Seismology Lectures

- Lecture 1 Equations and Waves (BR)
- Lecture 2 Surface waves (GM)
- Lecture 3 Geophysical Inverse Problems (GM)
- Lecture 4 Receiver functions (AS)
- Lecture 5 Array Methods (AL)
- Lecture 6 Transition zone in the context (AD)

Receiver Functions

Anne Sheehan
University of Colorado, Boulder

CIDER Summer'06

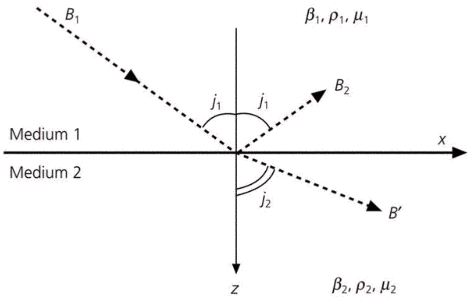
Receiver Functions: Outline

- P to S converted waves
- Deconvolution
- Stacking, moveout, receiver function ‘imaging’
- Examples - transition zone thickness near plumes, slabs, ‘normal’ mantle, global
- Complicated Earth - non planar layers, anisotropy
- Other body wave phases to image transition zone topography (SdS, PdP, ScS)

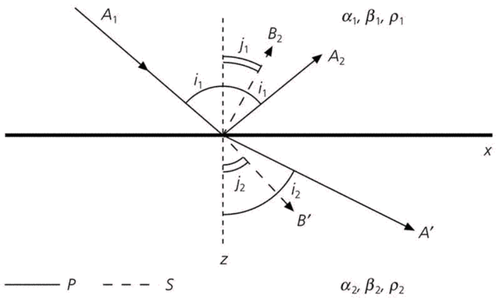
From Barbara’s lecture (Seismo 1, Tues Jul 18):

In general, some of the energy is transmitted, some reflected, and, in the P-SV case, some converted

SH case

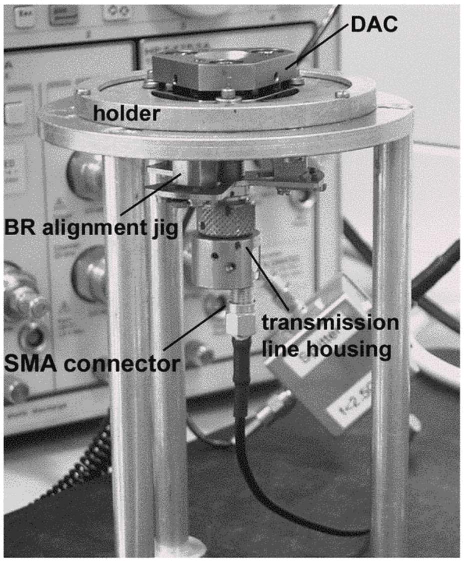


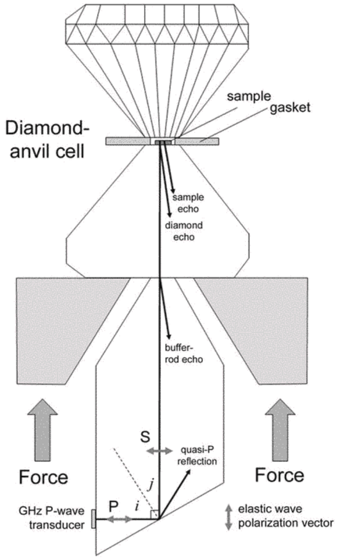
P-SV case



Angles of reflection/transmission depend only on velocities
Amplitudes depend on impedance (ρ v)

Use of seismic converted phases in a recent mineral physics experiment

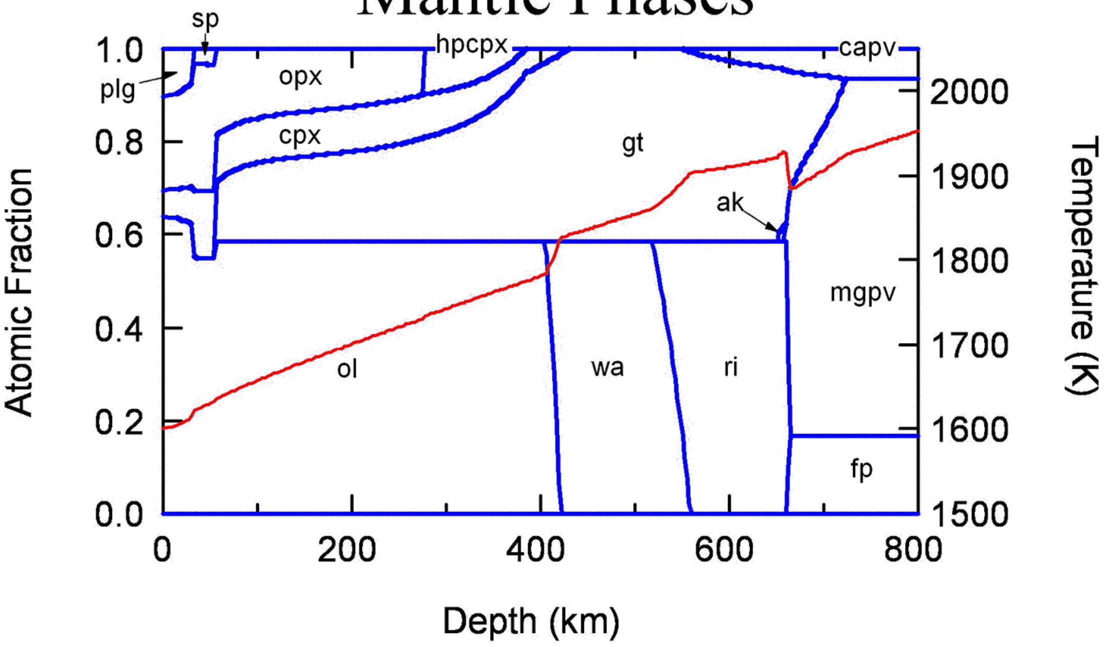




Jacobsen et al., 2005

Min Phys Lecture 1, Wed July 19:

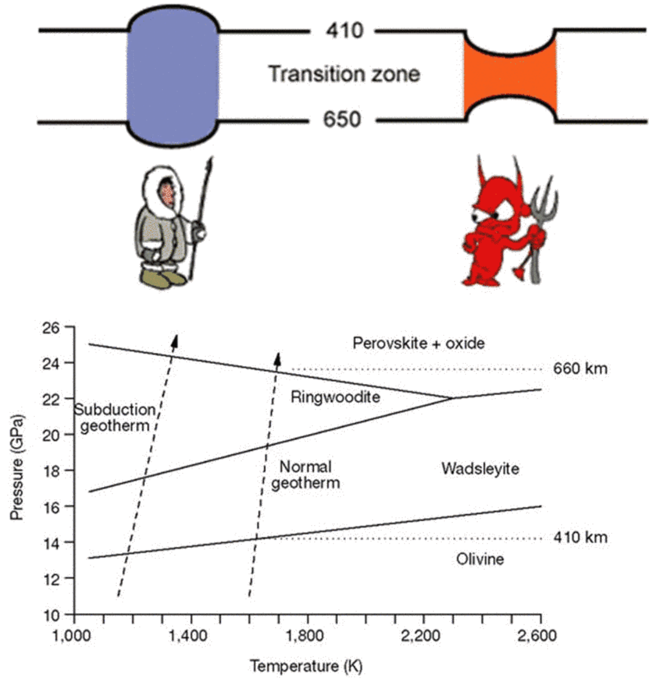
Mantle Phases

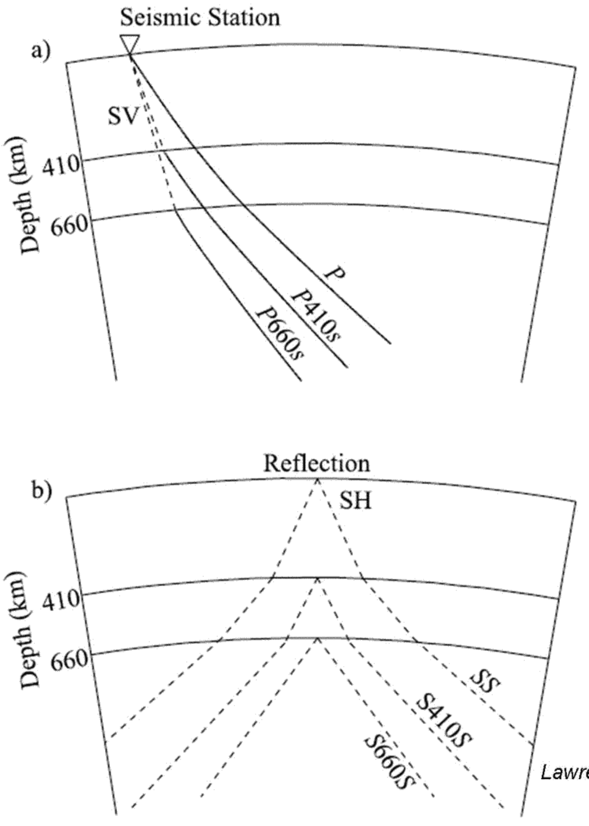


Wadsleyite (wa); Ringwoodite (ri); akimotoite (ak); Mg-perovskite (mgpv); Ca-perovskite (capv); Ferropericlase (fp)

Stixrude and Lithgow-Bertelloni, JGR (2005)

The Clapeyron Slope





Pds

Receiver function

SdS

Bottom converted precursors

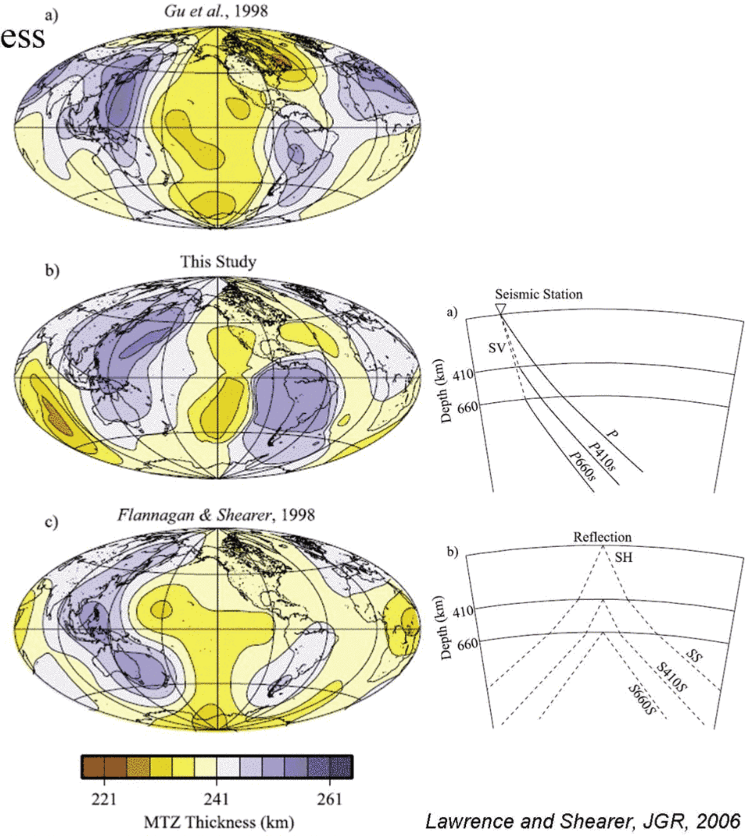
Lawrence and Shearer, JGR, 2006

Transition zone thickness

SdS (Harvard)

Pds
(receiver functions)

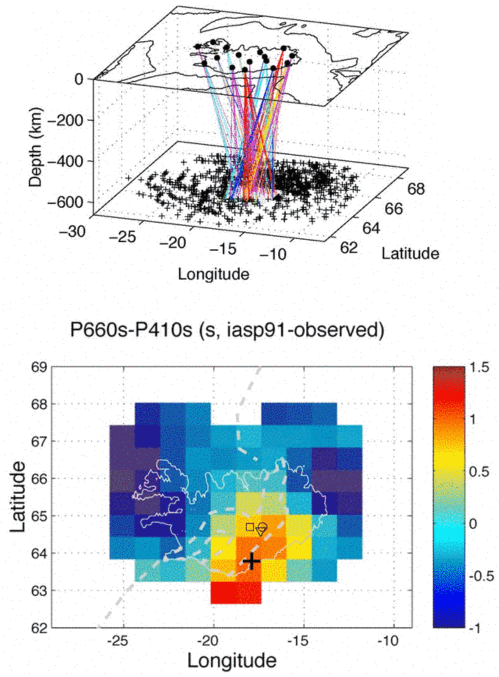
SdS (Scripps)
(SS precursors)

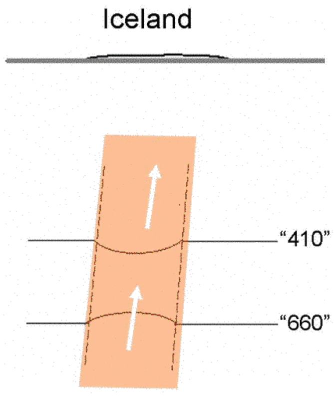


Lawrence and Shearer, JGR, 2006

Iceland transition zone thickness from receiver functions

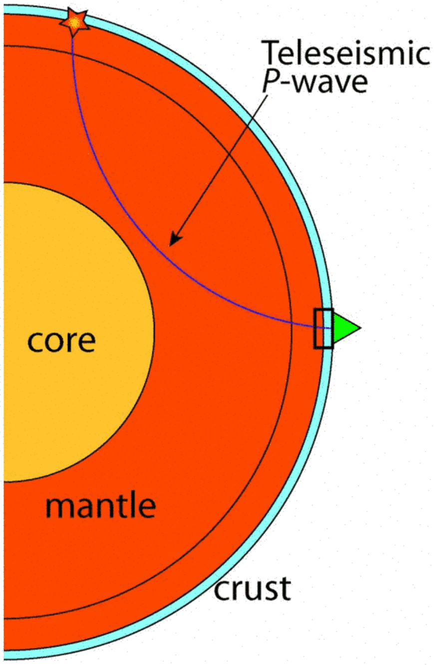
(Shown in Geodyn Lecture 2, Wed 7/19)

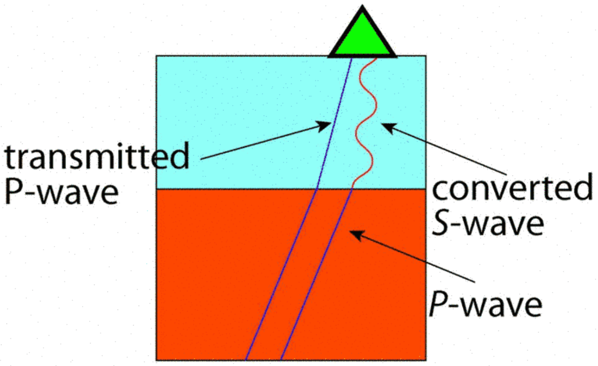


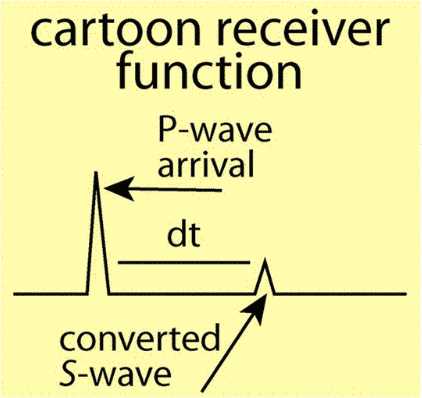


Shen et al.,Nature, 1998

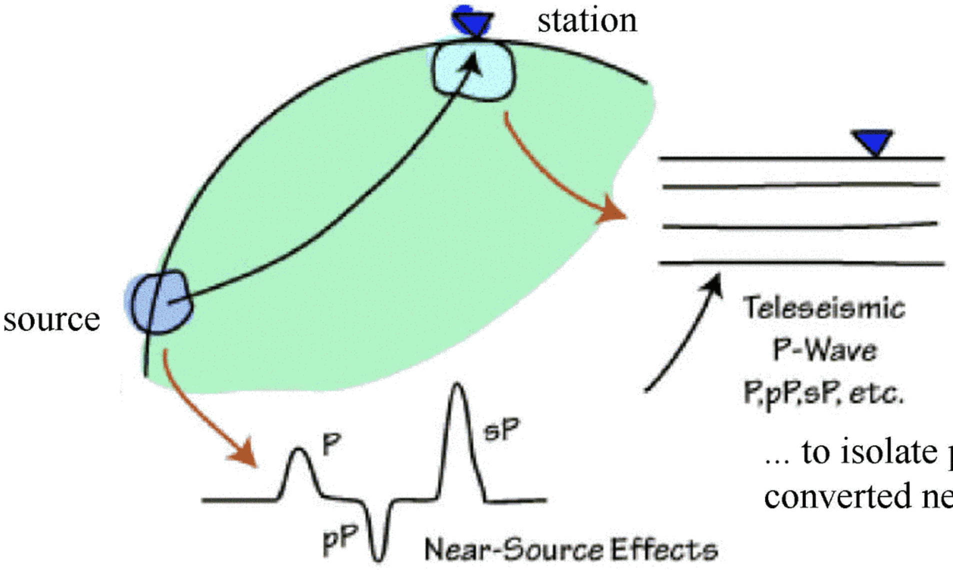
What is a receiver function?







unfortunately, incident P is not a nice simple bump:



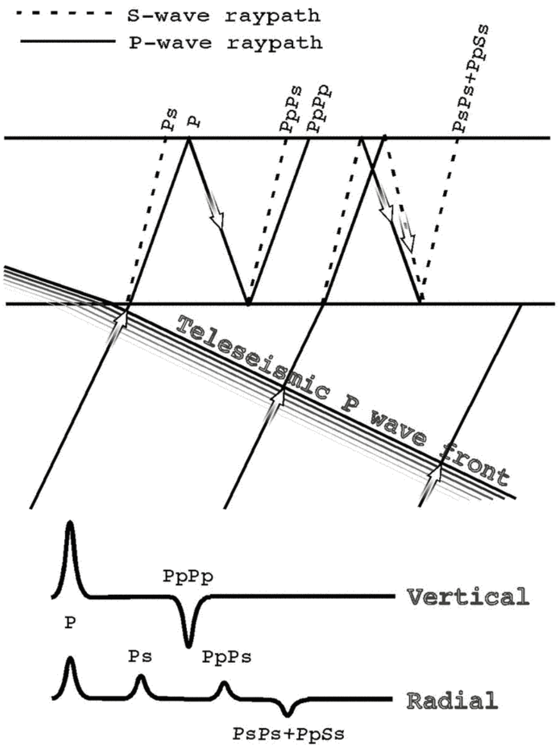
... to isolate phases
converted near station

need to remove these bits ...

receiver function animation
From Craig Jones

http://www.colorado.edu/GeolSci/Resources/

Teleseismic Converted Waves



What receiver functions are good for (roughly ranked)

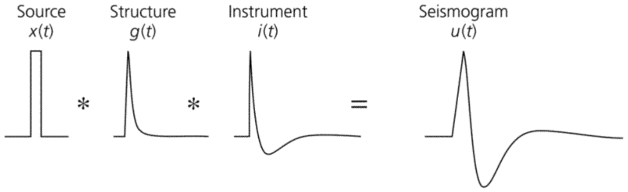
Sensitive to strong velocity contrasts (discontinuities) in the Earth -
    early receiver function studies focused on the Moho

Can provide detailed maps of discontinuity topography
    (but limited to where you have stations)
    better at relative depths than absolute depths
    better at transition zone thickness than 410 and 660 absolute depths

Determine depth to the discontinuity by assuming a velocity model
    Vp/Vs
    upper mantle heterogeneity
    transition zone heterogeneity

Huge velocity-discontinuity depth tradeoff
    Inverting for velocity using receiver functions is nonunique

Amplitudes more difficult than timing
    difficulties - variation with azimuth, incidence angle, stacking



Theoretical Displacement Response for a P plane wave

$D_v(t) = I(t)*S(t)*E_v(t)$ (vertical)

$D_r(t) = I(t)*S(t)*E_r(t)$ (radial)

