

## Simulation of Ductile Yield and Plastic Deformation: Size/Rate/Temperature Effects

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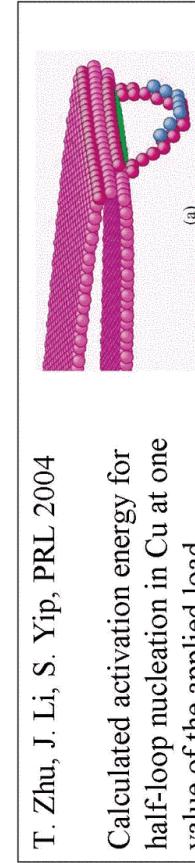
Supported by NSF-DMR and NIST-CTCMMS

### Questions about dislocation nucleation at a crack tip:

Should we expect Arrhenius behavior?

Is there only one relevant energy barrier?

How does the activation energy depend on stress intensity factor?



Our approach:

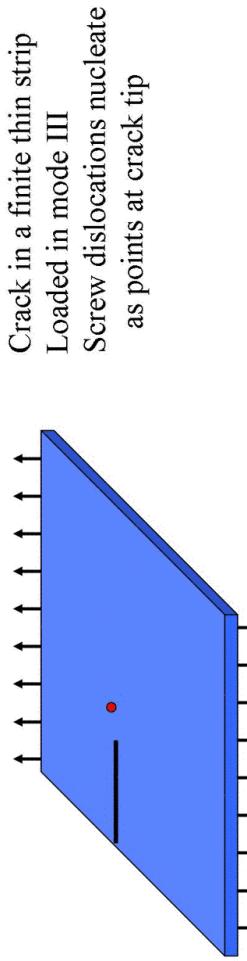
Simulate repeated emission events at different values of T, load

Determine activation energy from an Arrhenius plot

Includes effects of both nucleation and mobility

Strategy: simulate in 2-d, then compare with analytical solution

Consider problem in two dimensions...



Crack in a finite thin strip  
Loaded in mode III  
Screw dislocations nucleate  
as points at crack tip

Apply subcritical load  
→ at T=0, no nucleation  
→ at T>0, thermal activation

Dislocations nucleate at crack tip, annihilate on opposite free surface

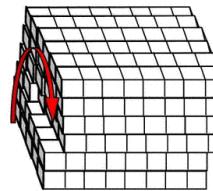
### Idealized molecular dynamics:

- antiplane displacements only- 2-d strain field  $z(x,y)$
- idealized simple potential between columns

$$H = \sum_{\langle i,j \rangle} -K_{ij} \cos\left(\frac{z_i - z_j}{a}\right) + \frac{1}{2} m \sum_i \left( \frac{dz_i}{dt} \right)^2$$

Potential

Kinetic



2-d strain field

Screw dislocation

System can only contain straight screw dislocations;  
no loops or half-loops

Screw dislocation nucleates as straight line

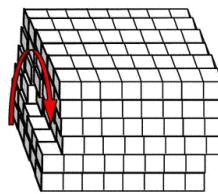
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**Potential**      **Kinetic**

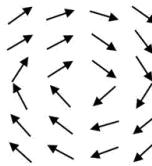
This is just the XY “rotor” model

$$H = \sum_{\langle i,j \rangle} -K_{ij} \cos(\theta_i - \theta_j) + \frac{1}{2} I \sum_i \left( \frac{d\theta}{dt} \right)^2$$

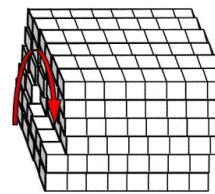


Screw dislocation

Column height  $z_i$  maps to XY spin orientation  $\theta_i$



Screw dislocation maps to a vortex



Screw dislocation

**Idealized molecular dynamics:**

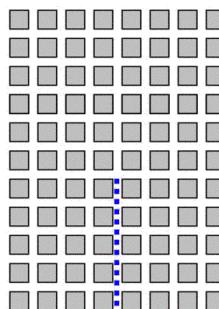
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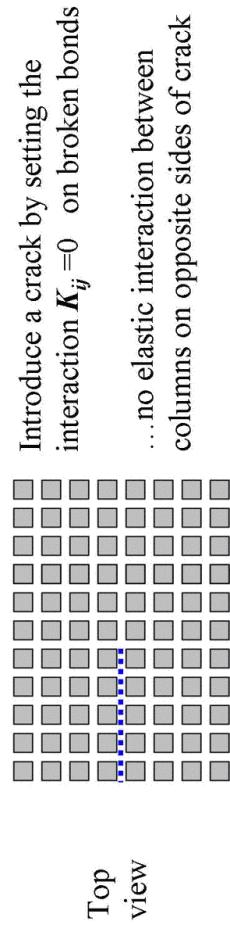


