Partial Slip and Fracture

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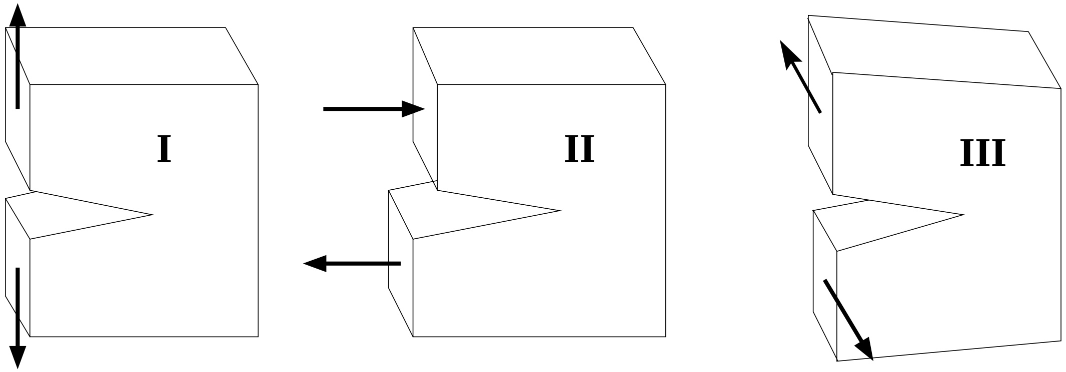
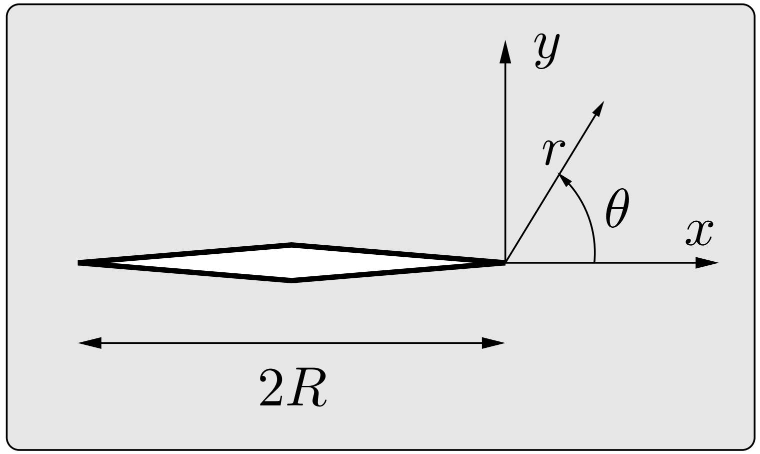
Near tip behavior





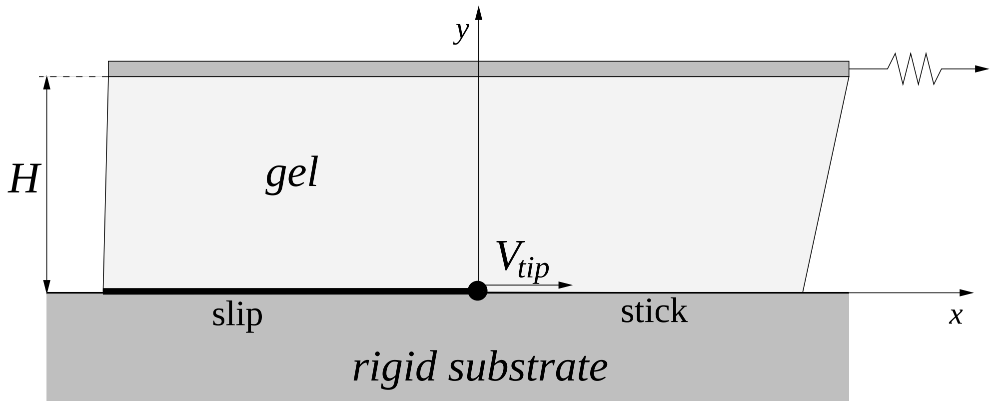
$$\sigma_{ij} = \frac{K}{r^{1/2}} f_{ij}(\theta)$$

with a *universal* function $f_{ij}(\theta)$ for each loading mode.

stress intensity factor: $K \sim PR^{1/2}$ contains the full information about the crack.