$D_t(t) = I(t)*S(t)*E_t(t)$ (transverse)

Displacement Response

Instrument Impulse Response

Source Time Function

Structure Impulse Response

(Receiver Function)

Convolution

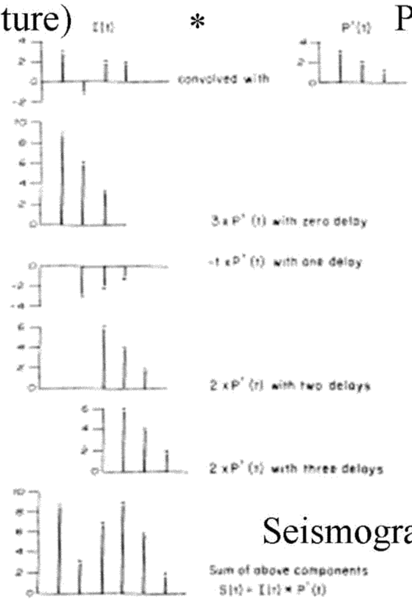
$$S_1 = a_1 p_1$$

$$S_2 = a_1 p_2 + a_2 p_1 \qquad S_i = \sum_{j=1}^{i} a_j p_{i-j+1} \qquad (4.17)$$

$$S_3 = a_1 p_3 + a_2 p_2 + a_3 p_1$$

This process is known as *convolution*, and symbolically we write

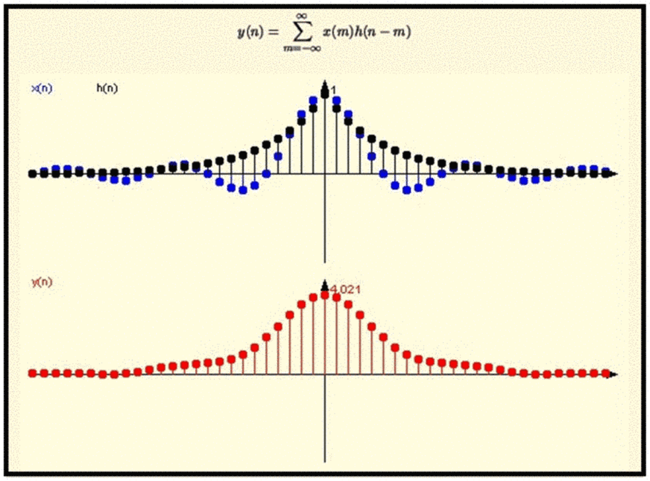
A (structure) * P (source)



Seismogram (A*p = S)

FIGURE 4.7 The procedure in a simple convolution.

$$(f * g)(t) = \int f(\tau) g(t-\tau)\, d\tau$$



http://www-es.fernuni-hagen.de/JAVA/DisFaltung/convol.html

Input Filter Output

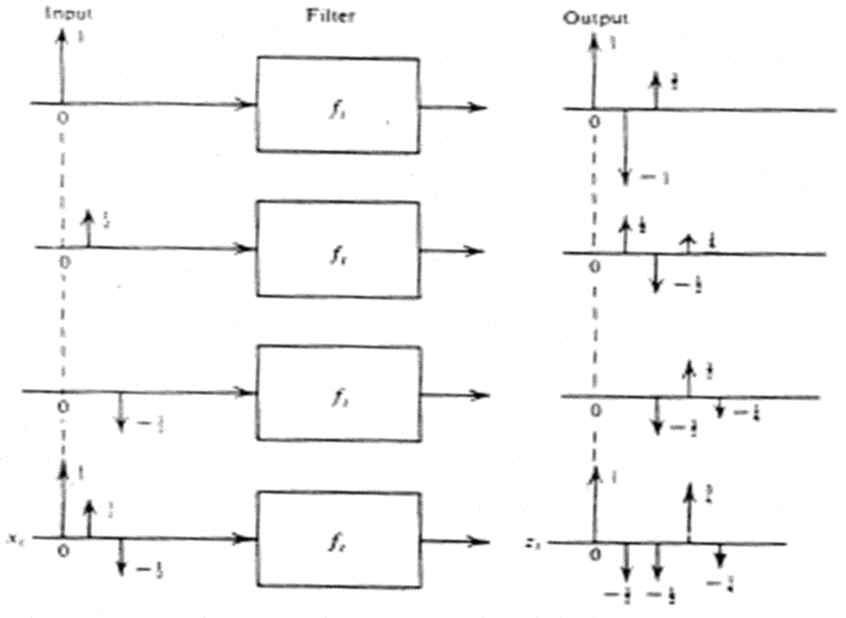


Figure 4.77. Filtering as an example of convolution.

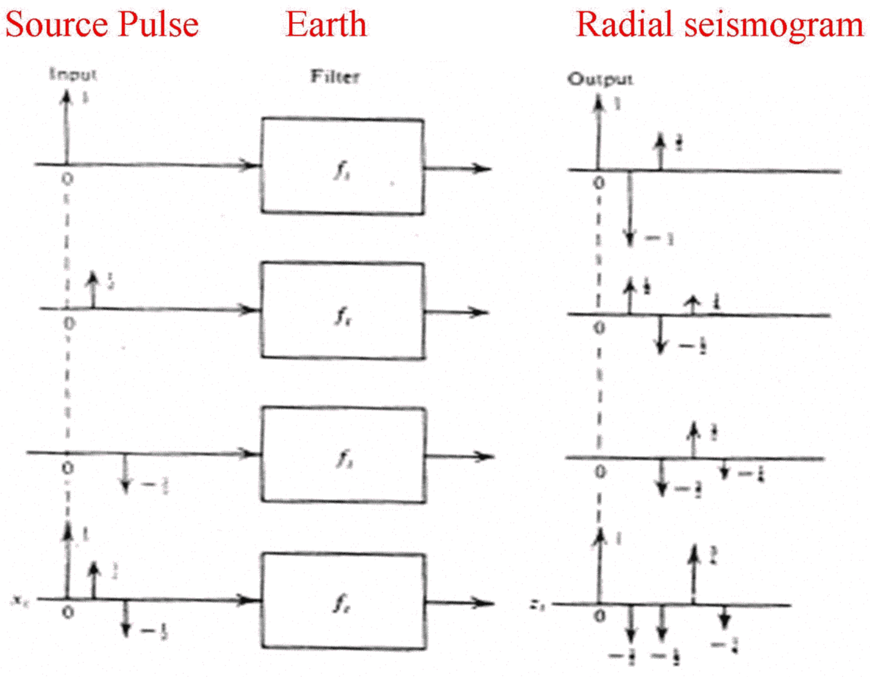
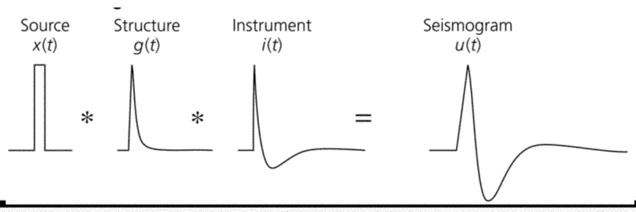


Figure 4.77. Filtering as an example of convolution.



Theoretical Displacement Response for a P plane wave

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$D_t(t) = I(t)*S(t)*E_t(t)$ (transverse)

Displacement Response

Instrument Impulse Response

Source Time Function

Structure Impulse Response
(Receiver Function)

- Receiver Function Construction

Receiver Function Construction

- Assumption: using nearly vertically incident events, the vertical component approximates the source function convolved with the instrument response

$$D_v(t) = I(t)*S(t)$$

Receiver Function Construction

- In the frequency domain, E_r and E_t can be simply calculated

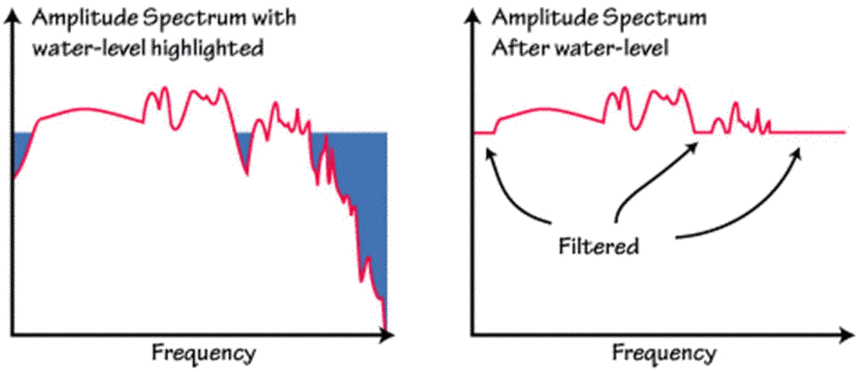
$$E_r(\omega) = \frac{D_r(\omega)}{I(\omega)S(\omega)} = \frac{D_r(\omega)}{D_v(\omega)}$$

$$E_t(\omega) = \frac{D_t(\omega)}{I(\omega)S(\omega)} = \frac{D_t(\omega)}{D_v(\omega)}$$

- this implies that $D_v(t)*E_r(t) = D_r(t)$

But spectral division is not very stable…

- Water level deconvolution in frequency domain



Chuck Ammon's online receiver function tutorial:
http://eqseis.geosc.psu.edu/~cammon/HTML/RftnDocs/rftn01.html

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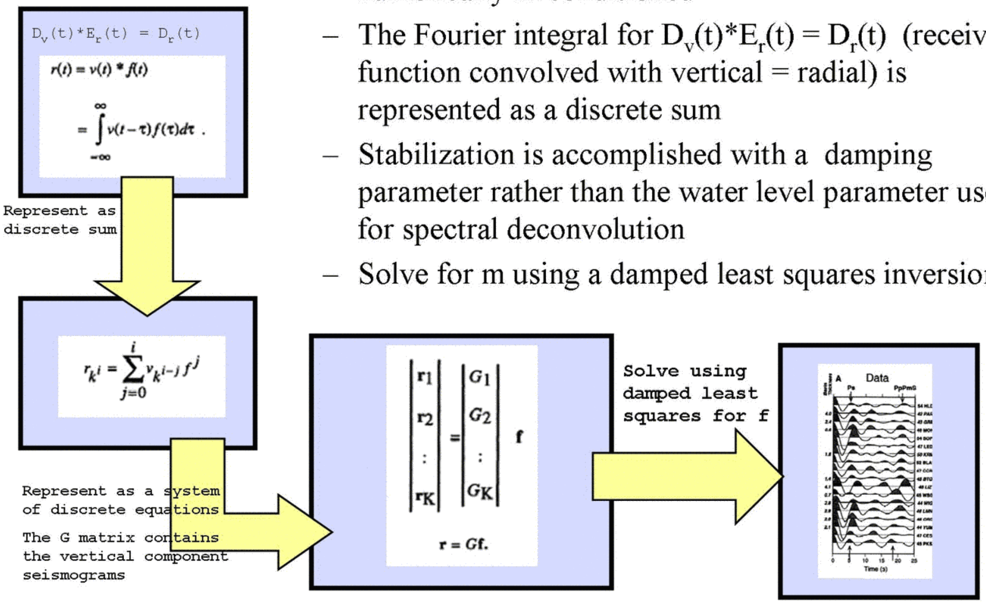
- Time Domain Deconvolution
  - Band-limited and nonstationary seismic signals make deconvolution using discrete Fourier transforms numerically ill conditioned
  - The Fourier integral for $D_v(t)*E_r(t) = D_r(t)$ (receiver function convolved with vertical = radial) is represented as a discrete sum
  - Stabilization is accomplished with a damping parameter rather than the water level parameter used for spectral deconvolution
  - Solve for m using a damped least squares inversion



$D_v(t)*E_r(t) = D_r(t)$

$$r(t) = v(t) * f(t) = \int_{-\infty}^{\infty} v(t-\tau) f(\tau) d\tau \, .$$

Represent as discrete sum

$$r_k{}^i = \sum_{j=0}^{i} v_{k^{i-j}} f^j$$

Represent as a system of discrete equations

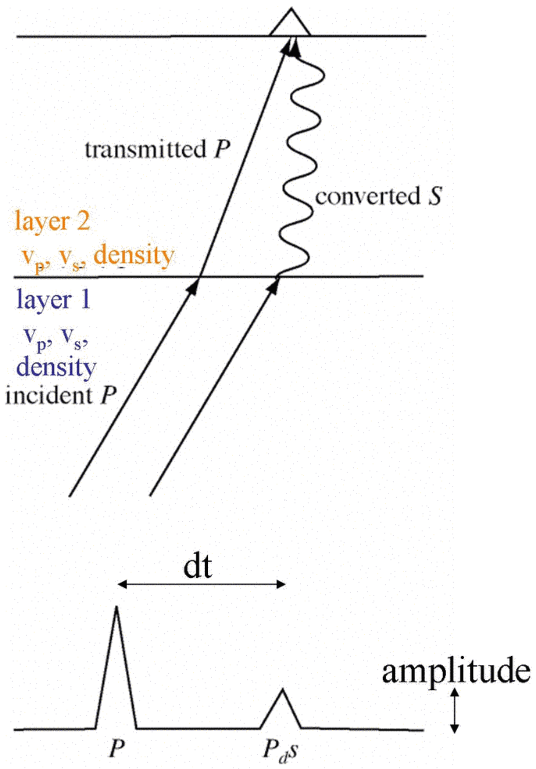
The G matrix contains the vertical component seismograms

$$\begin{vmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{vmatrix} = \begin{vmatrix} G_1 \\ G_2 \\ \vdots \\ G_K \end{vmatrix} \mathbf{f}$$

$$\mathbf{r} = G\mathbf{f}.$$

Solve using damped least squares for f

Sheehan et al., JGR, 1995



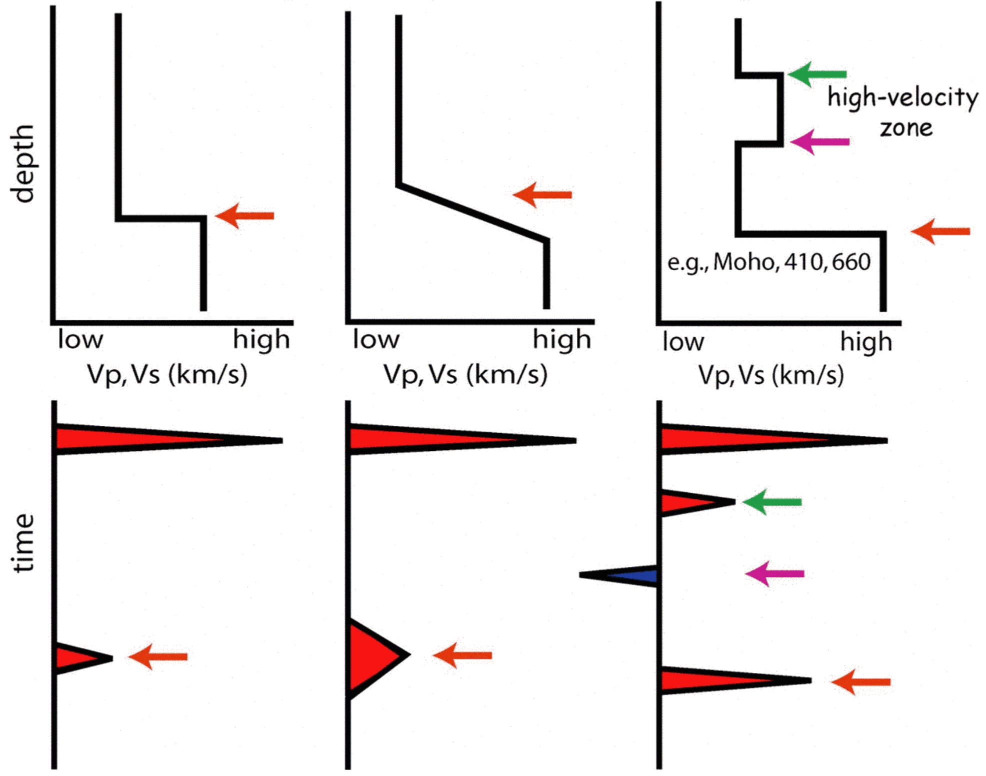
Sensitivity 1:

delay time dt depends on depth of interface and v_p, v_s

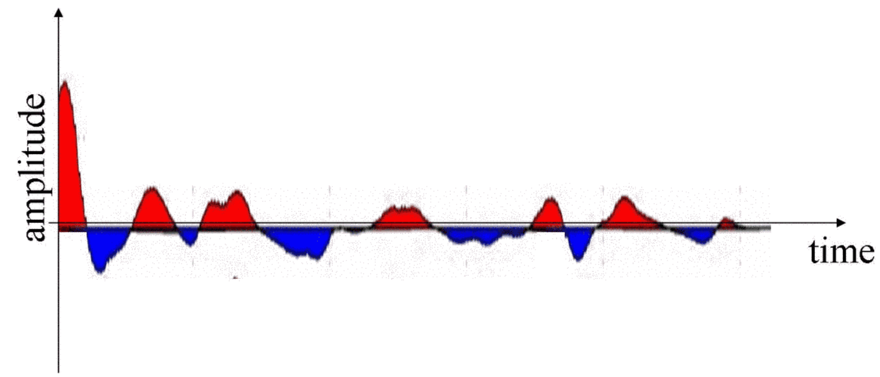
amplitude depends on velocity contrast (mostly) and density contrast (weakly) at the interface

converted arrival:
"+" bump = bottom slow, top fast
"-" bump = bottom fast, top slow

Sensitivity 2: sharp discontinuities v. gradients

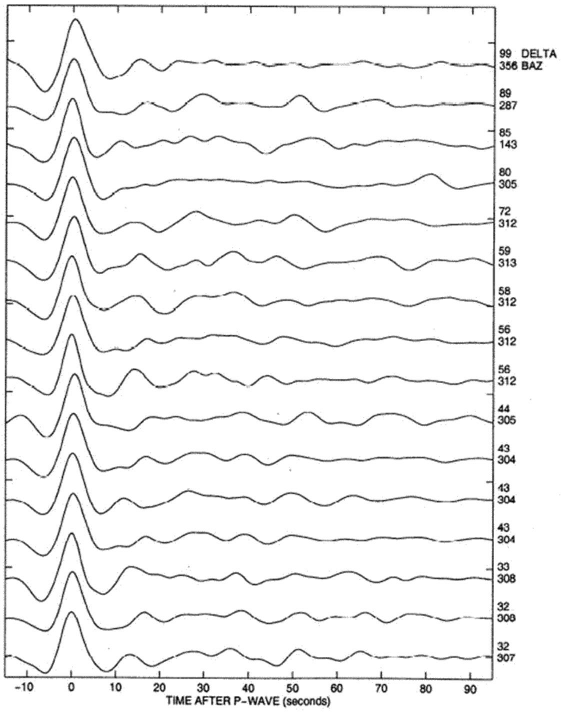


A single receiver function - hard to interpret

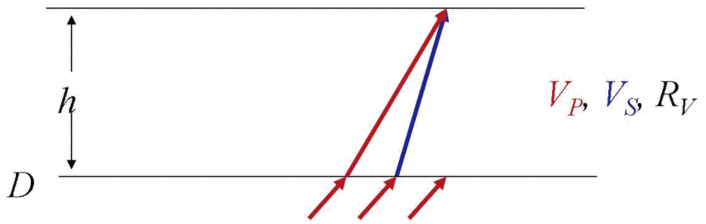


one receiver function per earthquake
-function of slowness (incidence angle)
-function of backazimuth (unless flat layered isotropic case)

Stacking: Enhance small signals



Arrival time of a non-vertically incident $P_d s$ wave



$$\Delta T_{P_d s} = \int_D^0 \sqrt{V_S(z)^{-2} - p^2} - \sqrt{V_P(z)^{-2} - p^2}\, dz$$