Top  
view

Introduce a crack by setting the interaction  $K_{ij} = 0$  on broken bonds  
...no elastic interaction between columns on opposite sides of crack

- Derive eqn. of motion from Hamiltonian
- Load forces applied at edges of cell
- Apply thermostat on edges of cell
- Other bonds cannot break

- Crack can only emit pure screw dislocations  
 → Crack cannot extend  
 → Pure, ideal ductile response

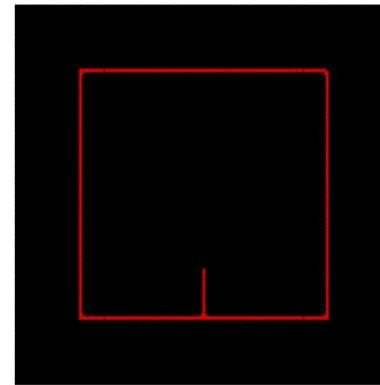


100 x 100, crack length = 20

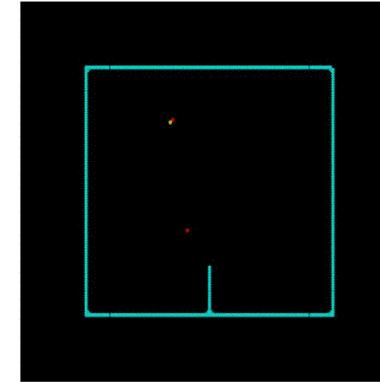
Low temperature

Defects nucleate one at a time at the crack tip

High temperature  
 (Note:  $T_{KT} = 0.895$ )  
 Nucleation rate limited by mobility  
 Occasional nucleation at wall

