Unscreened Coulomb interactions

Dispersion $\omega(q) = [v + u\ln(1/bq)]\, q$

Spread $l(t) \sim ut$ **Equilibration time** $t_{\mathrm{eq}} \sim (\hbar/eV) \cdot (v/u)$

What is the equilibrium state?

Characterise via one-electron correlations

Calculate $$G(x,t) = \langle \psi^\dagger(x,t)\psi(0,t) \rangle$$

in thermal state $$G(x,t) = [-2i\beta\hbar v \sinh(\pi[x+i0]/\beta\hbar v)]^{-1}$$

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Find $G(x,t) \propto \langle \exp(i\int \mathrm{d}y\, K(x,t;y)\rho(y))\rangle$

Scaling form for kernel at long times

$$K(x, x/2 - vt + \xi) \sim F(x/l(t), \xi/l(t))$$

with $l(t) \sim$ **spread** **e.g. for single edge** $l(t) = b(vt/b)^{1/3}$

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Hence simple long-time limit

Find $G(x,t) \propto \exp(-\int \mathrm{d}y\, C(x,y)\langle \rho(0)\rho(y)\rangle)$

with $C(x,y) = \pi^2(|x+y| + |x-y| - 2|y|)$ **indept of** $U(x)$

Comparison with thermal state

Short-distance correlations

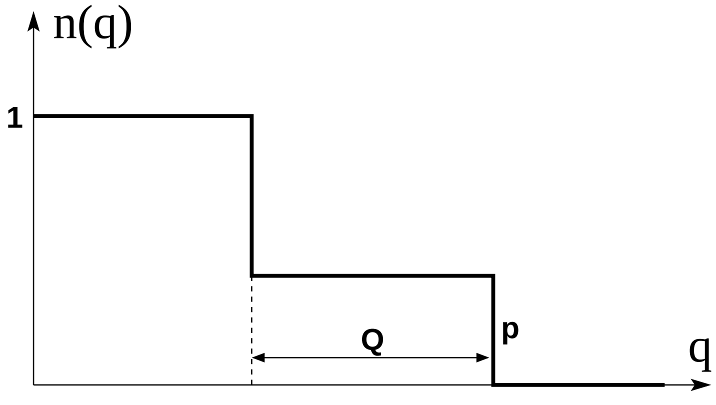
As in thermal state at same energy density

Long-distance correlations

$G(x,t) \sim \exp(-\alpha|x|)$ with α not fixed by energy density

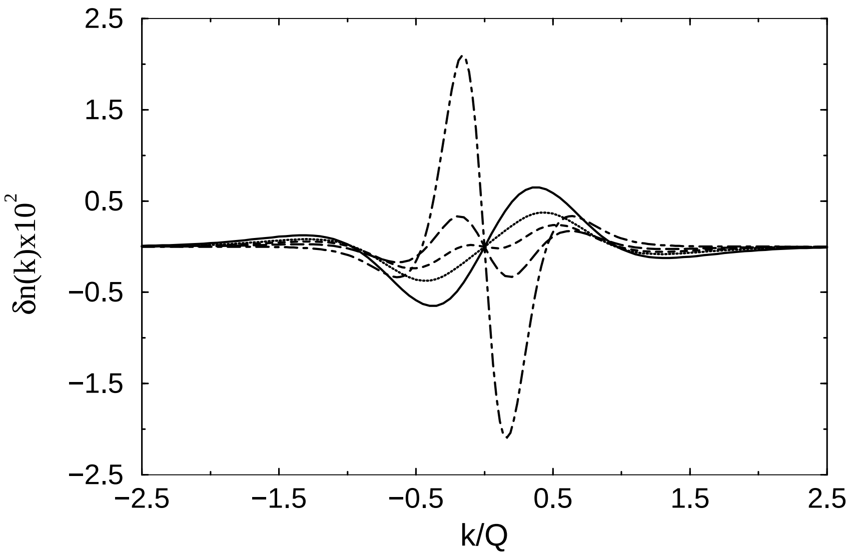
Example

Initial momentum distribution



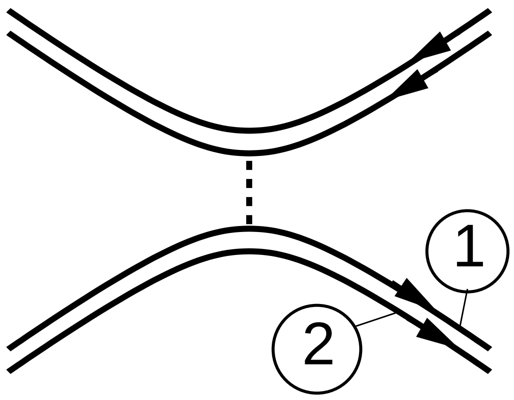
Difference from thermal in steady state

$p = 0.1, 0.2, 0.25, 0.3, 0.5$



Equilibration with two edge modes

Initial State



$$|\psi(t{=}0)\rangle = |\psi_{\mathrm{channel\,1}}\rangle \bigotimes |\psi_{\mathrm{channel\,2}}\rangle$$

Hamiltonian

$$H = \sum_k \hbar k \left[v_1\, a_k^\dagger a_k + v_2\, b_k^\dagger b_k + g(a_k b_k^\dagger + a_k^\dagger b_k) \right]$$

$$H = \sum_k \hbar k \left[v^{(+)} \alpha_k^\dagger \alpha_k + v^{(-)} \beta_k^\dagger \beta_k \right]$$

Mixing angle $\alpha_k = \cos\theta \ a_k + \sin\theta \ b_k \qquad \tan 2\theta = g/2\hbar(v_1 - v_2)$

Results for $\nu = 2$

Calculate $G_n(x,t) = \langle \psi_n^\dagger(x,t)\psi_n(0,t)\rangle \qquad n = 1,2$

In steady state:

Thermal at short distances, but with two effective temperatures

For channel 1 $T_{\mathrm{steady}} = \left[f\, T^2_{\mathrm{initial}\,1} + (1-f)T^2_{\mathrm{initial}\,2}\right]^{1/2}$

$$f = 1 - \tfrac{1}{2}\sin^2 2\theta$$

Long-distance form

$$G_n(x,t) \sim \exp(-\alpha_n|x|)$$

Interchannel equilibration: • **not complete** $\alpha_1 \neq \alpha_2$

• **not thermal – independent** α_1, α_2 **and** T_{steady} **for each channel**

Summary - Relaxation in QH edges

‘Quantum quench’ on isolated edge is useful caricature

of experiment with two edges coupled at QPC

- **Interactions bring system into non-thermal steady state**
- **At $\nu = 1$ steady state is indept of interactions**
- **Correlation function in steady state is**

  functional of initial momentum distribution
- **At $\nu = 2$ steady state depends on**

  coupling between channels and initial momentum distribution
- **At $\nu = 2$ no equipartition of energy between channels**