Need to know V_P,V_S, and (p), or slowness, of the incident P-wave

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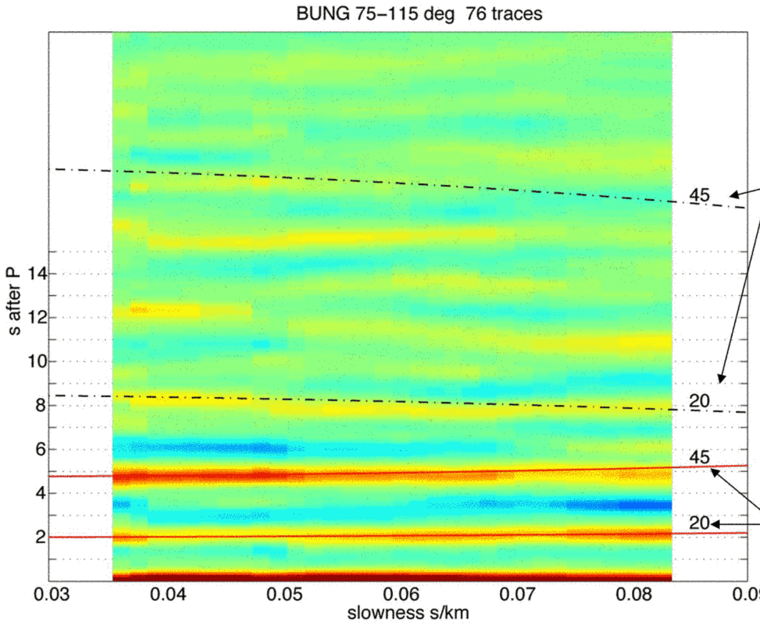
receiver function moveout animation
From Craig Jones

http://www.colorado.edu/GeolSci/Resources/

Receiver function moveout plot

(Many receiver functions plotted as a function of distance from station)



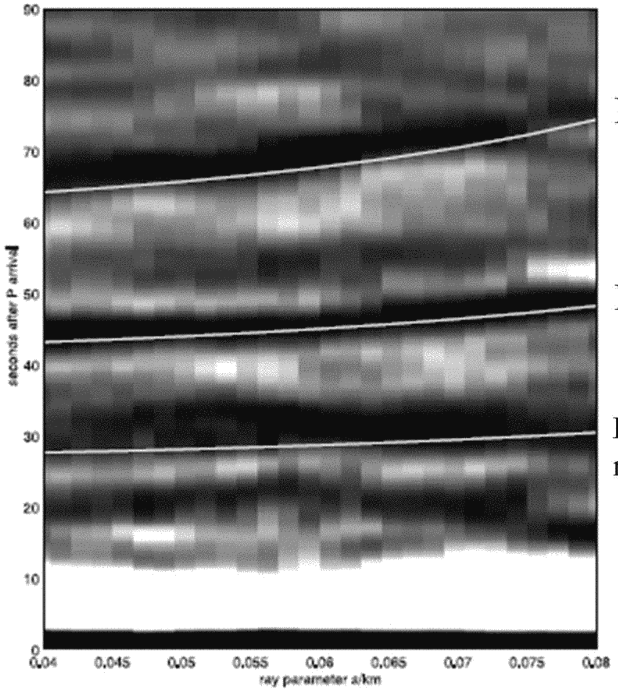
Time, s

Ray parameter

multiples

depth of modelled discontinuity (km)

Receiver function moveout curve



90 s

Time, s

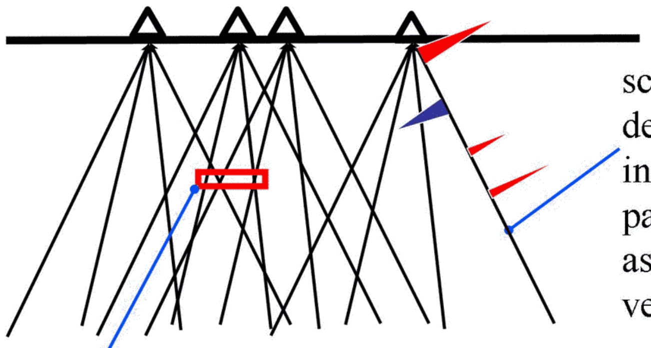
Ray parameter

P660s

P410s

P250s?? Moveout not convincing

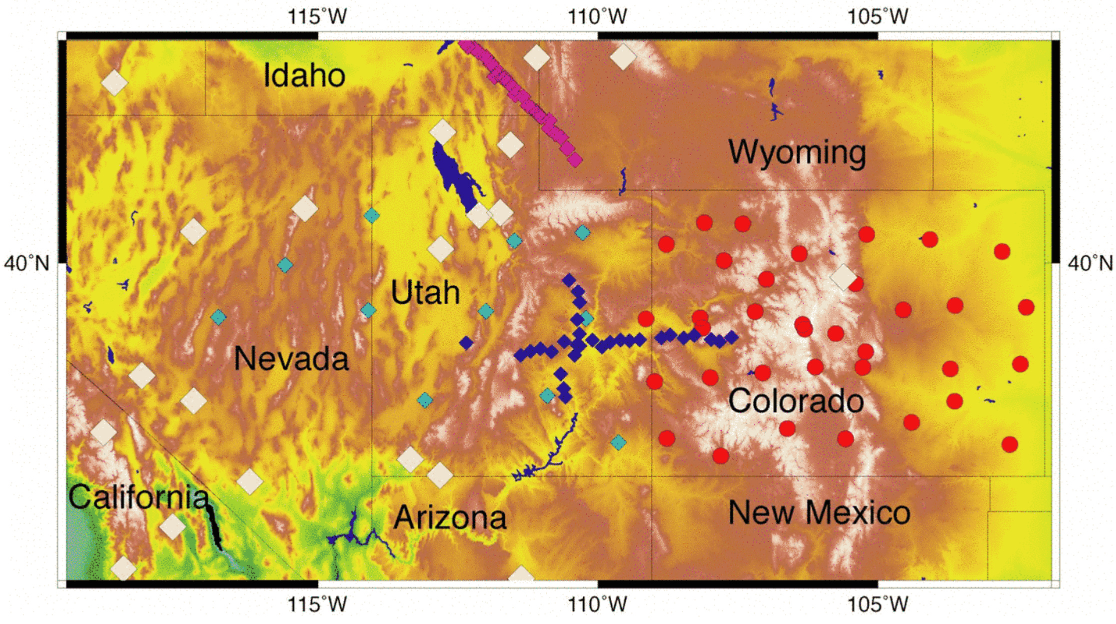
Common conversion point (CCP) stacking

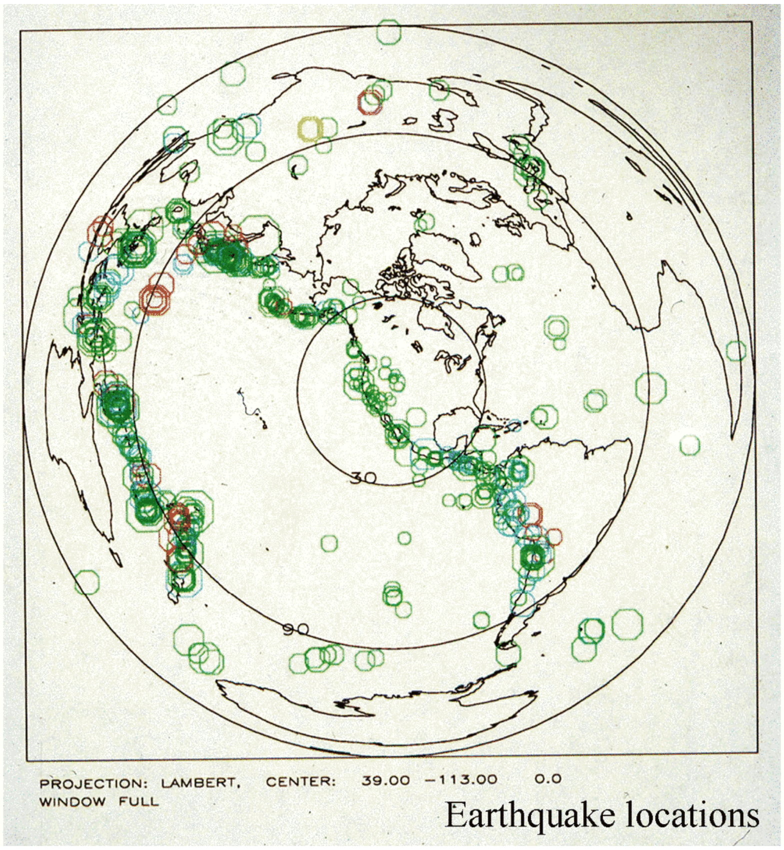


scale time to
depth along
incoming ray
paths with an
assumed
velocity model

stack all receiver
functions within
common conversion
point bin

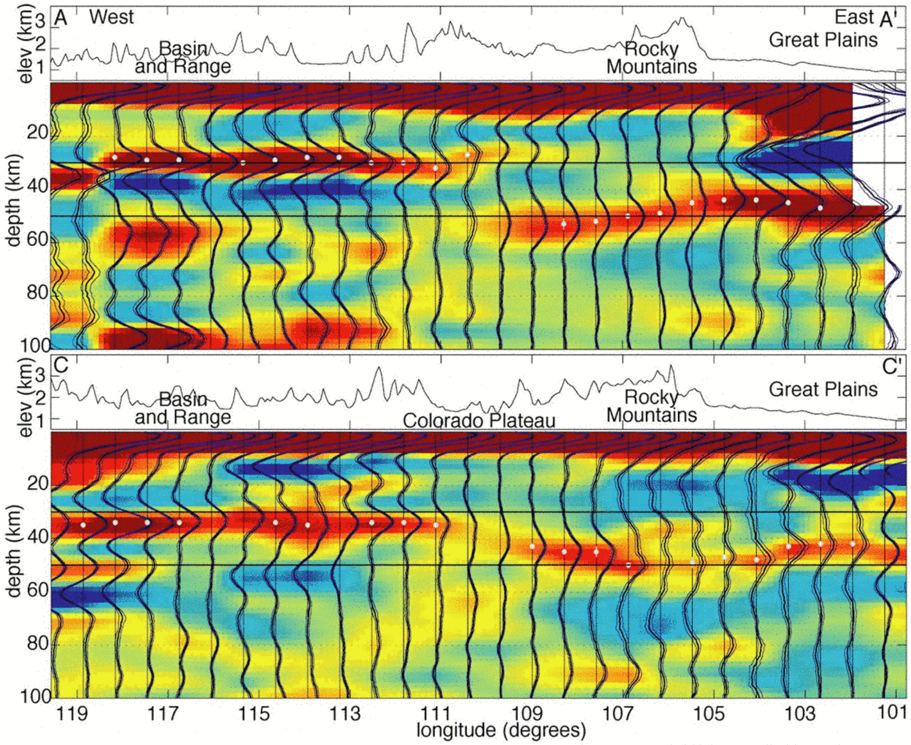
Western United States portable seismic stations





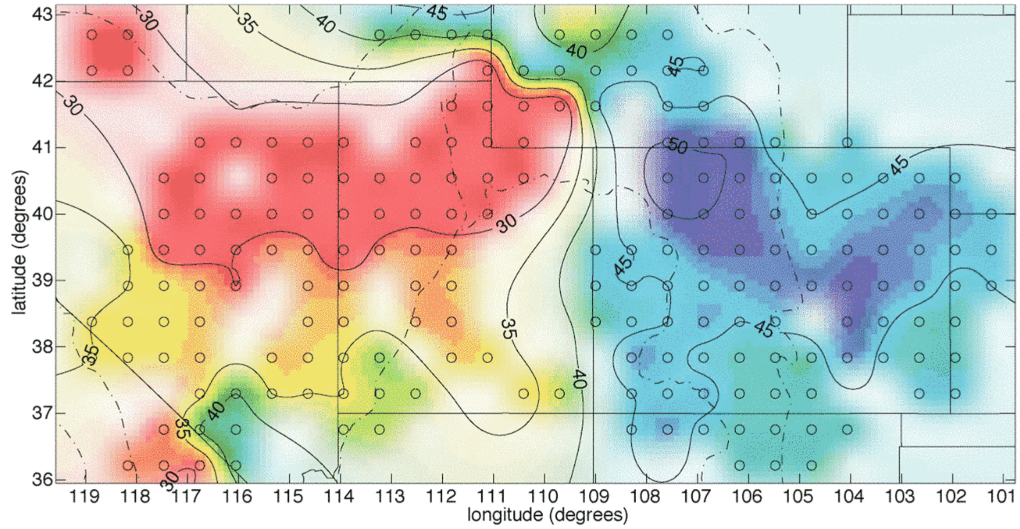
Earthquake locations

CRUSTAL Receiver function profiles across the Western United States



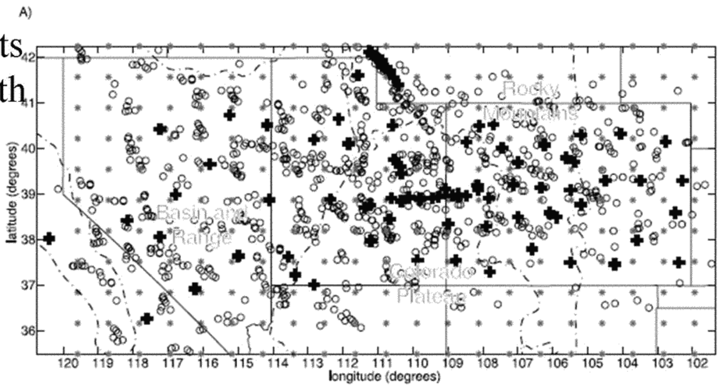
Gilbert & Sheehan, 2004

Western United States crustal thicknesses from receiver functions

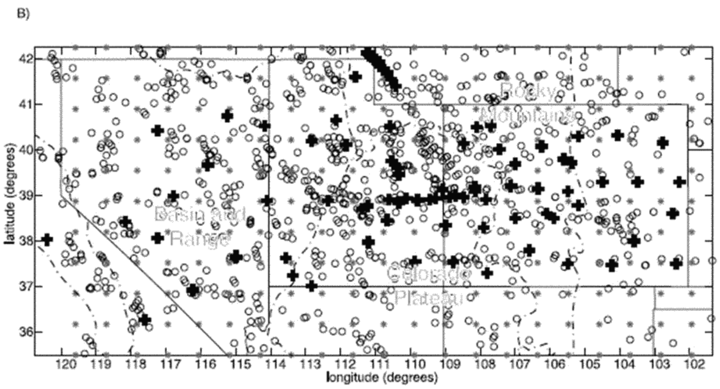


Gilbert & Sheehan, 2004

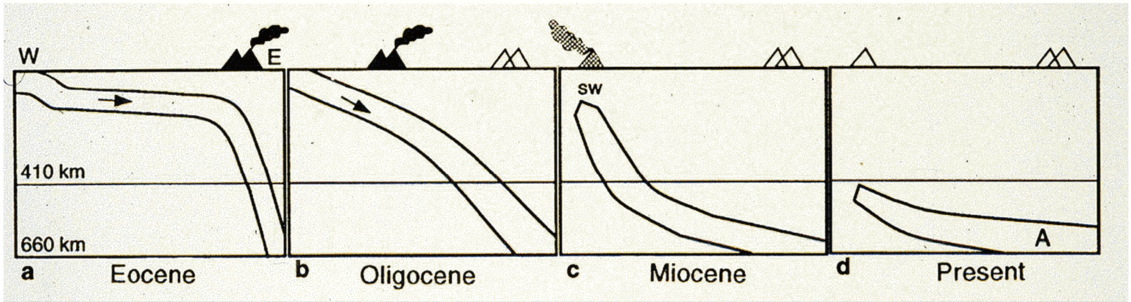
Piercing points
At 410 beneath
WUS



Piercing points
At 660

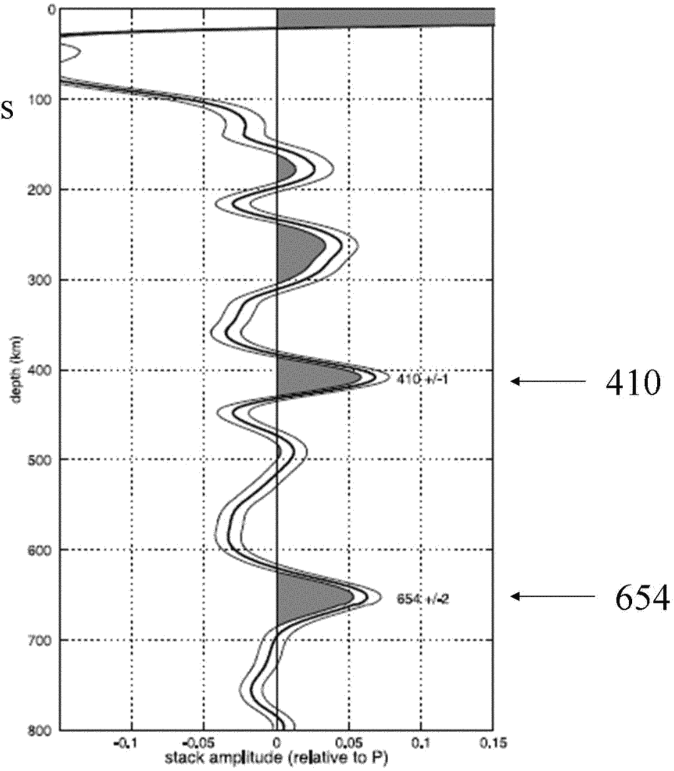


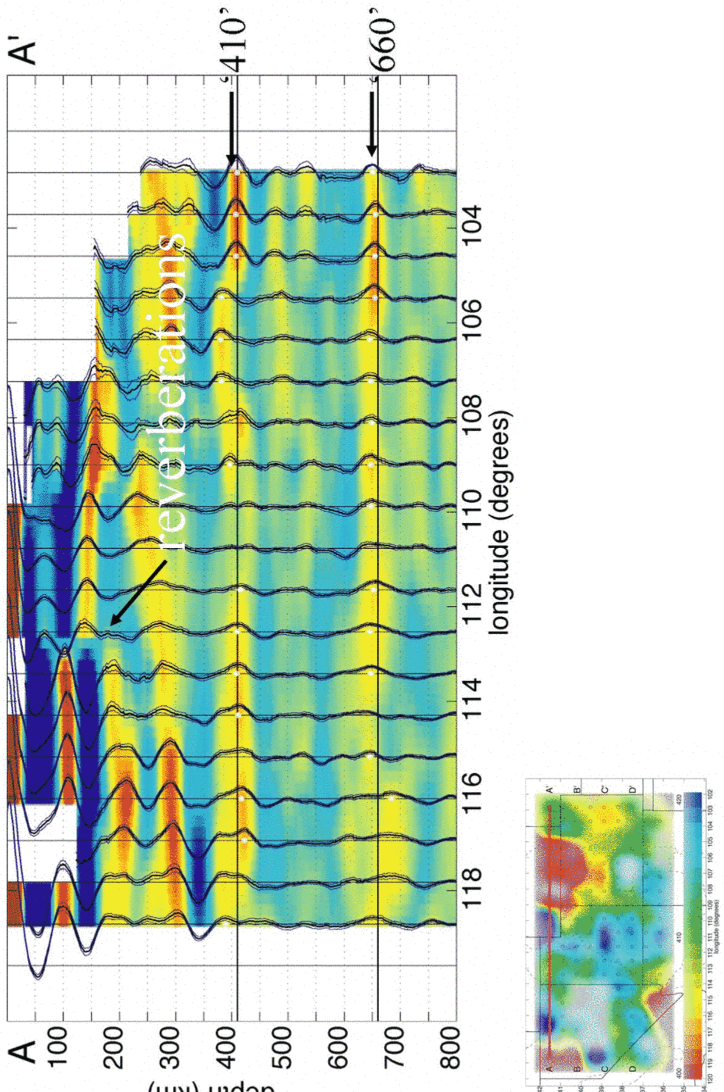
Subduction beneath Western North America - last 60 Million years