Griffith equilibrium: $K^2/E \sim \alpha \Rightarrow$ selection of R

Theoretical model



- Relaxation of the shear stress with the slipping region advance
- Two different states of the gel-glass surface
- Fracture surface energy $\gamma > 0$ independent of V_{tip}
- Linear elasticity, displacement field $\boldsymbol{u} = (u_x, u_y)$

Slip:

$u_y = 0$ and given σ_{xy} as a function of $\dot{u}_x$

$$x \to -\infty : u_{xy} = 0, \sigma_{xy} = 0$$

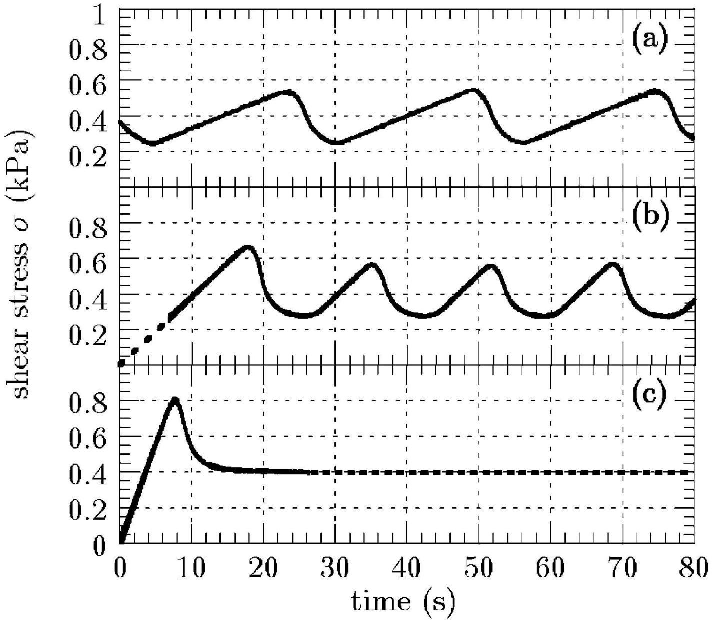
Stick:

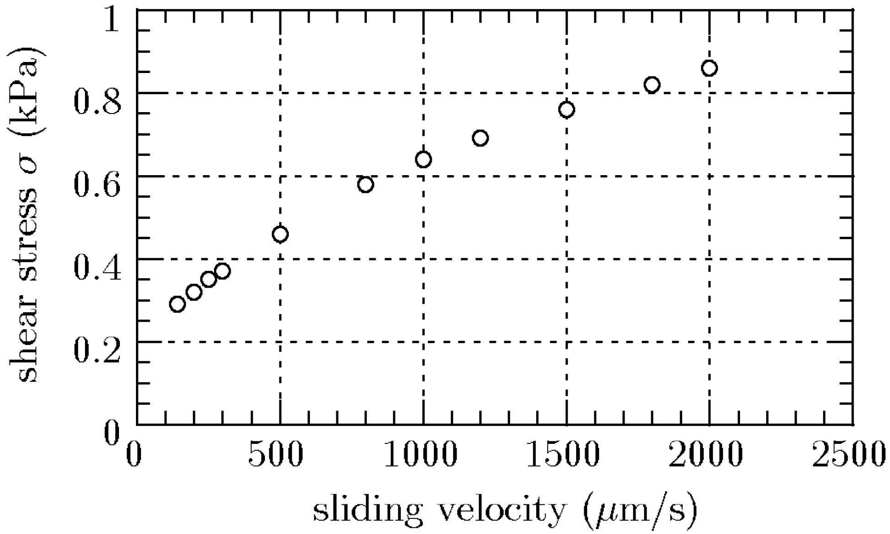
$u_y = 0$ and $u_x = 0$

$$x \to +\infty : u_{xy} \to u_{xy}^{\infty}, \sigma_{xy} \to \sigma_{xy}^{\infty} = 2\mu u_{xy}^{\infty}$$

