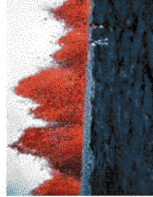
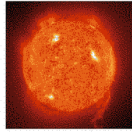





# Heat budget & Thermal Evolution

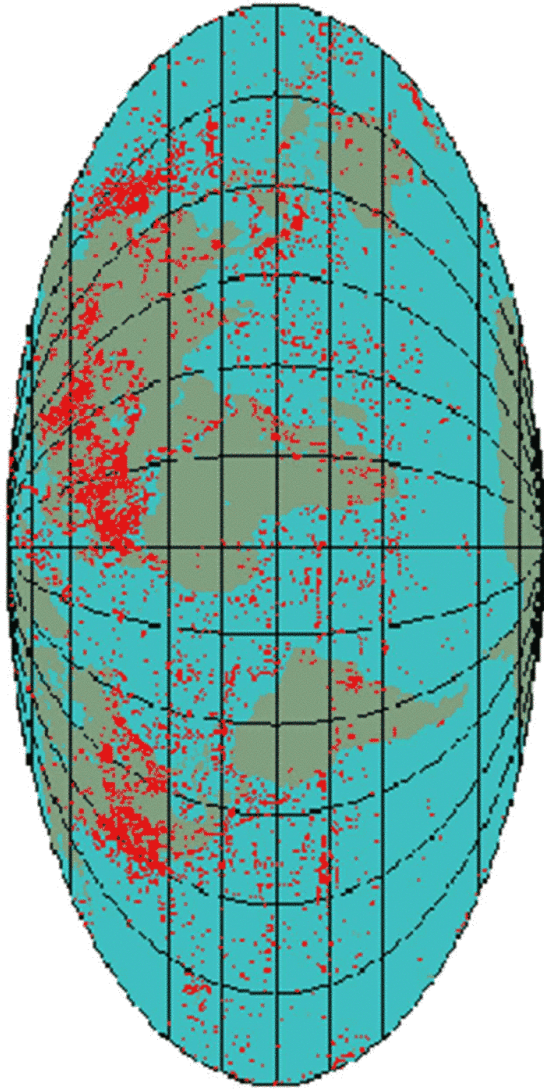
Louise Kellogg, University of California, Davis

- Sources of heat in the Earth
- Transport of heat
- Aspects of thermal evolution
- Can a planetary perspective help?

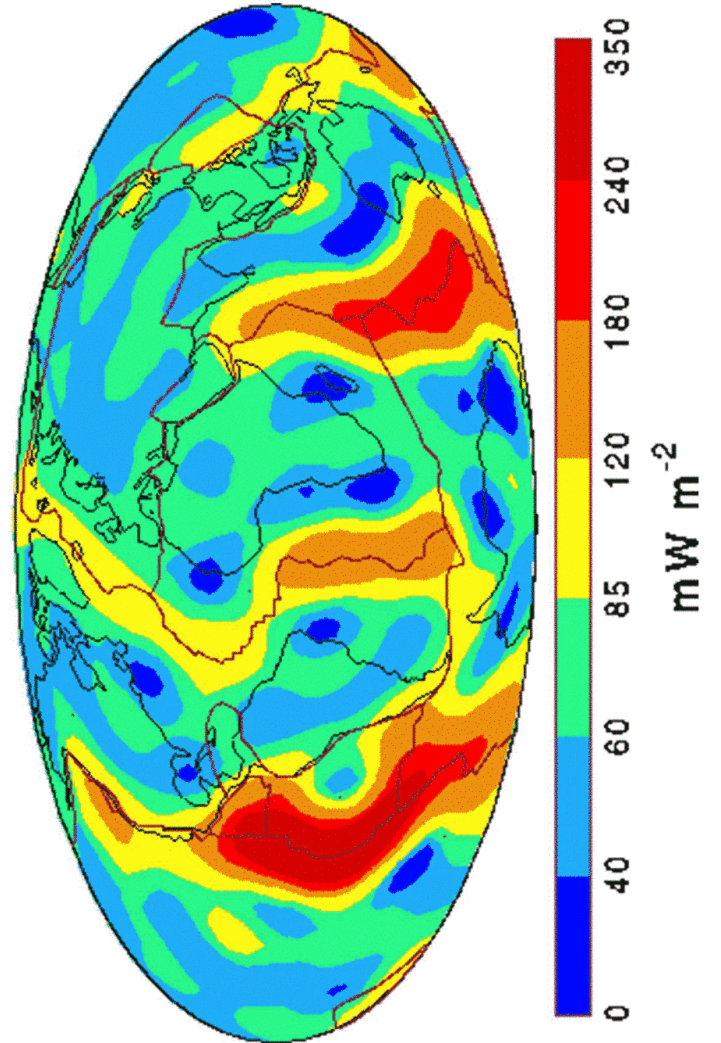
## Some powerful numbers...