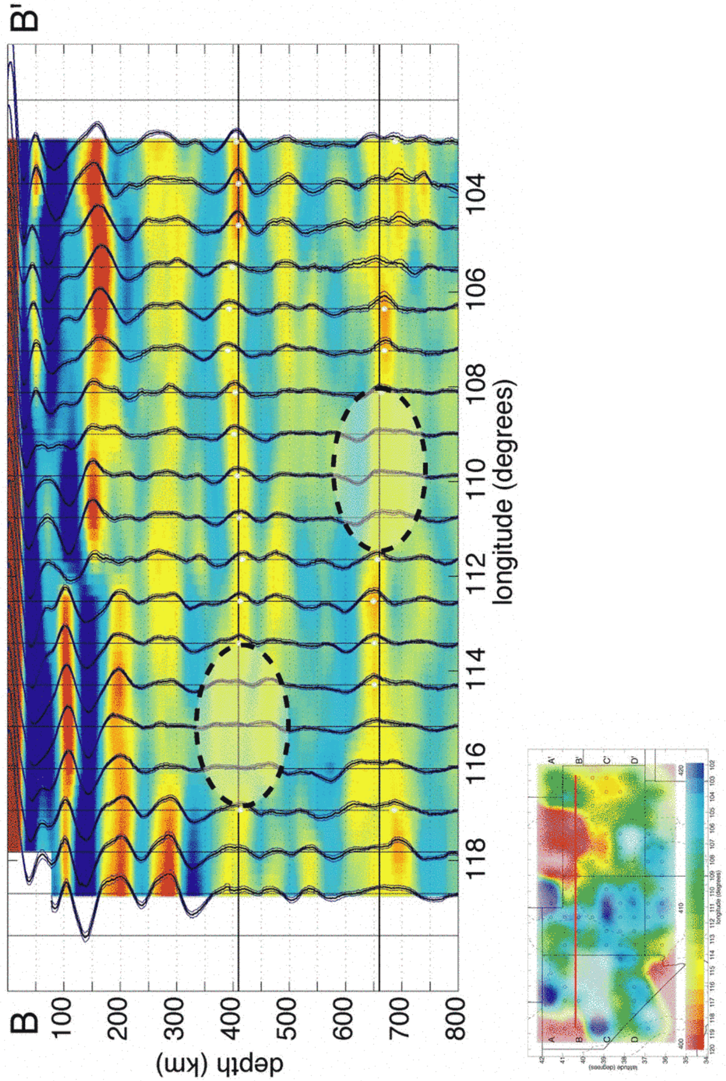


Van der Lee and Nolet, 1997

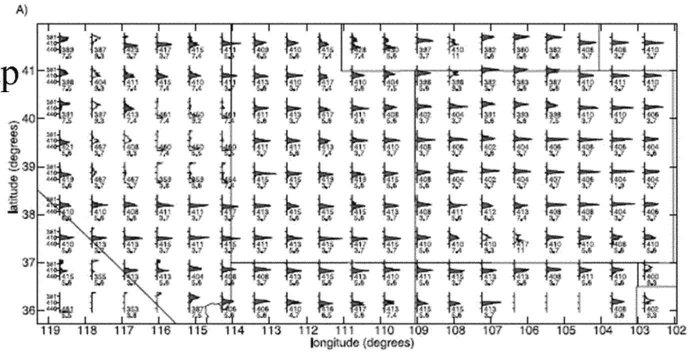
Stack of all
Western US
Receiver functions



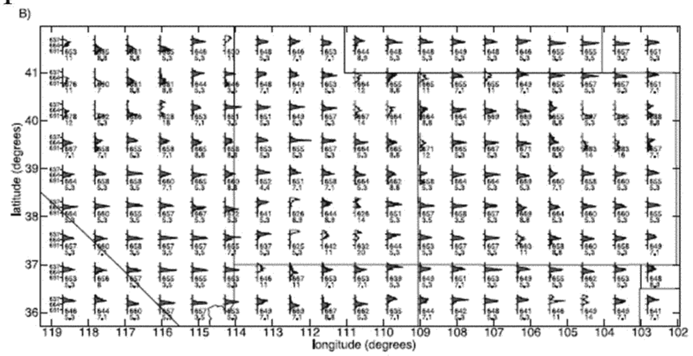

A
A'
depth (km)
100
200
300
400
500
600
700
800
'410'
'660'
reverberations
longitude (degrees)
104
106
108
110
112
114
116
118


B
B'
depth (km)
100
200
300
400
500
600
700
800
longitude (degrees)
104
106
108
110
112
114
116
118

410 bootstrap

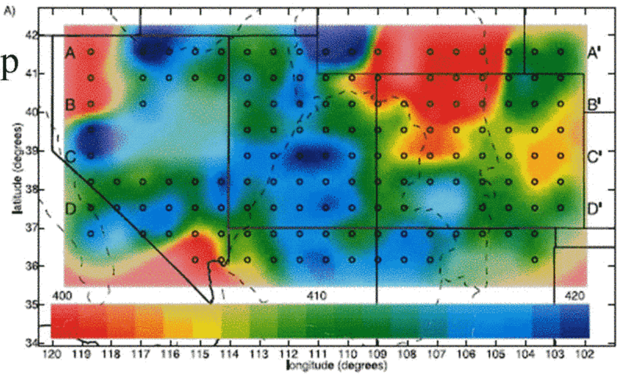


660 bootstrap

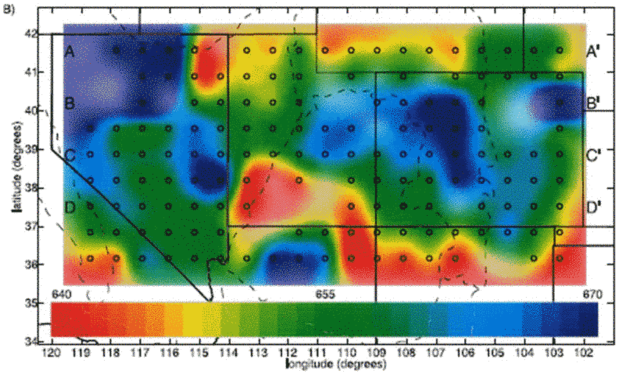


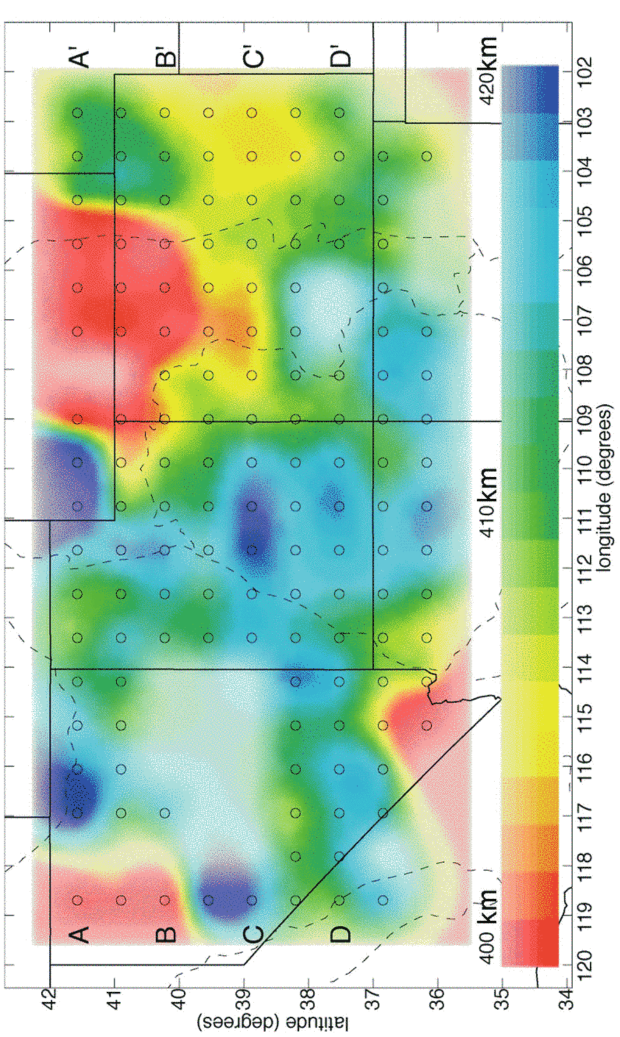
410 contour map

Red=shallow
Blue=deep

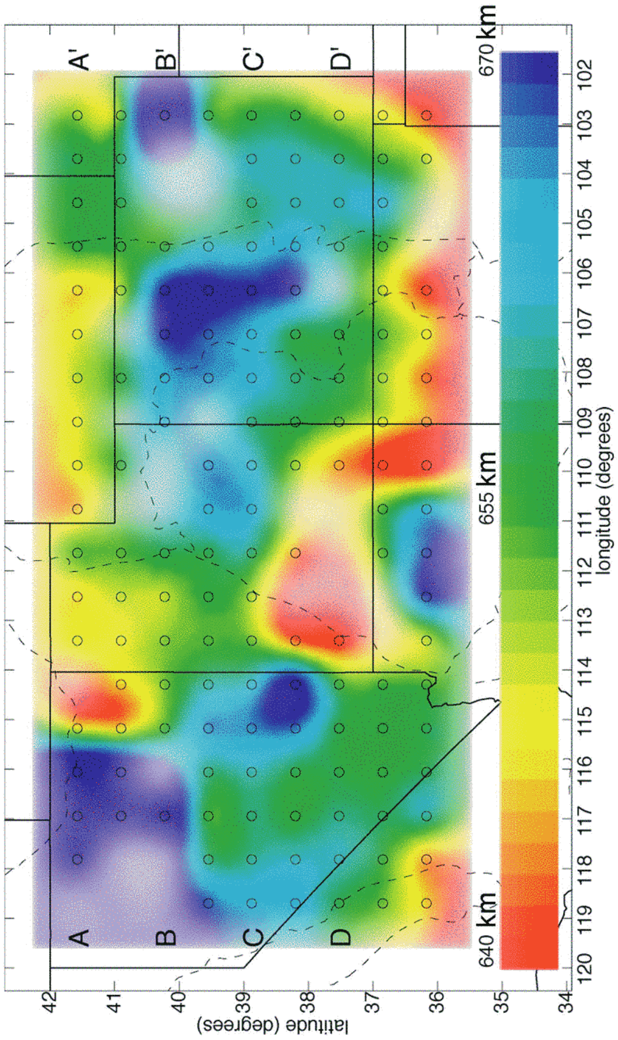


660



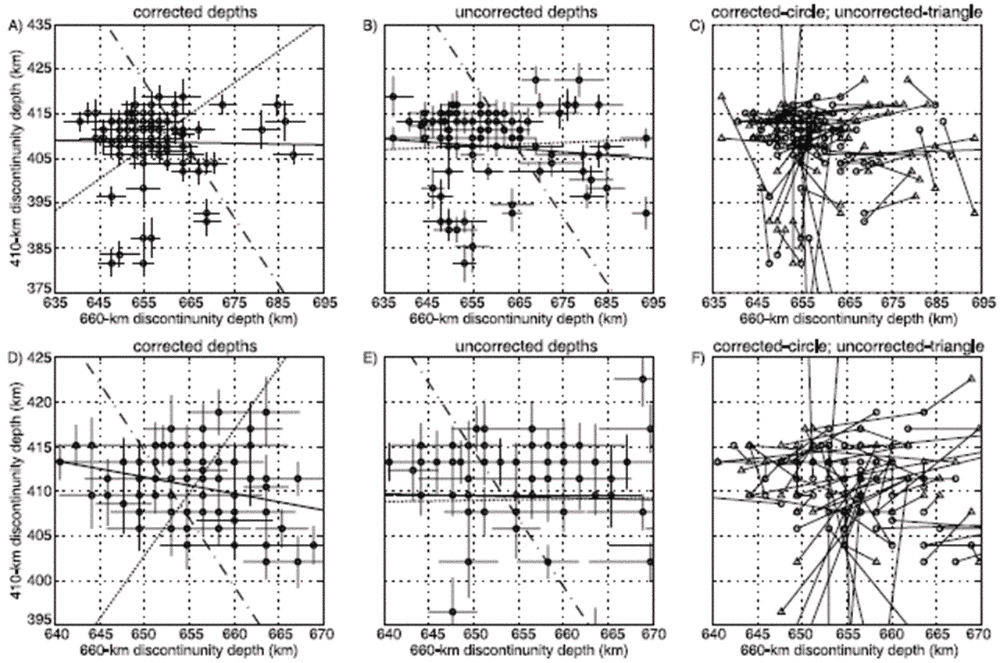


map of 410-km discontinuity depth



map of 660-km discontinuity depth

410 and 660 depths not correlated beneath Western US



→ Purely thermal interpretation of discontinuity topography doesn’t work here

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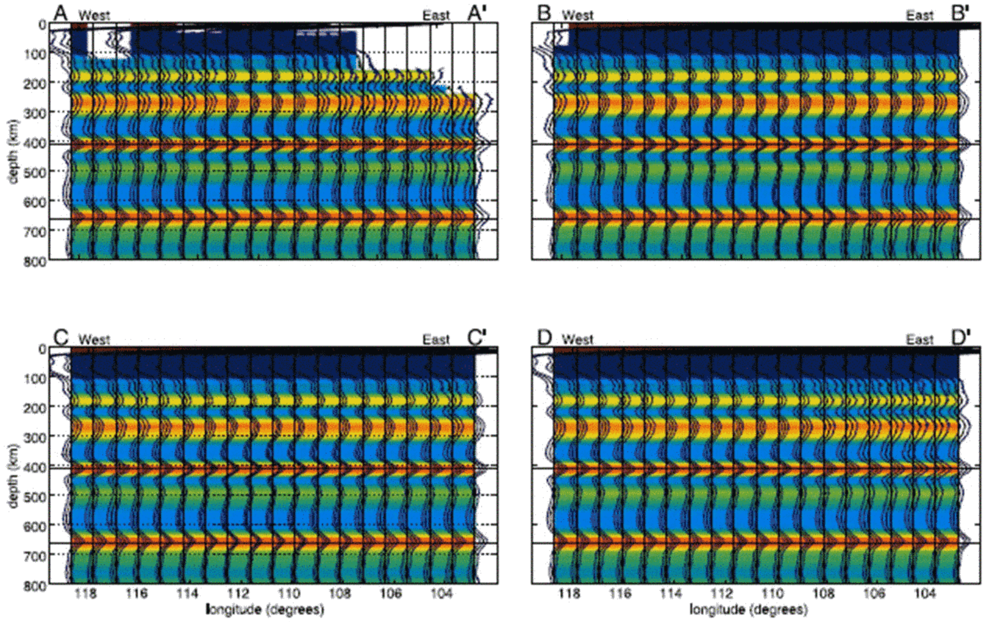
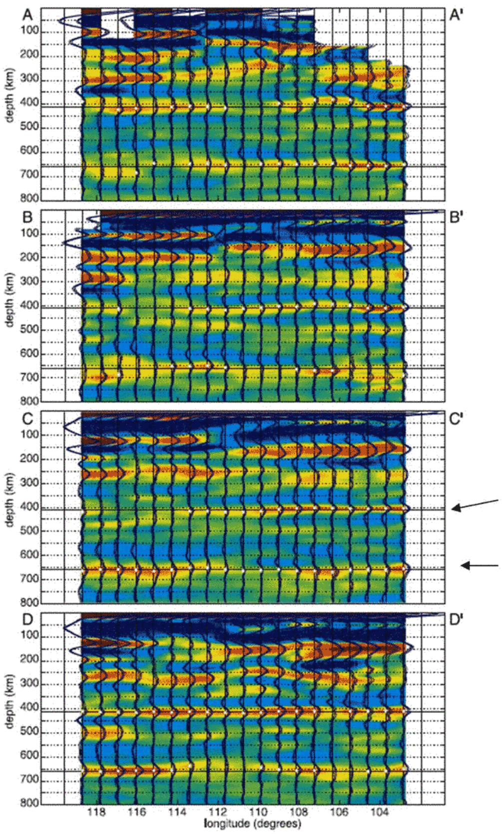
Random geographic stacks



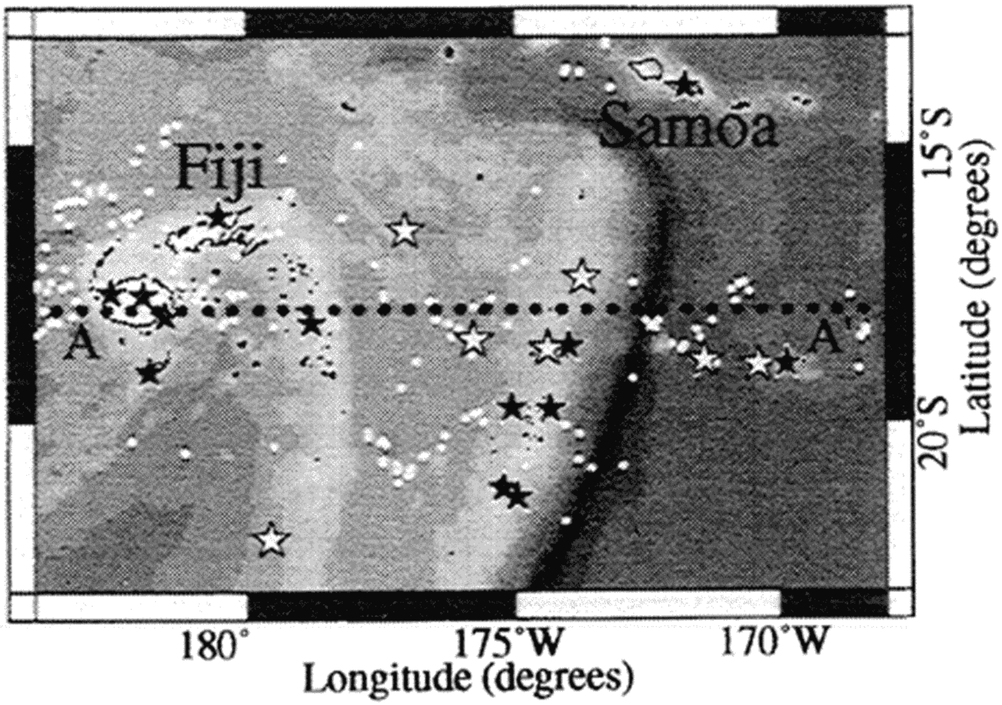
Cross sections
of Western US
receiver functions



410 km

660 km

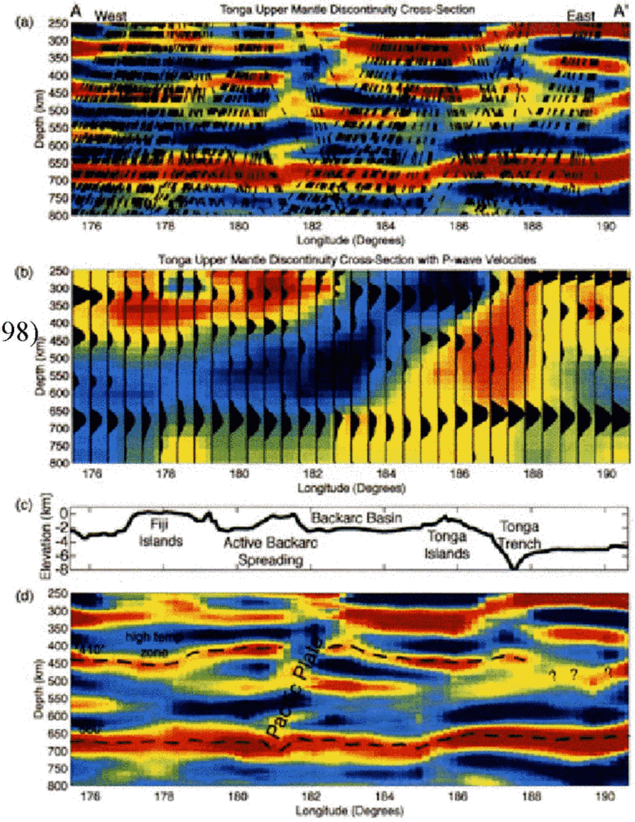
Slabs and plumes



Tonga

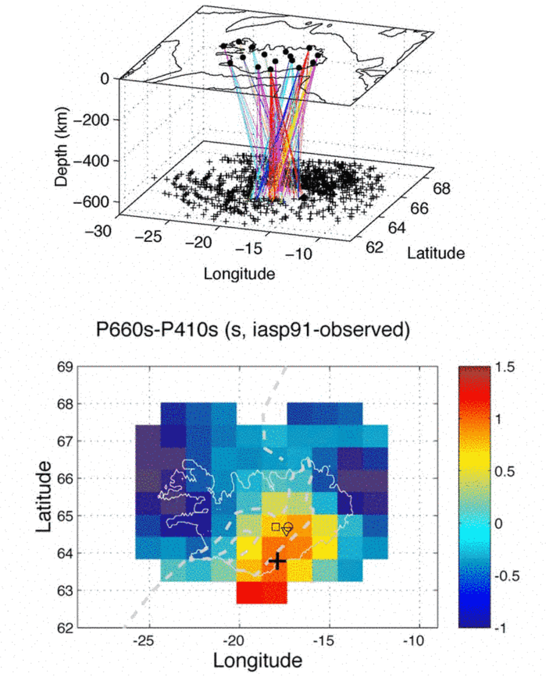
Tomography
(Bijwaard et al., 1998)