Experimental results

(T.Baumberger, C.Caroli, O.Ronsin, PRL **88***, 075509, (2002))*





Two dynamical behaviors:

- $V < V_c$: stick slip (**a,b**)
- $V > V_c$: uniform slip (**c**)

Griffith threshold, uniform slipping

Fracture surface energy γ

Energy balance $\Longrightarrow$ crack propagating with $V_{tip} > 0$ if

$$\Delta \equiv \frac{(\sigma_{xy}^{\infty})^2 H}{2\mu\gamma} > 1$$

Finite drag force is necessary for slipping

Stable steady sliding $\Rightarrow$ $\Delta \geq 1$, uniform $\sigma_{xy} \geq \sqrt{2\gamma\mu/H}$

Suppose linear viscous friction at $x < 0$: $\sigma_{xy} = \alpha \dot{u}_x$,

$$\sigma_{xy} = \alpha V$$

Uniform slipping is stable against healing (sticking) if the drag speed $V > V_c$,

$$V_c = S\sqrt{\frac{2\gamma}{\mu H}}, \qquad S = \frac{\mu}{\alpha}$$

Exact solution

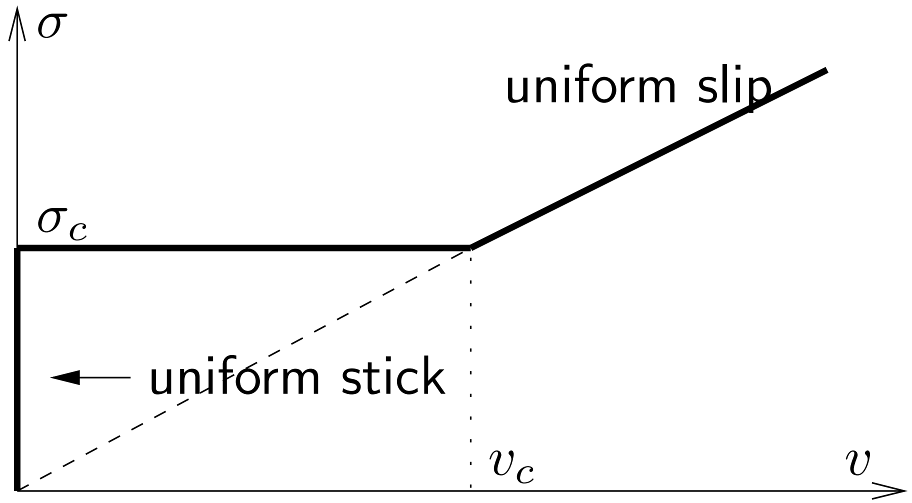
- Viscous friction: $\sigma_{xy} = \alpha \dot{u}_x$ at $x < 0,\ y = 0$
- Uniform solution with $\lambda = 1/2 + \varepsilon$:

$$u_x = A\,\mathrm{Re}\left[y(x+iy)^{\lambda-1} - i\,(3-4\nu)\,(x+iy)^{\lambda}/\lambda\right]$$
$$u_y = A\,\mathrm{Re}\left[iy(x+iy)^{\lambda-1}\right]$$

$$\varepsilon \approx \frac{1}{2\pi}\frac{3-4\nu}{1-\nu}\frac{V_{tip}}{S}$$

- Energy balance $\Rightarrow V_{tip} = 2\pi\frac{1-\nu}{3-4\nu}\frac{\ln\Delta}{\ln(H/a)}S\,, \quad \varepsilon \ll 1$

