A generalized law for aftershock behavior in a damage rheology model

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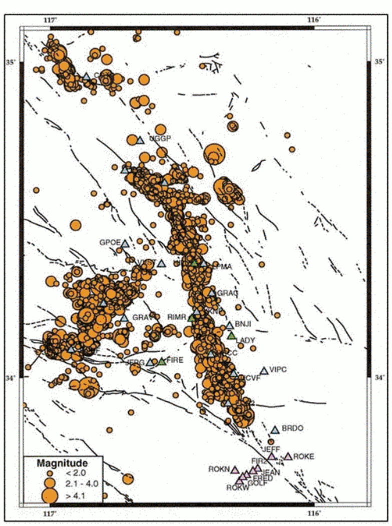
Outline

- Brief background on aftershocks
- Brief background on the employed damage rheology
- 1-D Analytical results on aftershocks
- 3-D Numerical results on aftershocks
- Discussion and Conclusions

Main observed features of aftershock sequences:

1. Aftershocks occur around the mainshock rupture zone



5. Aftershocks behavior is NOT universal!

2. Aftershock decay rates can be described by the modified Omori law:

$$\Delta N/\Delta t = K(c + t)^{-p}$$

However, aftershock decay rates can also be fitted with exponential and other functions (e.g., Kisslinger, 1996).

3. The frequency-size statistics of aftershocks follow the GR relation:

$$\log N(M) = a - bM$$

4. The largest aftershock magnitude is typically about 1-1.5 units below that of the mainshock (Båth law).

Existing aftershock models:

- Migration of pore fluids (e.g., Nur and Booker, 1972)
- Stress corrosion (e.g., Yamashita and Knopoff, 1987)
- Criticality (e.g., Bak et al., 1987; Amit et al., 2005)
- Rate- and state-dependent friction (Dieterich, 1994)
- Fault patches governed by dislocation creep (Zöller et al., 2005).

Is the problem solved?

The above models focus primarily on rates.

Some are "conceptual" rather than quantitative.

None explains properties (1)-(5), including the observed spatio-temporal variability, in terms of basic geological and physical properties.

This is done here with a damage rheology framework and realistic model of the lithosphere.

Non-linear Continuum Damage Rheology

The employed damage rheology accounts for 3 universal aspects of rock deformation under large strain:

- **Mechanical aspect:**

  The elastic moduli depend on the density of microcracks (damage)

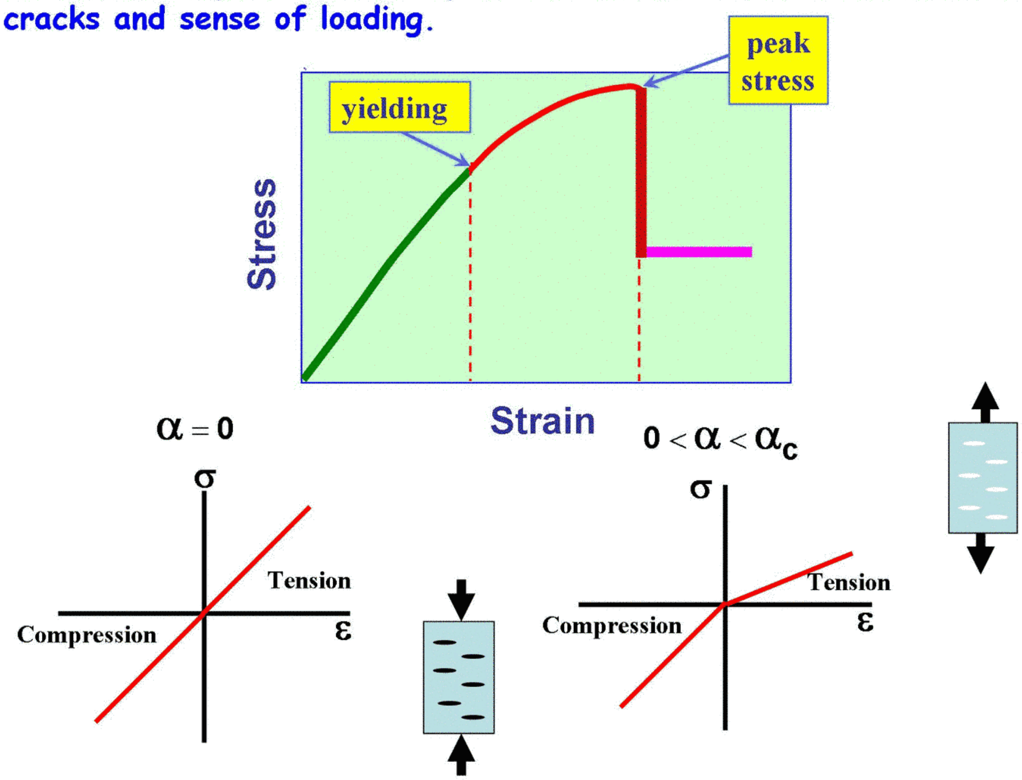
- **Kinetic aspect:**

  The microcrack density (damage) evolves with ongoing deformation

- **Dynamic aspect:**

  Brittle instability at a critical level of damage (when the energy function losses convexity & dynamic weakening correction)

Mechanical aspect: sensitivity of the elastic moduli to distributed cracks and sense of loading.



This is accounted for by generalizing the strain energy function of a deforming solid

The elastic energy U is written as:

$$U = \frac{1}{\rho}\left(\frac{\lambda}{2} I_1^2 + \mu I_2 - \gamma I_1 \sqrt{I_2}\right) \qquad \xi = \frac{I_1}{\sqrt{I_2}}$$

Where λ and μ are Lame constants; γ is an additional elastic modulus

$I_1 = \varepsilon_{kk}$

$I_2 = \varepsilon_{ij}\varepsilon_{ij}$

$$\sigma_{ij} = \rho \frac{\partial U}{\partial \varepsilon_{ij}} = \left(\lambda - \gamma \frac{\sqrt{I_2}}{I_1}\right) I_1 \delta_{ij} + \left(2\mu - \gamma \frac{I_1}{\sqrt{I_2}}\right) \varepsilon_{ij}$$

Origin of the generalized energy function

Consider a general strain energy function having any second-order term of the type

$$U \propto I_1^{2x} \cdot I_2^{1-x} \qquad \text{with} \quad 0 < x < 1,\ I_1 = \varepsilon_{kk},\ I_2 = \varepsilon_{ij}\varepsilon_{ij}$$

The limit values $x = 0$ and $x = 1$ are associated with the standard 2 Hookean terms.