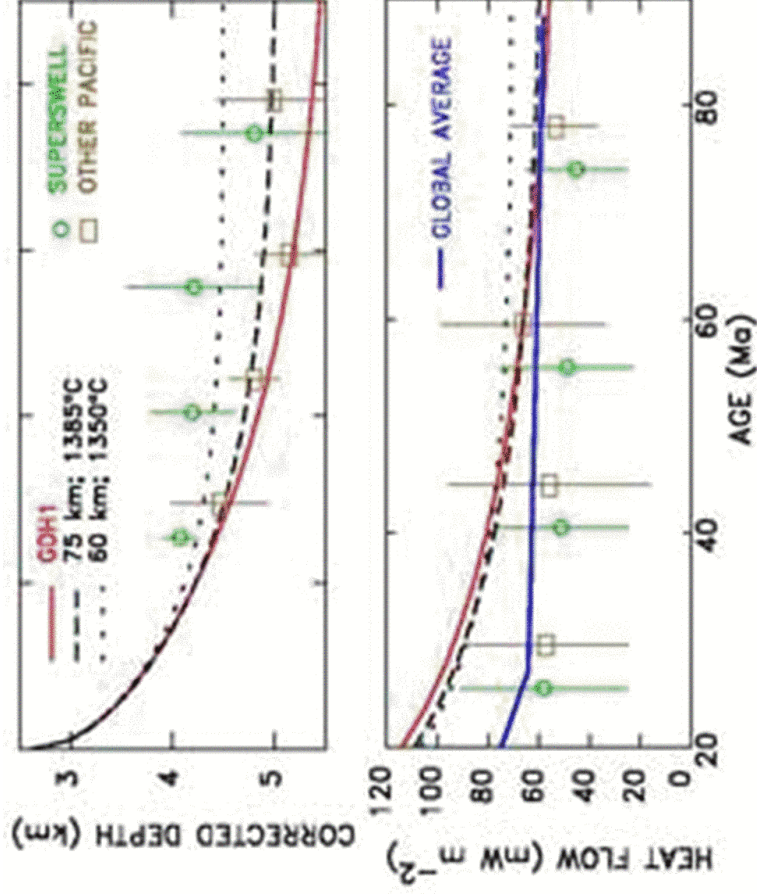
	Total global heat flow:	44 TW
	Sun: Solar energy received (and re-radiated)	180,000 TW
	Tides: Dissipation of Earth's rotational energy	3 TW
	Earthquakes: Elastic wave energy released	0.03 TW
	Humans: World power production	12 TW

# Heat Flow Sites



# Heat Flow





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### Mantle Convection: Basic Version of Equations

Conservation of mass:  $\nabla \cdot \vec{u} = 0$

Momentum:  $-\nabla P + \nabla \cdot \tau + Ra \hat{T} \hat{k} = 0$

Heat flow:  $\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla^2 T + Ra_H / Ra$

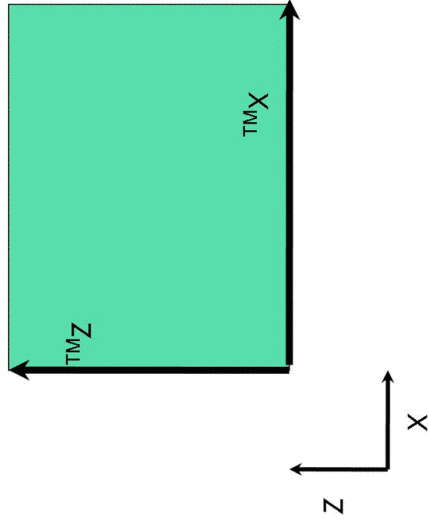
Viscosity varies with temperature and depth:  $\mu = \mu_0 e^{-\frac{E^* + Vz}{RT}}$

In this version, two parameters are of primary importance:

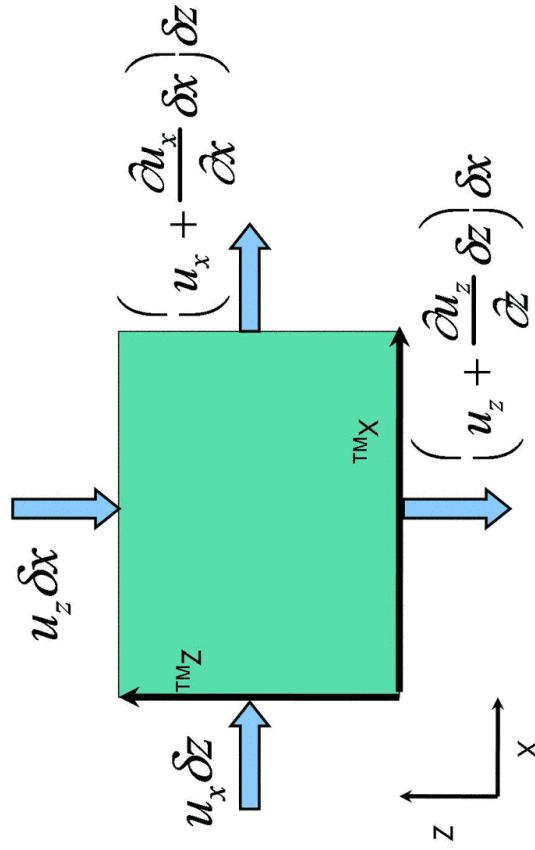
$$Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa \mu_0} \qquad Ra_H = \frac{\rho_m^2 g \alpha H d^5}{\kappa \mu_0}$$