Phase diagram: Velocity vs. shear stress



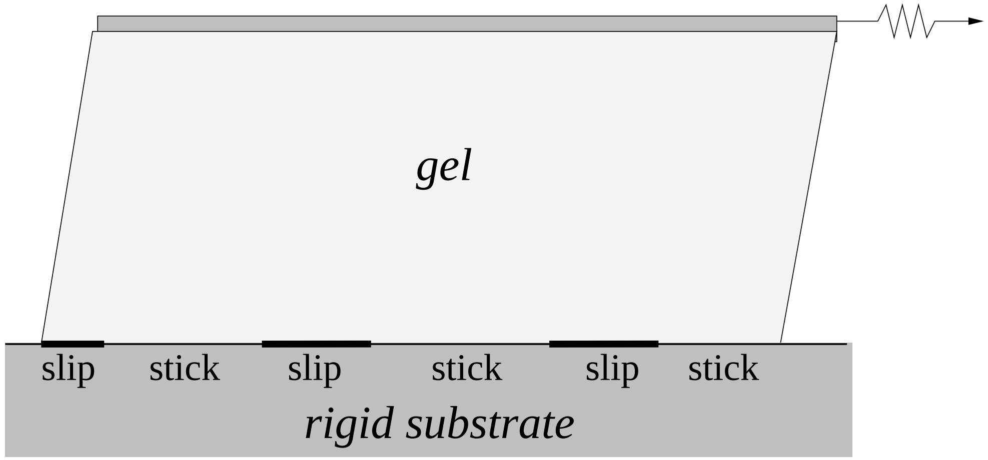
Given velocity

- Uniform slip for $v > v_c$
- Stick-slip motion for $v < v_c$
- Average shear stress: $\sigma = \sigma_c$

Given stress

- Uniform stick for $\sigma < \sigma_c$
- Uniform slip for $\sigma > \sigma_c$

Periodic stick slip motion



Pulling velocity v
Crack velocity c
Average Stress σ
Periodicity Λ
Fraction of slip phase η

- Only two equations to determine four parameters for given pulling velocity v

  $\Rightarrow$ two degrees of freedom

Solution of the elastic problem

Far enough from the crack tips, one can find an approximate solution of the elasticity problem.

In the stick region $|x| < \lambda_{st}/2$,

$$u_x = V_0 \frac{(H-y)x}{H} + \frac{\sigma_0 y}{\mu}\,, \qquad u_y = V_0 \frac{y(y-H)}{4(1-\nu)H}\,,$$

$$\sigma_{xy} = \sigma_0 - \mu \frac{x}{H} V_0\,,$$

where

$$V_0 = v/V_{tip}$$

In the slip region shear stress is relaxed to

$$\sigma_{sl} = \mu v/S$$

Energy fluxes into the crack edges and equations of motion

We can find the energy fluxes in the similar way as it was done for the semi-infinite slip .

$$\gamma = \frac{\mu H}{2}\left(\frac{\sigma_0}{\mu} + \frac{\lambda_{st}}{2H}V_0\right)^2\left(\frac{H}{a}\right)^{-\epsilon},$$

$$\gamma = \frac{\mu H}{2}\left(\frac{\sigma_0}{\mu} - \frac{\lambda_{st}}{2H}V_0\right)^2\left(\frac{H}{a}\right)^{\epsilon}$$