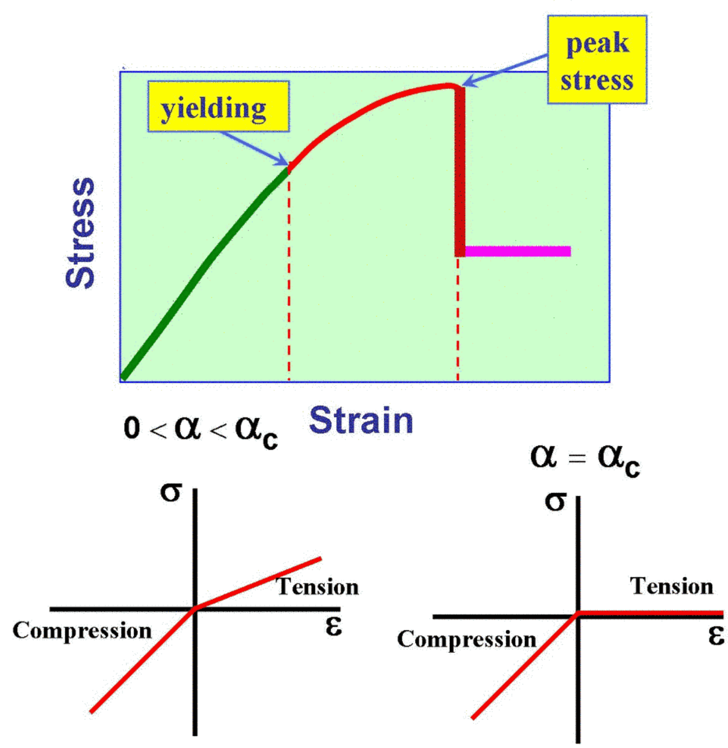
The relation between the mean stress (σ_{kk}) and volumetric deformation (I_1) for the assumed general form is

$$\sigma_{kk} = \rho \frac{\partial U}{\partial I_1} \sim 2x I_1^{2x-1} I_2^{1-x} + \frac{2}{3}(1-x) I_1^{2x+1} I_2^{-x}$$

The first term has a nonphysical singularity for $0 < x < 1/2$. In addition, for $x > 1/2$ the mean stress is zero ($\sigma_{kk} = 0$) for zero volumetric strain ($I_1 = 0$). This is not compatible with material dilation under shear loading.

Thus the only exponent (other than the classical 0 and 1 values) associated with realistic rock deformation is the $x = 1/2$ value represented by the third term in our energy function.

Kinetic aspect associated with damage evolution



This is accounted for by making the elastic moduli functions of a damage state variable $\alpha(x, y, z, t)$, representing crack density in a unit volume, and deriving an evolution equation for α.

Thermodynamics

Free energy of a solid, F, is

$$F = F(T, \varepsilon_{ij}, \alpha)$$

T – temperature, ε_{ij} – elastic strain tensor,
α – scalar damage parameter

Energy balance

$$\frac{dU}{dt} = \frac{d}{dt}(F + TS) = \frac{1}{\rho}\sigma_{ij}e_{ij} - \nabla_i J_i$$

Entropy balance

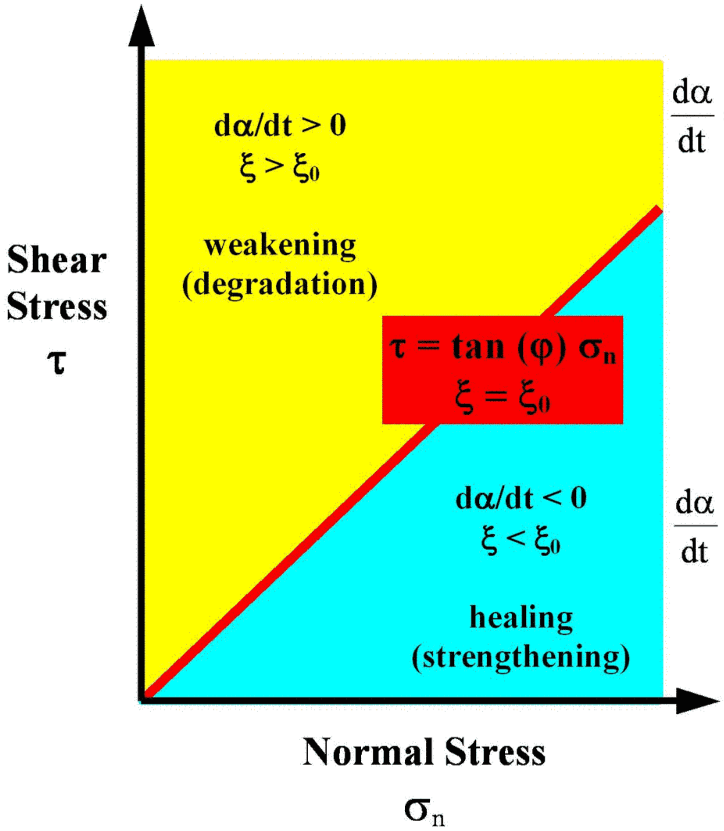
$$\frac{dS}{dt} = -\nabla_i\left(\frac{J_i}{T}\right) + \Gamma$$

Gibbs equation

$$dF = -SdT + \frac{\partial F}{\partial \varepsilon_{ij}} d\varepsilon_{ij} + \frac{\partial F}{\partial \alpha} d\alpha$$

The internal entropy production rate per unit mass, Γ, is:

$$\Gamma = -\frac{J_i}{\rho T^2}\nabla_i T + \frac{1}{T}\sigma_{ij}e_{ij} - \frac{1}{T}\frac{\partial F}{\partial \alpha}\frac{d\alpha}{dt} \geq 0$$