# Conservation of Mass Equation

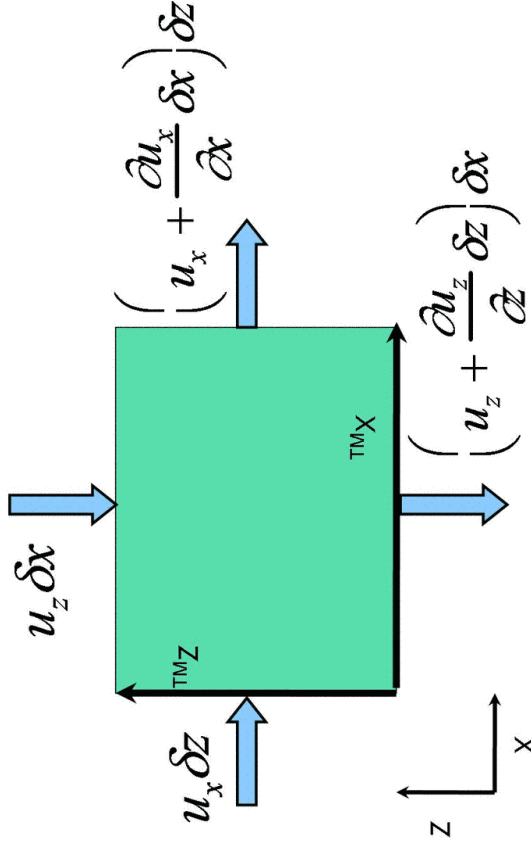
Consider a particle of fluid



For an incompressible fluid,  
flow into the box balances the flow out

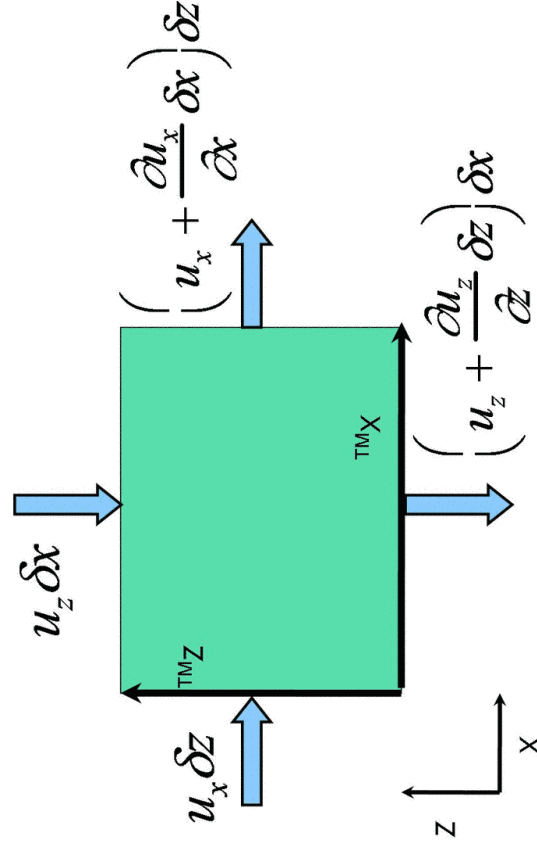


$$\left( u_x + \frac{\partial u_x}{\partial x} \delta x \right) \delta z - u_x \delta z + \left( u_z + \frac{\partial u_z}{\partial z} \delta z \right) \delta x - u_z \delta x = 0$$



### Conservation of Mass Equation

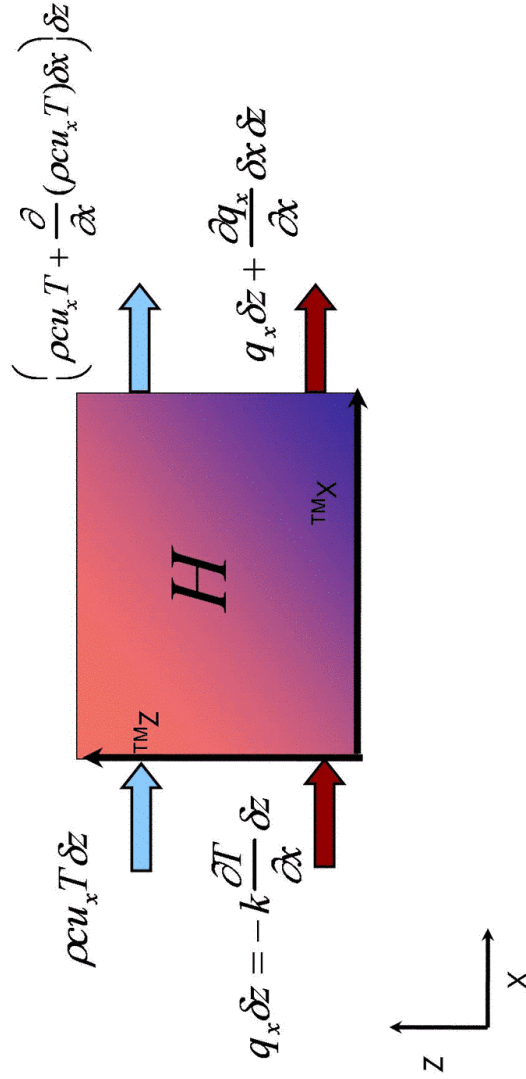
$$\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = 0 \quad \text{that is, } \nabla \cdot \vec{u} = 0$$





### Conservation of Energy (heat flow)

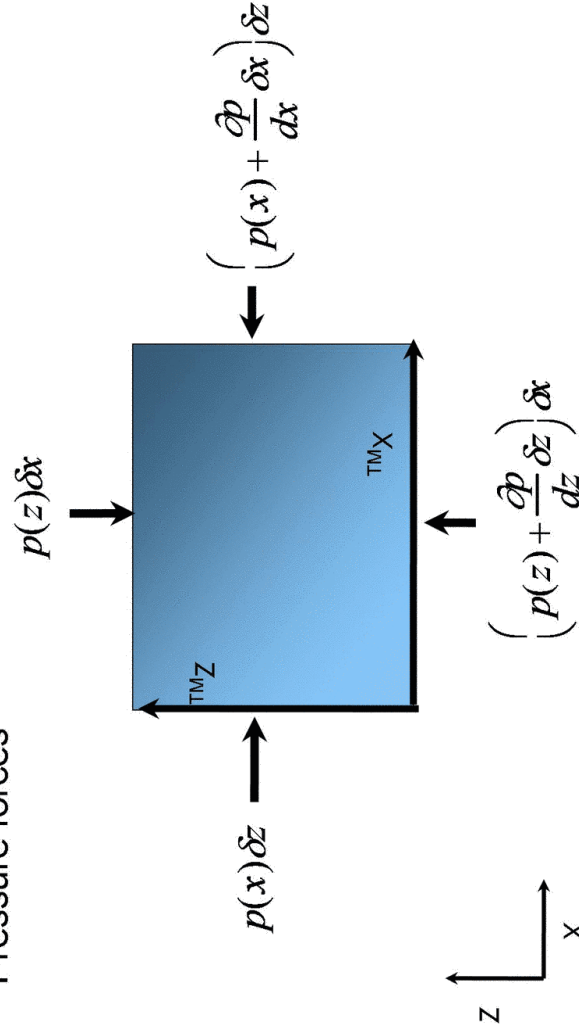
$$\frac{\partial T}{\partial t} + \bar{u} \cdot \nabla T = \nabla^2 T + Ra_H / Ra$$