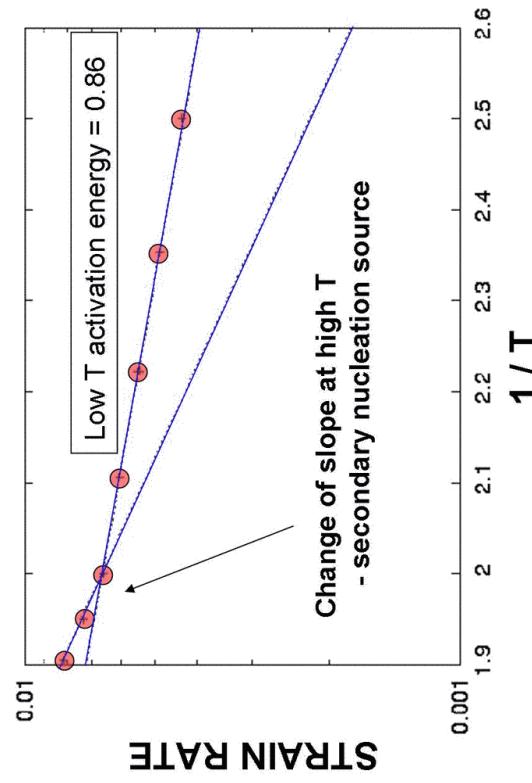


$T = 0.35$



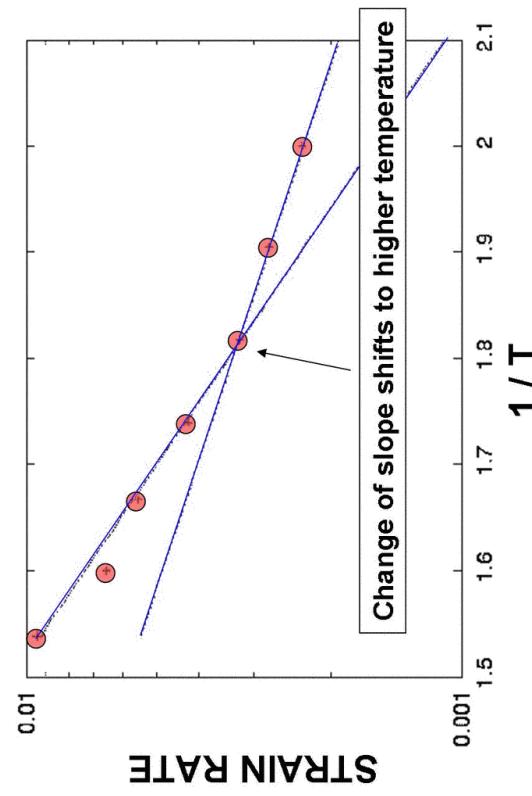
$T = 0.5$

Arrhenius plot shows thermal activation at low T



Load=0.05

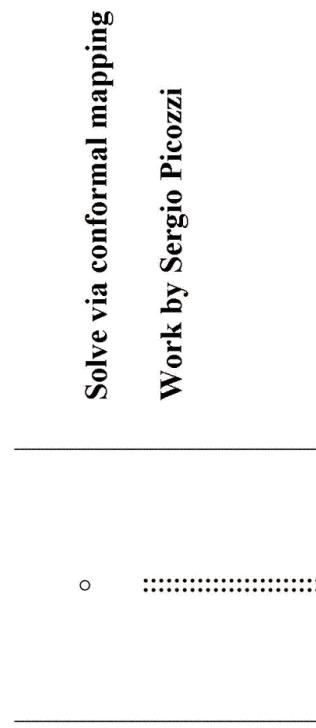
Reduce load by 20%, low-T activation energy increases  
by a factor of 2 (from 0.86 to 1.87 energy units)



Load=0.04

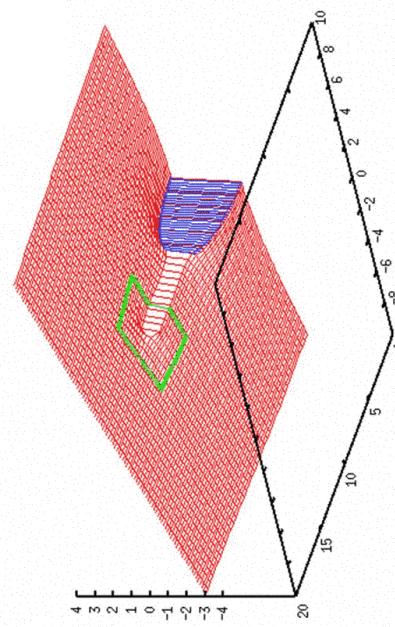
## 2-d analytical calculation: dislocation escape from crack tip

Need to know the effective potential experienced by a screw dislocation near a crack tip at Temperature=0, under shear load



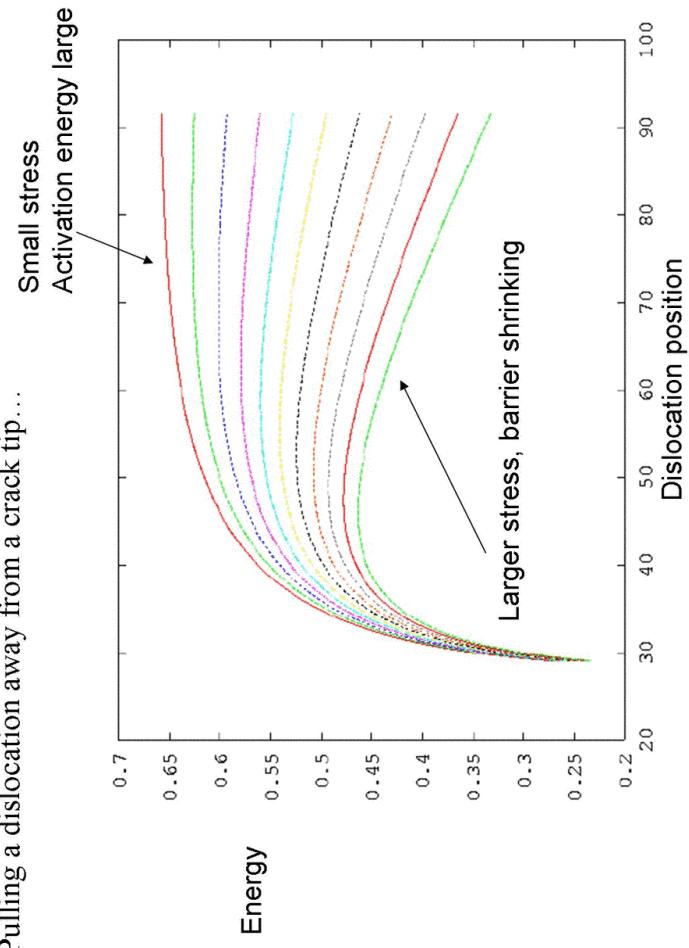
Solve via conformal mapping

Work by Sergio Picozzi



Analytical solution for screw dislocation near a crack tip in a strip with a sharp crack—from conformal mapping calculation

Pulling a dislocation away from a crack tip...