$$\frac{d\alpha}{dt} = C_d \cdot I_2(\xi - \xi_0)$$

$$\xi = \frac{I_1}{\sqrt{I_2}}$$ **Strain invariant ratio**

$I_1 = \varepsilon_{kk}$

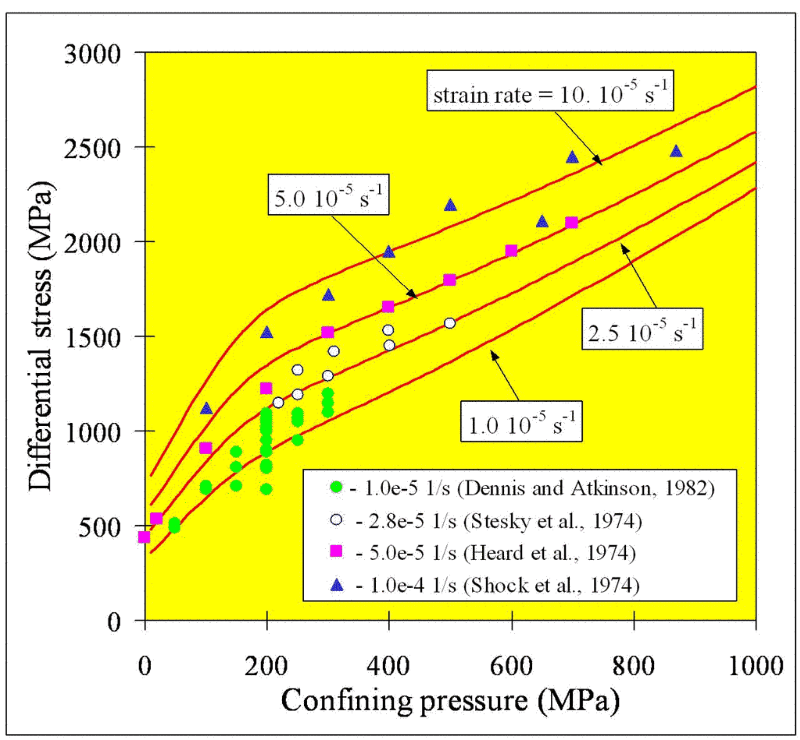
$I_2 = \varepsilon_{ij}\varepsilon_{ij}$

$$\frac{d\alpha}{dt} = C_1 \cdot \exp\left(\frac{\alpha}{C_2}\right) \cdot I_2(\xi - \xi_0)$$

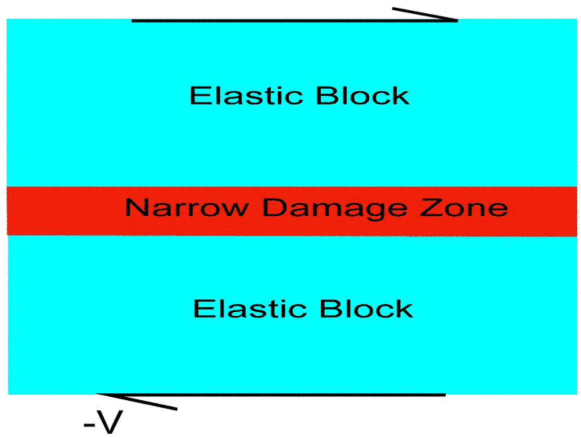
Fracturing intact rock experiments constrain parameters C_d and ξ_0.

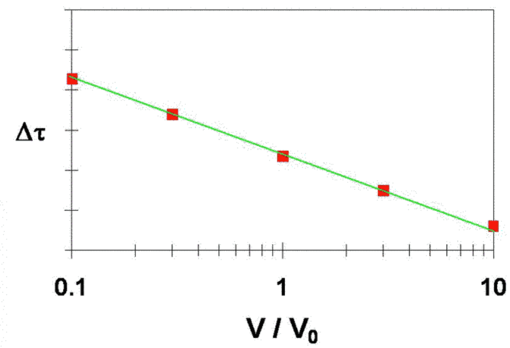
Lyakhovsky *et al.* (JGR, 97; GJI, 05)

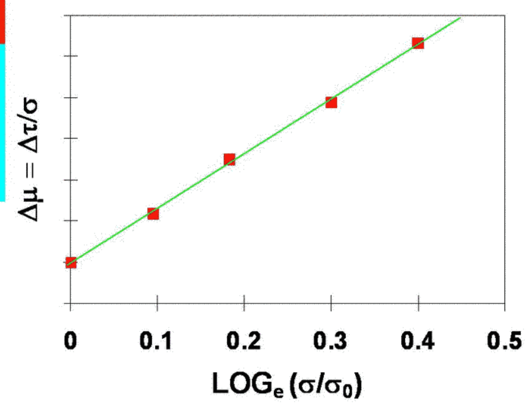
Ultimate stress for Westerly Granite



Rate- and state-dependent friction experiments constrain parameters c_1 and c_2.