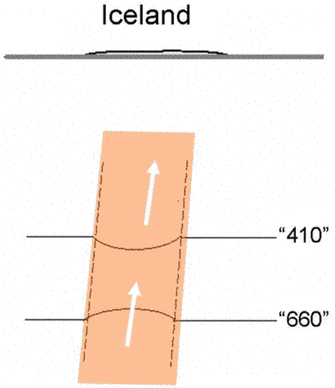
Receiver
functions



Gilbert et al,
GRL, 2001

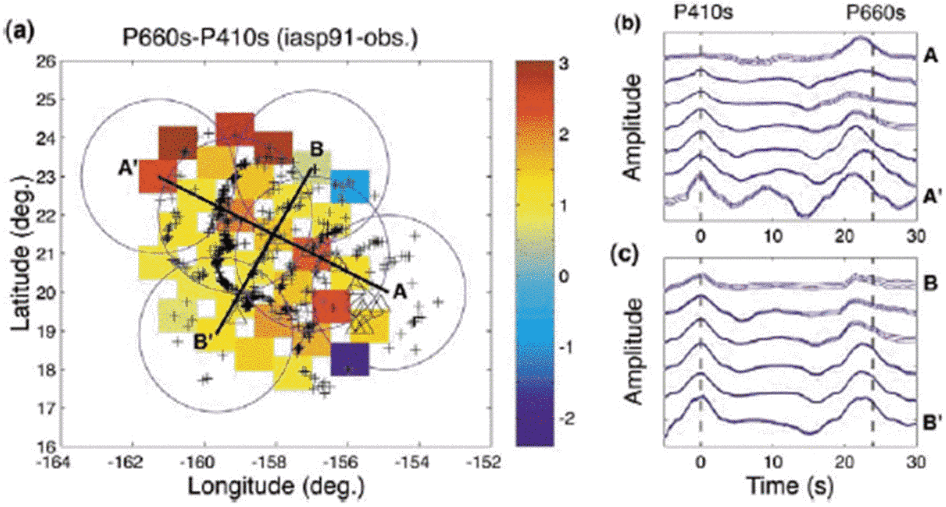
Iceland



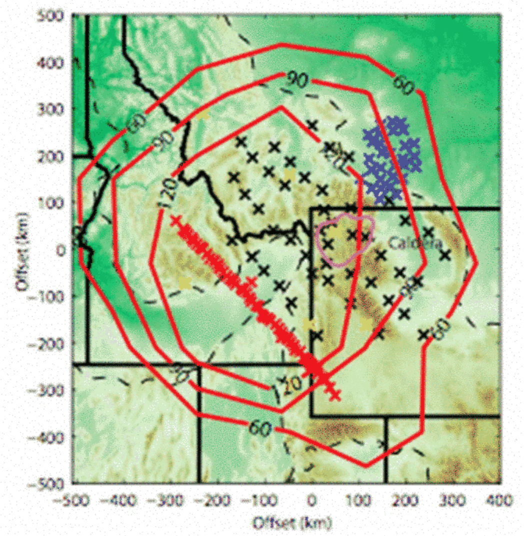


Shen et al., 1998; 2002

Hawaii



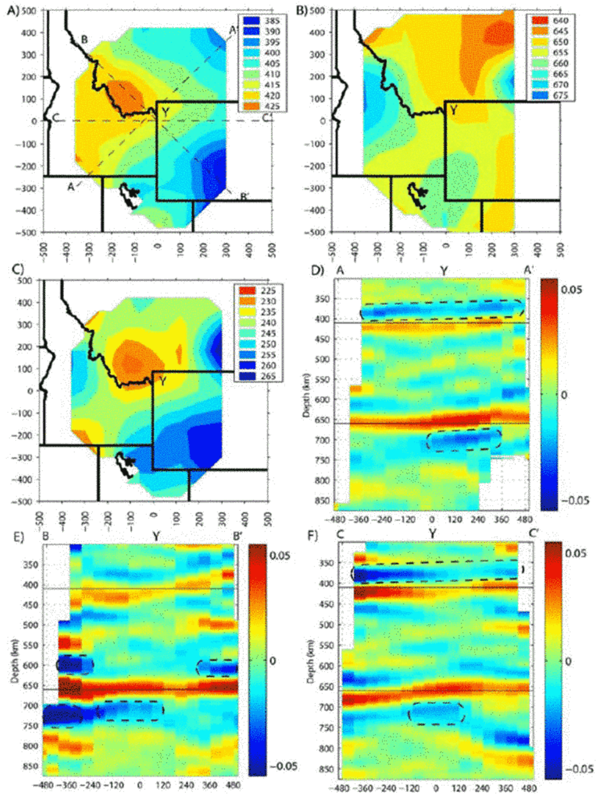
Shen et al., EPSL, 2003



Fee and Dueker, GRL, 2004

Yellowstone
410 depressed, 660 flat

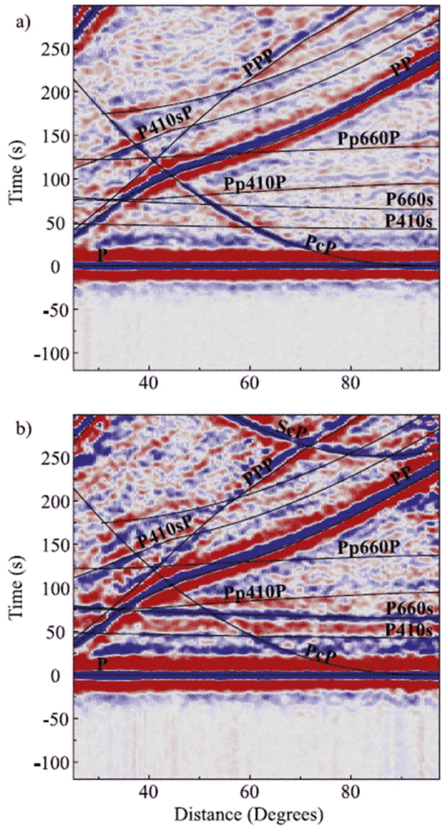
Yellowstone



Fee and Dueker, GRL, 2004

Global

Stack of >22,000
P waves



Vertical P stack

Radial P stack

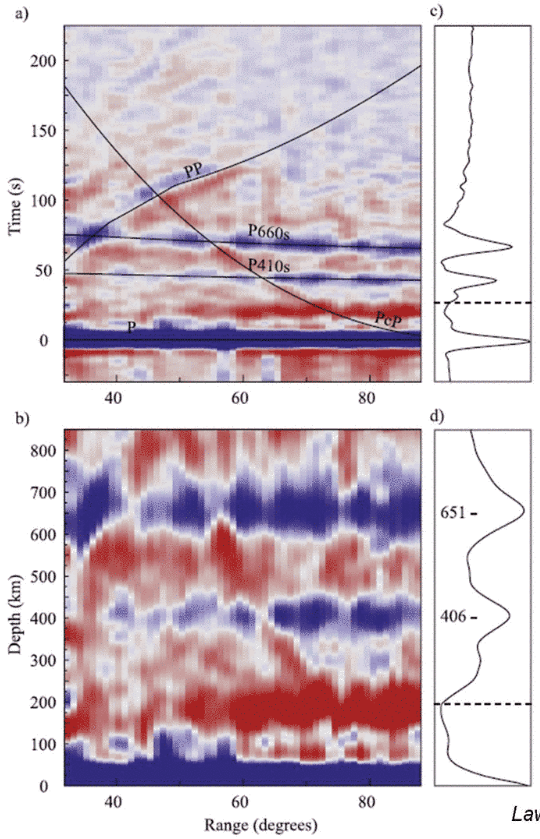
Black lines are predicted
Arrivals from PREM

Lawrence and Shearer, JGR, 2006

Receiver functions

Distance vs. time

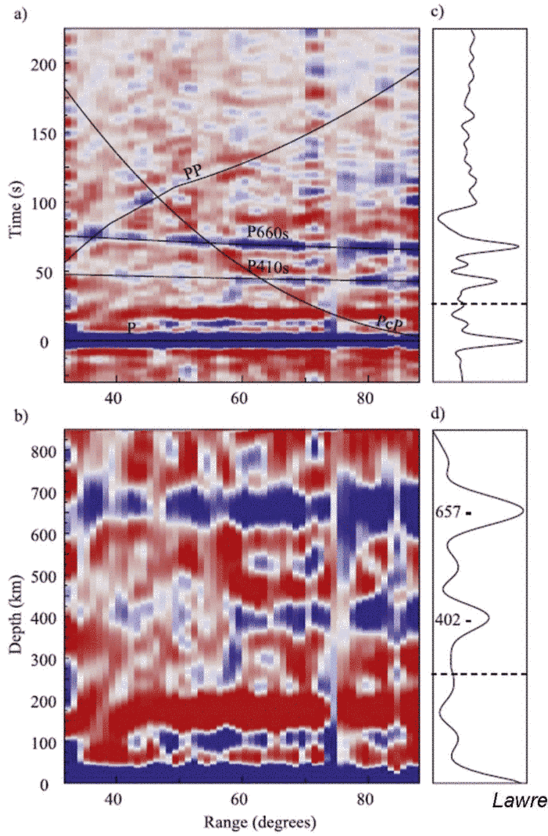
Distance vs. depth



Lawrence and Shearer, JGR, 2006

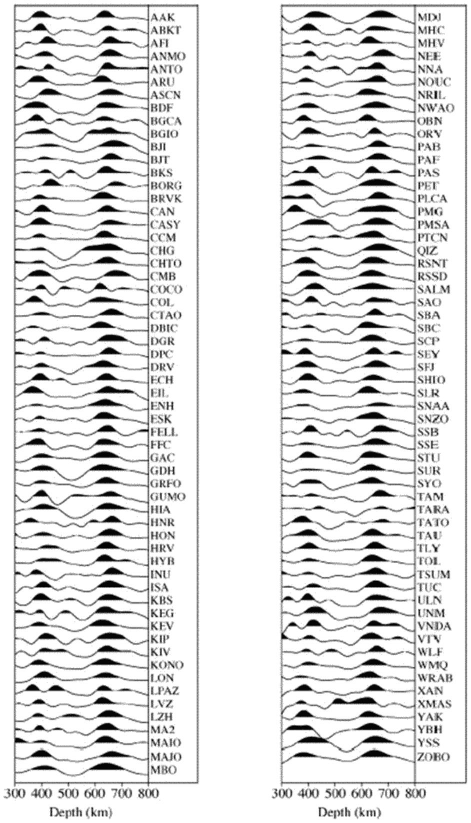
CTAO Receiver functions vs. time

CTAO Receiver functions vs. depth

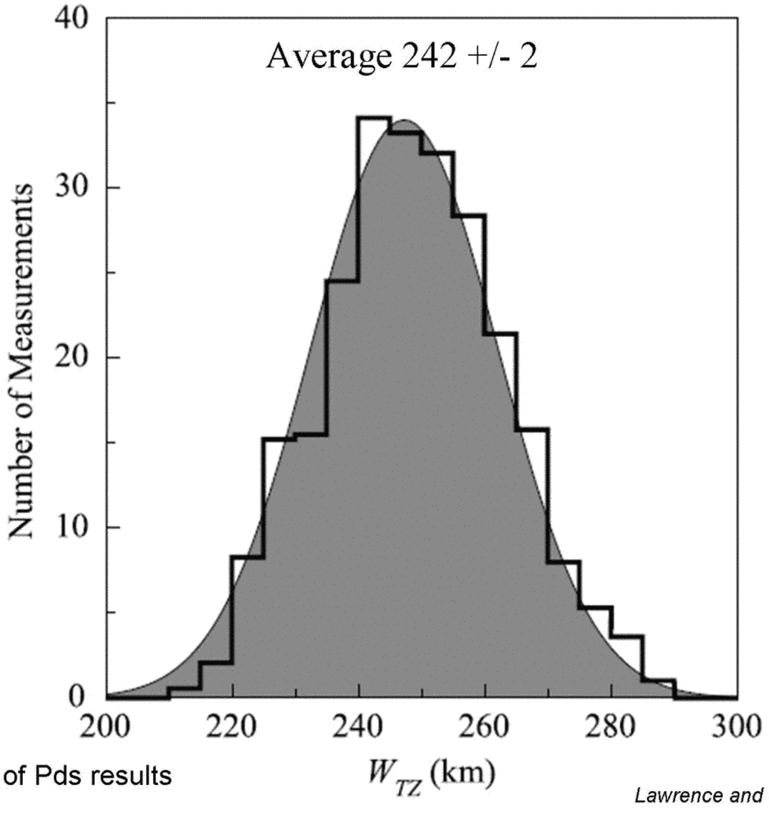


Lawrence and Shearer, JGR, 2006

Pds stacks for
118 stations



Lawrence and Shearer, JGR, 2006



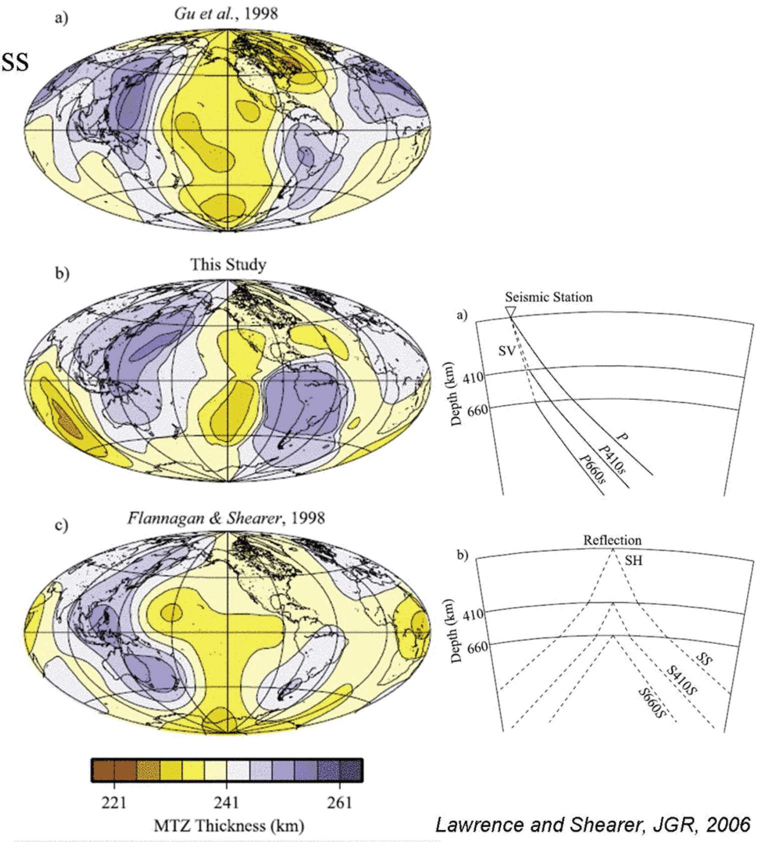
Distribution of Pds results

Lawrence and Shearer, JGR, 2006

Transition zone thickness

Pds
(receiver functions)

SdS
(SS precursors)



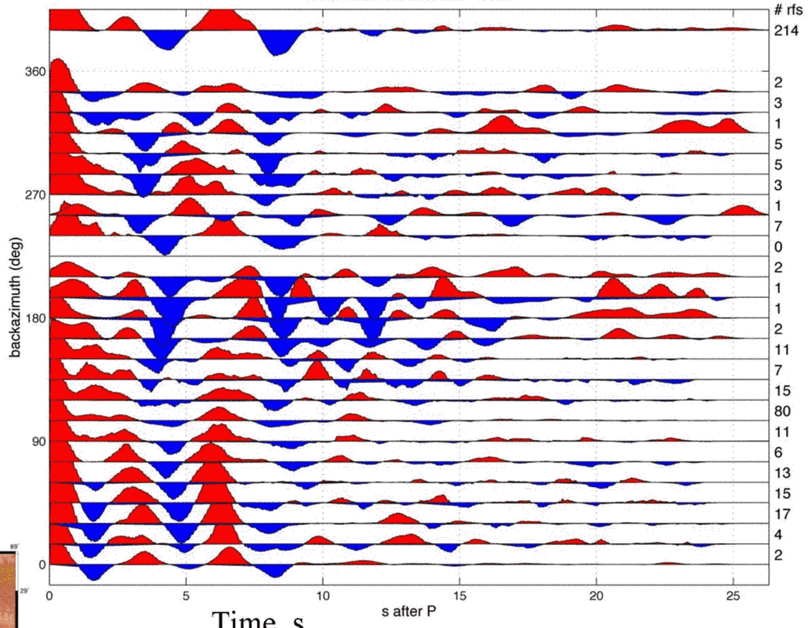
Lawrence and Shearer, JGR, 2006

Complicated Earth

Anisotropy (e.g. Vinnik)

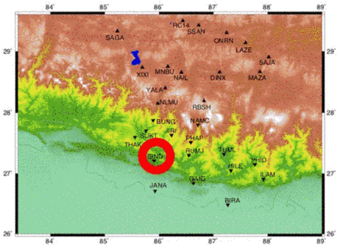
Non planar discontinuities (e.g. Levander)

Azimuthal variation in Himalayan crustal receiver functions



backazimuth

Time, s

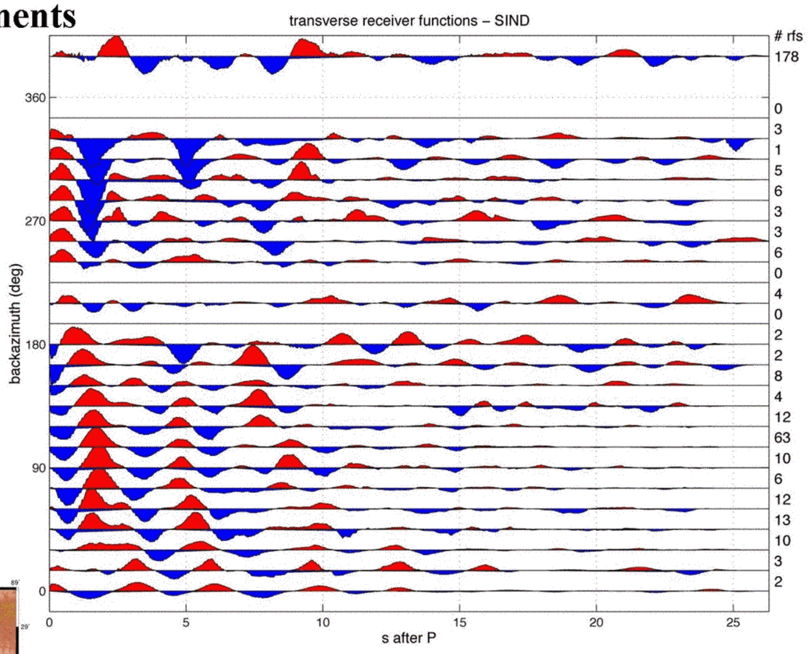


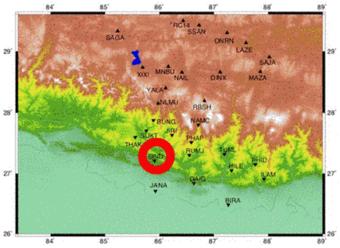
arrival time/polarity variation with backazimuth

(corrections for slowness + elevation applied)