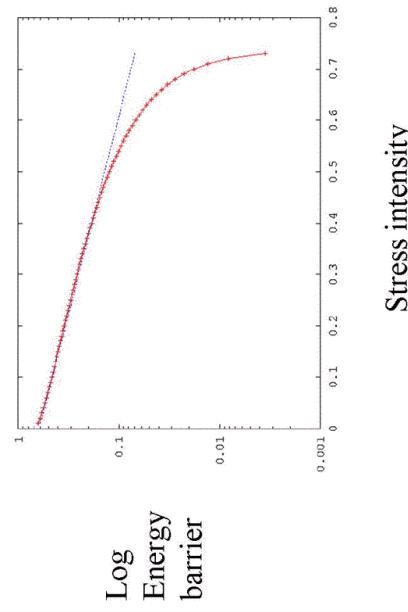
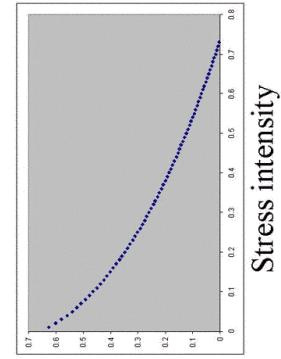


For large enough stress, barrier vanishes entirely.

Result from analytic calculation:

Barrier drops exponentially with load  
Energy barrier

- activation volume not well defined
- stress at crack tip also not well defined!



What we learned:

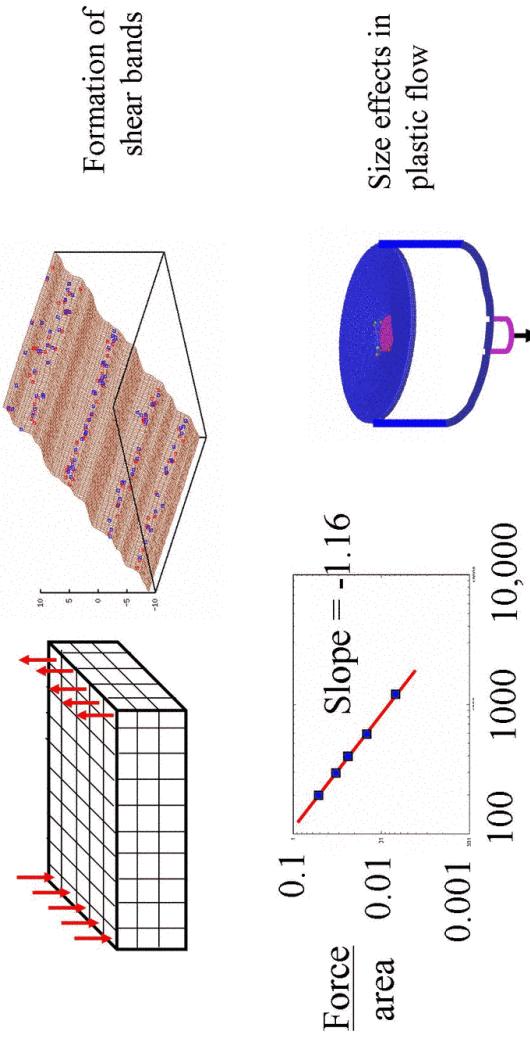
- 2-d dislocation nucleation rate at a crack tip shows Arrhenius behavior; slope changes when secondary source activated
- Activation energy highly sensitive to applied stress intensity factor  
... increased by factor of 2 when load reduced by 20%
- Concept of activation volume not well defined  
(Not big surprise; stress at crack tip not well defined either)
- Analytical calculation suggests that energy barrier drops exponentially with applied load

Still to figure out:

Does this tell us anything new about nucleation in 3-d???

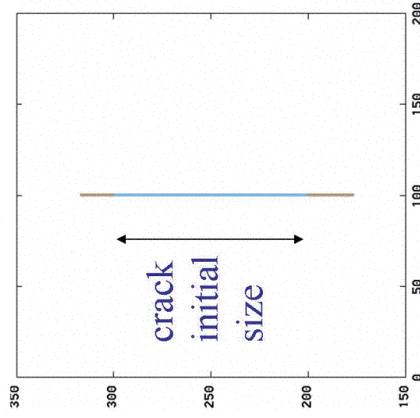
Good model for nucleating vortices from a notch in a superconducting thin film?

