Earthscope Imaging Science & CIG Seismology Workshop

- Imaging methods
- Forward modeling methods
  - Presentations
  - Tutorials
- Washington University, St. Louis
- Oct 31-Nov 2, 2006
- Look for an announcement on the IRIS & CIG webpages

Array Methods for Image Formation

Outline

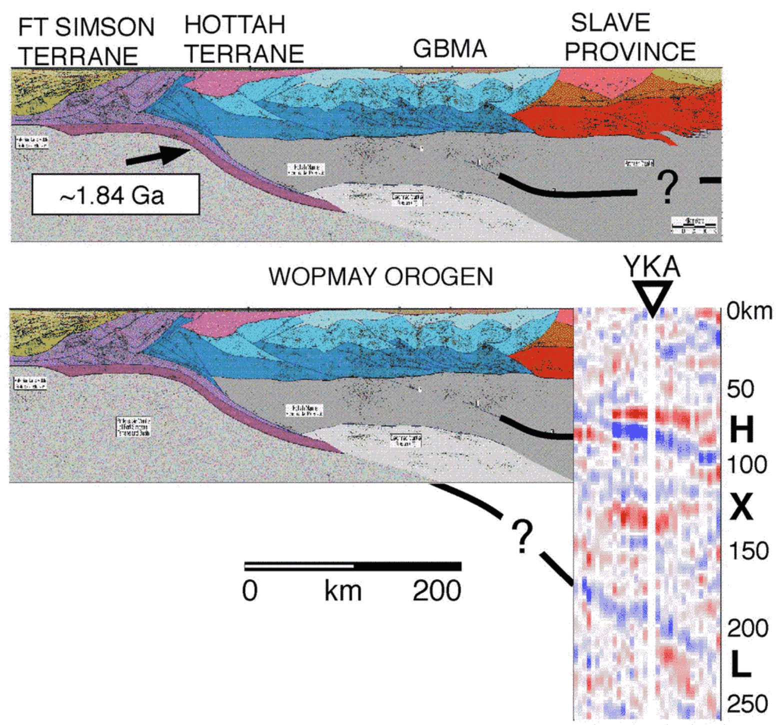
1. CCP stacking versus scattered wave imaging
2. Born Scattering = Single scattering
3. Diffraction integrals & Kirchhoff Depth Migration
4. Limitations
5. Examples
6. Inversion versus migration
7. Tutorial: www.kitp.ucsb.edu/~earth06/SeismologyTutorial.zip

SNORCLE

Subduction forming the Proterozoic continental mantle lithosphere:

Lateral advection of mass

Cook et al., (1999)
Bostock (1998)



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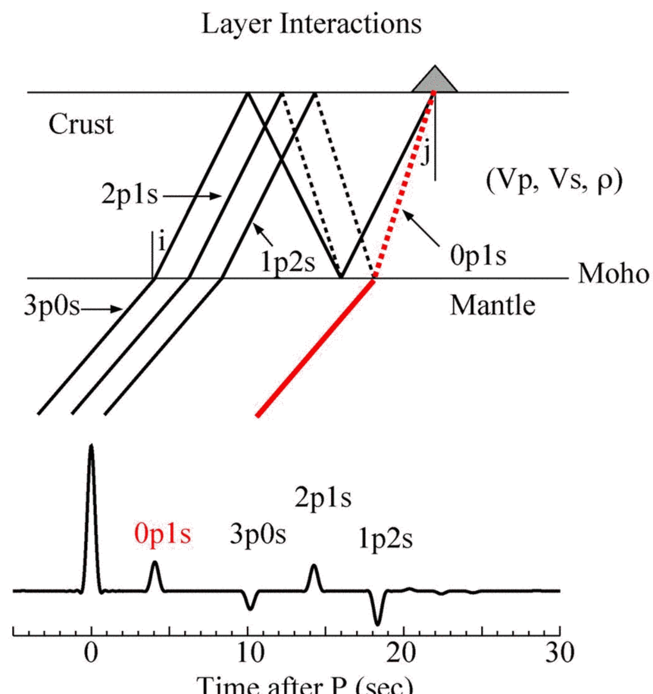
Direct imaging with seismic waves

Converted Wave Imaging from a continuous interface: Receiver Functions (RF)

Images based on this model of 1 D scattering are referred to as common conversion point stacks or

CCP images

These are analogs of seismic reflection images, most sensitive to **Shear velocity perturbations**



From Niu and James, 2002, EPSL

Two classes of scattered wave imaging systems

1. *Incoherent imaging systems* which are frequency and amplitude sensitive. Examples are
   1. Photography
   2. Military Sonar
   3. Some deep crustal reflection seismology
   4. Sumatra earthquake source by *Ishii et al. 2005*
2. *Coherent imaging systems* use phase coherence and are therefore sensitive to frequency, amplitude, and phase. Examples are
   1. Medical ultrasound imaging
   2. Military sonar
   3. Exploration seismology

See *Blackledge, 1989, Quantitative Coherent Imaging: Theory and Applications*

Resolution Considerations

- Earthquakes are the only inexpensive energy source able to investigate the mantle
- Coherent Scattered Wave Imaging provides about an order of magnitude better resolution than travel time tomography. For wavelength λ
- $R_{scat} \sim \lambda/2$ versus $R_{tomo} \sim (\lambda L)^{1/2}$
- For a normalized wavelength and path of 100
- $R_{scat} \sim 0.5$ versus $R_{tomo} \sim 10$

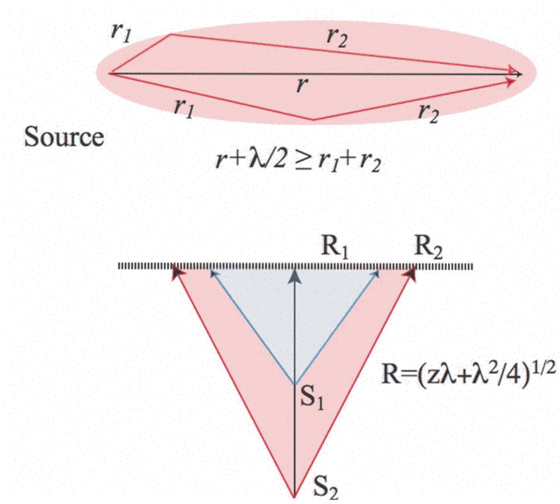
Fresnel Zones and other measures of wave sampling

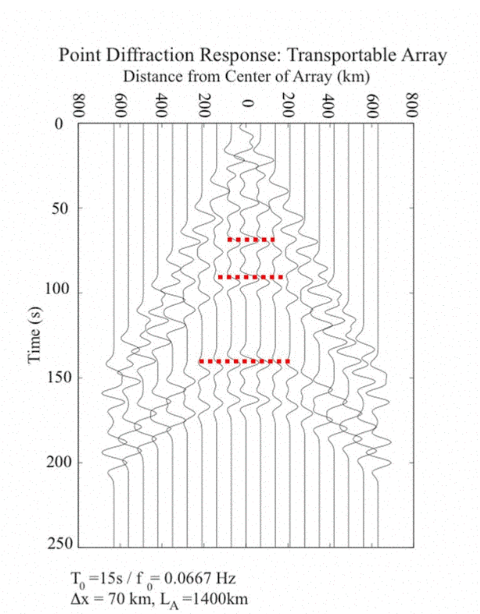
- Fresnel zones
- Banana donuts
- Wavepaths

Are all about the same thing

They are means to estimate the sampling volume for waves of finite-frequency, i.e. not a ray, but a wave.

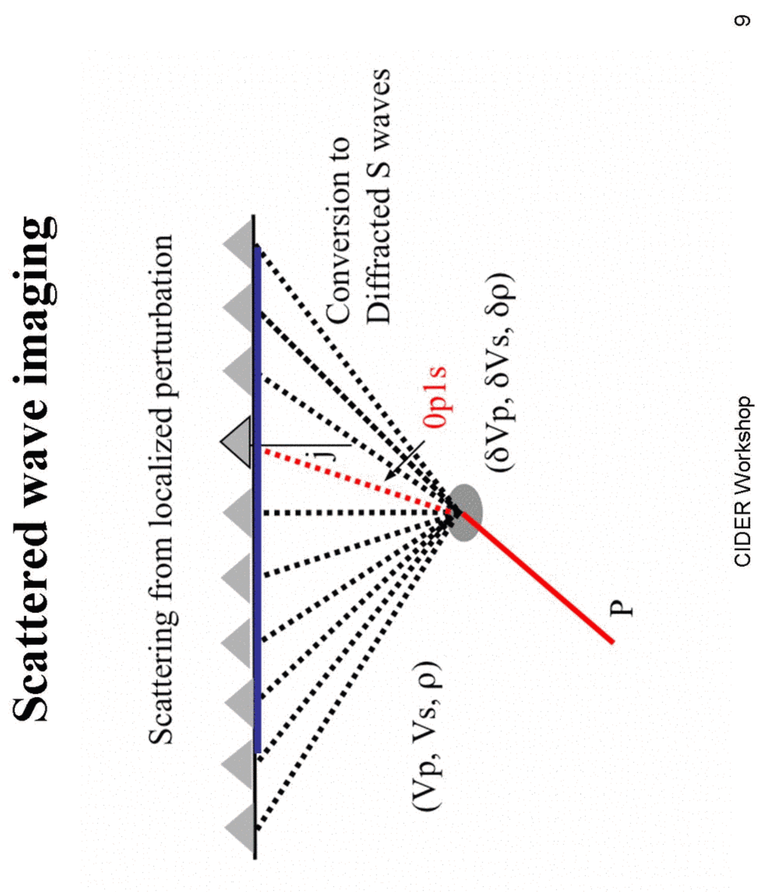
Flatté et al., *Sound Transmission Through a Fluctuating Ocean*





Fresnel zone increases with depth and with period

For depropagation the Fresnel zone has to be sampled completely in a Fourier sense


Scattered wave imaging
Scattering from localized perturbation
Conversion to
Diffracted S waves
j
0p1s
(Vp, Vs, ρ)
(δVp, δVs, δρ)
P
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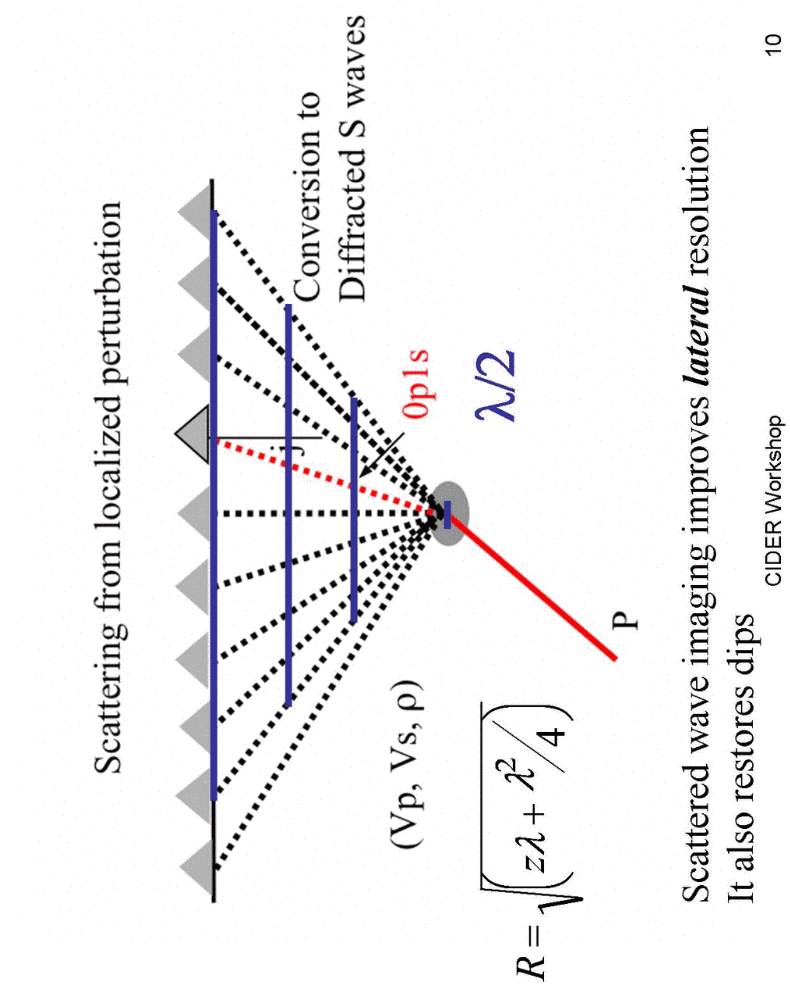
Scattering from localized perturbation
Conversion to
Diffracted S waves
j
0p1s
λ/2
(Vp, Vs, ρ)
P
$R=\sqrt{\left(z\lambda+\lambda^2/4\right)}$
Scattered wave imaging improves *lateral* resolution
It also restores dips
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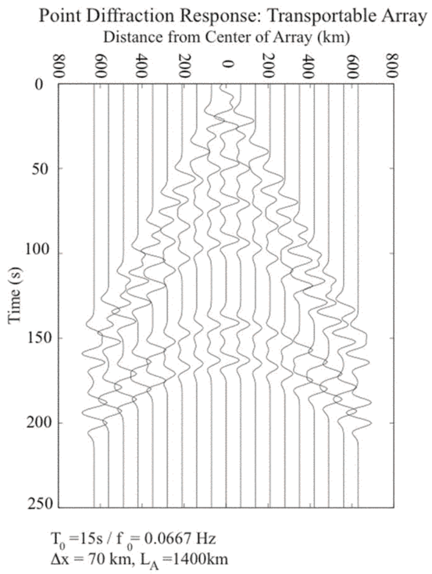
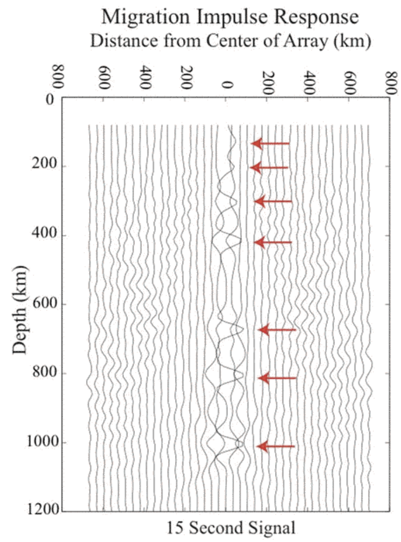
USArray: Bigfoot