### Momentum (force balance)

$$-\nabla P + \nabla \cdot \tau + RaT\hat{k} = 0 \quad Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa \mu_0}$$

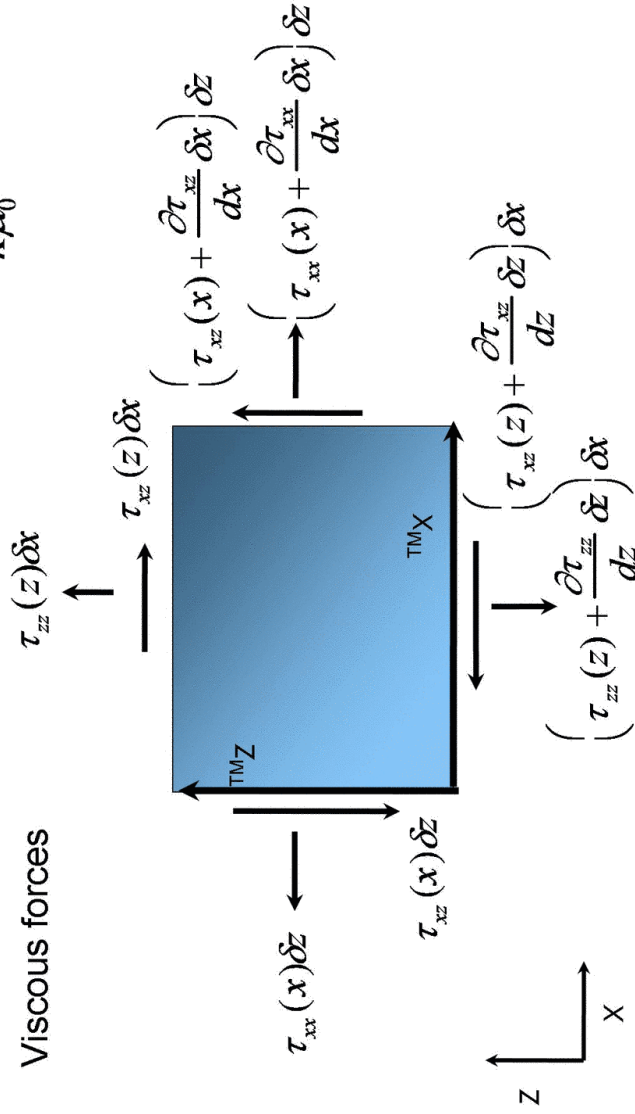
Pressure forces



### Momentum (force balance)

$$-\nabla P + \nabla \cdot \tau + RaT\hat{k} = 0 \quad Ra = \frac{\rho_m g \alpha \Delta T d^3}{K\mu_0}$$

Viscous forces

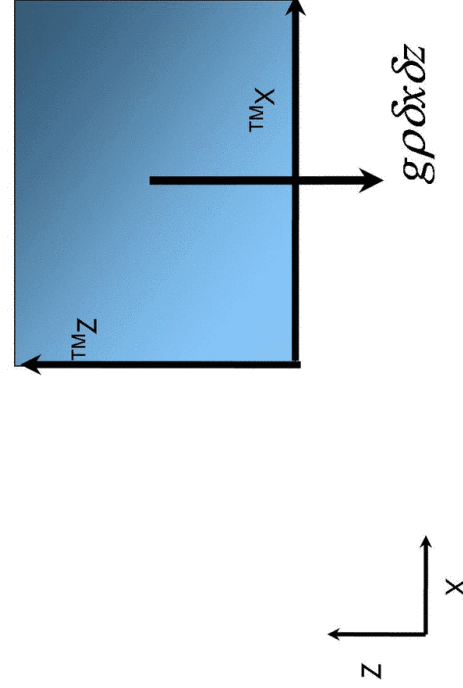


### Momentum (force balance)

$$-\nabla P + \nabla \cdot \tau + RaT\hat{k} = 0 \quad Ra = \frac{\rho_m g \alpha \Delta T d^3}{K\mu_0}$$

Body forces (gravity)

$$\rho(T) = \rho_0(1 - \alpha(T - T_0))$$



### Mantle Convection: Basic Version of Equations

Conservation of mass:  $\nabla \cdot \vec{u} = 0$

Momentum:  $-\nabla P + \nabla \cdot \tau + Ra\hat{T}\hat{k} = 0$

Heat flow:  $\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla^2 T + Ra_H / Ra$

Viscosity varies with temperature and depth:  $\mu = \mu_0 e^{-\frac{E^* + Vz}{RT}}$

Two parameters are of primary importance:

$$Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa \mu_0}$$

$$Ra_H = \frac{\rho_m^2 g \alpha H d^5}{k \kappa \mu_0}$$

Numerical methods discretize the region of interest

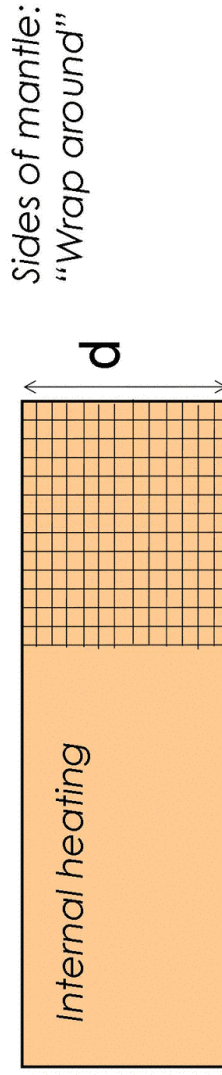
$$Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa \mu_0}$$

Top of mantle:

Traction-free surface

Constant temperature

$$Ra_H = \frac{\rho_m^2 g \alpha H d^5}{k \kappa \mu_0}$$



Core-mantle boundary:

Traction-free surface

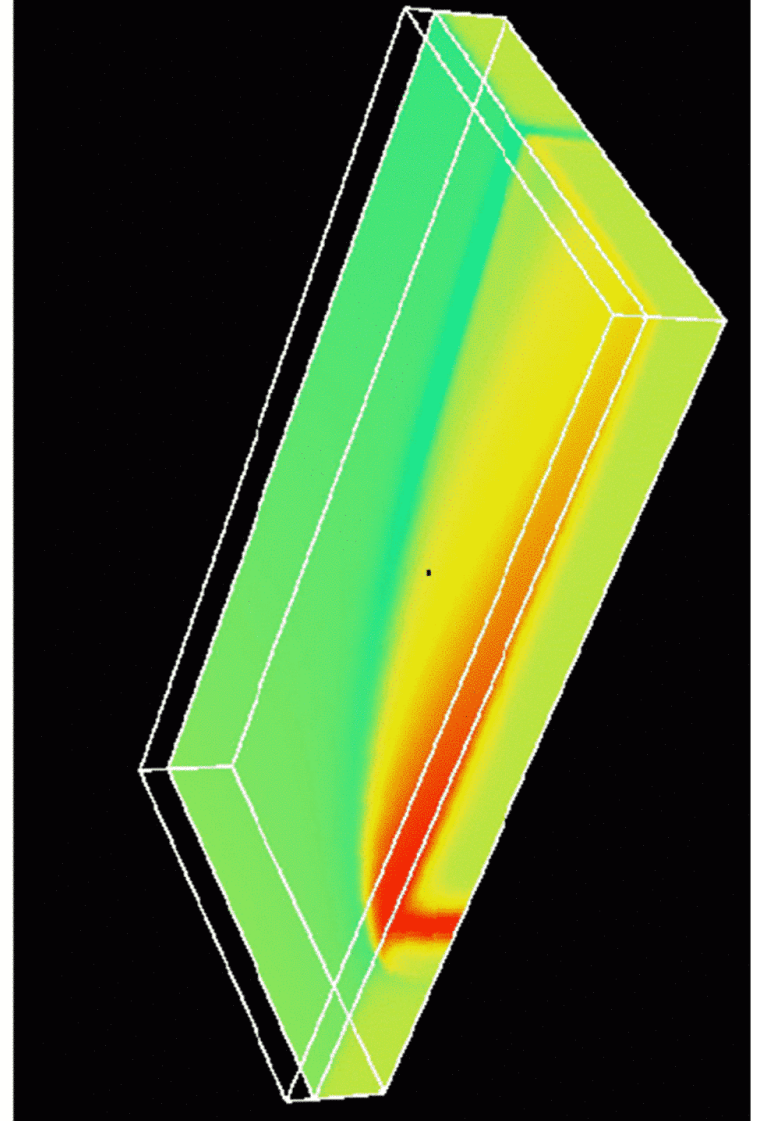
Constant temperature (or constant heat flux)



# A simple plume

QuickTime™ and a  
Graphics decompressor  
are needed to see this picture.

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## Heat sources from the Earth's interior

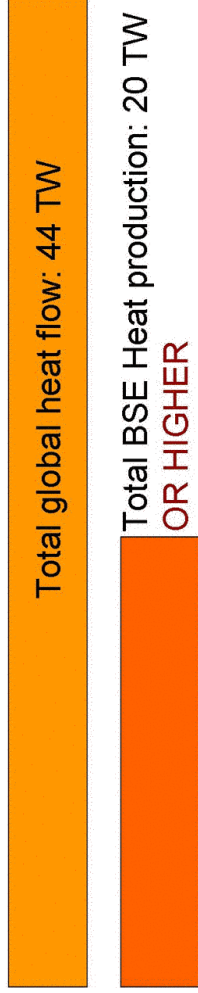
Parent	Daughter	half-life	heat production
238 U	206 Pb	4.49 By	94   w/kg of U
235 U	207 Pb	0.704 By	570
232 Th	208 Pb	14.0 By	26.6
40 K	40 Ar	1.25 By	27.9

## Heat budget of the Earth (all values given in terawatts)

Total global heat flow: 44 TW

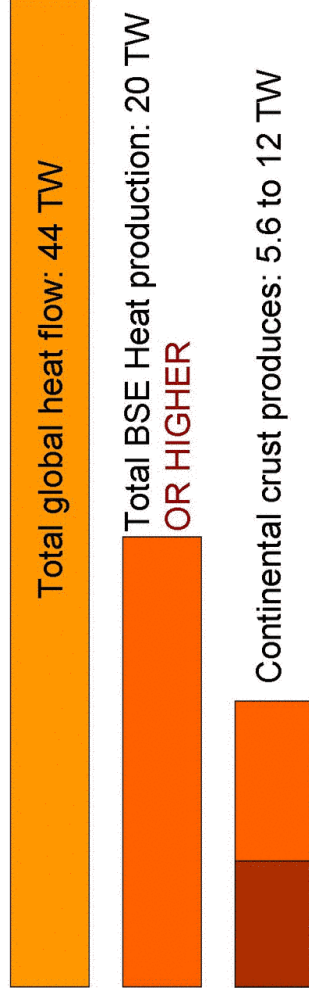
Stein, 1995

## Heat budget of the Earth (all values given in terawatts)



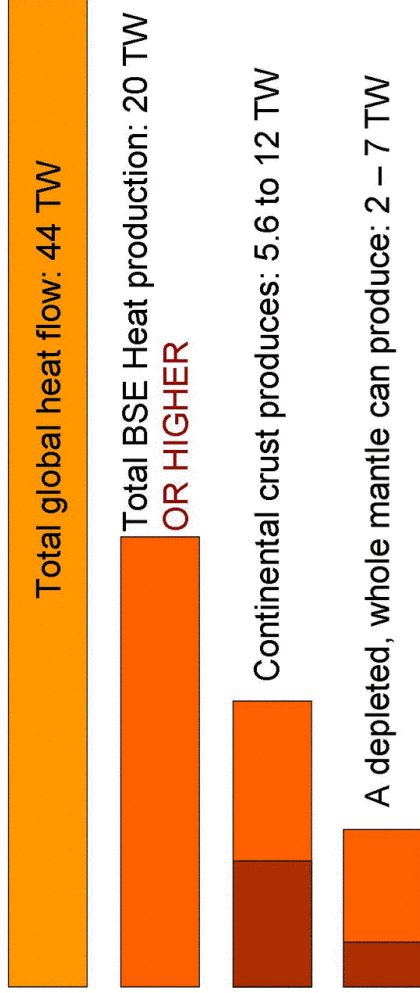
McDonough & Sun, 1995  
Van Schmus 1995  
Turcotte et al. 2001

