Failure time, Critical Behaviour and Activation Processes in Crack Formation
Models and experiments

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Motivations:

- May simple models describe the formation of a crack?
  *(percolation models, SOC, etc.)*
- Is the crack formation similar to a critical phenomenon?
- If yes, what are the experimental conditions?
- Is the life time of a sample submitted to a tensile stress predictable?
OUTLINE

1) **Microcrack localization and statistical analysis**
   - Experimental apparatus
   - Comparison with percolation models

2) **Sample life time**
   - Activation models for crack formation
   - Comparison with experiments
   - Problems

3) **The thermally activated fiber bundle model**
   - Properties and results:
     * The disorder strongly reduces the sample life time but it increases the life time predictability.
     * The Omori law and the divergence of the damage rate are natural consequences of this simple model.

   **L. Vanel talk 2 weeks ago**

4) **Extension of the fiber bundle in 2D**
   - The slow propagation of a single crack.
   - The crack tip equation of motion

5) **Comparison with a 2D experiment.**

6) **Conclusions**

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**Experimental Set up**

What do we measure?

- The applied pressure and the sample strain
  A feedback loop allows us to impose either the stress or the strain.

- Acoustic emissions of microcracks.

- Microcrack localization

- Acoustic energy.

- X-Ray Tomography

**Materials:**

chipboard wood panel, fiber glass, PU foams

Young modulus

<table>
<thead>
<tr>
<th>Material</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>wood panels</td>
<td>$2 \times 10^8$ N/m²</td>
</tr>
<tr>
<td>fiber glass</td>
<td>$10^{10}$ N/m²</td>
</tr>
<tr>
<td>PU and epoxy solids foams</td>
<td>$10^9$ N/m²</td>
</tr>
</tbody>
</table>
Experimental Apparatus

Pressure versus strain

- Chipboard wood panel
- Wood imposed pressure
- Wood imposed strain
- Fiber glass imposed pressure
EVENTS LOCALISATION FOR INCREASING PRESSURE

applied pressure: $P = A \cdot t$

From a) to e) the applied pressure increases
In f) the total number of events is shown.
Micro Crack Localisation

SAMPLE

LOCALIZED MICROCRACKS

SUM OF FIGURES A and B

Acoustic energy versus time

A linear ramp of pressure (strain) is applied to the sample

\[ P = A \cdot t \]

\[ E/E_{\max} \]

\[ \tau \] is the sample life time and

\[ E = \int_0^t \varepsilon(t') \, dt' \]
Event distribution during the pressure ramp

Uniaxial tensile test under X-ray tomography of PU solid foams

S. Deschanel, G. Vigier (INSA de Lyon)
Time evolution

Uniaxial tensile test (1)

Initial state

8.6% strain

Cumulative number of acoustic emissions

0
10
20
30
40

0 1000 3000 5000 7000

Stress (Mpa)

time (s)

12.6% strain

16.6% strain
Uniaxial tensile test (2)

16.6% strain

Cumulative number of acoustic emissions

18.5% strain

21.5% strain

Acoustic Localisation and tomography
**Acoustic energy versus time**

A linear ramp of pressure (strain) is applied to the sample

\[ P = A \cdot t \]

\[ E/E_{\text{max}} \]

\[ t/\tau \]

\[ \tau \] is the sample life time and

\[ E = \int_0^t \epsilon(t') \, dt' \]

**Acoustic energy versus reduced time**

imposed pressure ramp

\[ \sim \left( \frac{\tau - t}{\tau} \right)^{-\alpha} \]

\[ \alpha = 0.22 \text{ for wood} \]

\[ \alpha = 0.27 \text{ for fibeglass} \]

imposed strain ramp

No critical behaviour
Summary of the experimental results

1) Experimental data show a strong analogy between crack formation in composite materials and percolation models.

2) When the system is driven at imposed pressure a critical behavior is found near the critical pressure.

3) These models do not seem to have a good predicatability of the life time of the sample:
   - the critical divergence appear very close to \( \tau \)
   - The localisation is hard to detect in a quantitative way (minimum spanning tree, localisation entropy)

May life time be predicted?

Activation processes and crack formation


The main physical hypothesis behind is that the macroscopic failure of a material is produced by a thermal activation of micro cracks. Specifically

\[
\tau = \text{sample life time} = \exp\left(\frac{W}{kT}\right)
\]

W=activation energy, \( k = \)Boltzmann constant, \( T = \)temperature

How \( W \) can be computed for a crack?