$\Delta x = 70$ km, $7s < T < 30s$

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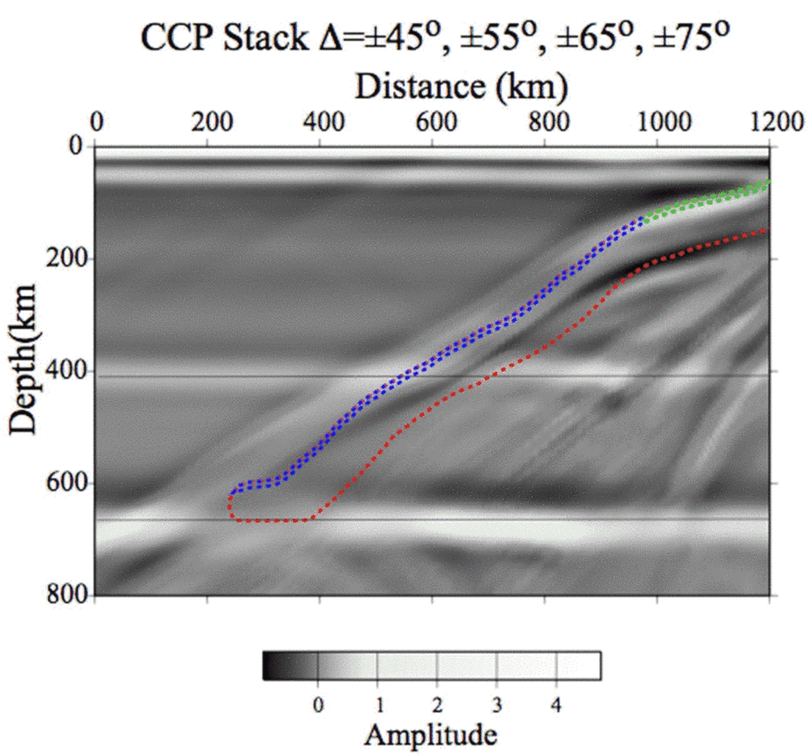
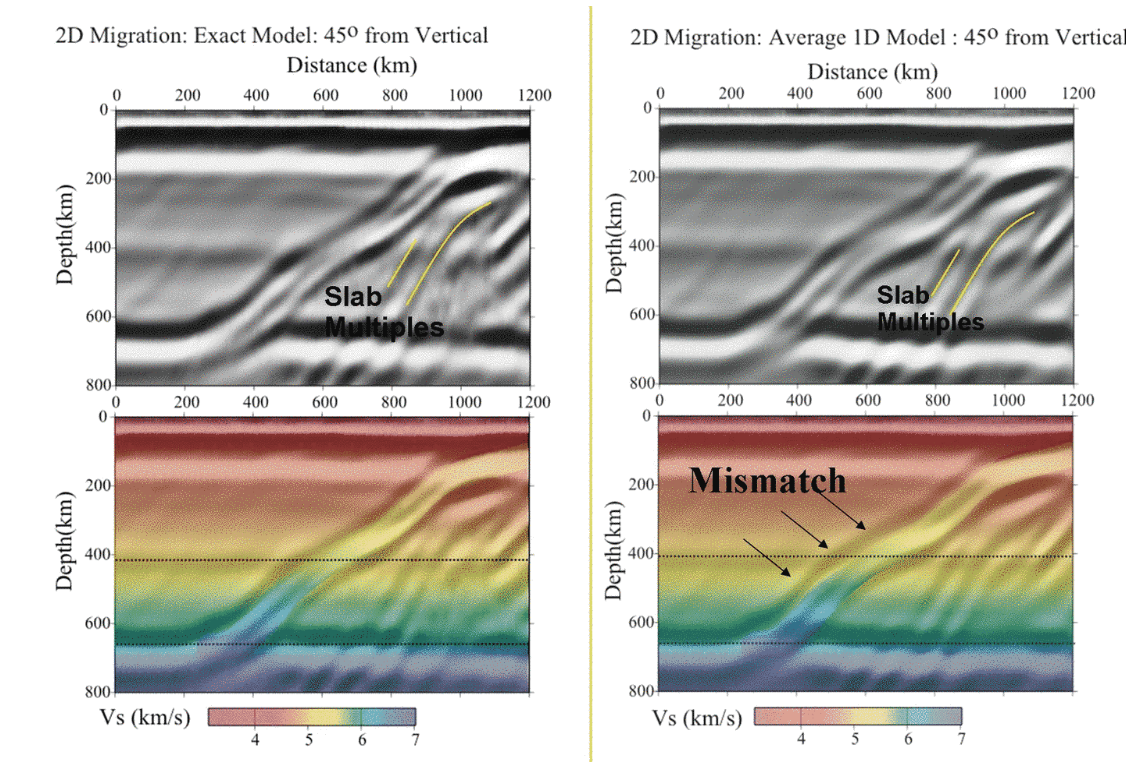




S. Ham, unpublished

- The receiver function:
  - Approximately isolates the SV wave from the P wave
  - Reduces a vector system to a scalar system
  - Allows use of a scalar imaging equation with P and S calculated separately for single scattering: From Barbara's lecture:

$$\vec{u}^P = \nabla \phi$$

$$\vec{u}^{SV} = \nabla \times \vec{\Psi} : \vec{\Psi} = (0, \psi, 0)$$

Three elements of an imaging algorithm

1. A scattering model
2. Wave (de)propagator
   - *Diffraction integrals (Wilson et al., 2005)*
   - *One-way* and *two-way finite-difference operators*
   - *Generalized Radon Transforms (Bostock et al., 2001)*
   - *Fourier transforms* in space and/or time with phase shifting
3. A focusing criteria known as an ***imaging condition***

Depropagator: assume smooth coefficients
i.e., v(x,y,z) varies smoothly. Let L be the wave equation
Operator for the smooth medium

$$LG = \delta(t)\delta(x - x_x)$$

$$LG = 1$$

$$G = L^{-1}$$

The Green's function is the inverse operator to the wave equation
For a general source:

$$Lu = s(\omega)$$

$$u = L^{-1}s(\omega)$$

$$u = Gs(\omega)$$

The Born Approximation

Perturbing the velocity field perturbs the wavefield
c= c+δc

$$[L + \delta L](u + \delta u) = s(\omega)$$

$$Lu + L\delta u + \delta L u = s(\omega) + O(\delta^2)$$

$$Lu \cong s$$

$$L\delta u \cong -\delta L u$$

$$\delta u = -L^{-1}\delta L u$$

Born Scattering

Assume a constant density scalar wave equation with a smoothly varying velocity field *c(x)*

$$\frac{1}{c^2(x)}\frac{\partial^2 U(x,t)}{\partial t^2} - \nabla^2 U(x,t) = f(t)\delta(x - x_s)$$

$$c(x) = \sqrt{\kappa(x)/\rho_o}$$

$$x \in R^n : n = 1,2,3$$

Perturb the velocity field

$$c(x) = c_o(x) + \delta c(x)$$

$$U(x,t) = U_o(x,t) + \delta U(x,t)$$

Born Scattering: Solve for the perturbed field to first order

$$\frac{1}{c^2}\frac{\partial^2 \delta U}{\partial t^2} - \nabla^2 \delta U = \frac{2\delta c}{c^3}\frac{\partial^2 U(x,t)}{\partial t^2}$$

The *total field* consists of two parts, the response to the smooth medium

$$\frac{1}{c^2(x)}\frac{\partial^2 U(x,t)}{\partial t^2} - \nabla^2 U(x,t) = f(t)\delta(x - x_s)$$

Plus the response to the perturbed medium $\boldsymbol{\delta U}$

The approximate solution satisfies two inhomogenous wave equations. Note that *energy is not conserved.*
It is the *scattered field* that we use for imaging

The scattering function for P to S conversion from a heterogeneity
Wu and Aki, 1985, Geophysics

$$U^{PS} = \frac{-V}{4\pi}\frac{\omega^2}{\alpha^2}\left(\frac{\alpha}{\beta}\right)^2 \frac{\exp(-i\omega(t - r/\beta))}{r}$$
$$\times\left[\frac{\delta Z_S}{Z_S}(\sin\vartheta - \frac{\beta}{\alpha}\sin 2\vartheta) - \frac{\delta\beta}{\beta}(\sin\vartheta + \frac{\beta}{\alpha}\sin 2\vartheta)\right]$$