## Heat budget of the Earth (all values given in terawatts)



Rudnick & Fountain, 1995  
Taylor & McLennan, 1995  
Gao et al., 1997  
Wedepohl, 1995

## Heat budget of the Earth (all values given in terawatts)



e.g. Stacey, 1992  
Zindler & Hart 1986

## Heat budget of the Earth (all values given in terawatts)



## Mantle viscosity is self-regulating

$$Mc \, dT/dt = MH - Aq$$

$$H = H_0 e^{-\lambda \tau}$$

$$\mu = \mu_0 e^{-\frac{E^* + Vz}{RT}}$$

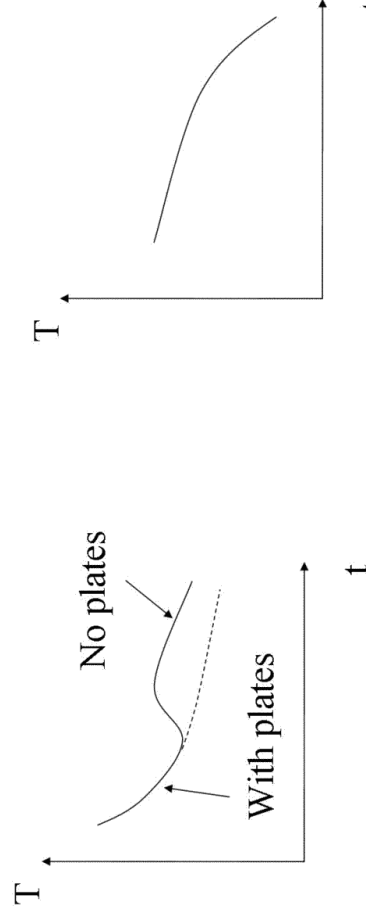
- As the mantle heats up, the viscosity drops
- More rapid convection cools the mantle
- Viscosity increases in a cooler mantle

$$dT/dt = - Aq/Mc(1 - Ur)$$

Where Ur, the Urey ratio, is MH/Aq

See, for example, Tozer, *Geophys. J. R. Astron. Soc.*, 9, 95-112 (1965) and *Phil Trans. Roy. Soc. London A*, 2588, 252-271 (1965), Schubert et al., *JGR* 85 2511-2518 (1980), Stevenson, *Comptes Rendues Geosciences* 335 99-111(2005)