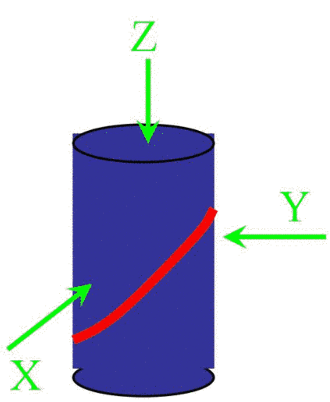


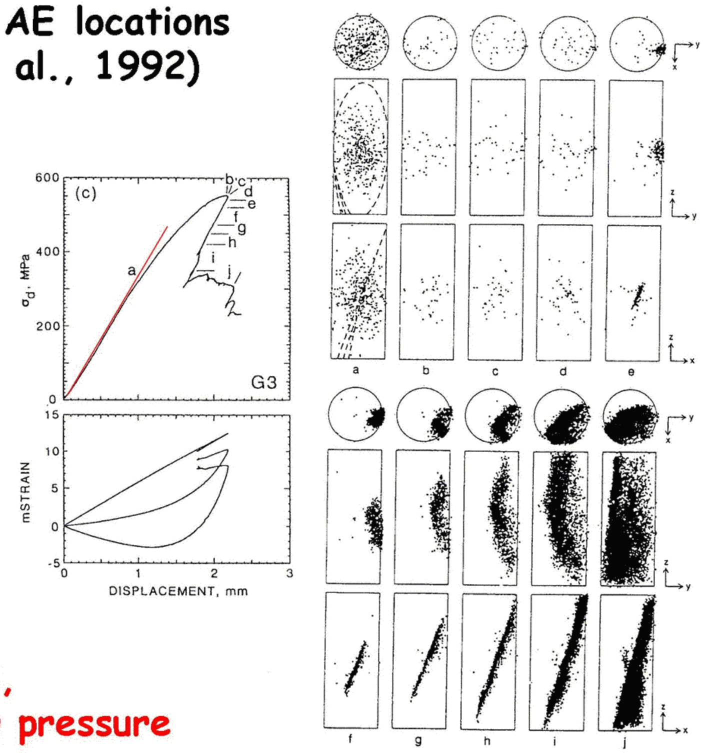




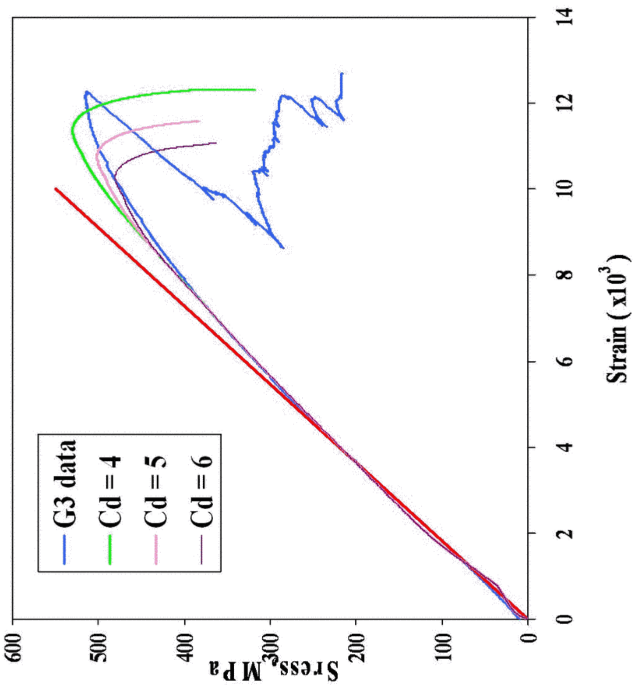
For details, see Lyakhovsky *et al.* (GJI, 2005)

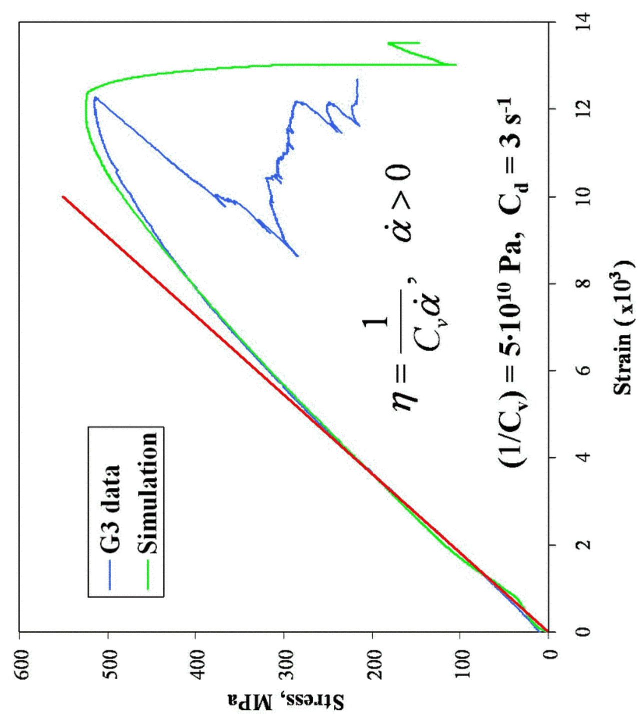
Stress-strain and AE locations for G3 (Lockner et al., 1992)

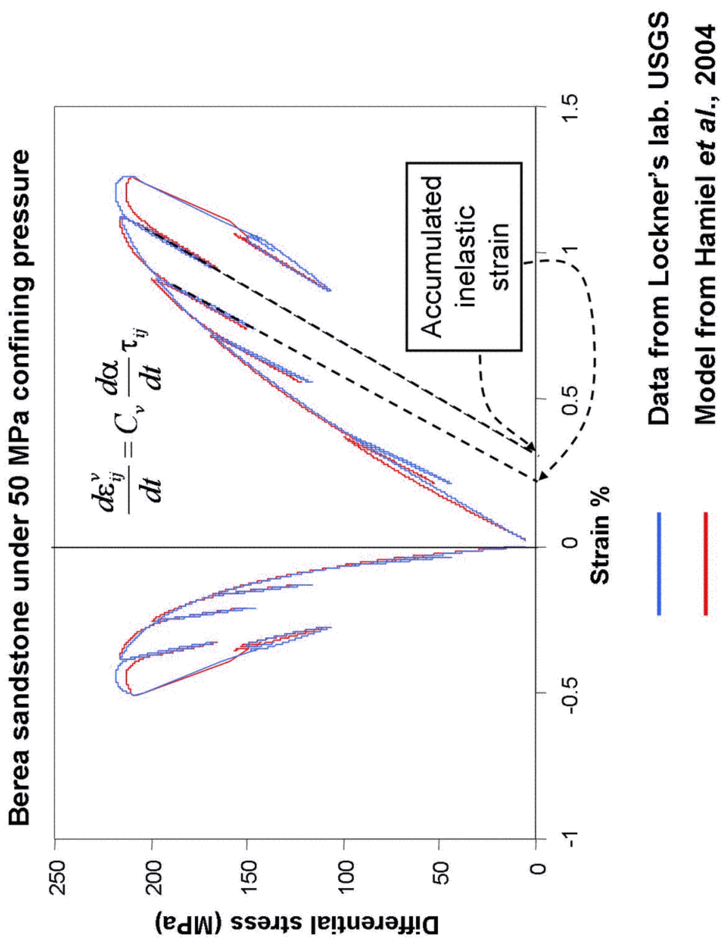




Westerly granite,
50 MPa confining pressure


G3 data
Cd = 4
Cd = 5
Cd = 6
Strain (x10^3)
Stress, M Pa


G3 data
Simulation
$\eta = \frac{1}{C_v \dot{\alpha}}, \quad \dot{\alpha} > 0$
$(1/C_v) = 5 \cdot 10^{10}$ Pa, $C_d = 3\ s^{-1}$
Strain (x10^3)
Stress, MPa



What about aftershocks?

Aftershocks: 1D analytical results for uniform deformation

For 1D deformation, the equation for positive damage evolution is

$$d\alpha/dt = C_d\,(\varepsilon^2-\varepsilon_0^2), \qquad (1)$$

where ε is the current strain and ε_0 separates degradation from healing.

The stress-strain relation in this case is

$$\sigma = 2\mu_0(1-\alpha)\varepsilon, \qquad (2)$$

where $\mu_0(1-\alpha)$ is the effective elastic modulus of a 1D damaged material with μ_0 being the initial modulus of the undamaged solid.

(Ben-Zion and Lyakhovsky [2002] showed analytically that these equations lead under constant stress loading to a power law time-to-failure relation with exponent 1/3 for a system-size brittle event).

For positive rate of damage evolution ($\varepsilon > \varepsilon_0$), we assume inelastic strain before macroscopic failure in the form

$$e = (C_v\, d\alpha/dt)\,\sigma \qquad (3)$$

For aftershocks, we consider material relaxation following a strain step. This corresponds to a situation with a boundary conditions of constant total strain.