---

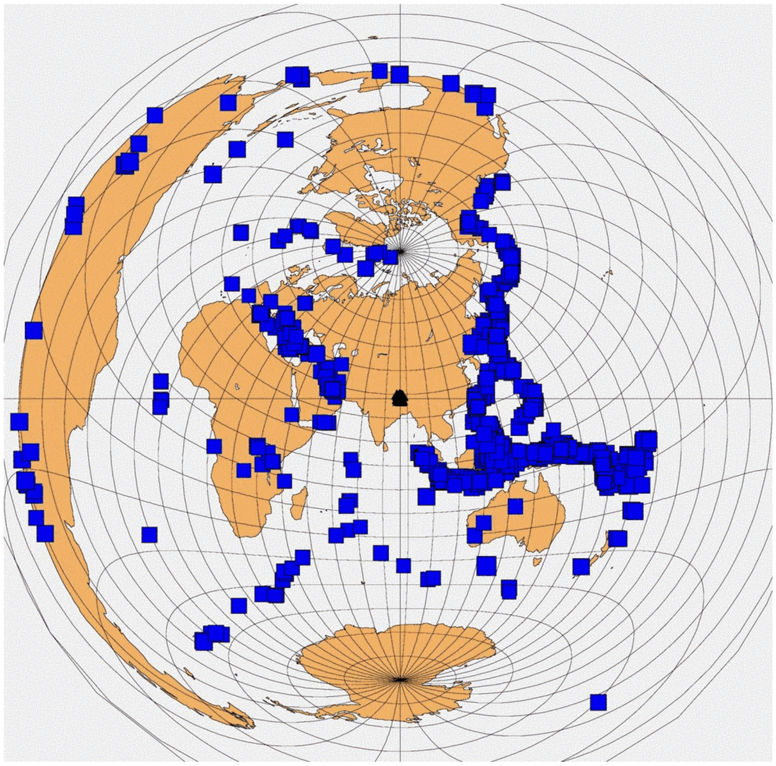
transverse components

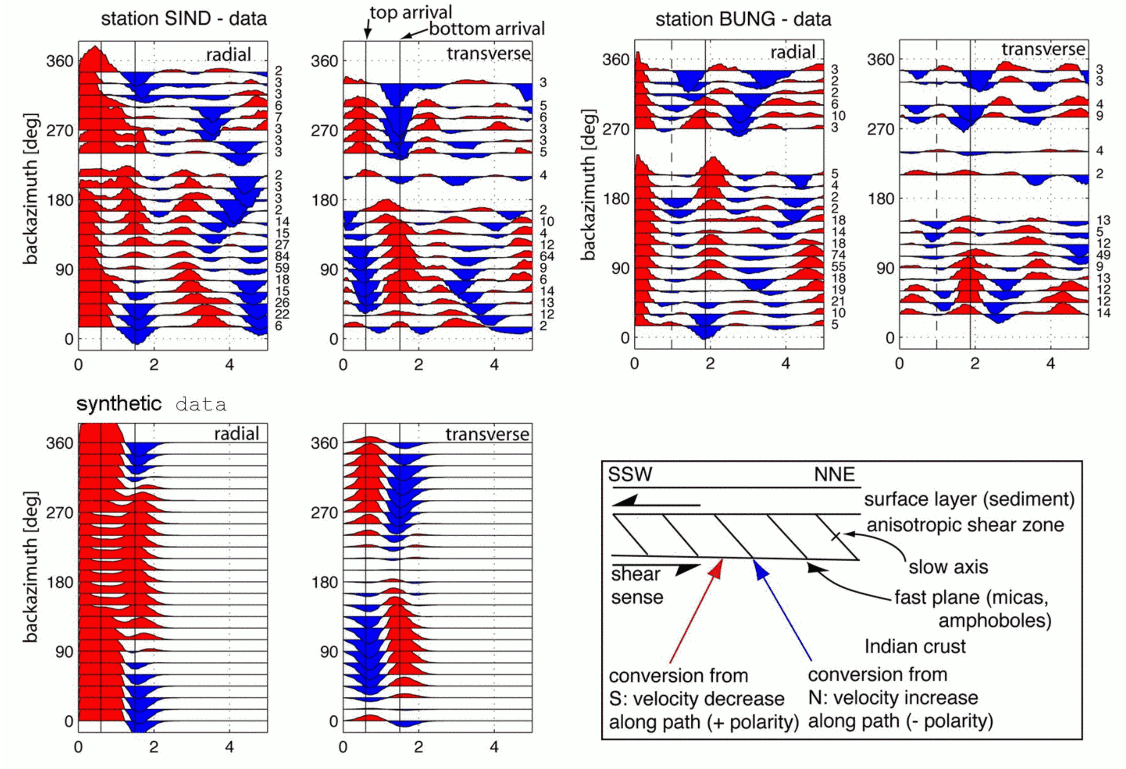




highly coherent transverse component receiver functions

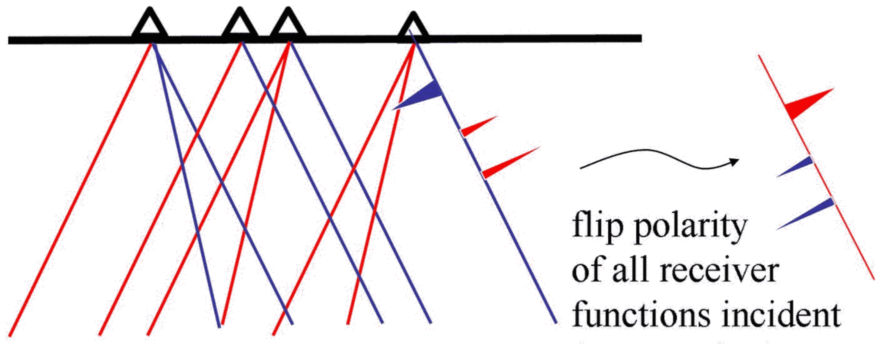
Himalayan study has good azimuthal coverage





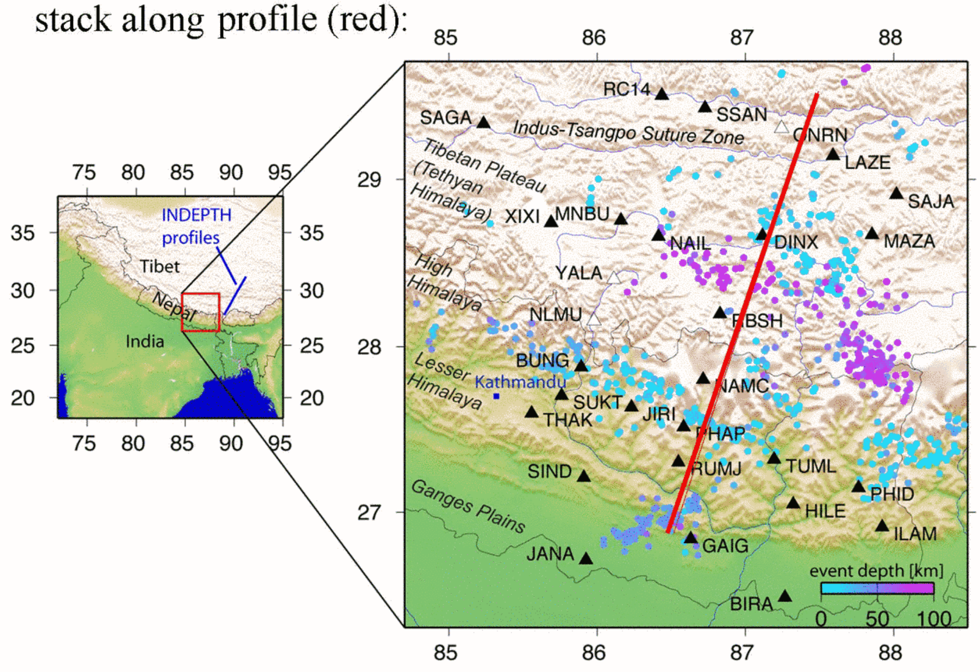
Schulte-Pelkum et al., 2005

Azimuthal difference stacking

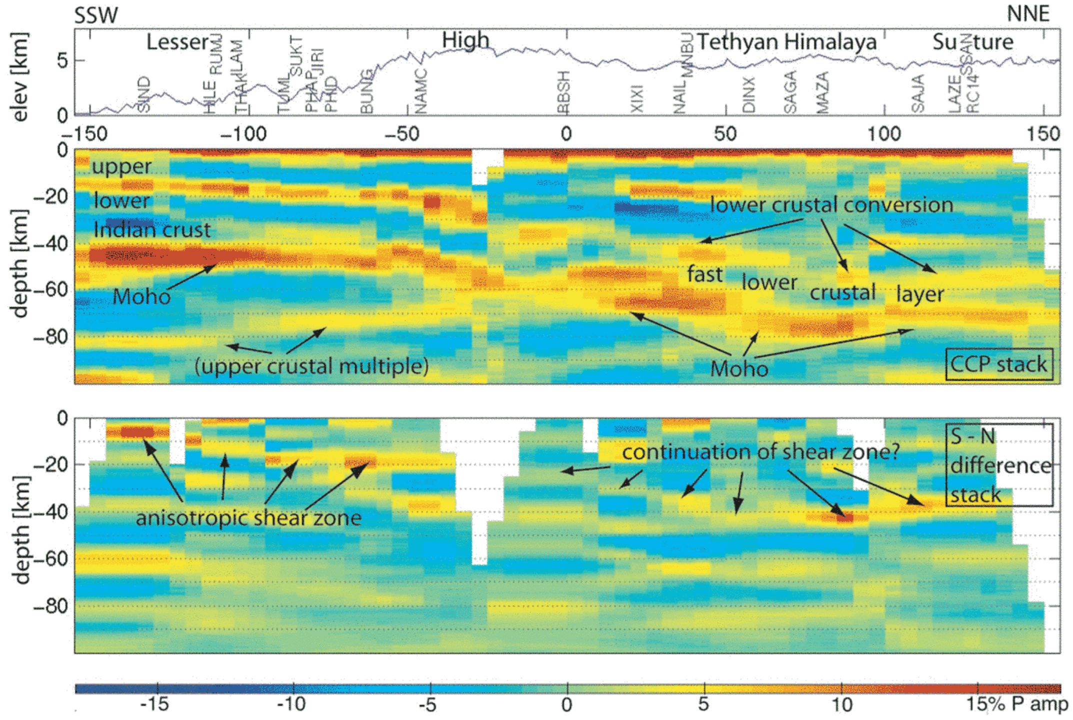


flip polarity of all receiver functions incident from northerly backazimuths before stacking

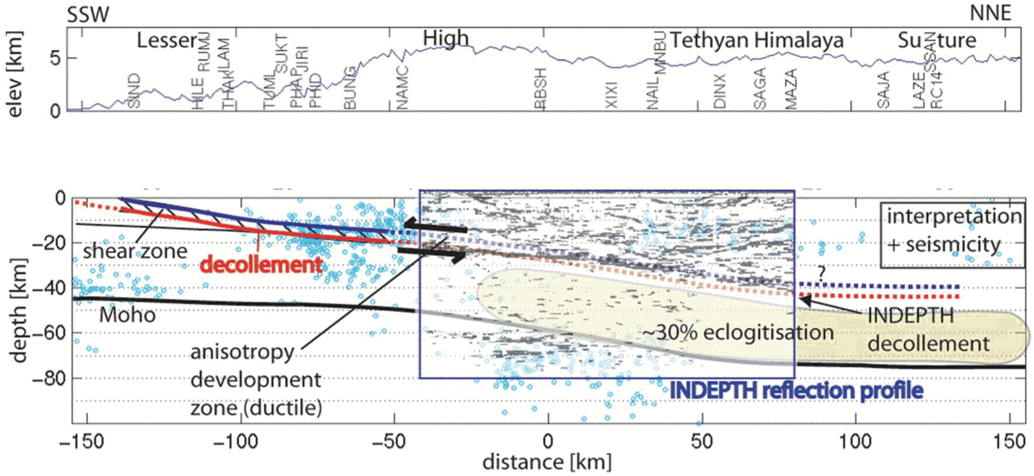
stack along profile (red):



-> new interface shows up in stack

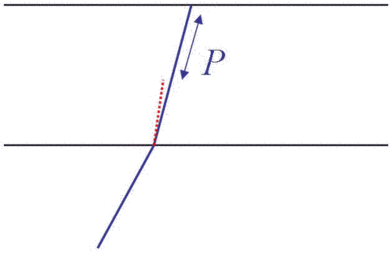


Schulte-Pelkum et al.. 2005

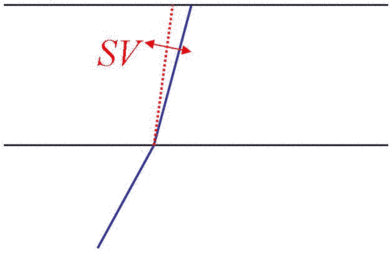


Schulte-Pelkum et al., 2005

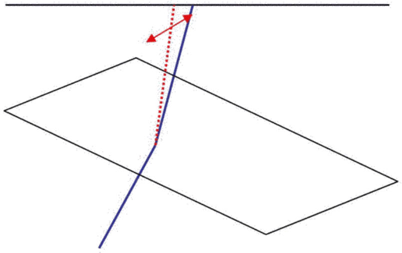
incident: steep *P*
mostly on vertical component



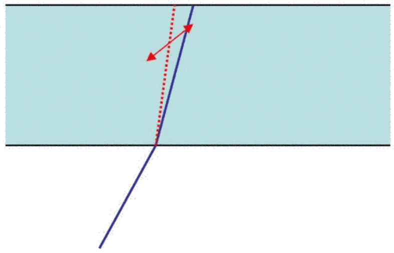
converted phase: *SV* (in plane)
mostly on radial component



out-of-plane *S* conversions
(on radial and transverse
components):

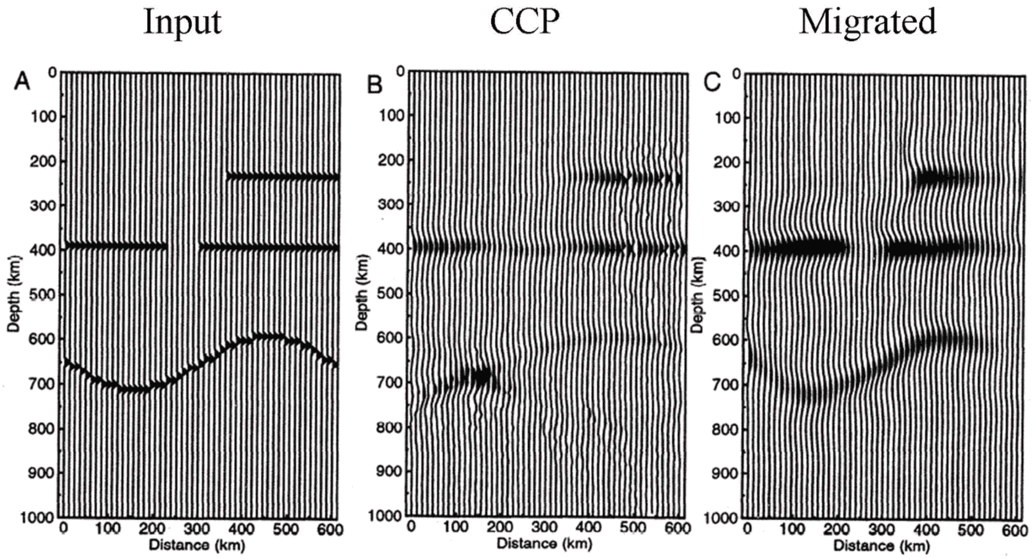


with dipping interface



with anisotropic layer

Artifacts from non planar and dipping interfaces
⟶ Stay tuned for Seismology Lecture 4!



Sheehan et al., 2000

Seismic impedance contrasts at the 410 km discontinuity

(From SdS and PdP, not receiver functions in this example)

Impedance = ρV

Seismic impedance contrast = $(\rho_1 V_1 - \rho_2 V_2)/\rho_1 V_1$

Summary from Chambers et al., 2005

	P	S
Chambers et al., 2005	.053+/-.005	.078+/-.006
Flanagan and Shearer, 1999	.085	.111
PREM	.077	.085

Seismic impedance contrasts at the 410 km discontinuity

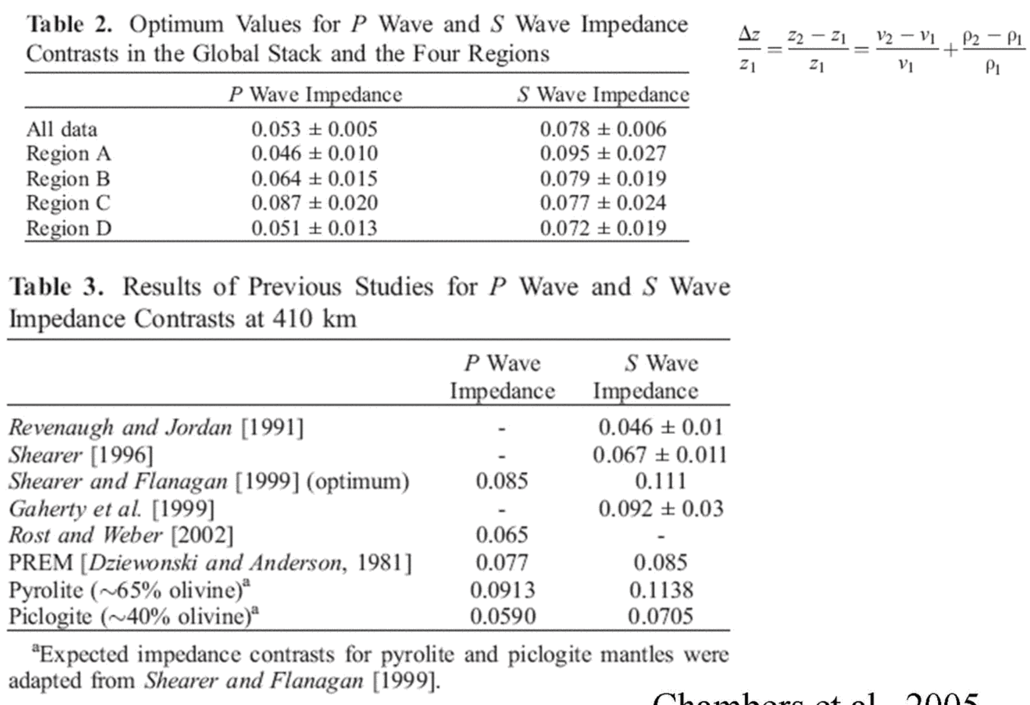


Table 2. Optimum Values for *P* Wave and *S* Wave Impedance Contrasts in the Global Stack and the Four Regions

	P Wave Impedance	*S* Wave Impedance
All data	0.053 ± 0.005	0.078 ± 0.006
Region A	0.046 ± 0.010	0.095 ± 0.027
Region B	0.064 ± 0.015	0.079 ± 0.019
Region C	0.087 ± 0.020	0.077 ± 0.024
Region D	0.051 ± 0.013	0.072 ± 0.019

$$\frac{\Delta z}{z_1} = \frac{z_2 - z_1}{z_1} = \frac{v_2 - v_1}{v_1} + \frac{\rho_2 - \rho_1}{\rho_1}$$

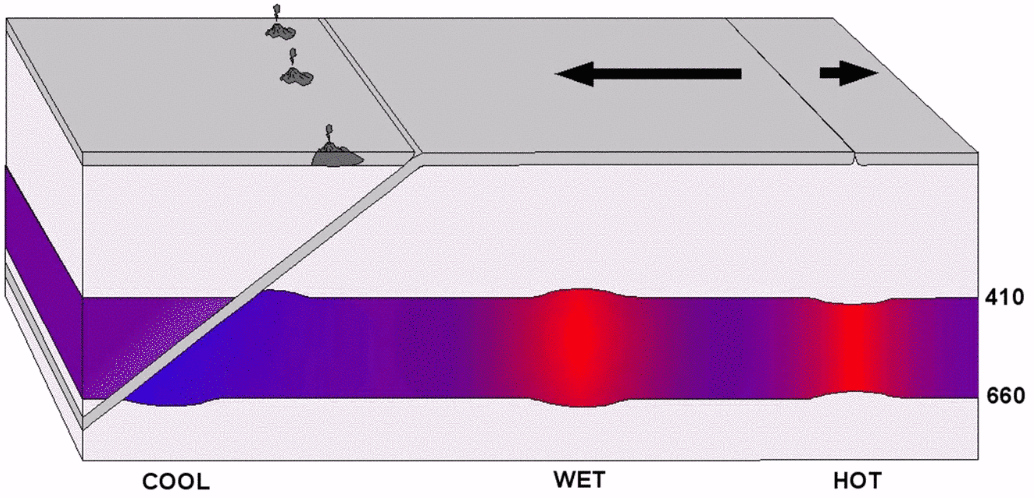
Table 3. Results of Previous Studies for *P* Wave and *S* Wave Impedance Contrasts at 410 km

	P Wave Impedance	*S* Wave Impedance
Revenaugh and Jordan [1991]	-	0.046 ± 0.01
Shearer [1996]	-	0.067 ± 0.011
Shearer and Flanagan [1999] (optimum)	0.085	0.111
Gaherty et al. [1999]	-	0.092 ± 0.03
Rost and Weber [2002]	0.065	-
PREM [*Dziewonski and Anderson*, 1981]	0.077	0.085
Pyrolite (~65% olivine)[a]	0.0913	0.1138
Piclogite (~40% olivine)[a]	0.0590	0.0705

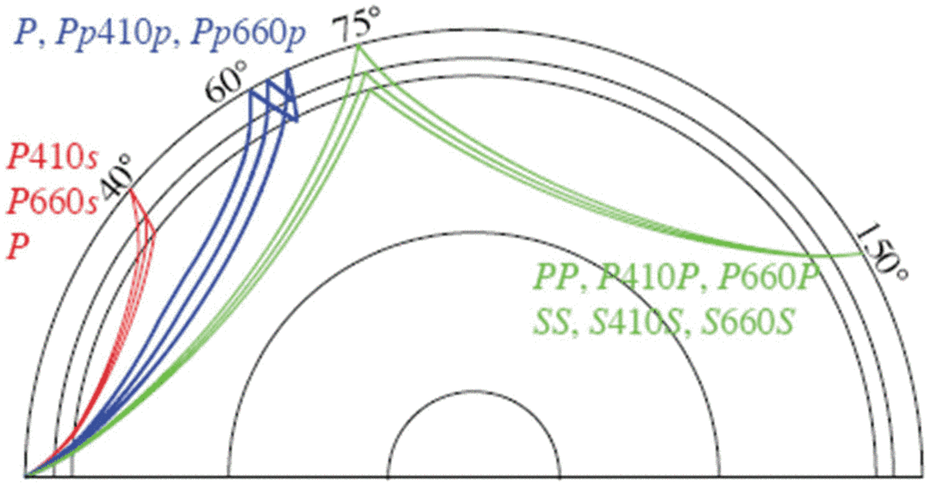
[a] Expected impedance contrasts for pyrolite and piclogite mantles were adapted from *Shearer and Flanagan* [1999].