where

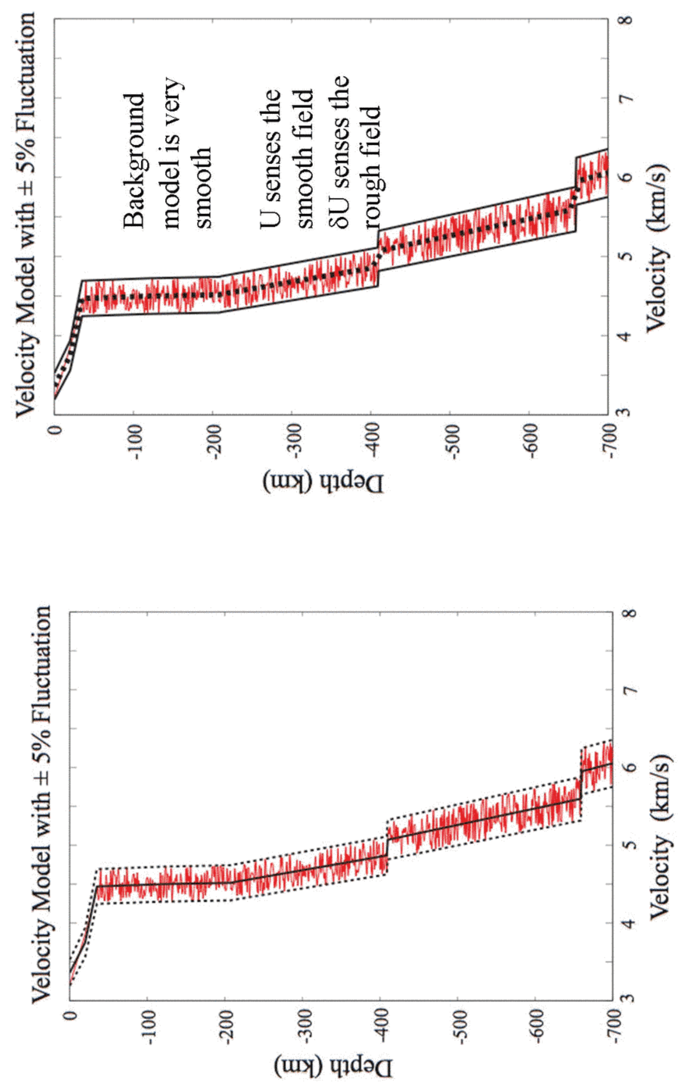
$Z_S = \rho\beta$ Shear wave impedance

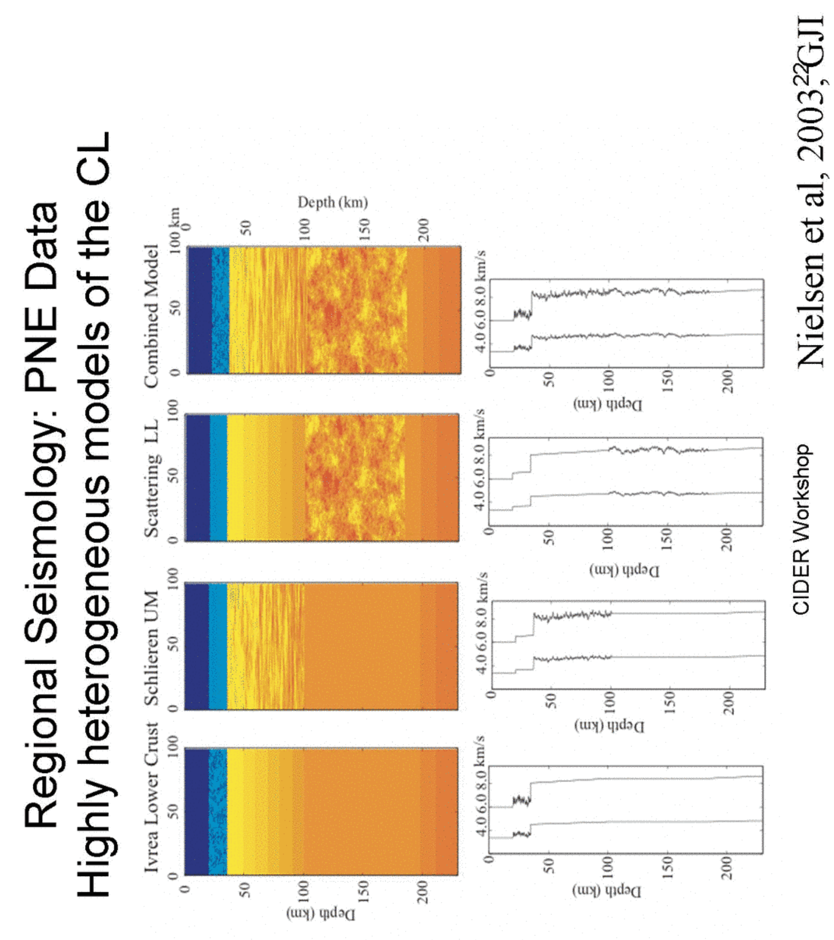
$$\frac{\delta Z_S}{Z_S} = \frac{\delta\rho}{\rho} + \frac{\delta\beta}{\beta}$$

$$-\omega^2 \Leftrightarrow \frac{\partial^2}{\partial t^2}$$

$$\frac{\exp(-i\omega(t - r/\beta))}{r} \Leftrightarrow \frac{\delta(t - r/\beta)}{r}$$

Fourier Transform relations


Velocity Model with ± 5% Fluctuation
Background model is very smooth
U senses the smooth field
δU senses the rough field
Depth (km)
Velocity (km/s)
Velocity Model with ± 5% Fluctuation
Depth (km)
Velocity (km/s)


Regional Seismology: PNE Data
Highly heterogeneous models of the CL
Ivrea Lower Crust
Schlieren UM
Scattering LL
Combined Model
Depth (km)
Nielsen et al, 2003[22], GJI
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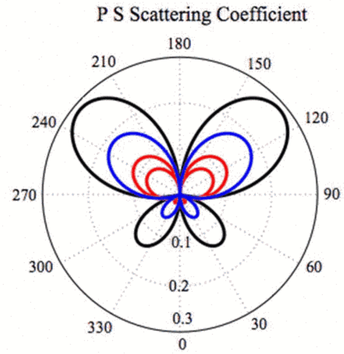
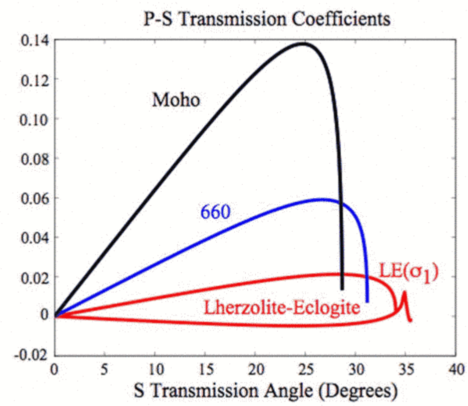
F6 LevNLC

Signal Detection:

Elastic P to S Scattering from a discrete heterogeneity
Wu and Aki, 1985, Geophysics



Specular P to S Conversion
Aki and Richards, 1980



Levander et al., 2006, *Tectonophysics*

Definition of an image, from *Scales (1995), Seismic Imaging*

$$I(x) = \vec{P}\vec{S}(x) = \int d\omega \left[F_1(\omega) \frac{u^{Scat}(x,\omega)}{u^{Inc}(x,\omega)} \right]$$

Recall from Anne Sheehan's lecture the definition of a receiver function

$$R_F(x_r,t) = \int d\omega \left[F_2(\omega) \frac{u^{SV}(x_r,\omega)}{u^{P}(x_r,\omega)} \right] \exp(-i\omega t)$$

A specific depropagator: Diffraction Integrals
Start with two scalar wave equations, one homogeneous:

$$\nabla^2 U(r,t) - \frac{1}{c^2(r)}\frac{\partial^2}{\partial t^2}U(r,t) = 0$$

and the other the equation for the Green's function: the response to a singularity in space and time:

$$\nabla^2 G(r,r_o,t) - \frac{1}{c^2(r)}\frac{\partial^2}{\partial t^2}G(r,r_o,t) = -4\pi\delta(t)\delta(r-r_o)$$

Fourier Transform with respect to time giving two Helmholtz equations

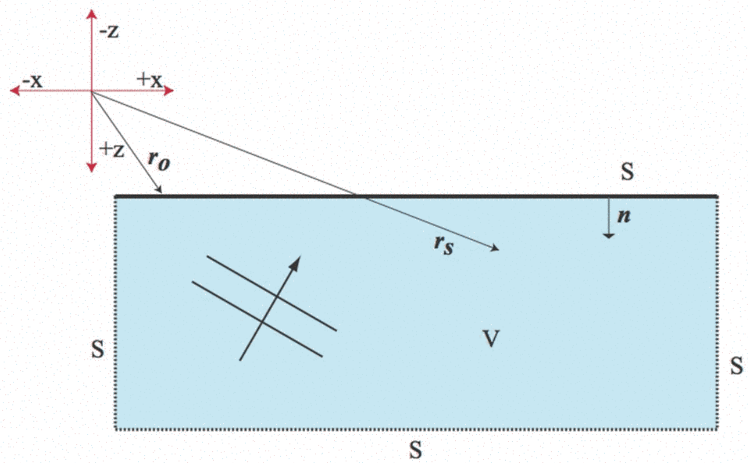
$$\nabla^2 U(r,\omega) + k^2 U(r,\omega) = 0$$

$$\nabla^2 G(r,r_s,\omega) + k^2 G(r,r_s,\omega) = -4\pi\delta(r-r_s)$$

where $k = \omega/c = \omega p$

Apply Green's Theorem, a form of representation theorem:

$$\iiint dV(r)\left[U(r,\omega)\nabla^2 G(r,r_s,\omega) - G(r,r_s,\omega)\nabla^2 U(r,\omega)\right]$$
$$= \iint dS_0(r_0)\left[U(r_0,\omega)\frac{\partial G(r_0,r_s,\omega)}{\partial n} - G(r_0,r_s,\omega)\frac{\partial U(r_0,\omega)}{\partial n}\right]$$



Use the definition of the 3D delta function to sift $U(r_s, \omega)$

$$U(r_s,\omega) = \frac{-1}{4\pi}\iint dS_0(r_0)[U(r_0,\omega)\frac{\partial G(r_0,r_s,\omega)}{\partial n} - G(r_0,r_s,\omega)\frac{\partial U(r_0,\omega)}{\partial n}]$$

Given measurements or estimates of two fields U and G, and their normal derivatives on a closed surface S defined by r_0, **we can predict the value of the field U anywhere within the volume, r_s.**

The integral has been widely used in diffraction theory and forms the basis of a class of seismic migration operators.

We need a Green's function

The constant velocity fress space Green's function is given by

$$G_{FS}(x,t;x_t) = \frac{\delta\left(t - |r - r_s|/c\right)}{4\pi|r - r_s|}$$

The free space asymptotic Green's function for variable velocity can be written as

$$G_{FS}(r,r_s,t) = A(r,r')\delta\left(t - \tau(r,r_s)\right)$$

$$G_{FS}(r,r_s,\omega) = A(r,r')\exp(i\omega\tau(r,r_s))$$

Here $\tau(r,r_s)$ is the solution to the *eikonal* equation:

$$|\nabla\tau(r,r_s)| = \frac{1}{c(r)}$$

And $A(r,r_s)$ is the solution to the transport equation:

$$2\nabla\tau\cdot\nabla A - \left(\nabla^2\tau\right)A = 0$$

The solution is referred to as a high-frequency or asymptotic solution. Both equations are solved numerically

We want a Green's function that vanishes on S_o, or one whose derivative vanishes on S_o

Recenter the coordinate system so that *z=0* lies in S
A Green's function which vanishes at *z=0* is given by:

$$G(r_o,r_s,\omega) = A(r_0,r_s)\exp(i\omega\tau(r_0,r_s)) - A(r_0,r_s')\exp(i\omega\tau(r_0,r_s'))$$

Where r'_s is the image around x of r_s. At *z=0*

$$\frac{\partial G}{\partial n} = 2\frac{\partial G_{FS}}{\partial n}\bigg|_{z=0}$$

This is also approximately true if S_o has topography with wavelength long compared to the incident field. This is called *Kirchhoff's approximation.*

The diffraction integral becomes

$$U(r,\omega) = \frac{-1}{2\pi}\int dS_0 U(r_0,\omega)[\frac{\partial A}{\partial n} + i\omega A\frac{\partial \tau}{\partial n}]\exp(i\omega\tau)$$

Assume that the phase fluctuations are greater than the amplitude fluctuations we can throw away the first term. This is a *far field approximation*:

$$U(r,\omega) = \frac{i\omega}{2\pi}\int dS_0 U(r_0,\omega)\frac{A(r_0,r)\cos\theta}{c(r_0)}\exp(i\omega\tau)$$

This is the *Rayleigh-Sommerfeld diffraction integral (Sommerfeld, 1932, Optics)*

If we use it to backward propagate the wavefield, rather than forward propagate it, it is an imaging integral.

Imaging Condition

The ***imaging condition*** is given by the time it took the source to arrive at the scatterer plus the time it took the scattered waves to arrive at the receiver. If we have an accurate description of *c(r)* then the waves are focused at the scattering point. (attributed to Jon Claerbout)

This is a big if.