In this case the rate of elastic strain relaxation is equal to the viscous strain rate,

$$2d\varepsilon/dt = -e \qquad (4)$$

Using this condition in (2) and (3) gives $\frac{d\varepsilon}{dt} = -C_v\mu_0(1-\alpha)\cdot\varepsilon\,\frac{d\alpha}{dt}$ (5)

and integrating (5) we get $\varepsilon = A\cdot exp\left[\frac{1}{2}R(1-\alpha)^2\right]$ (6)

where $R = \tau_d/\tau_M = \mu_0 C_v$ and $A = \varepsilon_s\cdot\exp\left[-\frac{1}{2}R(1-\alpha_s)^2\right]$ is integration constant with $\alpha = \alpha_s$ and $\varepsilon = \varepsilon_s$ for $t = 0$.

Using these results in (1) yields exponential damage evolution

$$\frac{d\alpha}{dt} = C_d\cdot\left\{\varepsilon_s^2\exp\left[R(1-\alpha)^2 - R(1-\alpha_s)^2\right] - \varepsilon_0^2\right\} \qquad (7)$$

Scaling the results to number of events N

Assuming that α is scaled linearly with the number of aftershocks N

$$\alpha = \alpha_s + \phi N \qquad (8)$$

we get

$$\phi \frac{dN}{dt} = C_d \cdot \left\{ \varepsilon_s^2 \exp\left[R(1-\alpha_s - \phi N)^2 - R(1-\alpha_s)^2 \right] - \varepsilon_0^2 \right\} \qquad (9)$$

If ϕN is small (generally true), so that $(\phi N)^2$ can be neglected

$$\phi \frac{dN}{dt} = C_d \cdot \left\{ \varepsilon_s^2 \exp\left[-2\phi N R(1-\alpha_s) \right] - \varepsilon_0^2 \right\} \qquad (10)$$

If also the initial strain induced by the mainshock is large enough so that

$$\varepsilon_0^2 << \varepsilon_s^2 \exp\left[-2\phi N R(1-\alpha_s) \right] \qquad (11)$$

the solution is (the modified Omori law)

$$\frac{dN}{dt} = \frac{C_d \varepsilon_s^2}{2\phi R(1-\alpha_s) C_d \varepsilon_s^2 t + \phi} \qquad (12)$$

For $t = 0$

$$\dot{N}_0 = \frac{C_d \varepsilon_s^2}{\phi}$$

so

$$\frac{dN}{dt} = \frac{\dot{N}_0}{2\phi R(1-\alpha_s)\dot{N}_0 t + 1} = \frac{\dot{N}_0}{2\phi R(1-\alpha_s)\dot{N}_0} \cdot \frac{1}{t + 1/2\phi R(1-\alpha_s)\dot{N}_0} \qquad (13)$$

The parameters of the modified Omori law are

$$dN/dt = K(c+t)^{-p}$$

$$k = \frac{1}{2\phi R(1-\alpha_s)}$$

$$c = \frac{1}{2\phi R(1-\alpha_s)\dot{N}_0} = \frac{k}{\dot{N}_0}$$

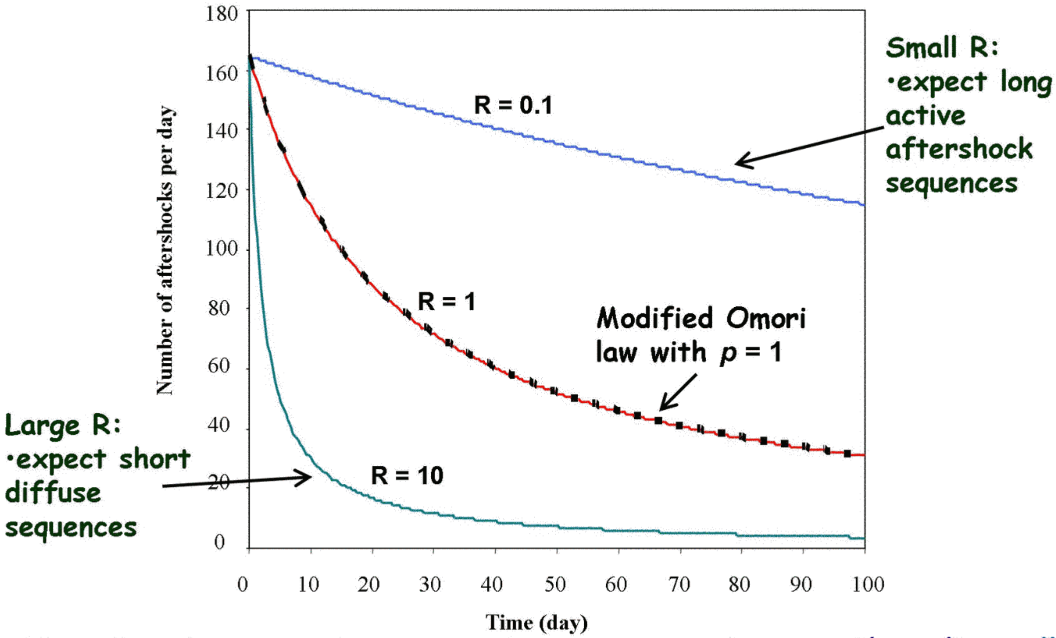
and $p = 1$

We now return to the general exponential equation (9) and examine analytical results first with ε_0=0, α_s=0 and then with finite small values.

$$\phi \frac{dN}{dt} = C_d \cdot \left\{ \varepsilon_s^2 \exp\left[R(1-\alpha_s - \phi N)^2 - R(1-\alpha_s)^2 \right] - \varepsilon_0^2 \right\} \qquad (9)$$

Events rate vs. time for several values of $R = \tau_d/\tau_M$ (with ε_0=0, α_s=0)

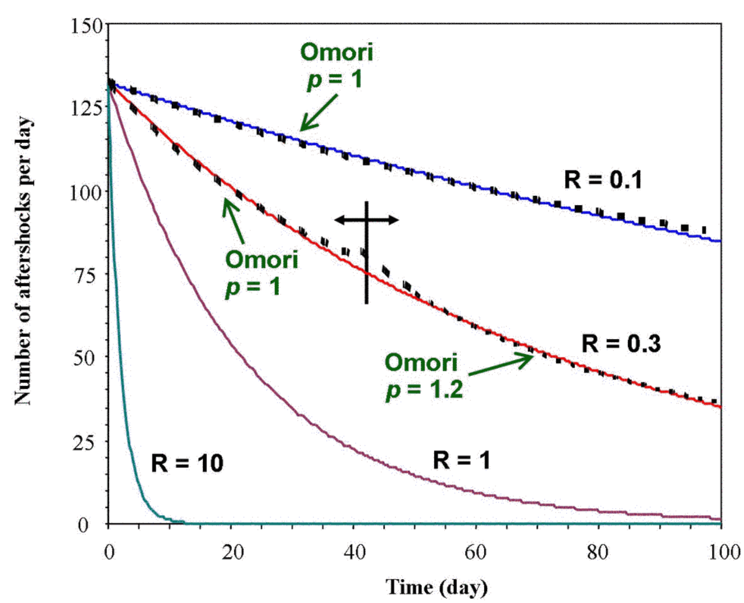
Material property $R = \dfrac{\text{Timescale of fracturing}}{\text{Timescale of stress relaxation}}$