Other fun things to do with this idealized model....

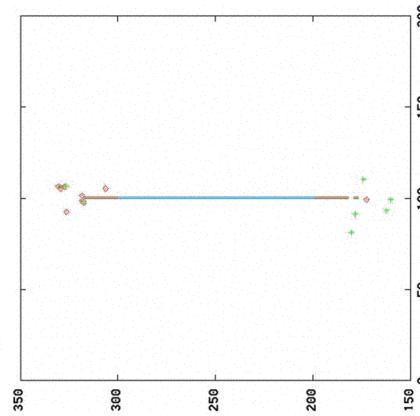


If add bond-breaking rules, apply constant strain rate, see brittle-ductile transition—“toy model”

Low T, high strain rate



High T, low strain rate

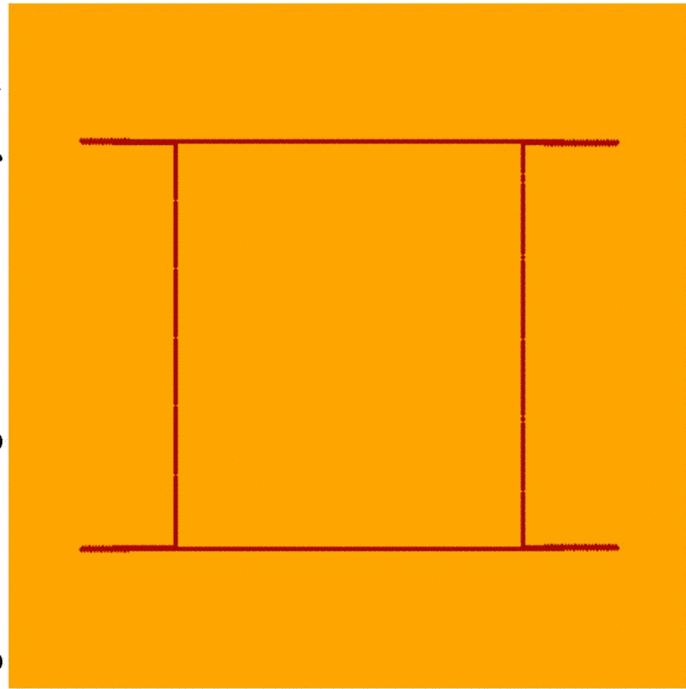


Brittle—just bond breaking

Ductile—crack arrests, emits dislocations; new cracks nucleate ahead of crack tip, coalesce