Using the Griffith criterion for the crack stability.
Activation model in 2D

\[ U = -\frac{\pi L^2 \sigma^2}{4Y} + 2\gamma L + U_0 \]

Griffith Potential energy

\[ U_G = \gamma L_G = \frac{4\gamma^2 Y}{\pi \sigma^2} \]

The time of life of the sample
time needed to nucleate a crack of critical length \( L_G \)

\[ \tau \propto \exp\left( \frac{W}{kT} \right) \]  
with \( W = U_G \)

Results of Pomeau's conjectures

2D

\[ \tau = \tau_0 \exp \left( \frac{P_o}{P} \right)^2 \]  
(verified in 2D crystals)

3D

\[ \tau = \tau_0 \exp \left( \frac{P_o}{P} \right)^4 \]  
with \( P_o^4 = \frac{\alpha \Gamma^3 Y^2}{K_B T} \)

We checked this idea in our experiment

Does this hold in a cumulative way and for a time dependent pressure? 

If pressure is time dependent then:

\[ 1 = \frac{1}{\tau_0} \int_0^\tau \exp \left[ - \left( \frac{P_o}{P} \right)^4 \right] dt \]
Dr. Sergio Ciliberto, ENS Lyon (KITP Earthquake 9-21-05) Failure Time, Critical Behaviour and Activation Processes in Cracks

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a) Imposed Pressure

\[ \tau = \tau_0 \exp \left( \frac{P_o}{P} \right)^4 \]

Life time versus \( P \)

- wood
  \( \tau_0 = 50 \text{ s} \)
  \( P_o = 0.63 \text{ atm} \)

- fiber glass
  \( \tau_0 = 44 \text{ s} \)
  \( P_o = 3 \text{ atm} \)

Exponential

Mori, Zurkov

b) Imposed Strain

Acoustic energy versus reduced time

\[ \alpha = 0.27 \]
Events distribution at imposed constant pressure

\[ (\delta t) \]

\[ \delta t (s) \]

\[ N (\varepsilon) \]

\[ \varepsilon \]

Breaking time as a function of the pressure ramp rate

\[ T = 380 \, K \]

\[ T = 300 \, K \]

The applied pressure is \( P = A_p \, t \)

Continuous lines are computed using:

\[
1 = \frac{1}{\tau_o} \int_0^T \exp \left[ -\left( \frac{P_o}{P} \right)^4 \right] \, dt
\]
OPEN PROBLEMS

1) Temperature has not a big influence on $\tau$
   For a temperature variation $\Delta T=80K$ the expected variation of $\tau$ is about:
   50% at the smallest pressure
   100% at the largest pressure

   To have $\Delta \tau/\tau <10\%$ for a $\Delta T \sim 80K$ one has to assume an effective temperature:

   $T_{\text{eff}} \sim 3000K$

   What is the origin of such a high temperature?

2) Very large accuracy on the life time estimation

2a) In activation process the time statistics is Poissonian whereas in our experiment it is Gaussian

2b) In many creep tests done in many materials (e.g. stainless steel) the life time of samples submitted to a tensile stress has fluctuations of about 100%.
   In our experiment the error is less than 10%.

   Where these big differences come from?

Discussion

The sample disorder may play an important role

- The original Pomeau description was based on a single crack.

- In a real sample the final crack is the sum over many microcracks. This process may produce a Gaussian statistic starting from non-Gaussian ones.

- This could also explain why in small samples the time is not reproducible.

A series of experiments done on samples of different sizes confirm this conjecture
Life time in small samples

\[ M. \text{Zei} \]

\[ \text{Distribution of } \tau \]

\[ N(\tau) \]

\[ 0 \quad 10 \quad 20 \quad 30 \quad 40 \]

\[ \tau \ (s) \]

\[ 0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \quad 300 \]

\[ 1 \]

\[ Q(\tau) \]

\[ 0.5 \quad 1 \]

\[ \tau \ (s) \]

Distribution of the lifetimes \( \tau \) of 100 samples in fiberglass broken with the BM (clamped edges) with a load \( P = 54 \text{ Kg} \). The sample's size are \( 22 \times 2 \times 0.2 \text{ cm} \).

a) The histogram of \( \tau \) shows that the distribution of lifetimes is not gaussian.
b) The cumulative distribution \( Q(\tau) = \int_{\tau_0}^{\tau} N(\tau) d\tau \) (solid line) is best fitted by the sum of two exponential terms (dotted line).