For an incident P-wave and a scattered S-wave, and an observation at $\boldsymbol{r_o}$, then the image would focus at point $\boldsymbol{r}$, when

$$0 = \tau_p(r, r_e) + \tau_s(r_o, r) - t_e(r_o, r_e)$$

Because the phase is zero at the imaging point, the inverse Fourier transform becomes a frequency sum

$$I(r) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\omega\, (i\omega) \int_{-L}^{L} dS_0 \exp(-i\omega(t - (\tau_p + \tau_s)))U(r_0, \omega) \frac{A(r, r_0)\cos\theta}{c(r_0)} \Bigg|_{t=\tau_p+\tau_s}$$

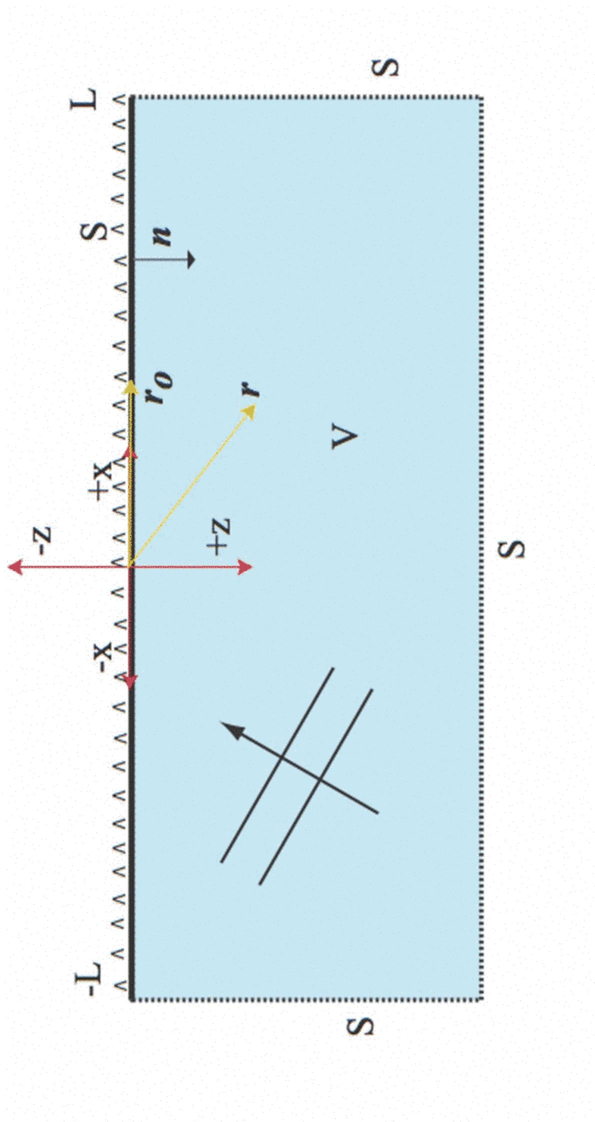
Recall that U is a receiver function and the desired image is

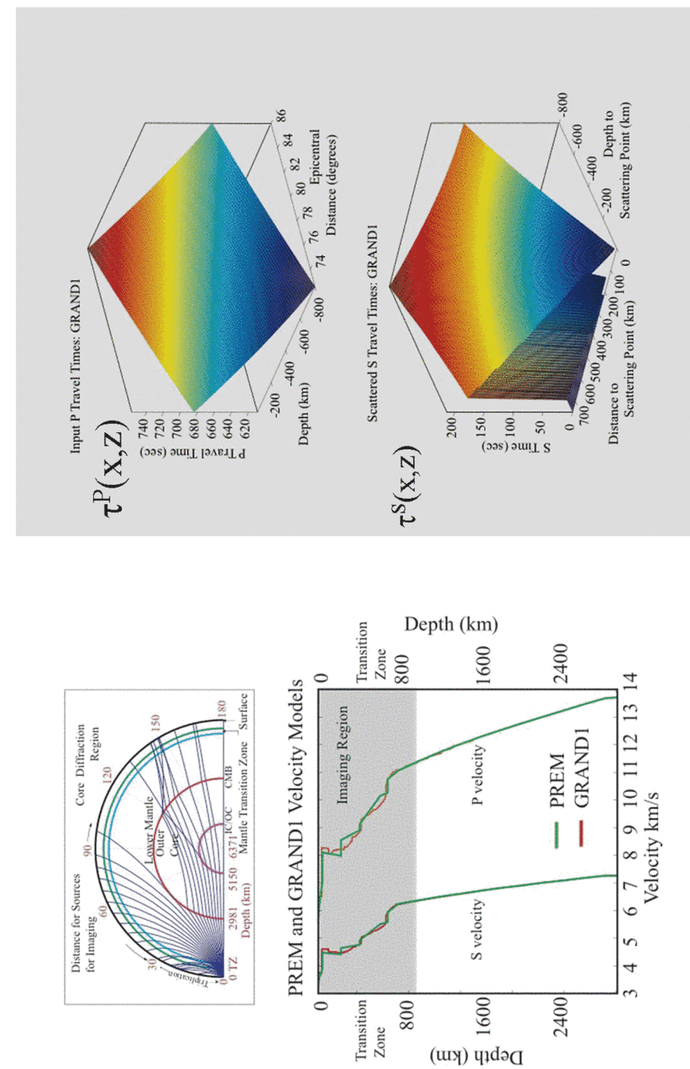
$$I(x) = \vec{P}\vec{S}(x) = \int d\omega \left[F_1(\omega) \frac{u^{SV}(x,\omega)}{u^P(x,\omega)} \right]$$

Define

$$S_{PS}(r) = A_S(r) / A_P(r)$$

$$I(r) = \vec{P}\vec{S}(r) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} d\omega(i\omega)\int_{-L}^{L} dS_0 \exp(-i\omega(t-(\tau_P+\tau_S)))R_F(r_0,\omega)\frac{A_S(r,r_0)\cos\theta(r_0)}{A_P(r,r_e)\beta(r_0)}\bigg|_{t=t_e}$$





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Sensitivities

- Spatial aliasing and Aperture
  - First Fresnel zone has to be well sampled in a Fourier sense

$$R_F \approx \sqrt{z_{target}\lambda}$$

$$\Delta x = \frac{\beta_{min}}{2 f_{max} \sin\theta}$$

  - Generally $2L > 2z_{target}$ We need lots of receivers

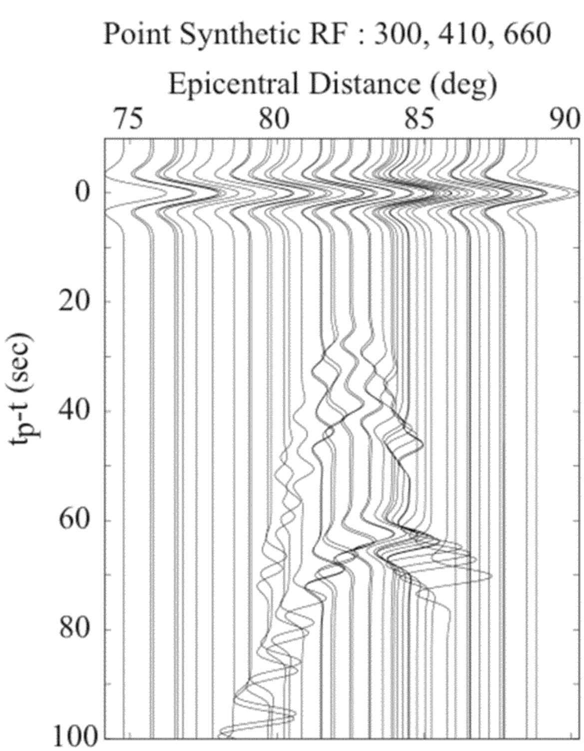
$$N_{instruments} = M * 2R_f / \Delta x$$

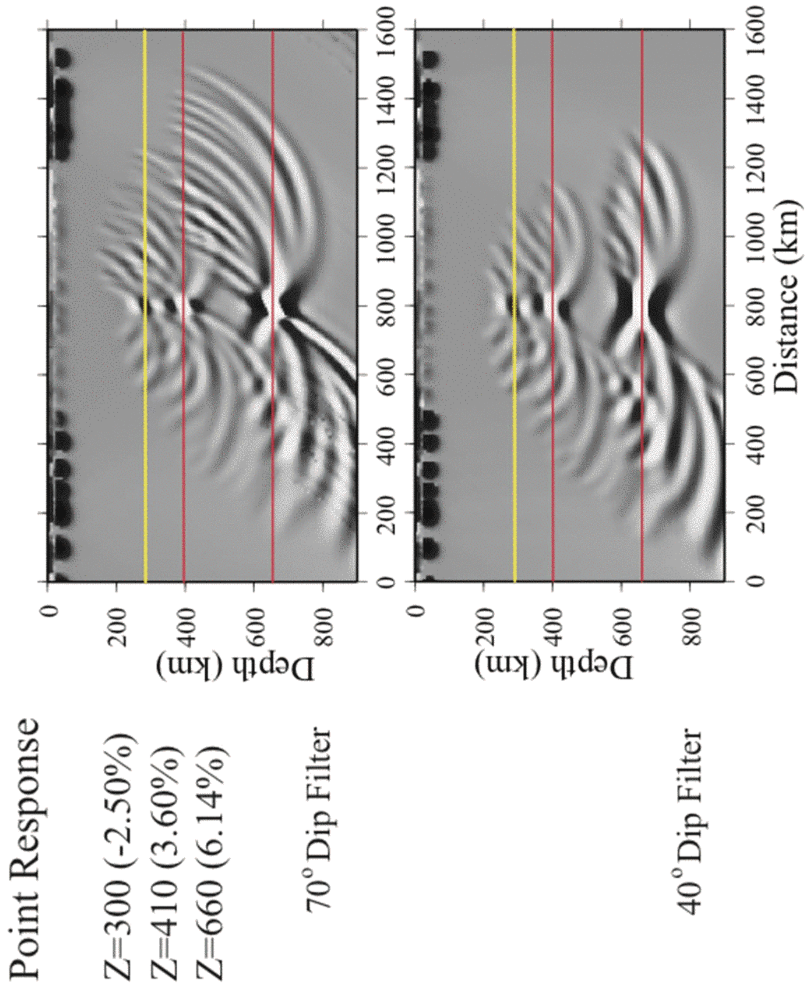
$$N_{instruments} = M * \frac{4 f_{max} \sin\theta_f \sqrt{z_{target}\lambda}}{\beta_{min}}$$