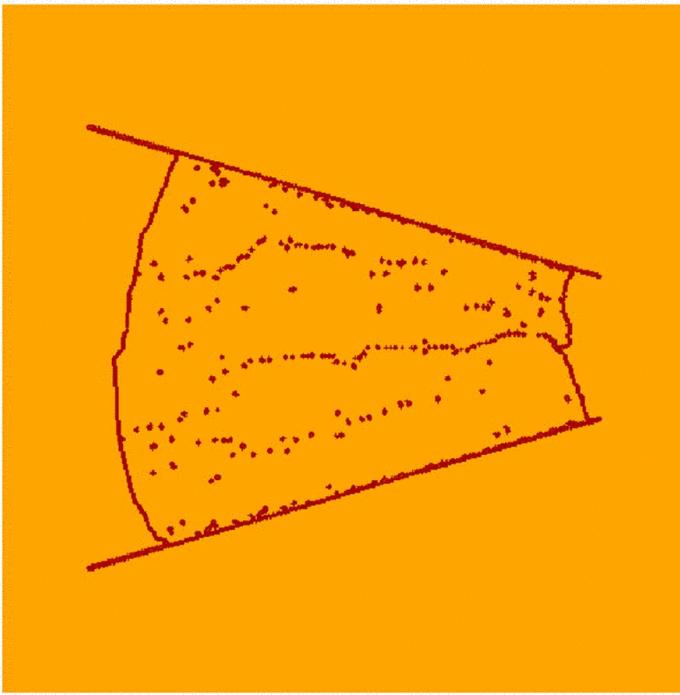
## Dislocation patterning in 2-d

Scott Weingarten-bending Lennard-Jones crystal, Monte Carlo



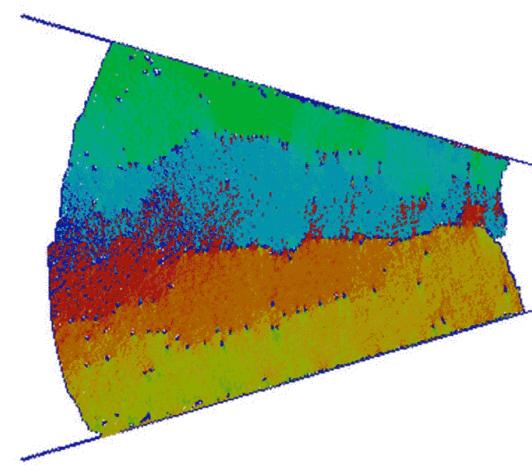
### Atomistic simulation of bending

73879 particles =  $270 \times 270 +$  walls, 2 million MC steps



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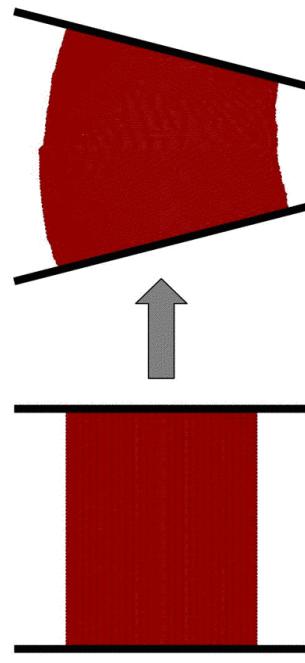
Final configuration

## Simulation Technique

- 2-D Lennard-Jones single crystal, triangular lattice
- Rigid walls rotate, apply compressive strain
- System evolves via off-lattice Monte Carlo (Metropolis algorithm)
- Effective strain rate depends on MC steps per bend increment

## Why Monte Carlo?

- System is always driven toward thermal equilibrium
- No thermal gradients or shockwaves
- No momentum, dislocation motion always overdamped

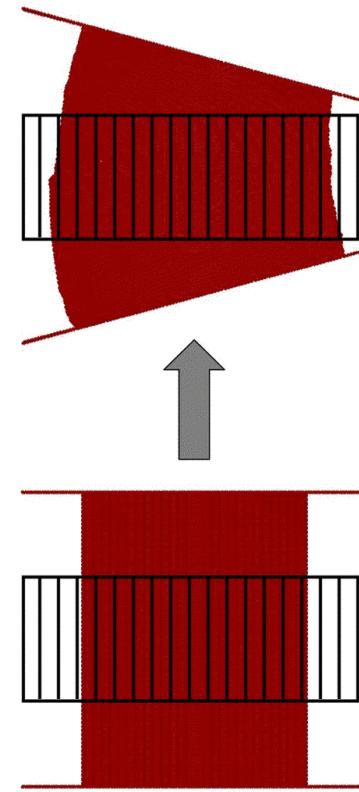


## Mechanical Response quantified via calculation of the Bending Moment

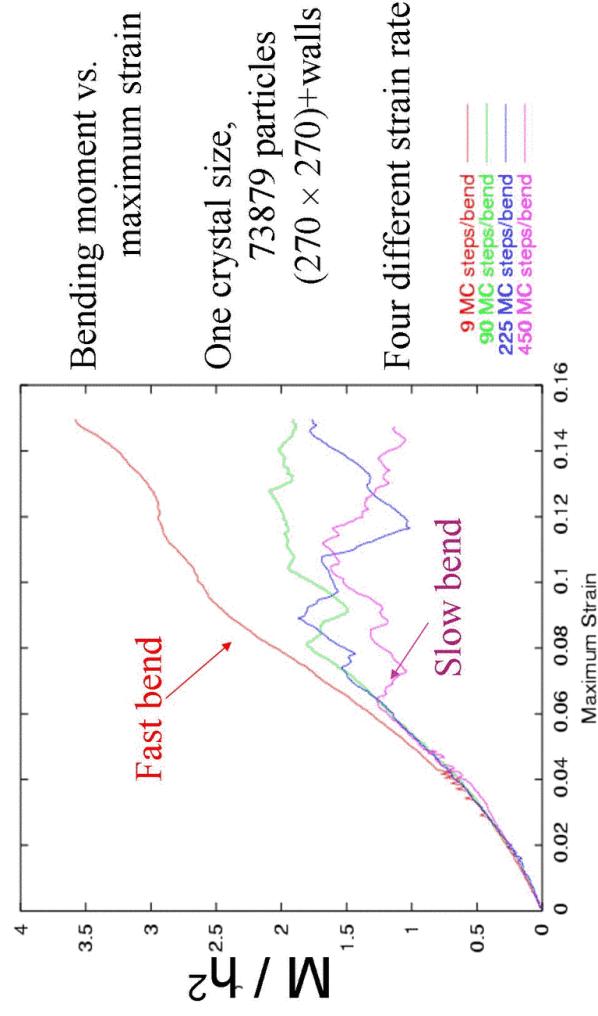
$$\text{Bending moment: } M = \int \sigma_{xx}(y) y dy$$