## Changes in Convective Style (existence or extent of plate tectonics) can have big effects on Thermal History

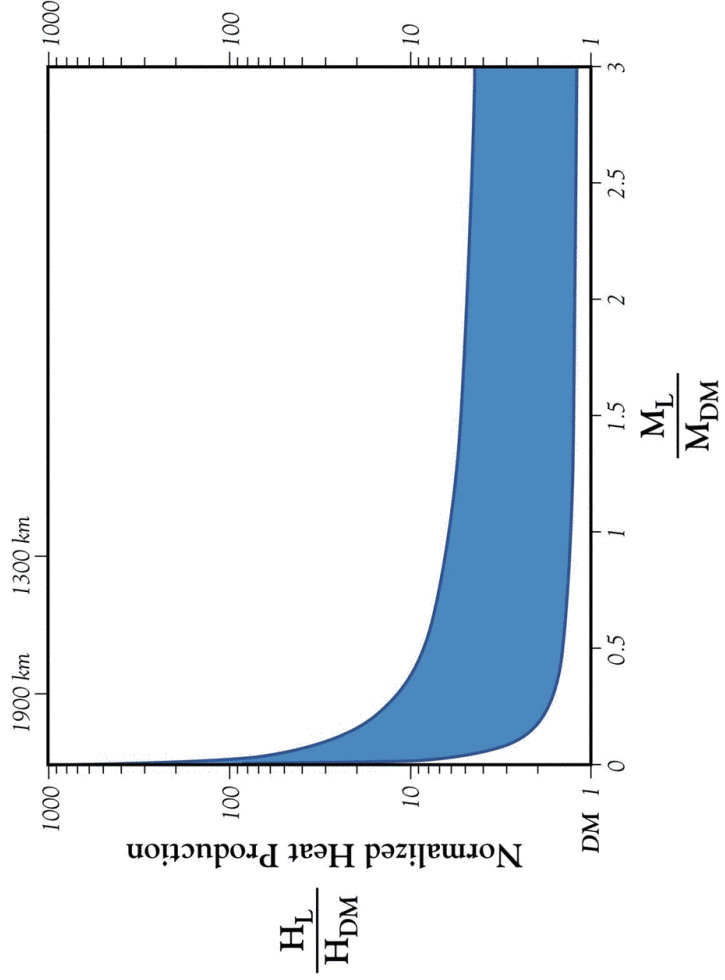


cf. Nimmo & Stevenson(2000) applied to Mars

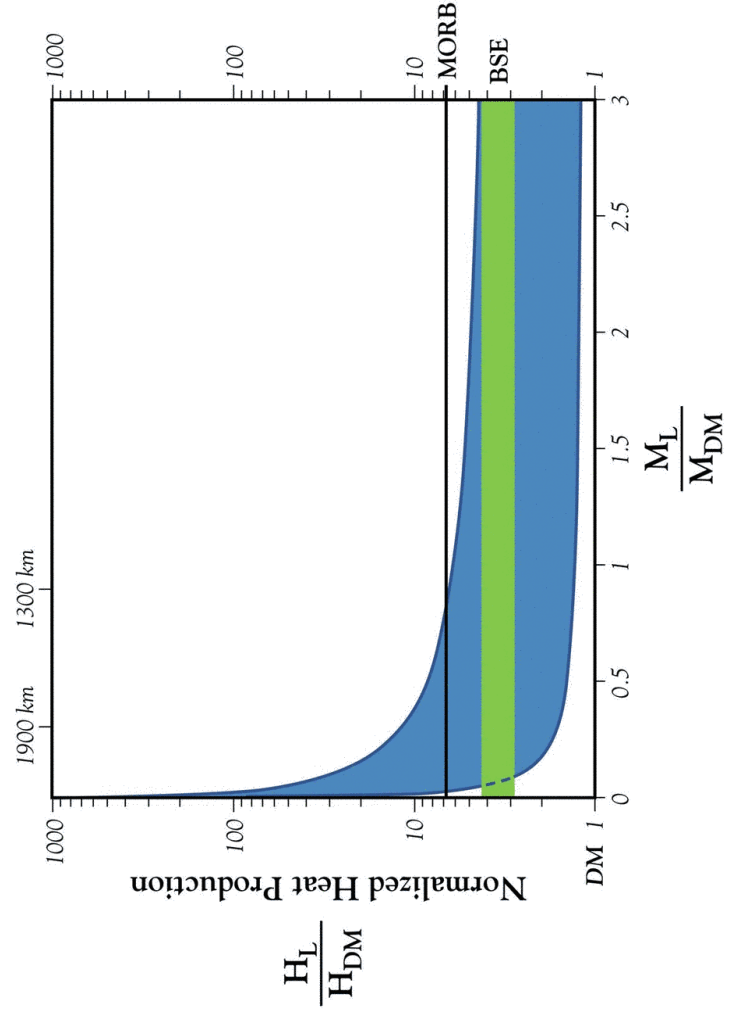
Notice the different curvature. cf. Korenaga, 2003. Also, Conrad & Hager, 1999, Solomatov (1995 onwards).