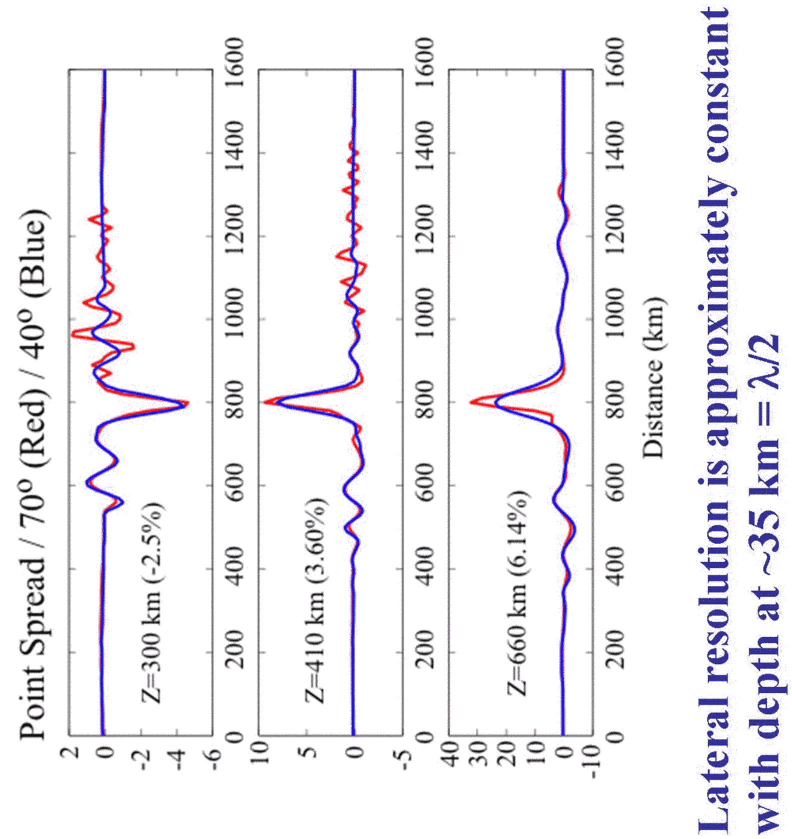
Synthetics
For 3 point
Scatterers

Kaapvaal
Geometry
Eq 98246

Z=300
Z=400
Z=660



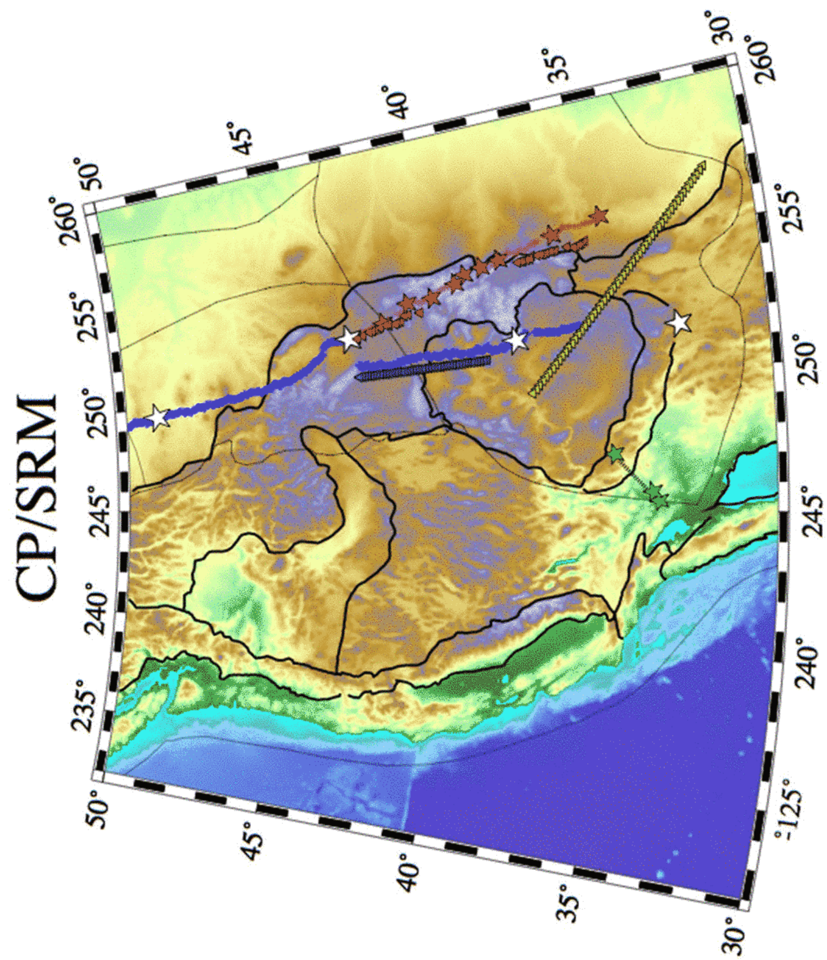

Point Response
Z=300 (-2.50%)
Z=410 (3.60%)
Z=660 (6.14%)
70° Dip Filter
40° Dip Filter
Depth (km)
Distance (km)

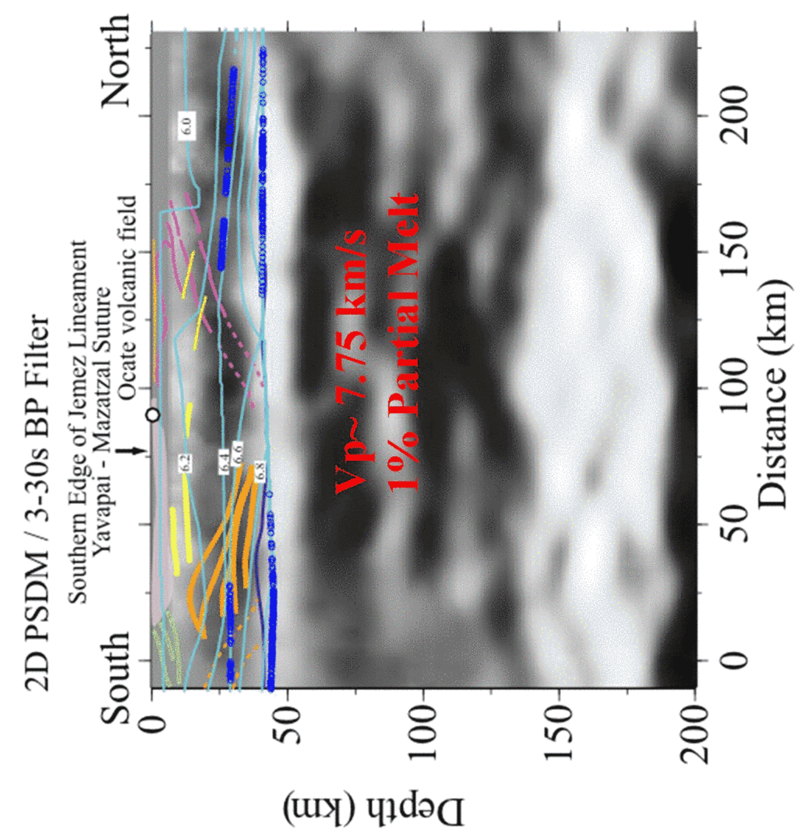

Point Spread / 70° (Red) / 40° (Blue)
Z=300 km (-2.5%)
Z=410 km (3.60%)
Z=660 km (6.14%)
Distance (km)
Lateral resolution is approximately constant
with depth at ~35 km = λ/2

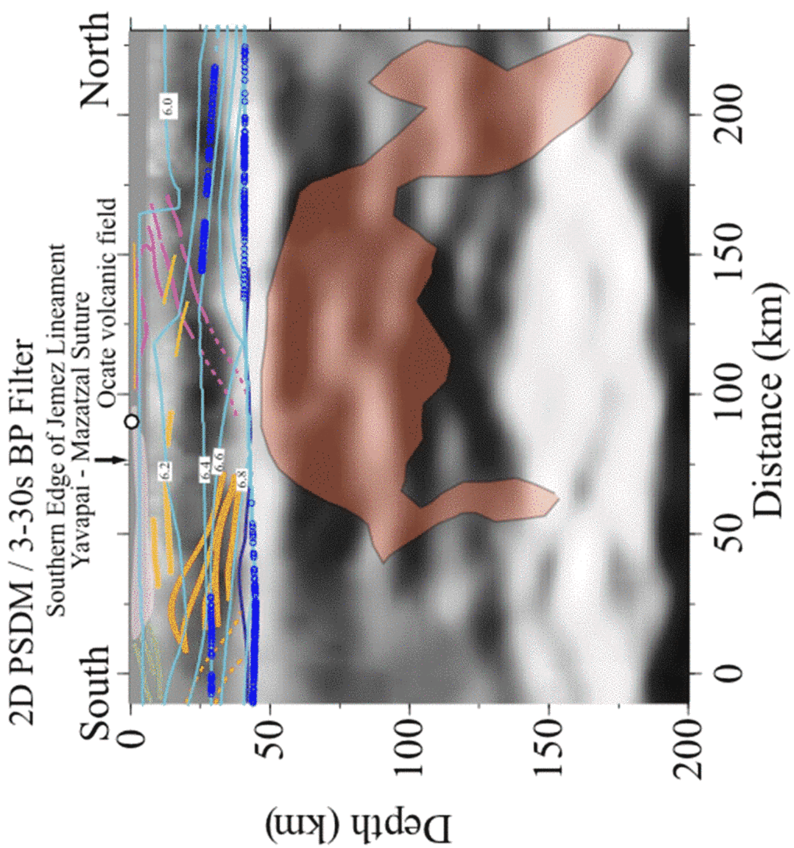
- Data redundancy
  - Redundancy builds Signal/(Random Noise) as $N^{0.5}$, where N is the number of data contributing at a given image point: $\sim N_{event} * N_{receiver}$
  - Events from different incidence angles, i.e. source distances, suppresses reverberations from dipping interfaces
  - Events from different incidence angles reconstruct a larger angular spectrum of the target

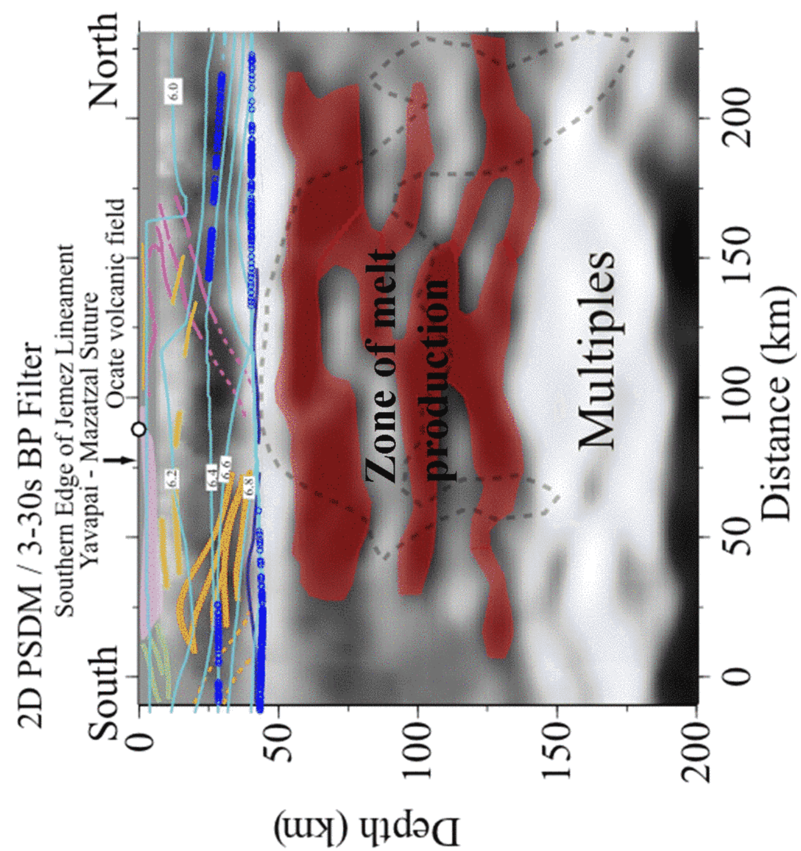
Other considerations

1. A good starting velocity model is required
   - Use the travel-time tomography model
2. Data preconditioning
   - DC removal
   - Frequency filter
   - Mute direct wave
   - Wavenumber filters (ouch!)


CP/SRM
50°
45°
40°
35°
30°
260°
255°
250°
245°
240°
235°
-125°


2D PSDM / 3-30s BP Filter
Southern Edge of Jemez Lineament
Yavapai - Mazatzal Suture
Ocate volcanic field
South
North
Vp~ 7.75 km/s
1% Partial Melt
Distance (km)
Depth (km)
0
50
100
150
200


2D PSDM / 3-30s BP Filter
Southern Edge of Jemez Lineament
Yavapai - Mazatzal Suture
Ocate volcanic field
South
North
Depth (km)
Distance (km)
0
50
100
150
200
6.0
6.2
6.4
6.6
6.8


2D PSDM / 3-30s BP Filter
Southern Edge of Jemez Lineament
Yavapai - Mazatzal Suture
Ocate volcanic field
South
North
Zone of melt production
Multiples
Depth (km)
Distance (km)
0
50
100
150
200
6.0
6.2
6.4
6.6
6.8