where

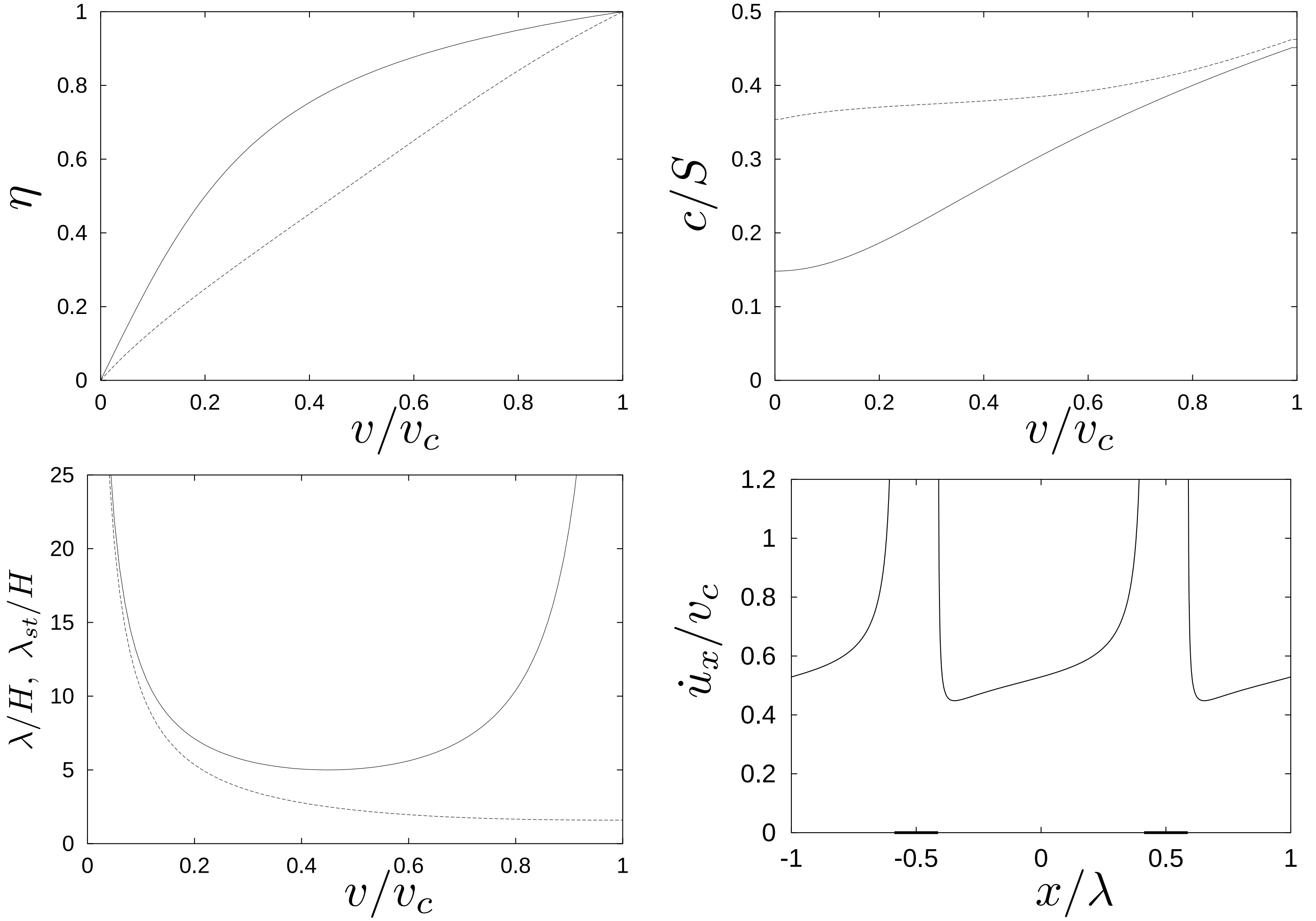
$$\epsilon \sim V_{tip}/S$$

This can be written down in terms of the average shear stress σ with help of the relation

$$\sigma_0 = \frac{\lambda}{\lambda_s t}\left(\sigma - \mu\frac{v}{S}\right).$$

Results

$$\sigma = \sigma_c, \qquad \min\{\Lambda_{stick}, \Lambda_{slip}\} \approx H.$$



Experimental results

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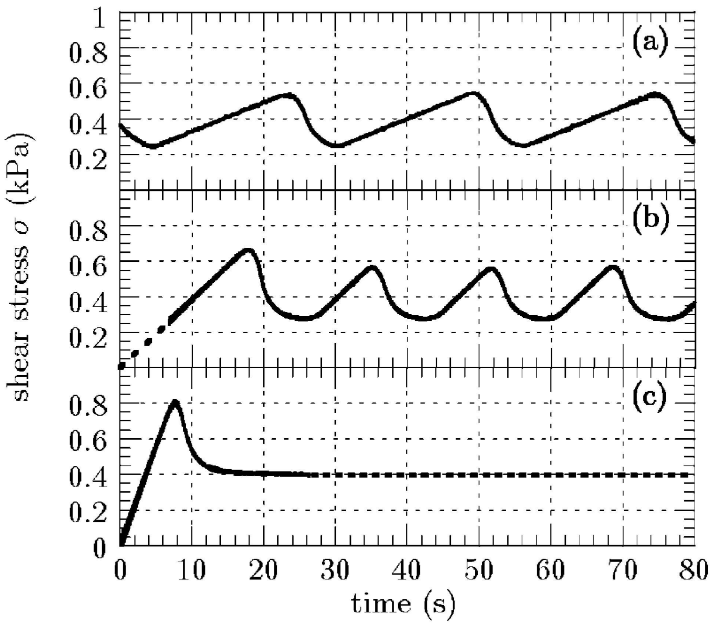