Stress calculated from interatomic forces in each layer, integrated numerically

Calculated using middle half of crystal, away from contact force:

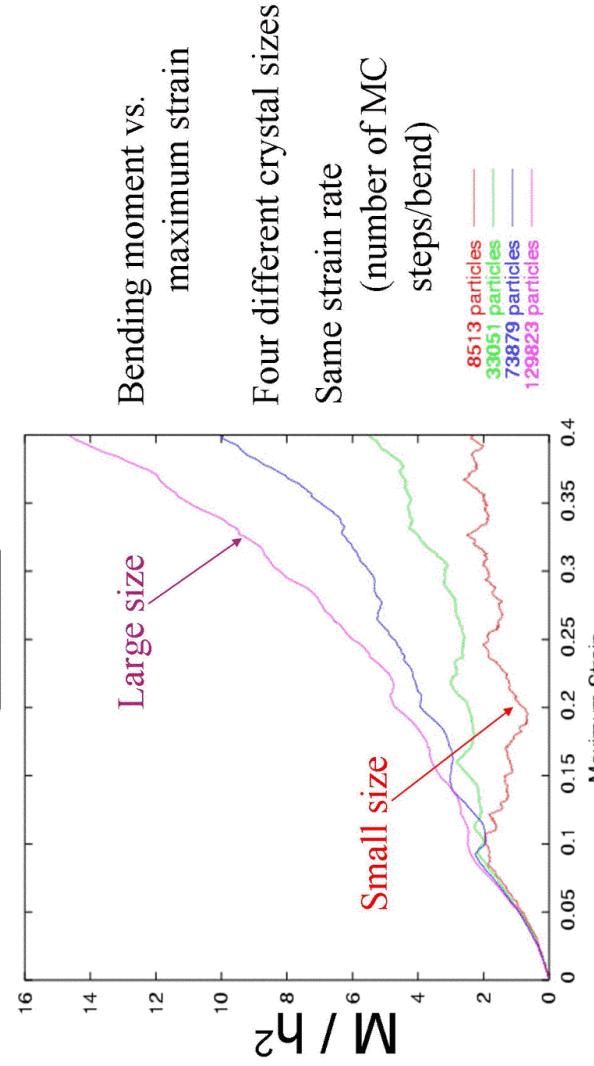


## Mechanical response depends on strain rate



Simulations approach (but don't reach) quasistatic limit

## Size effect at constant strain rate: Inverse Size Effect



Results in an apparent REVERSE size effect: larger  $\rightarrow$  stronger

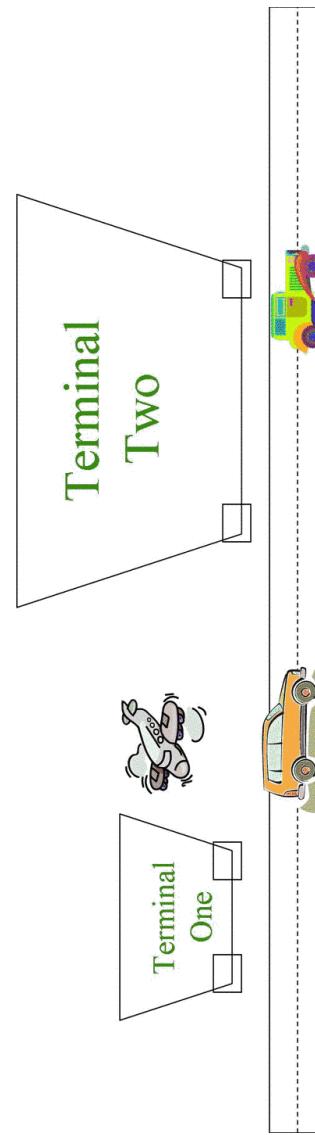
## Inverse size effect in single crystal