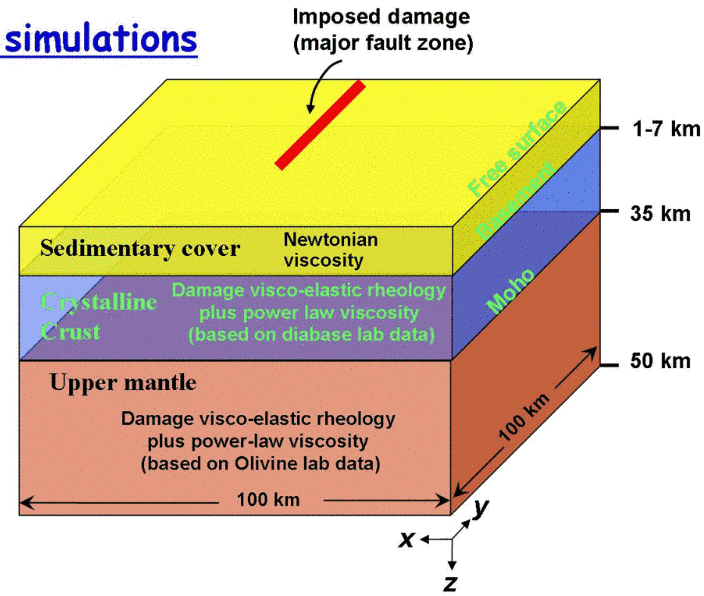
Changing the power-law parameters, we can fit the other lines !!!

Events rate vs. time for several values of $R = \tau_d/\tau_M$ (with finite ε_0,α_s)

Material property $R = \dfrac{\text{Timescale of fracturing}}{\text{Timescale of stress relaxation}}$

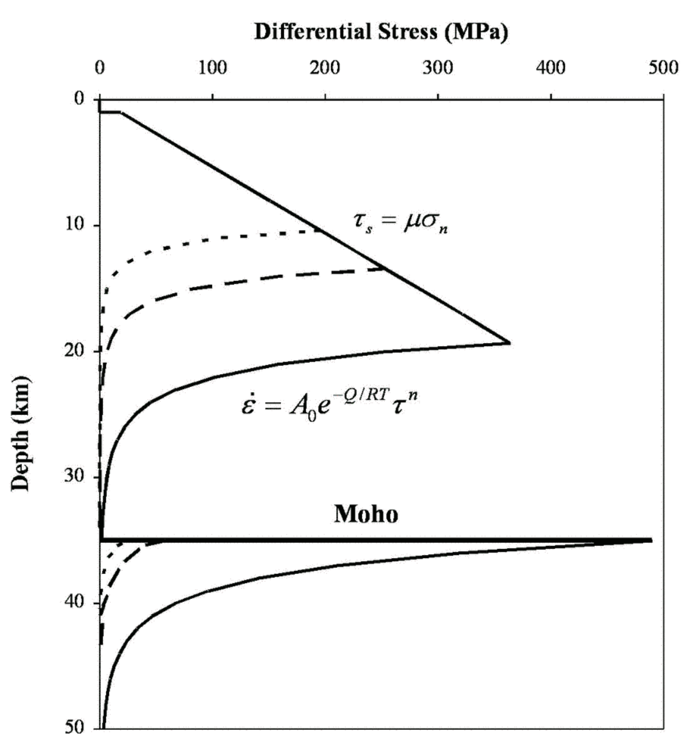


3-D numerical simulations



In each layer the strain is the sum of damage-elastic, damage-related inelastic, and ductile components: $\varepsilon_{ij}^{t} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{i} + \varepsilon_{ij}^{d}$

Initial stress = regional stress + imposed mainshock slip on a fault extending over 50 km ≤ y ≤ 150 km, 0 ≤ z ≤ 15 km with fixed boundaries



Initial regional stress for temperature gradients
20 °C/km – heavy line
30 °C/km – dash line
40 °C/km – dotted line

Strain rate = 10^{-15} 1/s

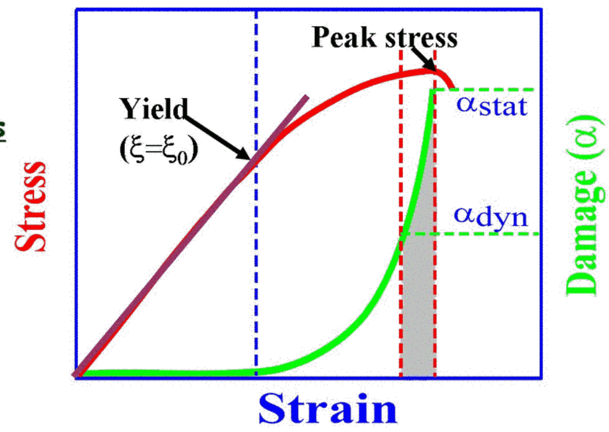
Brittle-ductile transition at 300 °C

Computational Issues:

- FE simulations with moving Lagrangian mesh (*FLAC* algo), using tetrahedral elements (~500m in seismogenic zone and ~5km below).
- The range of the simulated events governed by numerical elements size, large scale model dimensions, size of the imposed mainshock, & rheological parameters.
- Quasi-static procedure with a dynamic weakening correction.
- The size of the largest event is affected strongly by the dynamic weakening timescale τ_r.
- Earthquake magnitudes are calculated from the empirical potency-magnitude relation:

$$\log_{10} P_0 = 0.06M^2 + 0.98M - 4.87$$

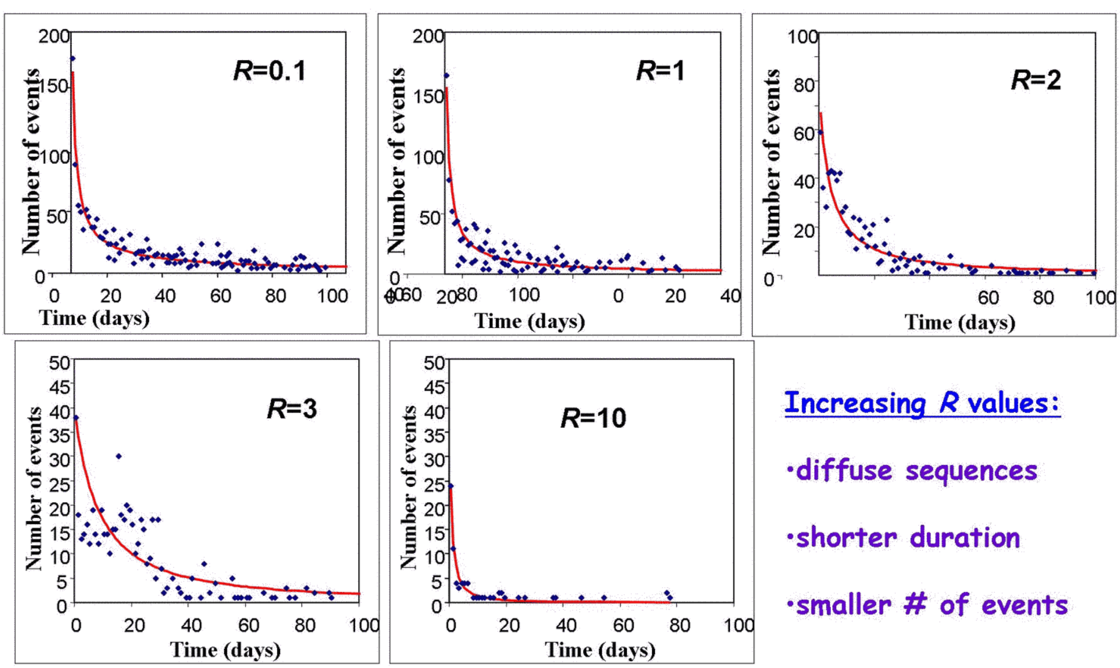
Dynamic weakening



$$\alpha_{dynamic} = \alpha_{static} - \sqrt{\tau_r \dot{\alpha}}$$

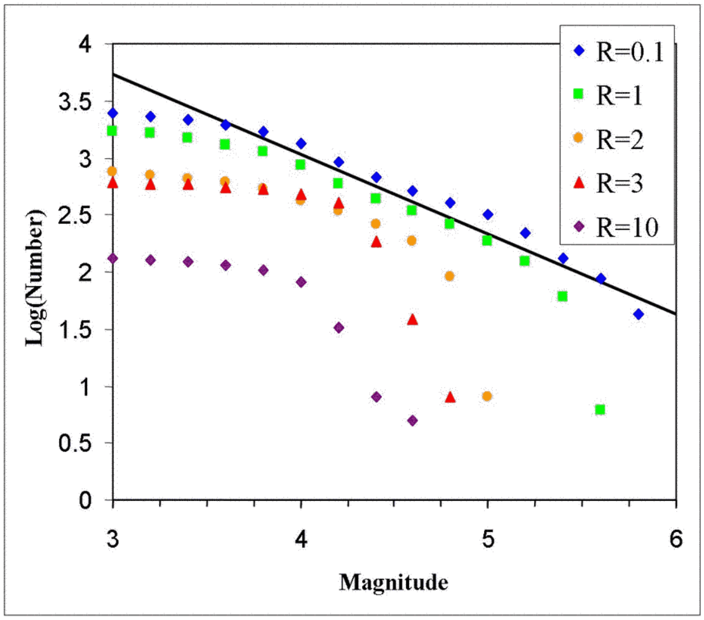
Simulations with fixed τ_r = 300 s

Effects of *R* (sediment thickness = 1 km, gradient 20 °C/km)



Increasing *R* values:

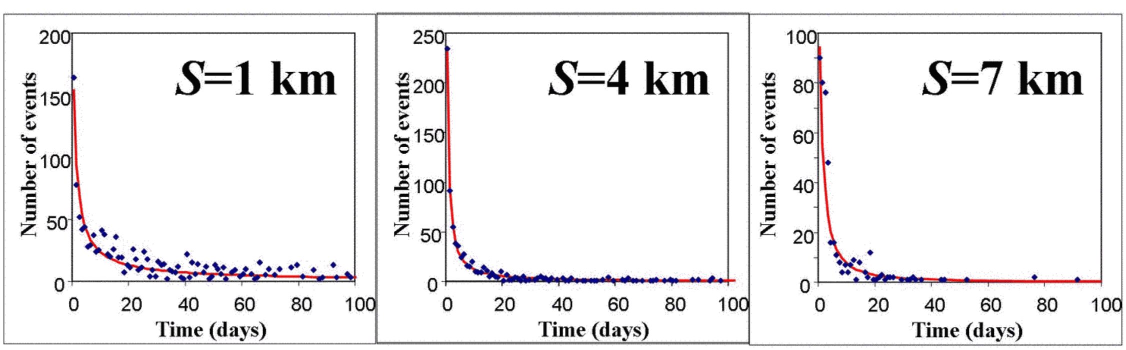
- diffuse sequences
- shorter duration
- smaller # of events



Small R values ($R < 1$): Power law frequency-size statistics

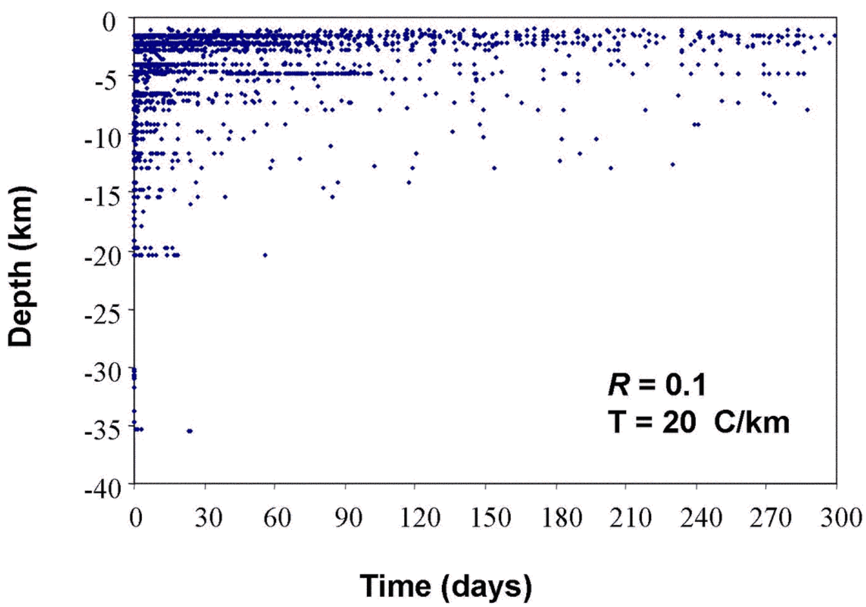
Large R values ($R > 3$): Narrow range of event sizes