Life time in small samples

\[ M. \text{Zei} \]

\[ \text{Failure time } \tau \text{ of the samples in fiberglass broken with the FM (clamped edges).} \]

The sample's size are \( 22 \times 2 \times 0.2 \text{ cm} \) (a) and \( 22 \times 1 \times 0.2 \text{ cm} \) (b). Each point represents the mean value of 20 measures. Lines represent the best fit with \( \tau = \tau_0 \exp \left( \frac{P}{P_0} \right)^2 \) (solid line), \( \tau = \tau_0 \exp \left( \frac{P}{P_0} \right)^4 \) (dashed-dotted line), \( \tau = A \exp(-bP) \) (dotted line), and \( \tau = A P^{-b} \) (dashed line).
Summary of experimental results

1) Experimental data show that the sample life time follows a well defined function of pressure. (Pomeau's model)

2) When the system is driven at constant pressure a critical divergence is found as a function of the reduced time

3) Event sizes and time delay between events are power law distributed.

4) The life time statistics depends on the system size

5) The life time does not depend on temperature

The thermally activated fiber bundle model

1) Each fiber is submitted to a local force $f_i = f_0/(1 - n/N)$, where $n$ is the number of broken fibers

2) Each fiber $i$ is characterized by a critical strength $f_c(i)$. If, at time $t$, $f_i > f_c(i)$ the fiber $i$ cracks.

3) The distribution $P(f_c)$ of $f_c$ is characterized by a variance $T_d$ and can be normal or uniform.

4) Each fiber is submitted to an additive random stress which follows a zero mean normal distribution of variance $T$. Thus the total force acting on each fiber is:

$$f_i = \frac{f_0}{1 - n/N} + \eta_0$$
The solution of the model (1)

We choose \( P(f_c) = \frac{1}{\sqrt{2\pi T_d}} \exp\left[-\frac{(f_c-1)^2}{2T_d}\right] \)

The relevant variables of the problem are:
The distribution of unbroken bonds \( Q(f_c, t) \) with initial condition \( Q(f_c, 0) = P(f_c) \).
The fraction of broken bonds is:

\[
\Phi(t) = \lim_{N \to \infty} \frac{n}{N} \equiv 1 - \int_{-\infty}^{\infty} Q(f_c, t) \, df_c
\]

The average force \( f_a \) exerted on each fiber is:

\[
f_a = \frac{f_0}{1 - \Phi}
\]

The probability of breaking the fiber of critical strength \( f_c \) is:

\[
G(f_c - f_a) = \frac{\gamma}{2} \left\{ 1 - \text{erf} \left[ -\frac{(f_c-f_a)}{2T} \right] \right\}
\]

for small \( T \)

\[
G(f_c - f_a) = \frac{\gamma}{f_c-f_a} \sqrt{\frac{T}{2\pi}} \exp\left[-\frac{(f_c-f_a)^2}{2T}\right]
\]

The solution of the model (2)

The dynamical equation for \( Q(f_c, t) \) is

\[
\dot{Q}(f_c, t) = -Q(f_c, t)G(f_c - f_a)
\]

The difficulty of solving this model arises from the time dependence of \( f_a \) which is determined by the integral of \( Q \) over all \( f_c \) values.

\[
f_a = \frac{f_0}{1 - \Phi}
\]

with

\[
\Phi(t) = 1 - \int_{-\infty}^{\infty} Q(f_c, t) \, df_c
\]
**Numerical simulation**

Evolution of the fraction of broken bonds

\[ T_d = 10^{-3} \quad \text{and} \quad T = 8 \times 10^{-3} \]
\[ f_o = 0.6 \]

Evolution of \( Q(f_c, t) \)

\[ f_o = 0.6, T = 0.006 \quad \text{and} \quad T_d = 0.005 \]

\[ \Phi = 0 \]
\[ \Phi = 0.15 \]
\[ t = 0.9\tau \]

**Time evolution of Q**

We define:

\[ S(f_c, t) = \frac{Q(f_c, t)}{P(f_c)} \]

\[ T_d = 10^{-2}, T = 10^{-3} \quad \text{and} \quad f_o = 0.6 \]