RISTRA Receiver Functions

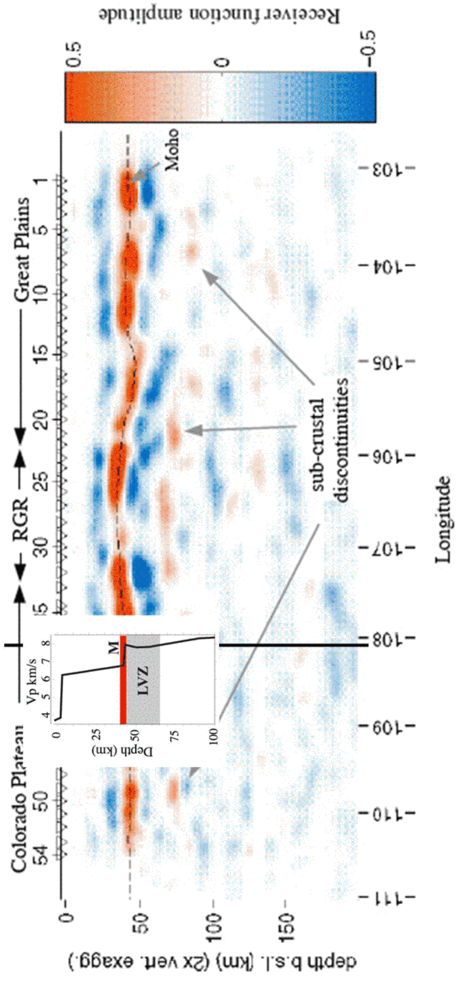


Figure 8: Wilson et al., JGR 2005

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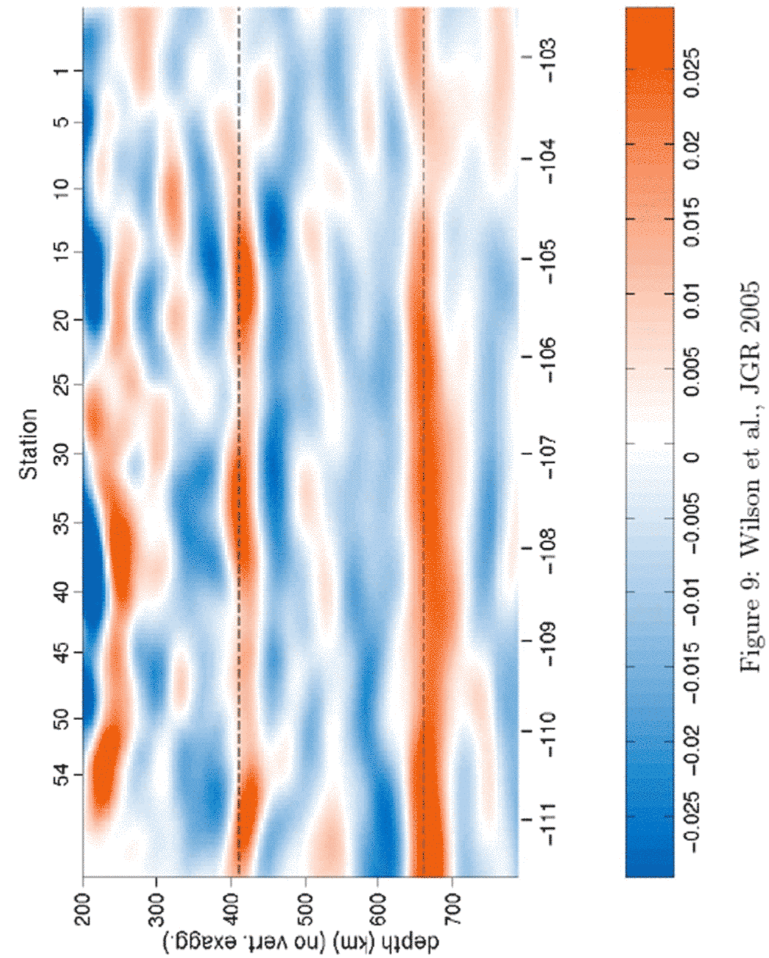
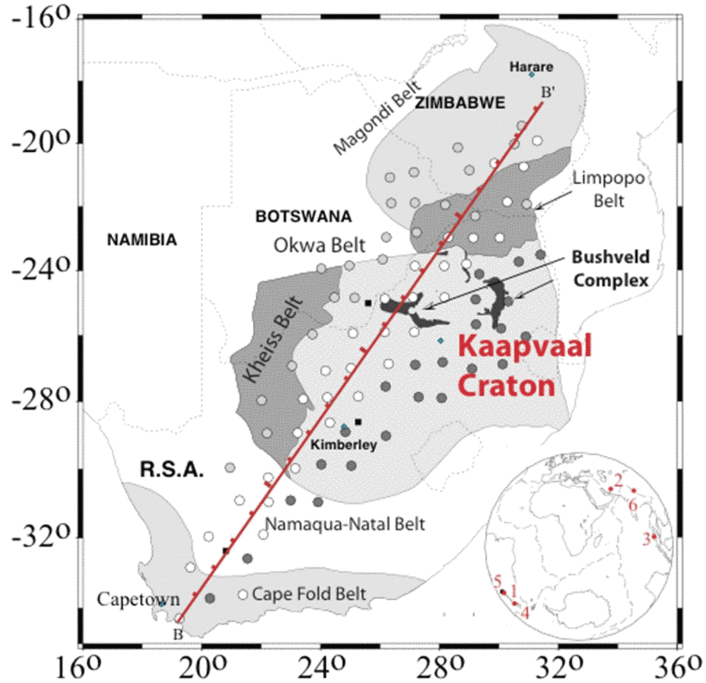