Chambers et al., 2005

Temperature or water? Both topography and velocity important.

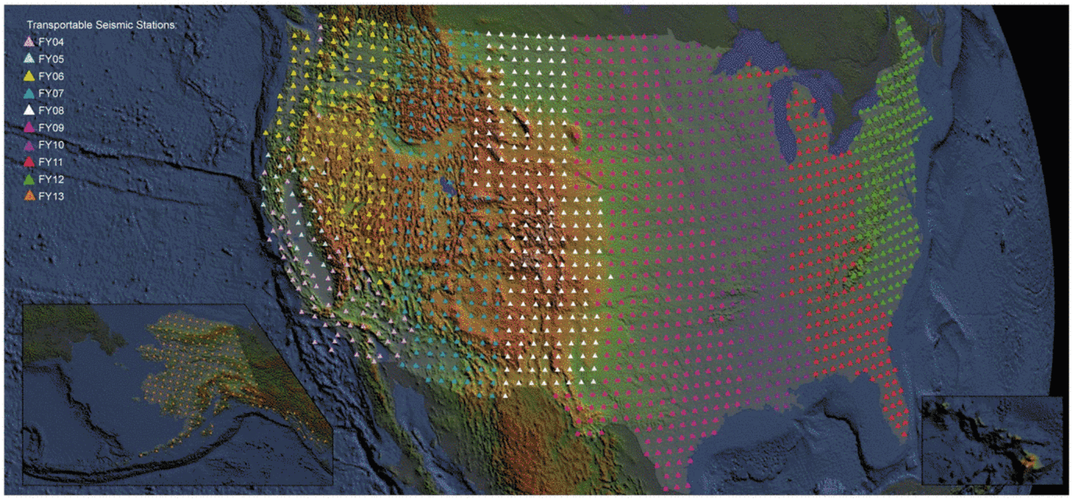


Smyth and Jacobsen, 2006, in press, Earth's Deep Water Cycle (S. van der Lee and S. D. Jacobsen, eds.) American Geophysical Union Monograph Series



Lawrence and Shearer, 2006b

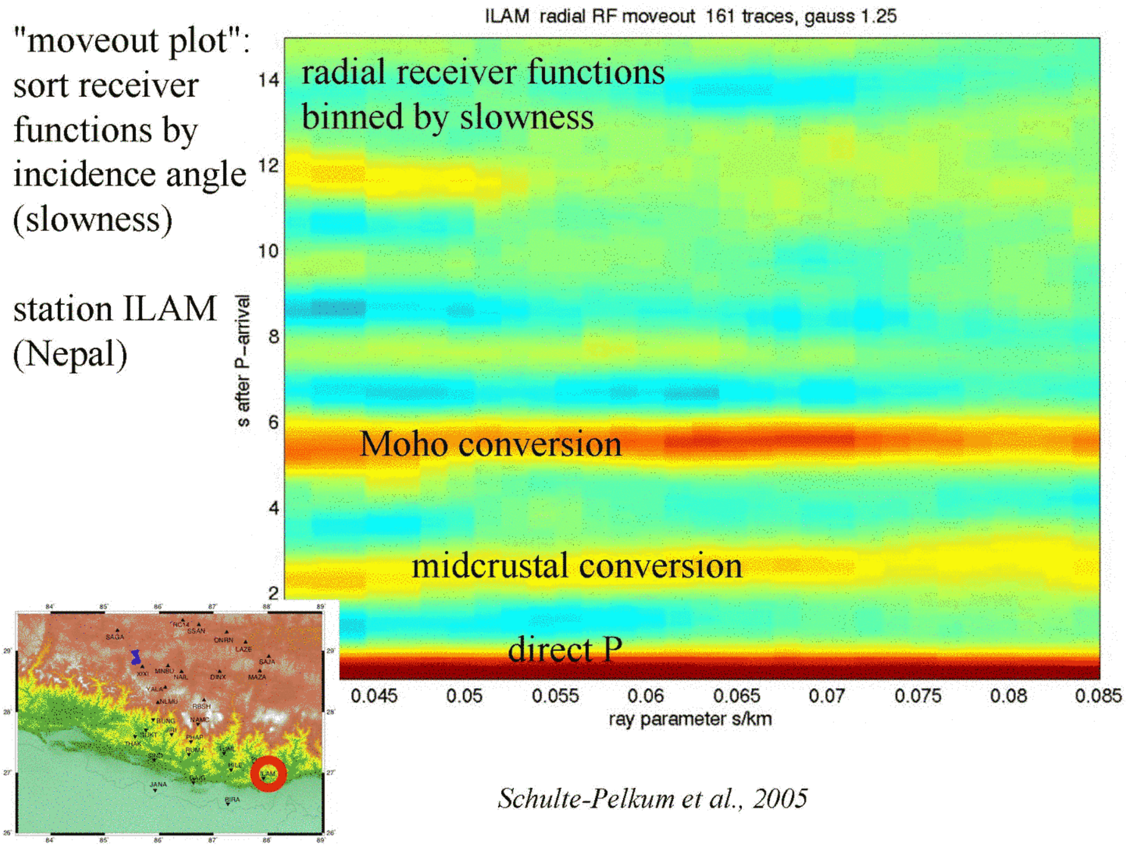
EARTHSCOPE: USArray



Extra slides

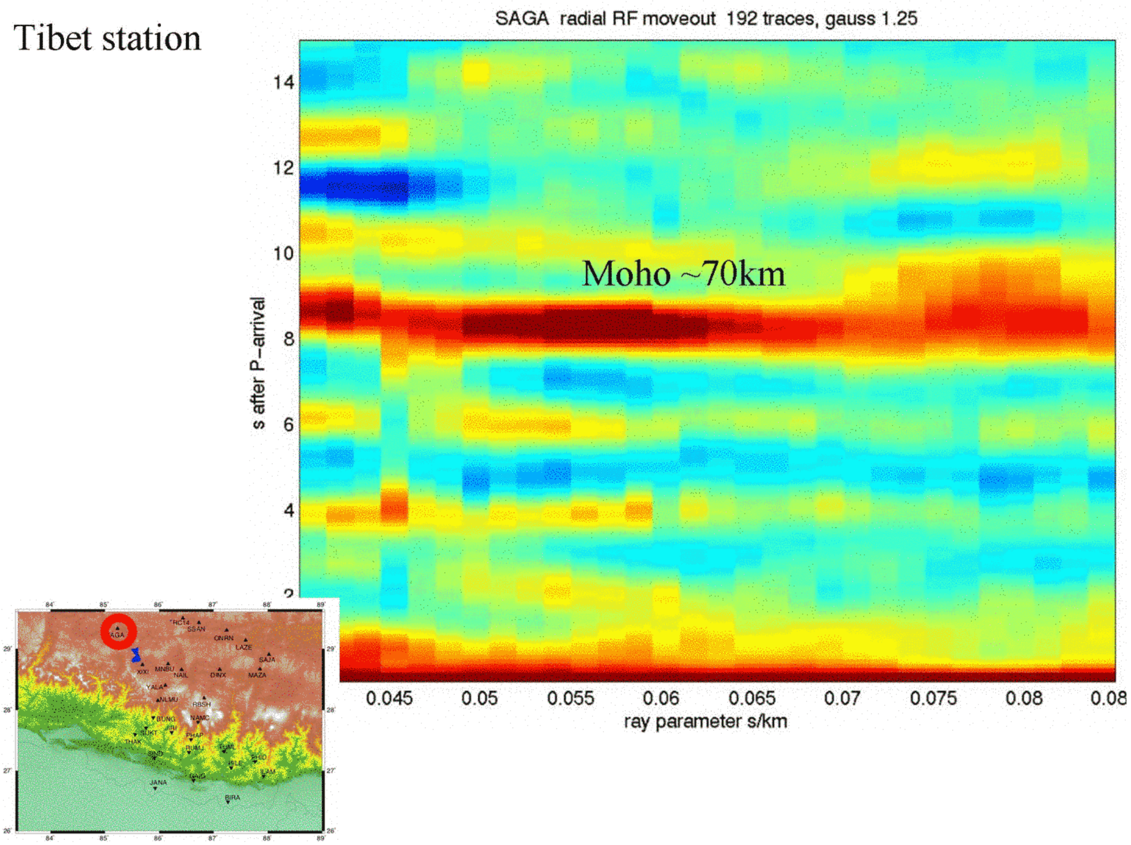
"moveout plot": sort receiver functions by incidence angle (slowness)

station ILAM (Nepal)



Schulte-Pelkum et al., 2005

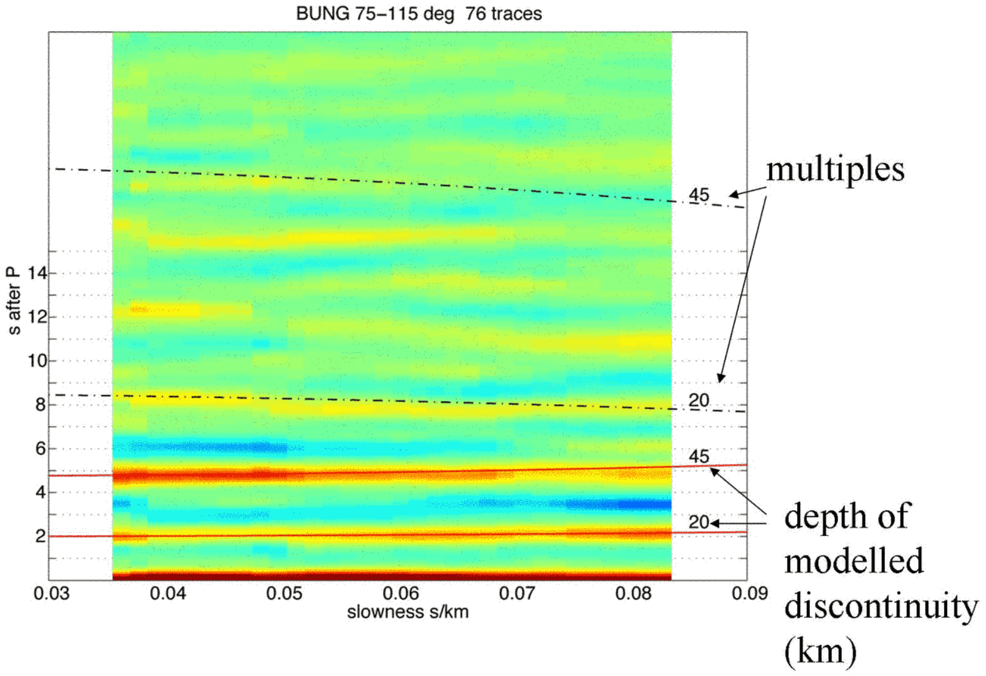
Tibet station

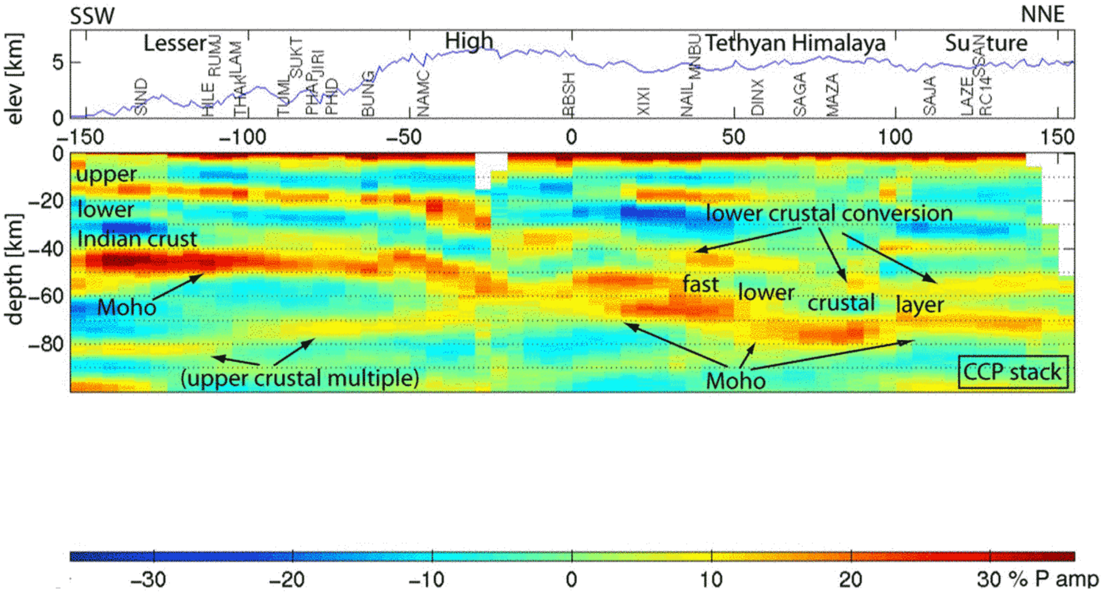


Receiver Function
Advantages & Disadvantages

- Advantages:
  - Sensitive to velocity discontinuities
  - Deconvolution removes source signature and isolates P-S conversions
  - Can provide detailed images of discontinuity topography
- Shortcomings
  - Velocity - depth tradeoff
  - Non planar interfaces
  - Absolute amplitude hard to resolve and interpret
  - Coverage limited to where you have seismic stations

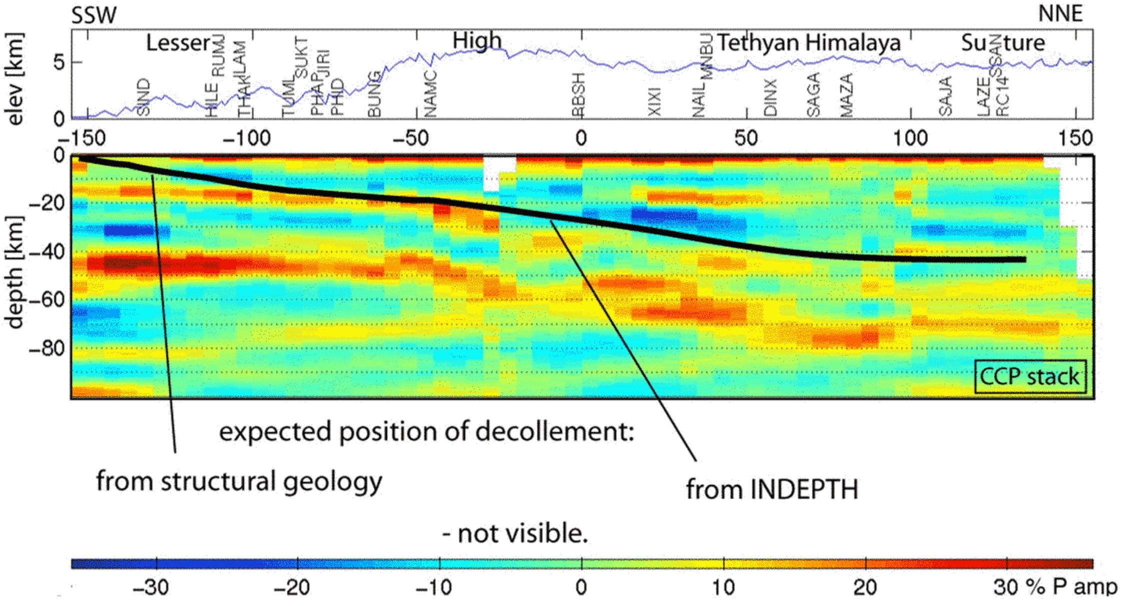
moveout plot for narrow azimuthal range





Schulte-Pelkum et al., 2005

but where is the decollement?



expected position of decollement:

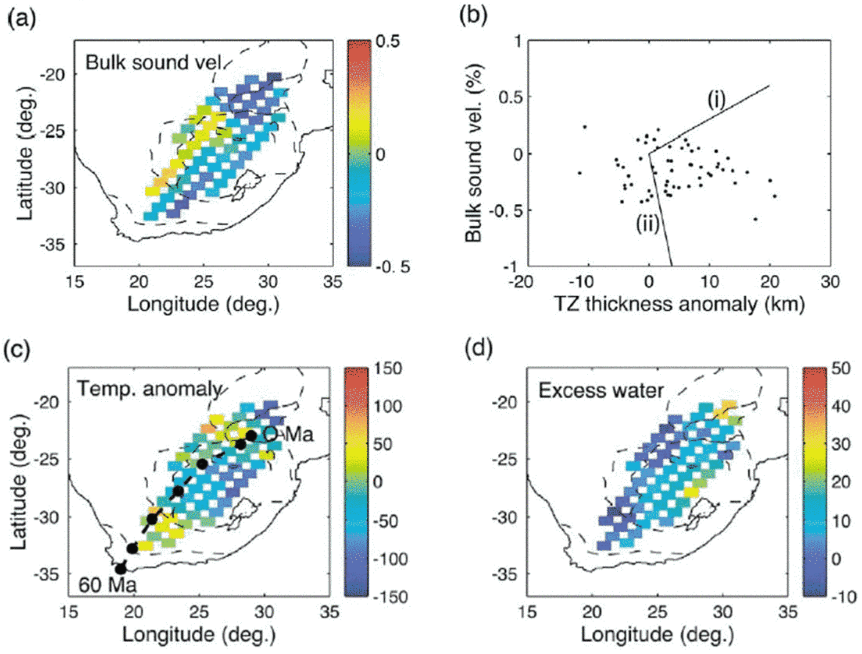
from structural geology

from INDEPTH

- not visible.