For \( T \to 0 \)

the Heaviside shape is a good approximation for \( S \)
Approximated analytical solution for large $t$

$$\ln\left(\frac{\tau - t}{\tau}\right) = -C \phi$$

$$t = \tau \left[1 - \exp(-\phi C)\right]$$

where

$$\tau \simeq \tau_0 \exp\left(\frac{(1 - f_0)^2}{2 T_{eff}}\right)$$

and

$$T_{eff} \simeq \frac{T}{\left(1 - \frac{\sqrt{\pi} \sigma_0}{2(1-f_0)}\right)^2}$$

with $\sigma_0 = \sqrt{T_d}$

$$\frac{d\phi}{dt} \propto \frac{1}{\tau - t}$$

Critical divergence of the damage rate
**Conclusions on the fiber bundle**

The thermally activated fiber bundle model presents interesting features:

* The disorder strongly reduces the sample life time but it increases the life time predictability. 
  *This property could explain our experimental results on life time*

* The Omori law and the divergence of the damage rate are natural consequences of this simple model.

**Problem**

The dependence on P is not the one predicted by Pomeau

Is this due to the 1D structure of the model?

**Extension of the thermally activated fiber bundle in 2D**

1) Noise induced dynamics of a single crack
   (L. Vanel talk 2 weeks ago)

2) Damage growth in a desordered 2D network

**Damage growth in a desordered 2D network**

In the ordered network:

\[ \tau = \tau_o \exp \left( \frac{U_o}{T} \right) \quad \text{with} \quad U_o = \frac{1}{2} (1 - I_o)^2 \]

In the disordered network:

\[ \tau = \tau \exp \left( \frac{U}{T} \right) \quad \text{with} \quad U = U_o \left( 1 - \frac{\alpha(I_o)}{\sqrt{2U_o}} \right) \]
Conclusions

Activation models of crack explain many experimental results.

In the case of damage growth they justify:

- The existence of an effective temperature (reduction of the energy barrier) induced by the sample disorder.
- The transition from a Poissonian to a Gaussian statistics of the lifetime, when the sample size is increased.
- The critical divergence of the damage rate close to $\tau$.

In the case of a single crack growth in a heterogenous material:

- The dynamics of the crack tip.
- The statistics of jumps.

Activation models can probably be used to predict the lifetime in large heterogenous structures.
Problems and perspectives

- The dependence in $P^{-4}$ of the life time of 3D heterogenous sample is not explained by our model.

- The role of disorder in samples has to be studied experimentally.

* work is in progress using solid foams and other materials

2D models

Griffith Potential energy

$$U = -\frac{\pi L^2 \sigma^2}{4Y} + 2\gamma L + U_0$$

$Y$ : Young modulus, $\gamma$ : surface energy
$U_0$ : elastic energy without fracture

The time of life of the sample time needed to nucleate a crack of critical length $L_G$

$$\tau \propto \exp\left(\frac{U_G}{kT}\right)$$

In the Pomeau model crack is reversible
**Irreversible crack**

**Hypothesis of the model**

a) The local breaking threshold is overcome by the fluctuations of the applied stress.

\[
\left( \Delta \sigma \right)^2 \propto kT
\]

\[k \text{ constante de Boltzmann,}
\]

\[T \text{ température}
\]

b) The damage processes is irreversible
c) The velocity \( V \) of the crack tip is proportional to the probability \( P \) of having a fluctuation \( K>Kc \)

**Analytical results**

*Time evolution of a crack of initial length \( L_i \):*

\[
t = \tau \left[ 1 - \exp \left( -\frac{L - L_i}{\zeta} \right) \right]
\]

**Time of life**

\[
\tau = \tau_0 \exp \left[ \frac{(\sigma_c - \sigma_i)^2 V}{2Y k_B T} \right]
\]

**Characteristic length**

\[
\zeta = \frac{2Y k_B T}{V} \frac{L_i}{\sigma_i (\sigma_c - \sigma_i)}
\]

**Numerical simulation on a spring network**

(S. Santucci et al, Europhysics Letters (2003))

Square network in antiplane deformation (same as fuse network)

Shearing load \( \sigma \) (mode III)

No disorder: constant spring breaking threshold \( \sigma_c \)

This is equivalent to a 2D fuse network driven at constant current
Results of the 2D numerical simulation

(S. Santucci et al, Europhysics Letters (2003))

Average over 50 realisations

\[
\frac{t}{\tau} = \lambda (L - L_i)/\xi
\]

\[
\frac{U_{pot}}{E_G} \approx \frac{L}{L_G} - 1
\]

Mean life time of the sample

\[
\delta U \propto (\sigma_c - \sigma_i)^2
\]

Potential energy \( U_{pot} \) in the lattice model

in agreement with expected Griffith energy!