Effect of Sediment thickness (R = 1, gradient 20 °C/km)



Increasing thickness of weak sediments: diffuse sequences, shorter duration, smaller number of events (similar to increasing R values)

Effect of thermal gradient and R (sediment layer 1 km)

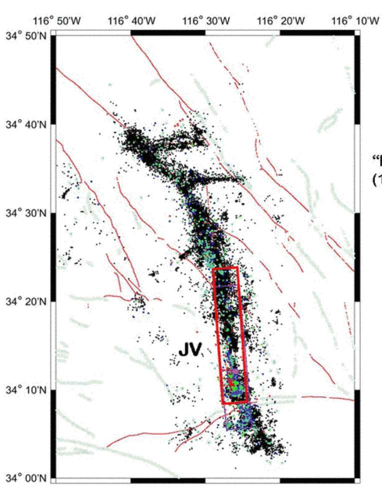


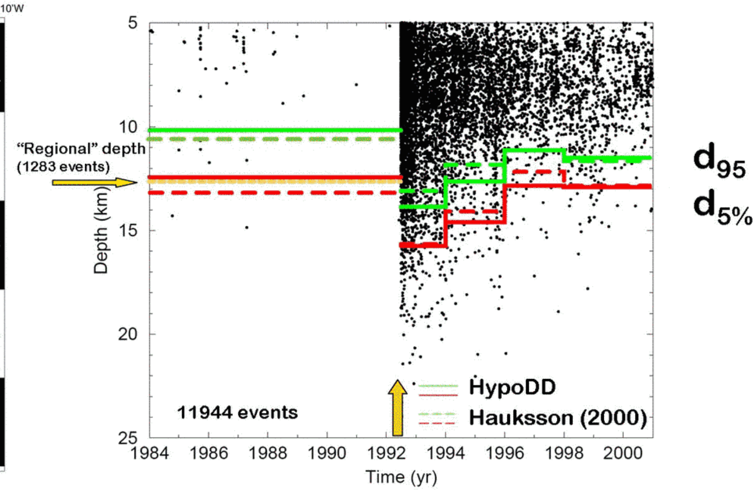
Increasing thermal gradient and/or R: thinner seismogenic zone

The maximum event depth decreases with time from the mainshock

Observed Depth Evolution of Landers aftershocks

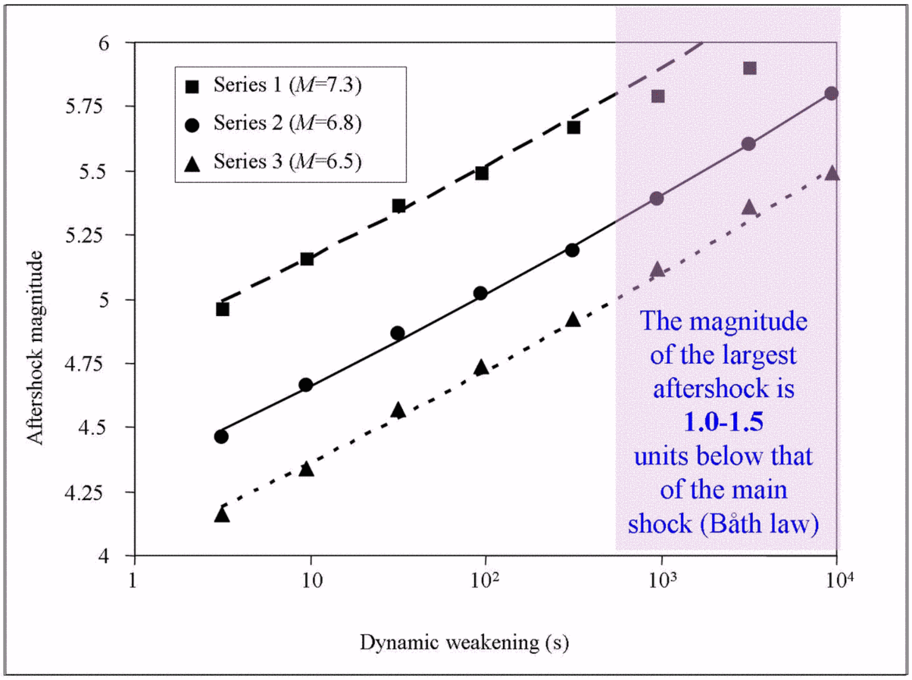
(Rolandone et al., 2004)





➡ Depth of seismic-aseismic transition increases following Landers EQ and then shallows by ≤ 3 km over the course of 4 yrs.

Analysis with variable τ_r for Båth law



The parameter *R* controls the partition of energy between seismic and aseismic components (degree of seismic coupling across a fault)

The brittle (seismic) component of deformation can be estimated as $\sigma = 2\mu \cdot \varepsilon_{seis}$

The rate of gradual inelastic strain can be estimated as $d\varepsilon_i / dt = -\dot{\alpha} C_v \sigma / 2$

The inelastic strain accumulation (aseismic creep) is $\varepsilon_i = C_v \sigma / 2$

$$\frac{\text{Seismic slip}}{\text{Total slip}} = \frac{\varepsilon_{seis}}{\varepsilon_{total}} = \frac{1}{1+R}$$

R	Slip ratio
0.1	90 %
1	50 %
2	33 %
10	10%

Main Conclusions

•Aftershocks decay rate may be governed by exponential rather than power law as is commonly believed (see also Dieterich, 1994; Kisslinger, 1996, Narteau et al., 2002)

•The key factor controlling aftershocks behavior is the ratio R of the timescale for brittle fracture evolution to viscous relaxation timescale.

•The material parameter R increases with increasing heat and fluids, and is inversely proportional to the degree of seismic coupling.

•Situations with $R \leq 1$, representing highly brittle cases, produce clear aftershock sequences that can be fitted well by the Omori power law relation with $p \approx 1$, and have power law frequency size statistics.

•Situations with $R >> 1$, representing stable cases with low seismic coupling, produce diffuse aftershock sequences & swarm-like behavior.

•Increasing thickness of weak sedimentary cover produce results that are similar to those associated with increasing R.

Thank you

Key References (on damage and evolution of earthquakes & faults):

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