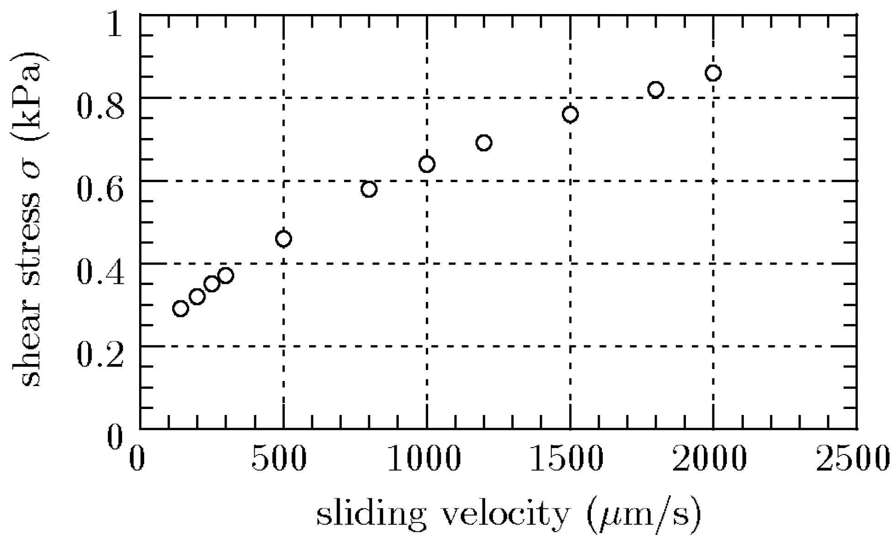

Two dynamical behaviors:

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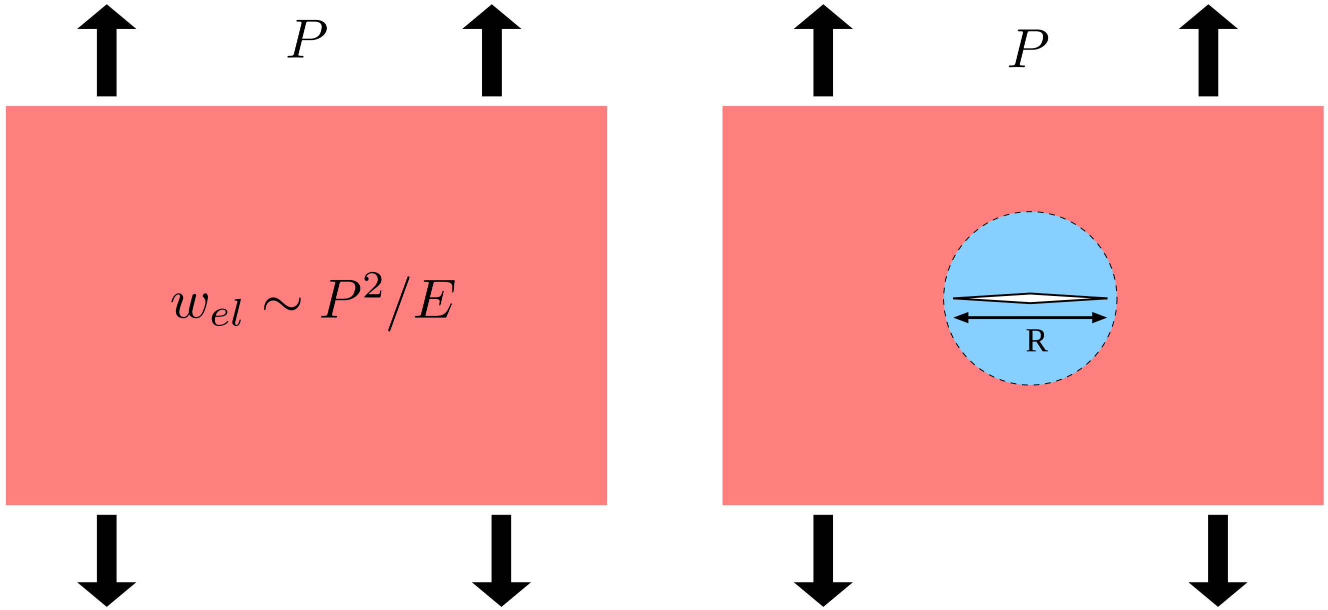
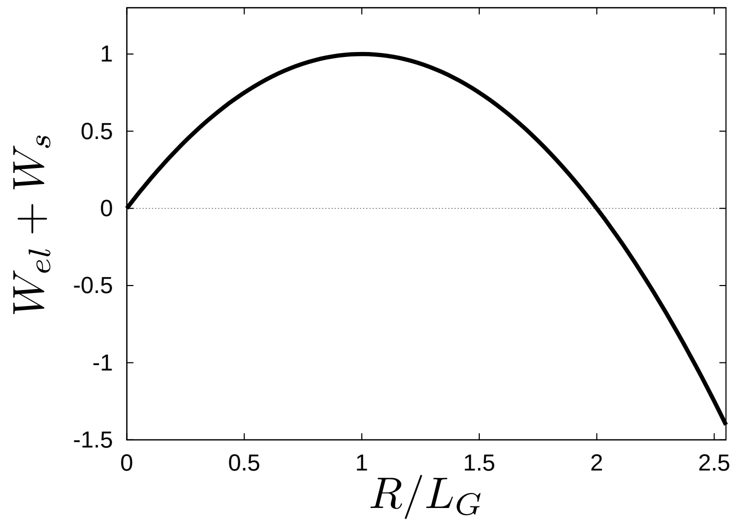
Theory of Cracks



- elastic relaxation in the area $\sim R^2$: $W_{el} \sim -\dfrac{P^2R^2}{E}$

- increase of surface energy: $W_s \sim \alpha R$



Griffith length: $L_G \sim \dfrac{E\alpha}{P^2}$

Approximate solution

Assume $\sigma_{xy} = 0$ at $x < 0,\ y = 0$
Quasistatic approximation $\Rightarrow$ singularity with $\lambda = 1/2$:

$$u_x = A\,\mathrm{Re}\left[y(x+iy)^{\lambda-1} - i\,(3-4\nu)\,(x+iy)^{\lambda}/\lambda\right]$$
$$u_y = A\,\mathrm{Re}\left[iy(x+iy)^{\lambda-1}\right]$$

(valid if $\sqrt{x^2+y^2} \lesssim H$)

Energy flow density $j_i = \sigma_{ik}\dot{u}_k + \frac{1}{2}\sigma_{jk}u_{jk}V^i_{tip}$

Elastic energy flux into the crack tip

$$J_0 = 2\pi\mu(3-4\nu)(1-\nu)V_{tip}A^2$$

equals the surface energy change

$$J_0 = \gamma V_{tip} \Rightarrow A = A(\gamma)$$

The global energy conservation law is

$$J_0 + J_d = \mu (u_{xy}^{\infty})^2 H V_{tip}$$

where J_d is the energy flux through the sliding surface (energy release due to the friction).

$$J_d = \int_{-\infty}^{0} dx \, \sigma_{xy} \dot{u}_x \, ,$$

the main logarithmic contribution is

$$J_d = \alpha (3 - 4\nu)^2 A^2 \ln \left(\frac{H}{a} \right)$$

At scales smaller than a linear theory (elasticity, viscous friction) is not valid.

$$V_{tip} = 2\pi \frac{1-\nu}{3-4\nu} \frac{\Delta - 1}{\ln(H/a)} S \, , \quad \Delta - 1 \ll 1$$