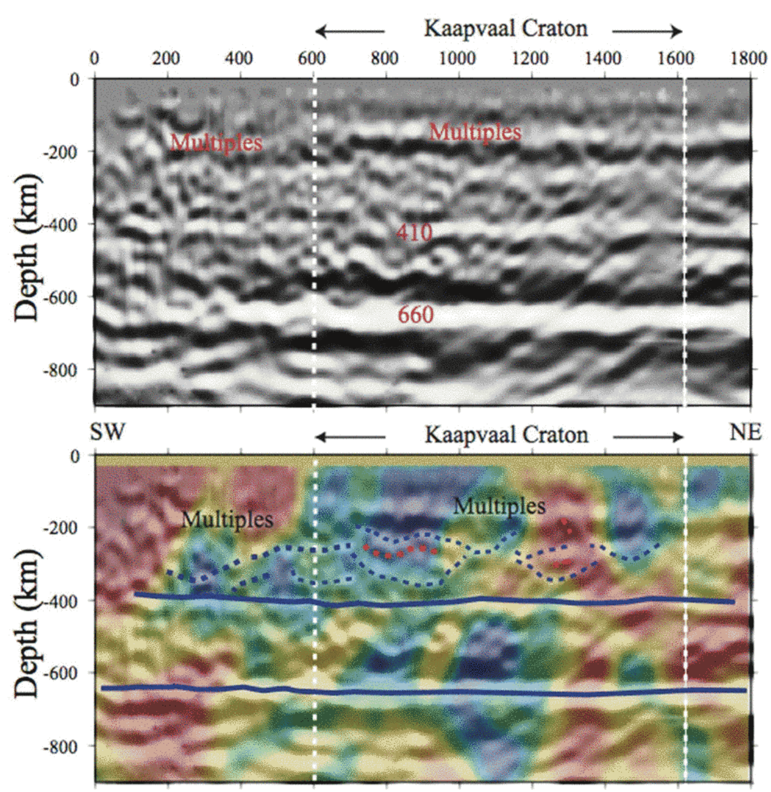
Figure 9: Wilson et al., JGR 2005

Example:
Kaapvaal
Craton

$\Delta x \sim 35$ km
$A_L \sim 20^o$

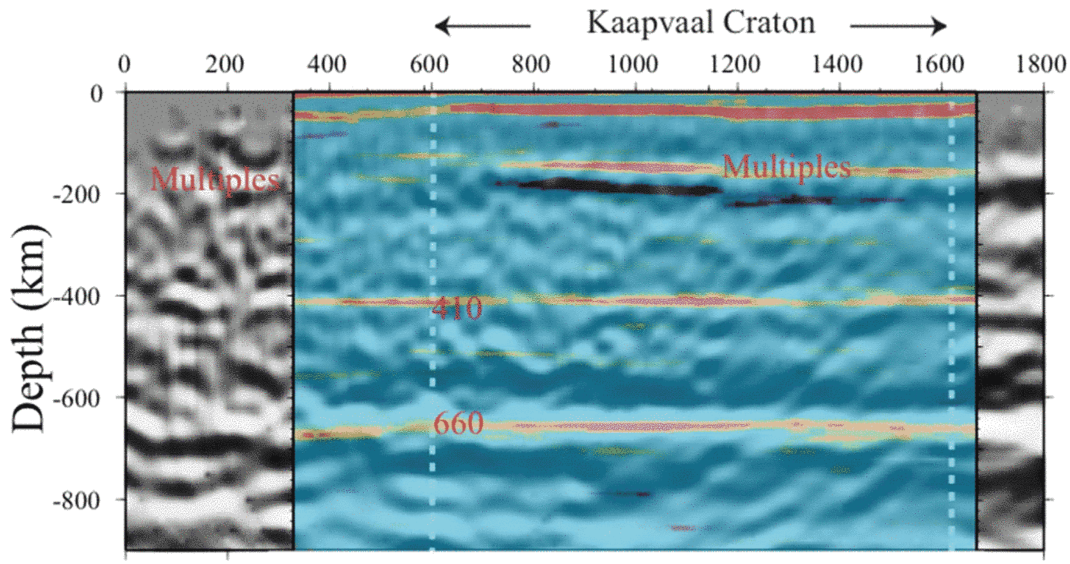
9 Eqs





Levander et al., 2005,
AGU Monograph

9 Eqs/0.033-0.15Hz/45deg



RF from Niu et al., 2004, *EPSL*

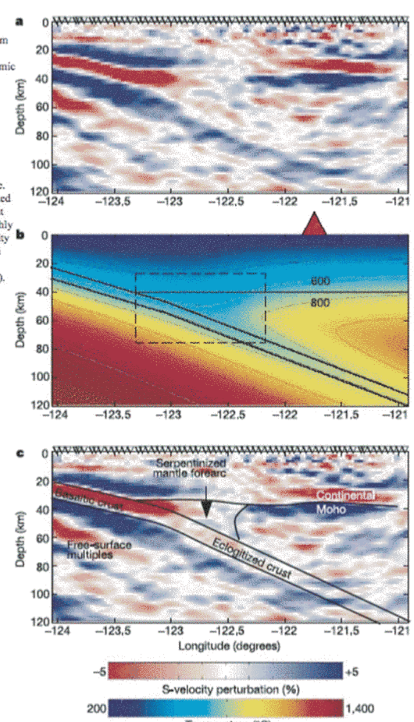
Bostock et al.,
2001, 2002

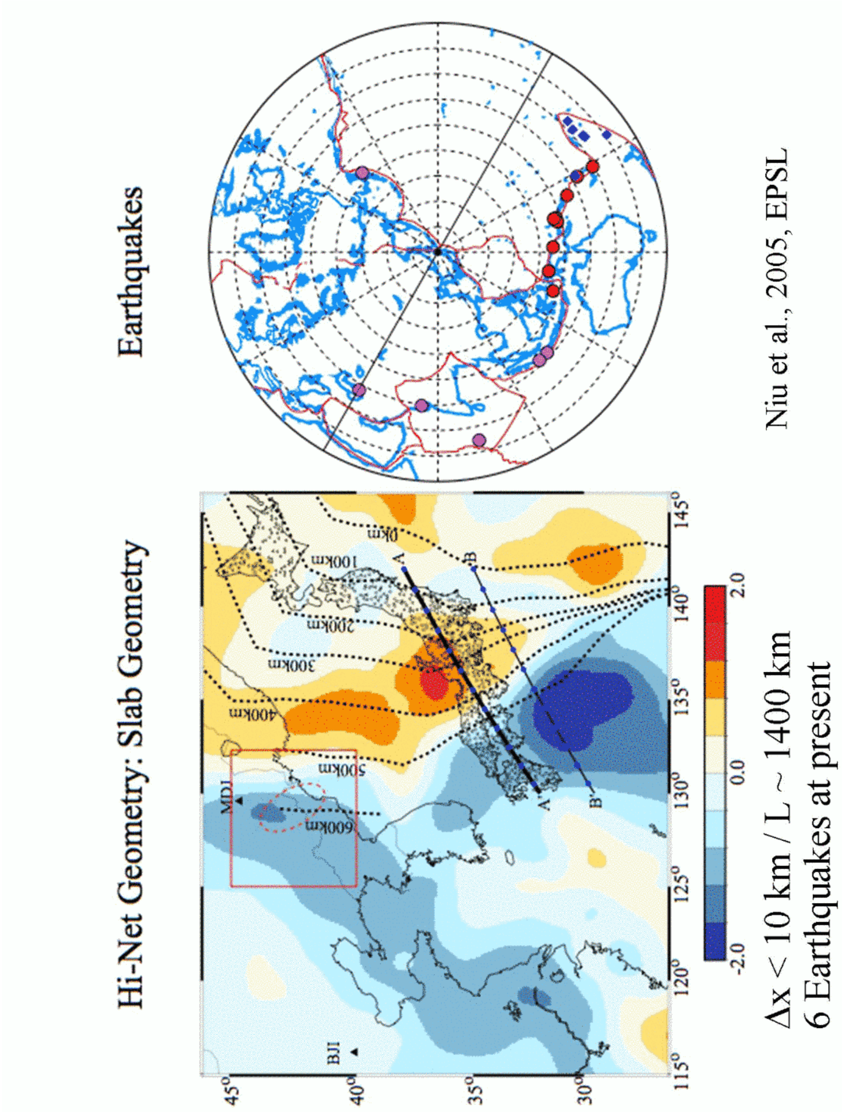
Generalized
Radon Transform

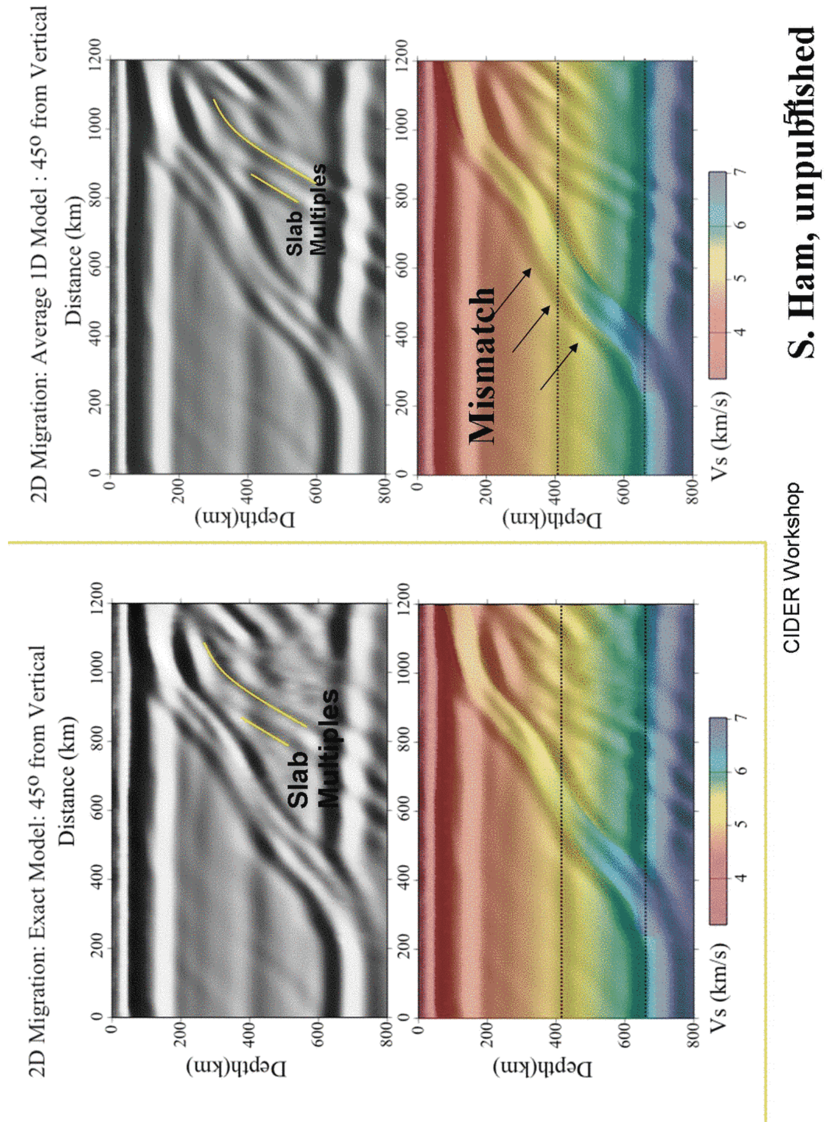
Figure 1: a) Generalized Radon Transform scattered wave image of the Cascadia subduction zone in Oregon from teleseismic data. The image shows S-wave velocity perturbation in percent from a 1-D backgroung model.

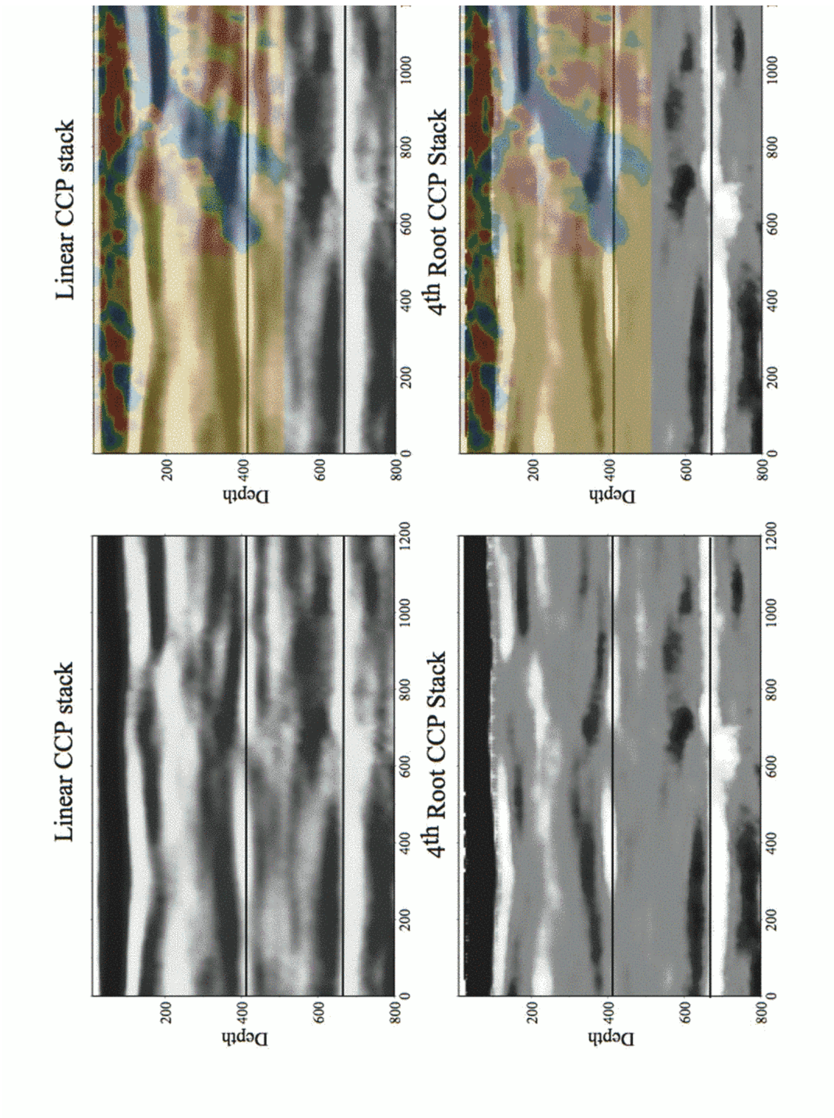
b) Cascadia subduction zone thermal structure.

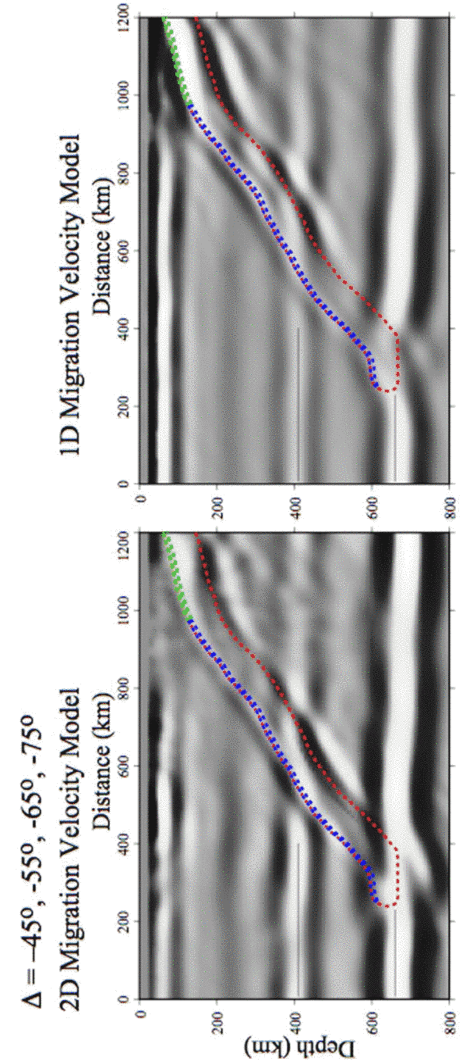
c) Interpretation of S-wave velocity perturbations in light of thermal structure. The authors interpret the absent or inverted continental Moho conversion from about -123° to -122.5° as resulting from a highly serpentinized, and therefore low S velocity upper mantle, and the loss of signal from the subducting oceanic crust as due to eclogitization (from Bostock et al., 2002).

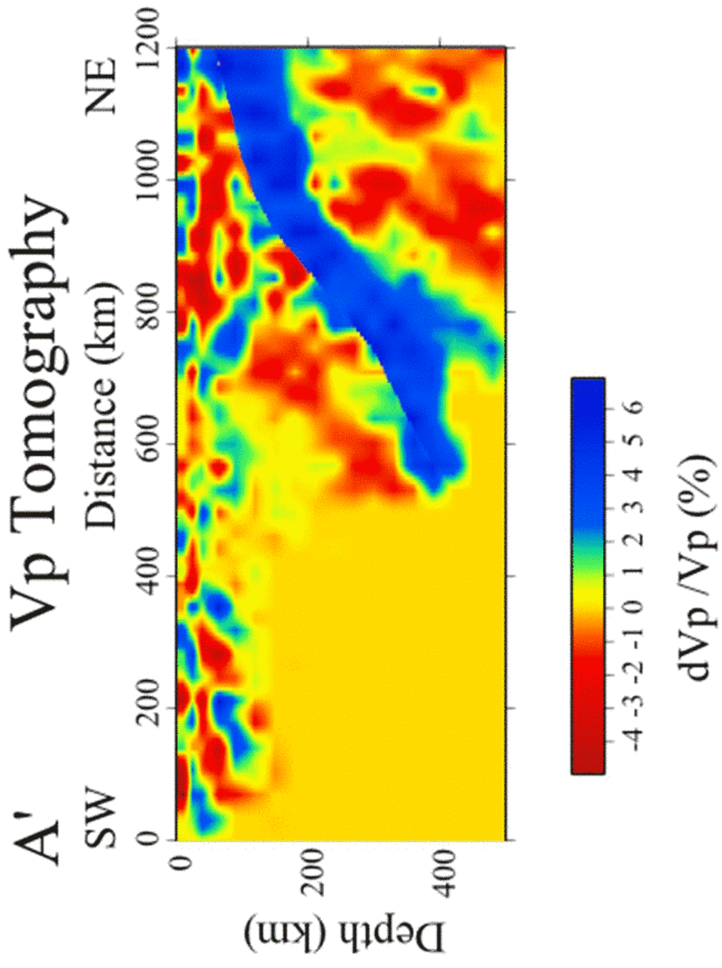


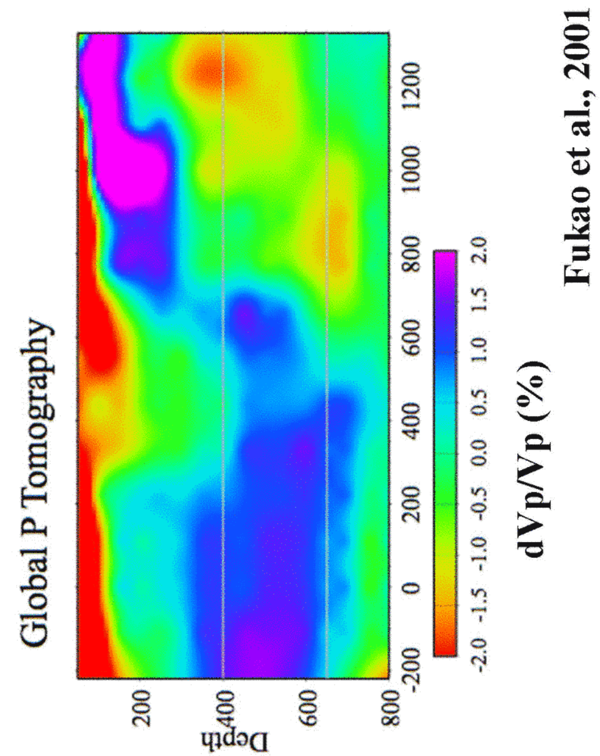

Hi-Net Geometry: Slab Geometry
Earthquakes
Niu et al., 2005, EPSL
Δx < 10 km / L ~ 1400 km
6 Earthquakes at present