If scale system size by a factor of 2 with same strain rate,  
...should have 4 times as many total dislocations

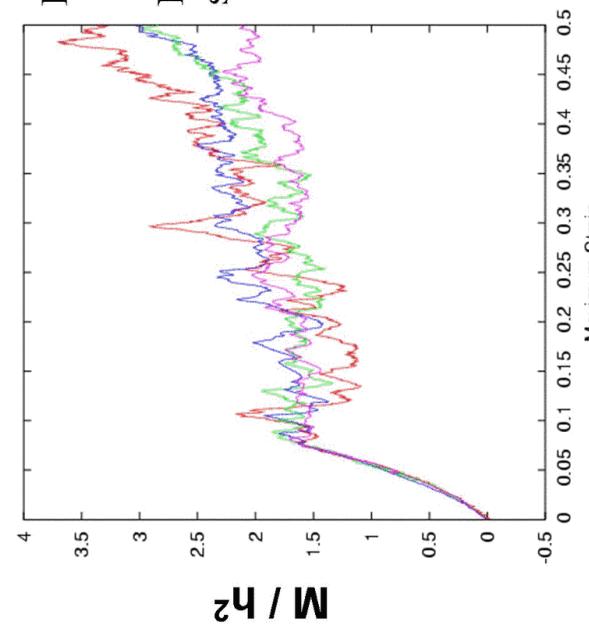
Dislocation nucleation rate at corners can't rise by a factor of 4

Larger system has too few dislocations → stress rises  
→ higher bending moment

### Inverse size effect due to dislocation “starvation”



**Size effect disappears when steps/bend scaled with area:  
increase atoms by 4, increase cpu time by 16**

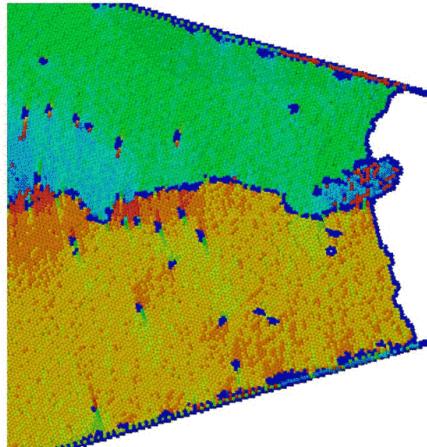
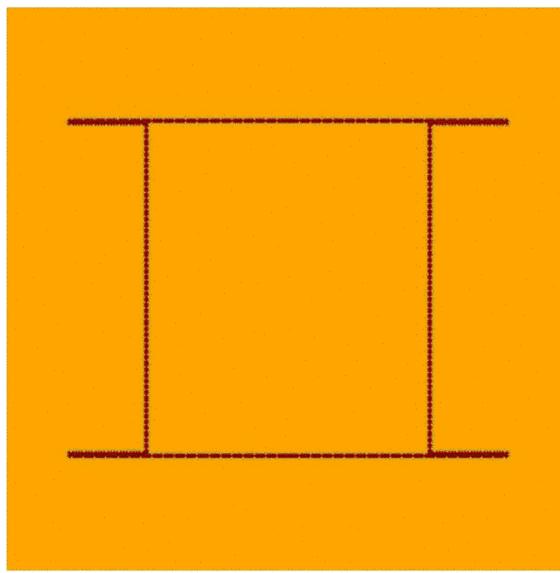


Bending moment vs.  
maximum strain  
Number of MC steps/bend  
scaled to area:

Particles	Steps/Bend
90 × 90	25
180 × 180	100
270 × 270	225
360 × 360	400

**No Size Effect!!!**

Grain ejection... hillock formation?



#### Mechanics of materials in 2-d

Even without entanglement and other 3-d effects, we get...

- Thermal activation at a crack tip
  - Arrhenius behavior
  - Change of slope with activation of secondary source
  - Analysis--Activation energy drops exponentially with load
- Dislocation patterning
  - shortage of sources  $\rightarrow$  reverse size effect
    - size effect goes away when strain rate scaled
- Grain boundary instability
  - hillock formation/grain ejection

Question for future work...

Is coalescence of dislocations into grain boundaries a phase transition?