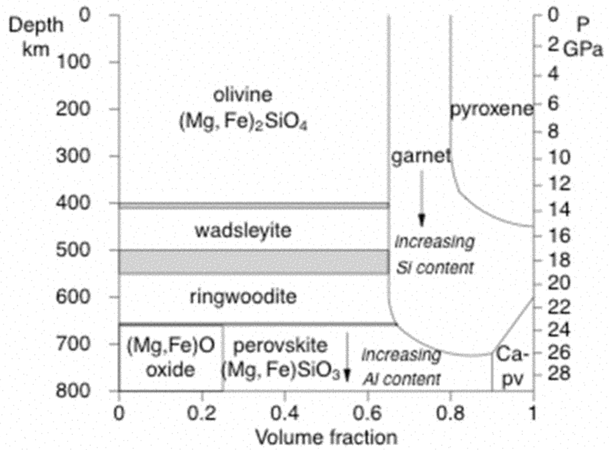
S. Africa craton

The transition zone beneath the cratons is relatively cold (ΔT ~ -100 K) and water enriched (up to 0.3-0.5 wt%)



Blum and Shen, 2004

Mineral Composition



(Helffrich, 2003)

Olivine Phases

$$(Mg,Fe)_2SiO_4 = (Mg,Fe)_2SiO_4$$

Olivine Wadsleyite

Pressure 13 - 14 GPa.
410 km.

$$(Mg,Fe)_2SiO_4 = (Mg,Fe)_2SiO_4$$

Wadsleyite Ringwoodite

Pressure 18 GPa.
520 km.

$$(Mg,Fe)_2SiO_4 = (Mg,Fe)SiO_3 + (Mg,Fe)O$$

Ringwoodite Peroskovite Magnesiowüstite

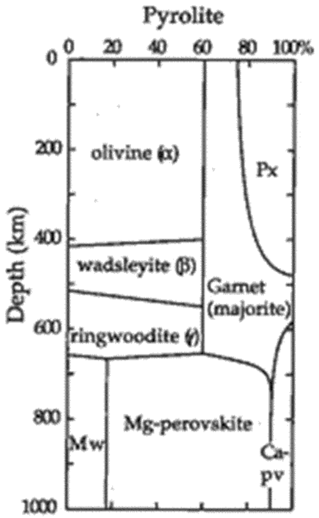
Pressure 23 GPa.
660 km.

Piroxene - Garnet Phases

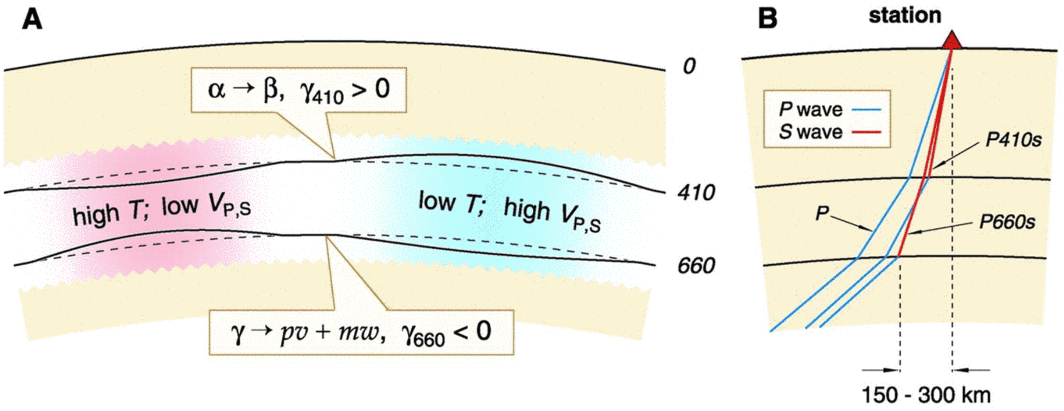
Transformation of non-olivine components are also important (30%).
This phase changes are gradual and lead to changes of slope of velocity.

Piroxene starts to dissolve into the garnet Structure at 350 - 500 km.

At about 580 km $CaSiO_3$ perovskovite Exsolves from garnet.

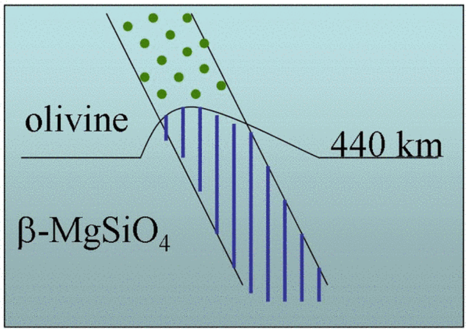


Clapeyron Slope

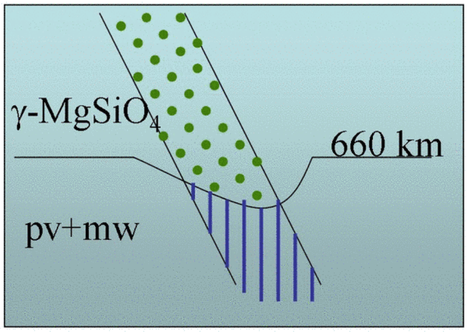


Clapeyron Slope 3

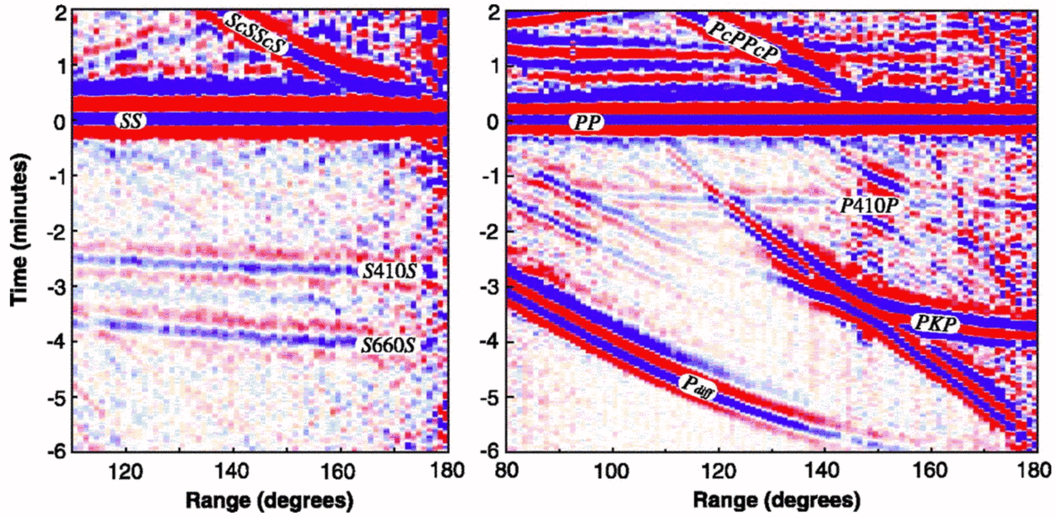
- Mineralogical reactions have characteristic P-T slopes.
- If a predictable change in mantle T is available, the depth of the phase transformation should also change.
- Many studies on transition zone topography.
- dT ~700K between slab and mantle should produce a -60km and +30km topography.



exothermic reaction

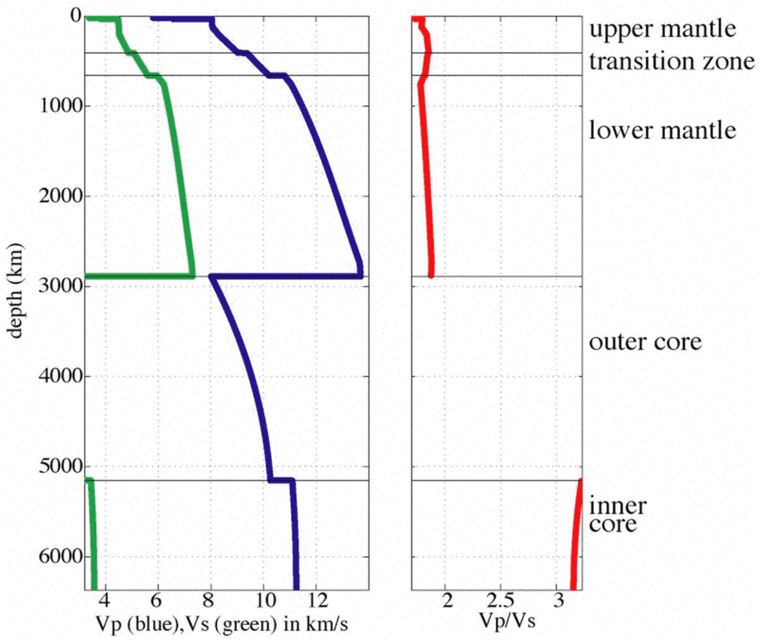


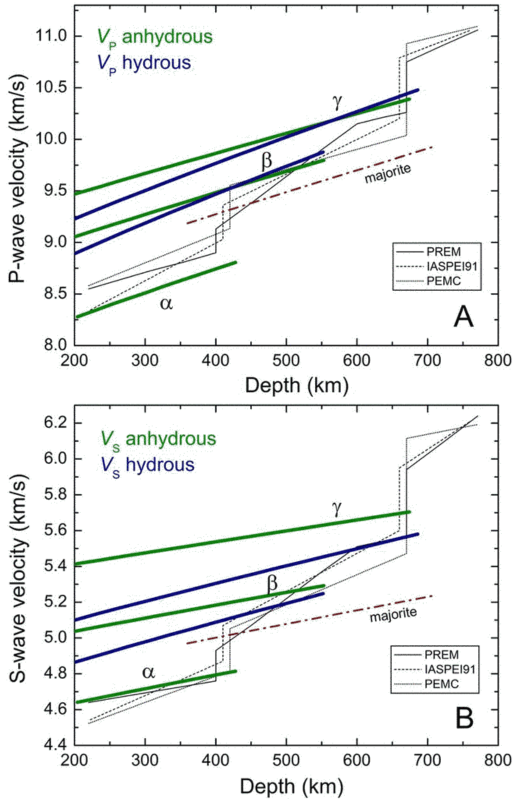
endothermic reaction

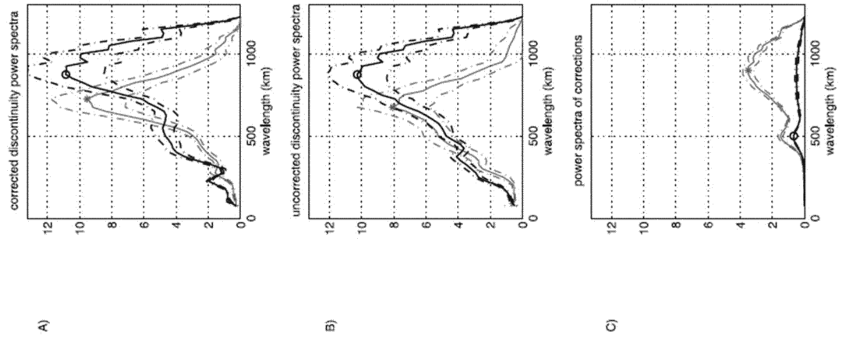


Stacks of *SS* (**left**) and *PP* (**right**) precursors derived from over 30,000 long-period seismograms. The precursors are underside reflections off the 410- and 660-km discontinuities that arrive several minutes before the main phase.

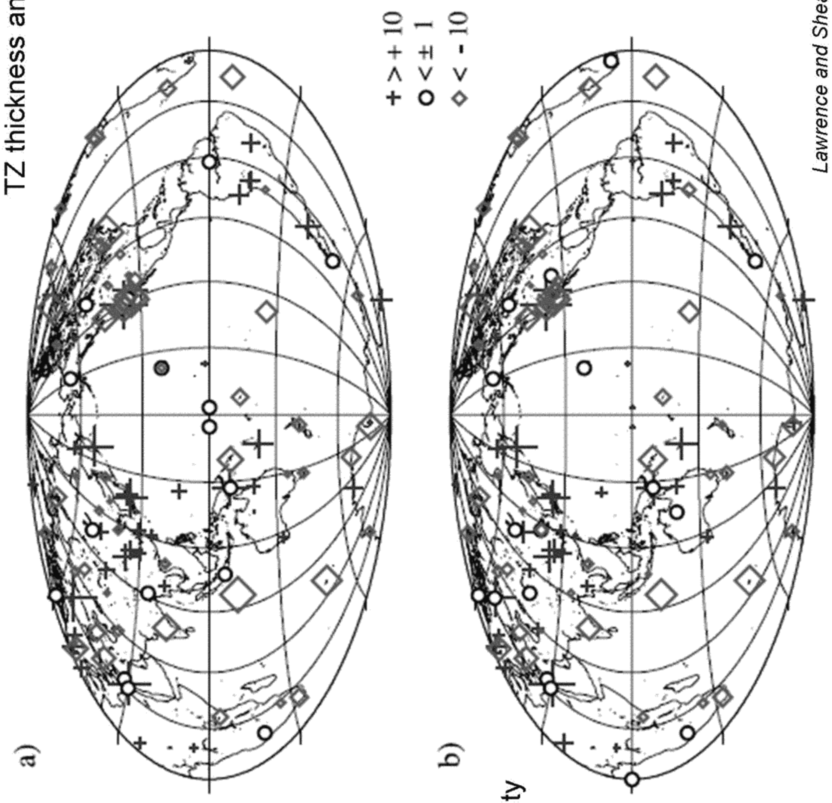
IASP reference seismic *P*- and *S*- wave velocity model







TZ thickness anomalies



raw

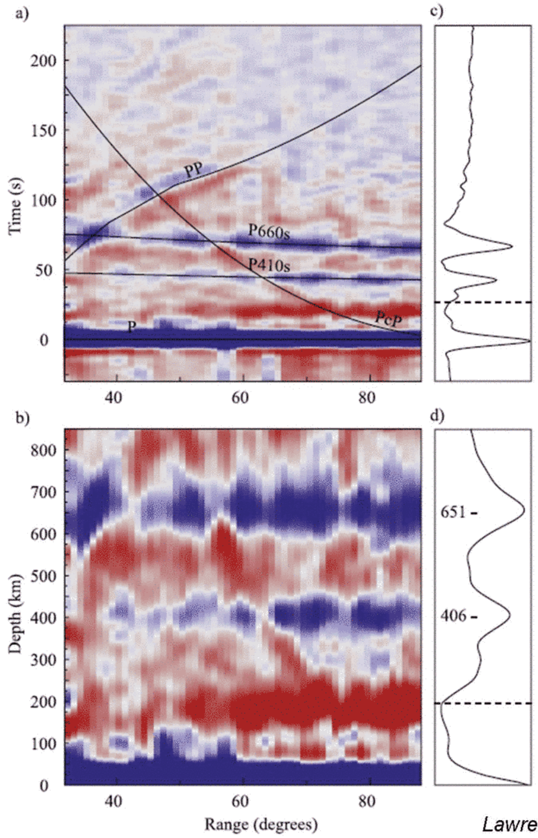
Heterogeneity corrected

Lawrence and Shearer, JGR, 2006

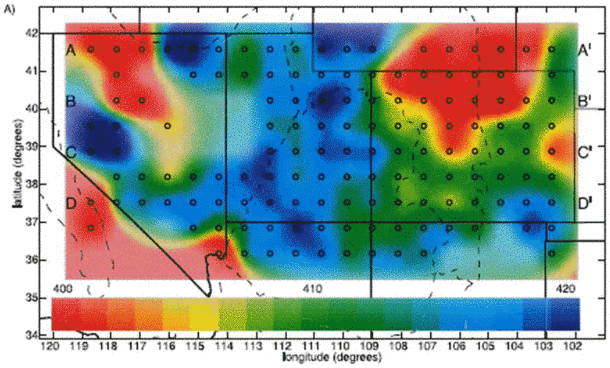
Receiver functions

Dist vs time

Dist vs depth



Lawrence and Shearer, JGR, 2006



No lat het corr

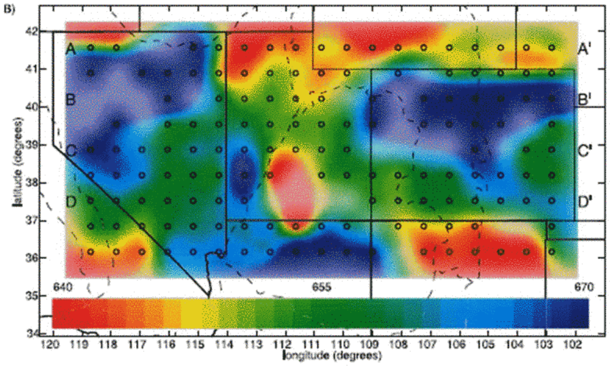
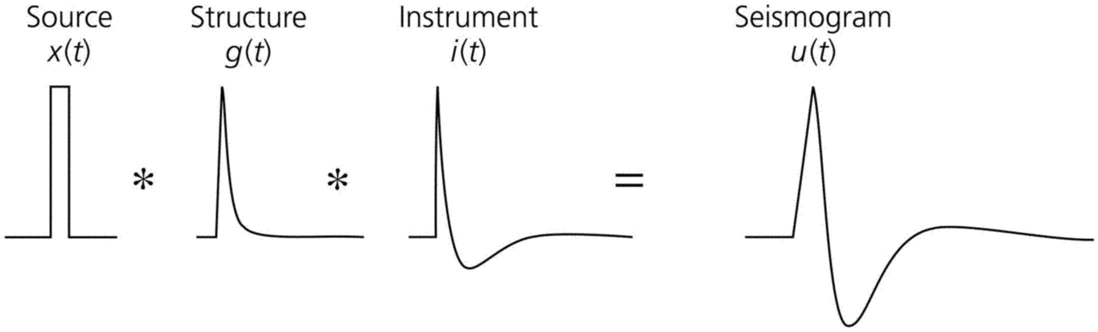


Figure 6.3-5: Seismogram as the convolution of the source, structure, and instrument signals.



Want to deconvolve source and instrument response so we are just left with the signal from structure

Deconvolution

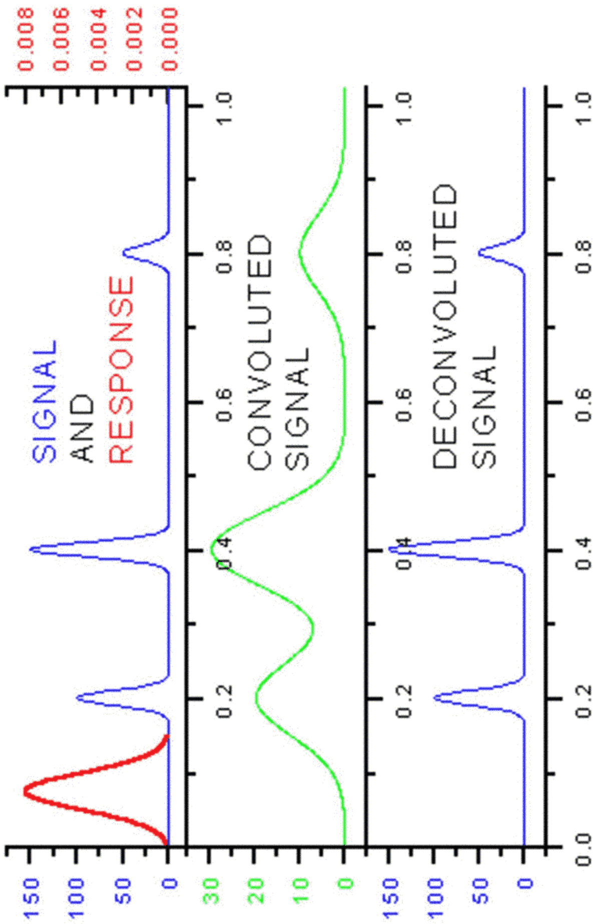
- Using Fourier analysis, deconvolution of linear system responses becomes a very simple problem of division in the frequency domain
- Solution in the frequency domain is converted to a solution in the time domain using the Fourier transform

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{iwt}d\omega$$

Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-iwt}dt$$

inverse Fourier transform


SIGNAL
AND
RESPONSE
CONVOLUTED
SIGNAL
DECONVOLUTED
SIGNAL
0.008
0.006
0.004
0.002
0.000
150
100
50
0
30
20
10
0
150
100
50
0
0.0
0.2
0.4
0.6
0.8
1.0