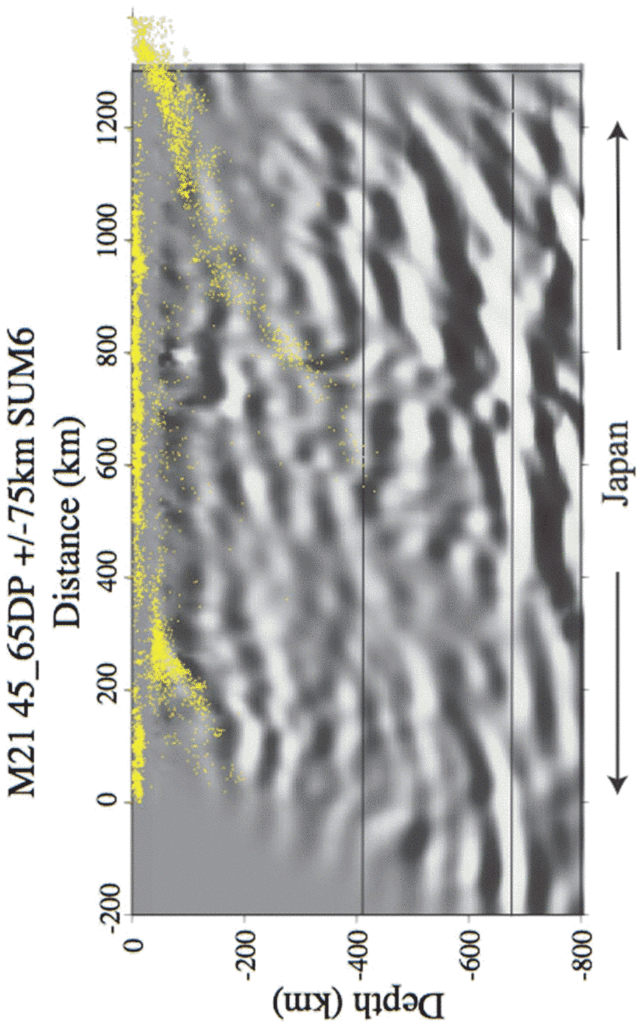

2D Migration: Exact Model: 45° from Vertical
2D Migration: Average 1D Model : 45° from Vertical
Distance (km)
Depth(km)
Slab Multiples
Mismatch
Vs (km/s)
CIDER Workshop
S. Ham, unpublished

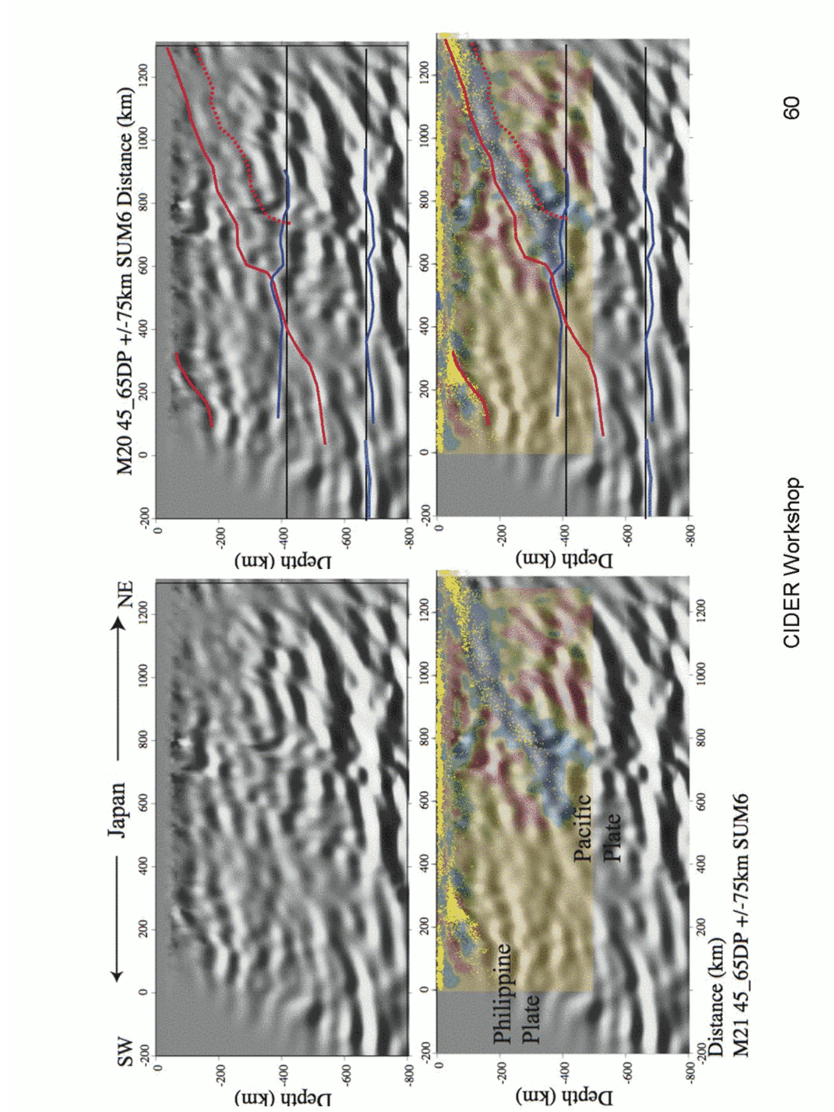

Linear CCP stack
Linear CCP stack
4th Root CCP Stack
4th Root CCP Stack
Depth
Depth
Depth
Depth

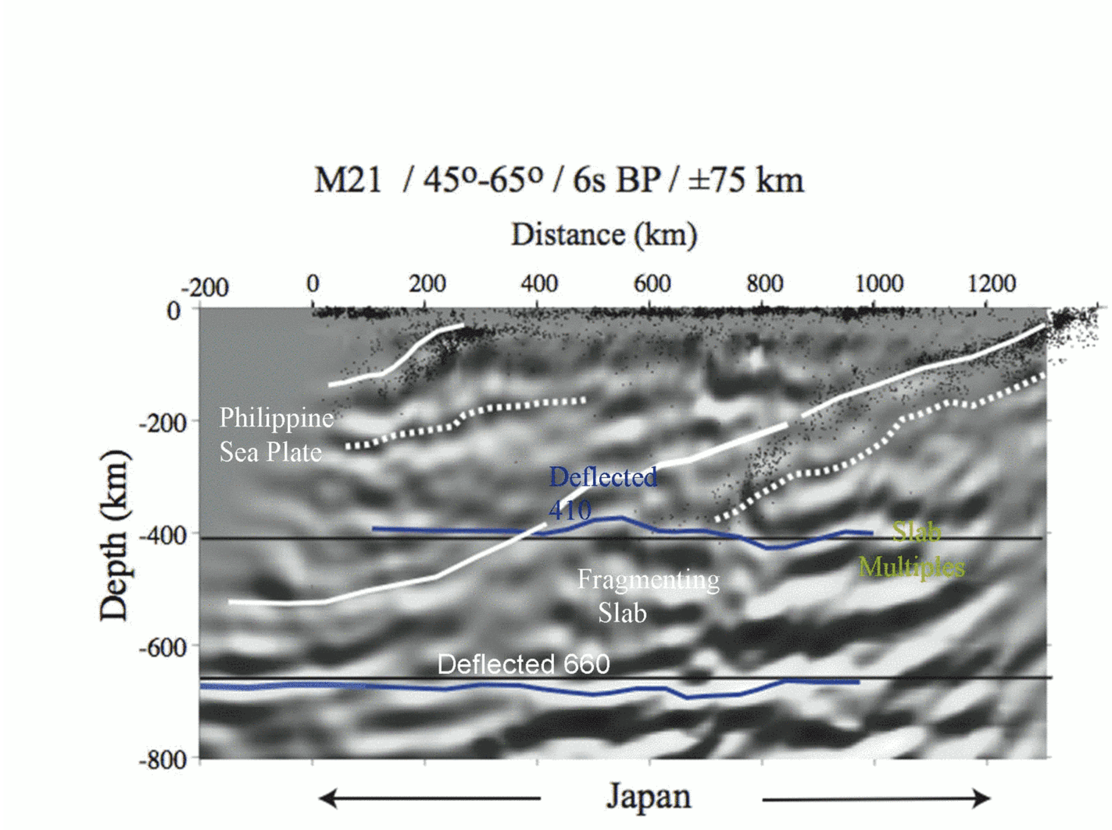

Δ = -45º, -55º, -65º, -75º
2D Migration Velocity Model
1D Migration Velocity Model
Distance (km)
Distance (km)
Depth (km)


A'
SW
NE
Vp Tomography
Distance (km)
0 200 400 600 800 1000 1200
Depth (km)
0 200 400
-4 -3 -2 -1 0 1 2 3 4 5 6
dVp /Vp (%)


Global P Tomography
Depth
200 400 600 800
-200 0 200 400 600 800 1000 1200
-2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0
dVp/Vp (%)
Fukao et al., 2001


M21 45_65DP +/-75km SUM6
Distance (km)
Depth (km)
Japan


SW
Japan
NE
M20 45_65DP +/-75km SUM6 Distance (km)
Depth (km)
Philippine Plate
Pacific Plate
Distance (km)
M21 45_65DP +/-75km SUM6
CIDER Workshop
60



Operator notation

$$Lu(x_r,t;x_s,t_s) = f(x_s,t)$$

$$LG(x_r,t-t_s;x_s) = s(t-t_s)\delta(x_s)$$

$$f(x_s,t) = \sum_{x_s} \delta(x-x_s)s(t-px_{1s}+q_z^p(x_{3s})\Delta x_{3s})$$

Another view of the Green's function

$$s(t) = \delta(t)$$

$$s(\omega) = 1$$

$$LG(x_r,\omega;x_s) = \delta(x-x_s)$$

$$\delta(x-x_s^i) = I; x_s^i = 1,2,\ldots,n$$

$$G(x_r,\omega;x_s) = L^{-1}I$$

The Green's function is the inverse operator to the wave equation

Waveform inversion: Pratt et al., 1998

$$[\nabla^2 + \frac{\omega^2}{c^2(x)}]\vec{p}(x,x_s,\omega) = f(x,x_s,\omega)$$

$$L\vec{p}(x,x_s\omega) = \vec{f}(x,x_s,\omega)$$

$$\vec{p}(x,x_s\omega) = L^{-1}\vec{f}(x,x_s,\omega)$$

The inverse of the wave operator ***L*** can be found using LU decomposition. We want to minimize the functional

$$2S(\vec{m}) = \delta\vec{d}(\vec{m})^T \delta\vec{d}(\vec{m})^*$$

$$\delta\vec{d}(\vec{m}) = \vec{d}_{obs} - \vec{d}_{syn}(\vec{m})$$

Where ***m*** is the model vector

This can be solved iteratively using gradient methods

$$\vec{m}^{(k+1)} = \vec{m}^{(k)} - \alpha\nabla_m S(\vec{m}^{(k)})$$

$$\vec{m}^{(k+1)} = \vec{m}^{(k)} + \alpha \,\mathrm{Re}\left\{\left[\frac{\partial\vec{p}}{\partial\vec{m}^{(k)}}\right]^T \delta\vec{d}(\vec{m}^{(k)})^*\right\}$$

$$\vec{m}^{(k+1)} = \vec{m}^{(k)} + \alpha \,\mathrm{Re}\left\{[J^{(k)}]^T \delta\vec{d}(\vec{m}^{(k)})^*\right\}$$

The Jacobian can be very expensive to calculate.

Note that the data residuals are conjugated, meaning that they are back-propagated

Pratt et al (1988) show that the Jacobian finds those parts of the first order scattered field that are present in the data but not in the model.

$$\frac{\partial L}{\partial m_i}\vec{p} + L\frac{\partial \vec{p}}{\partial m_i} = 0$$

$$\frac{\partial \vec{p}}{\partial m_i} = L^{-1}\left[\frac{\partial L}{\partial m_i}\vec{p}\right] = L^{-1}\vec{f}^{(i)}$$

$$J = L^{-1}[f^{(1)} \ldots\ldots f^{(n)}]$$

The Jacobian can be interpreted as a series of virtual sources applied at the *n* parameter locations

An efficient scheme for iteration is then

$$\vec{m}^{(k+1)} = \vec{m}^{(k)} + \alpha \,\mathrm{Re}\left\{[L^{-1}F]^T \delta\vec{d}(\vec{m}^{(k)})^* \right\}$$

$$\vec{m}^{(k+1)} = \vec{m}^{(k)} + \alpha \,\mathrm{Re}\left\{\left[F^{(k)}\right]^T \left[L^{-1}\right]^T \delta\vec{d}(\vec{m}^{(k)})^* \right\}$$

The first iterate of inversion for velocity perturbation is *prestack depth migration* (Lailly, 1983)

$$\delta\vec{m}^{(1)} = \alpha \,\mathrm{Re}\left\{\left[F^{(k)}\right]^T \left[L^{-1}\right]^T \delta\vec{d}(\vec{m}^{(0)})^* \right\}$$

$$\delta d(m^{(0)})^* = d^*_{obs}$$

With a lot of work, this reduces to something that looks very much like *prestack Kirchhoff depth migration*