Minimum energy cost: \( L \rightarrow L + 1 \)?
Potential energy $U_{pot}$ in the lattice model

"LATTICE TRAPPING": (R. Thomson, Solid State Physics (1986))

Dynamics controlled by thermal noise:
- $L < L_g$: crack closes
- $L_g < L < L_c$: irreversible crack growth
- $L_c < L$: fast rupture

Probability distribution $P_s$ of crack jump sizes $S$

(S. Santucci et al., PRL (2004))

Jump distribution $P_s \xrightarrow{\text{Distribution of stress fluctuations } G(\sigma_f)}$

- Mean growth velocity $V(L)$
  \[ V(L) \propto P(\sigma_f > \sigma_c) \]
  \[ V(L) = \frac{\langle s \rangle}{\tau_m} = \frac{\int s P_s ds}{\tau_m} \]

- Crack arrest mechanism
  
  \[ s = \frac{\delta U_f}{\delta U_c} \lambda = \frac{(\sigma_f - \sigma_m)^2}{(\sigma_c - \sigma_m)^2} \lambda \]

  \[ \text{distribution of crack jump sizes} \]
  \[ P_s = N(V, \lambda) s^{-3/2} e^{-s/\xi} \]

  Cut-off length $\xi = \lambda \frac{2Y k_B T}{(\sigma_c - \sigma_m)^2 V}$

  Similar to « critical point theory in percolation »
**Experiment in 2D**

Material: Sheets of fax paper under tensile stress (mode 1).

\[ \sigma_L = \sigma \sqrt{\frac{L}{\lambda}} = \frac{K}{\sqrt{\lambda}} \]

- **motor**
- **cutter**
- **Force gage**
- **sample**

Humidity at 5%

**Measures:**
- Applied strain and stress,
- Length of the crack as a function of time
- Time of life

**Crack Length versus time**

Evolution of two cracks having the same initial conditions

For \( L > L_c \) the crack propagates very fast

Extended study for:

\[ 1 \text{ cm} < L_i < 4 \text{ cm} \]

\[ 140 \text{ N} < F < 280 \text{ N} \]
Mean dynamics

\[ \frac{\langle t \rangle}{\tau} = \left[ 1 - \exp \left( -\frac{L - L_c}{\zeta} \right) \right] \]

Prediction of the model

\[ L_c = 1 \text{ cm} ; F_c = 270 \text{ N} \]

Average of 10 experiments

\[ \langle L_c \rangle \text{ is the critical length} \]

Agreement with the model

\[ \frac{\langle t \rangle}{\tau} = \left[ 1 - \exp \left( -\frac{L - L_c}{\zeta} \right) \right] \]

\[ L_c \text{ dependence on } \sigma \]

Comparison with

\[ L_G = \frac{4 \langle \gamma \rangle Y}{\pi \sigma^2} \]
Characteristic length

Prediction of the model:

\[
\zeta = \frac{2Yk_B T}{V} \frac{L_i}{\sigma_i (\sigma_c - \sigma_i)}
\]

Life Time

Prediction of the model:

\[
\tau = \tau_0 \exp \left( \frac{(\sigma_c - \sigma_i)^2 V}{2Yk_B T} \right)
\]
Life time

Prediction of the model: \[ \tau = \tau_0 \exp \left[ \frac{(\sigma_c - \sigma_i)^2 V}{2 Y k_B T} \right] \]

The activation model predicts the average dynamics of the crack. The jumps statistics can also be derived.

**Problem**

If \( \lambda \sim 50 \mu m \) (size of the fibers) the only free parameter of the fits is \( V \), which can be determined by the fit of \( \zeta \) and \( \tau \).

These two independent fits gives \( V^{1/3} \sim 1 \text{ nm} \)
Average growth dynamics in polycarbonate

\[
\frac{\langle t \rangle}{\tau} \neq \left[ 1 - \exp \left( -\frac{L-L_i}{\zeta} \right) \right]
\]

(Cortet et al, EPL 2005)