Images courtesy of D. Stevenson

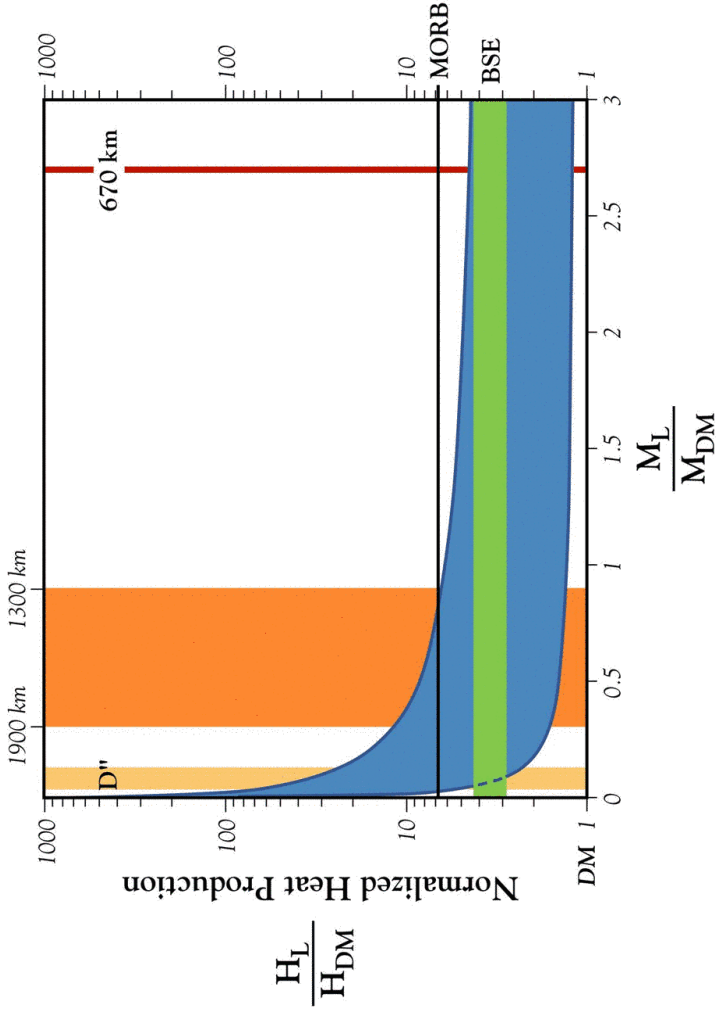




Ferrachat & Kellogg

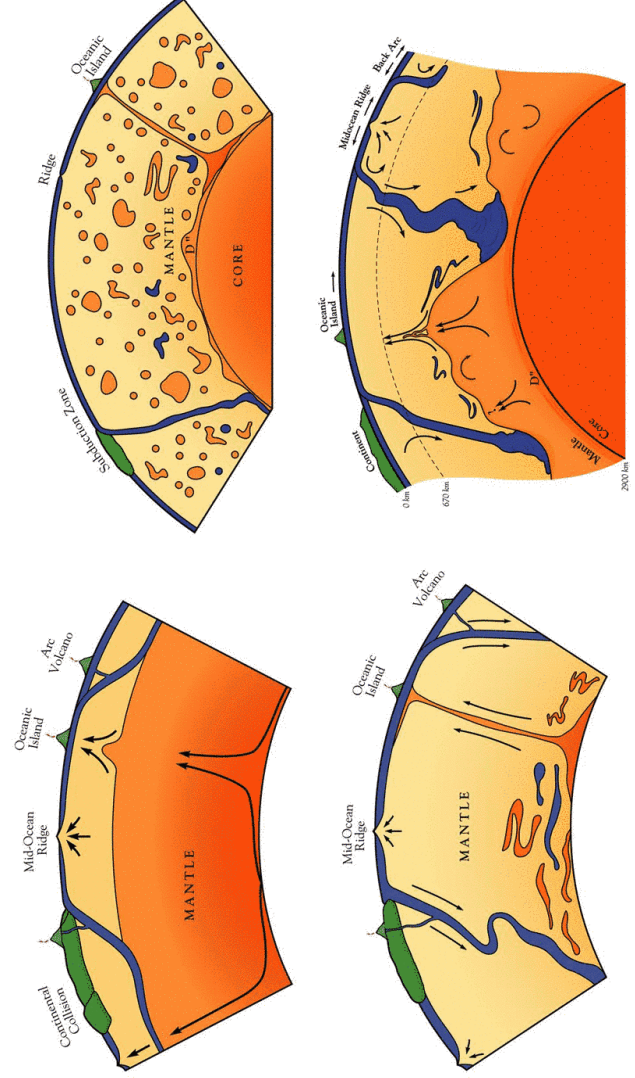


Ferrachat & Kellogg



Ferrachat & Kellogg

### A menagerie of models

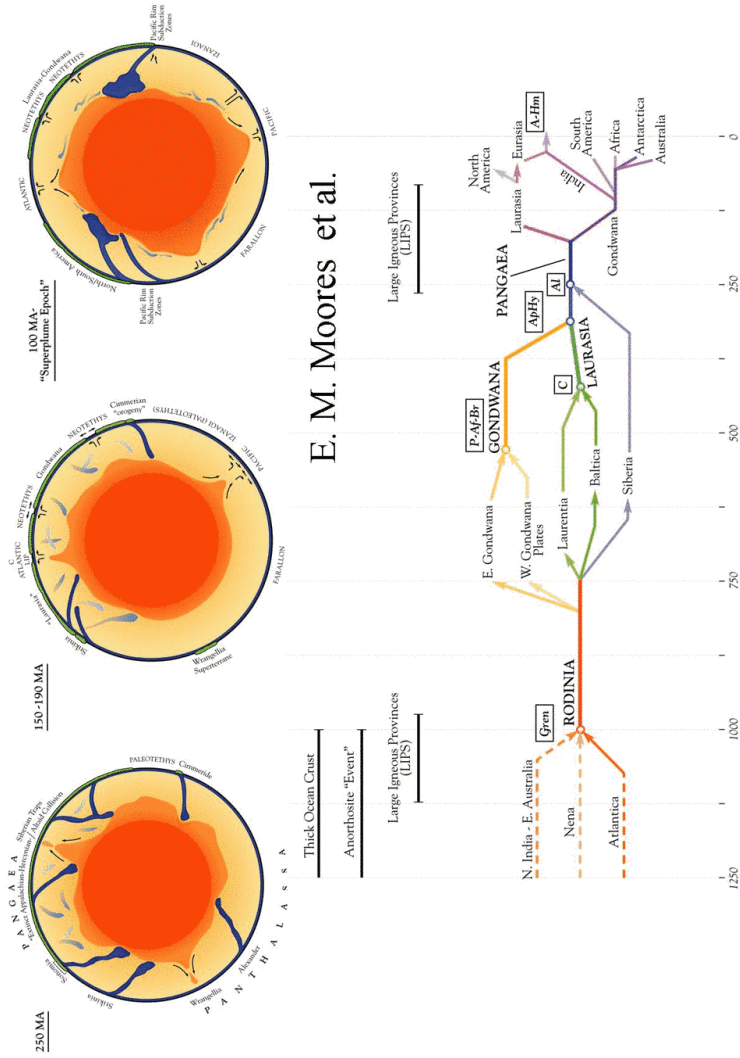


## Consider the Earth's mantle as a non-linear, chaotic system

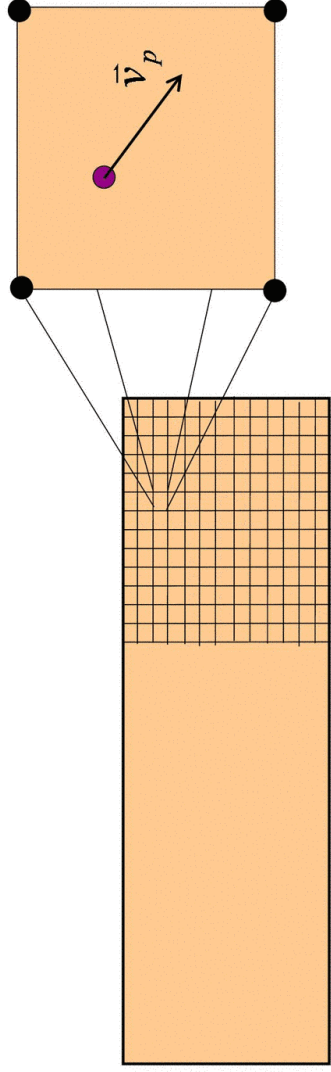
- Conservation of mass:  $\nabla \cdot \mathbf{u} = 0$
- Momentum:  $\nabla^2 \mathbf{u} - \nabla \cdot \boldsymbol{\tau} + RaT \hat{\mathbf{k}} = 0$  where  $\boldsymbol{\tau} = \mu \boldsymbol{\epsilon}$
- Heat flow:  $\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla(\kappa \nabla T)$
- Rayleigh Number -  $Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa_0 \mu_0} = 10^7$  - Referenced to Earth's near surface values
- Flow is driven by basal heating (T=1) and surface cooling (T=0)

See for example Stewart & Turcotte JGR, 94, 13707-13717 (1989)

## Thermal and dynamical evolution of the Earth?



We use passive “tracers” to track  
 trace elements  
 (such as strontium isotopes)



Example of mixing in 2-D  
 internally heated convection

QuickTime™ and a  
 Sorenson Video decompressor  
 are needed to see this picture.



300,000 particles  
 Starting position

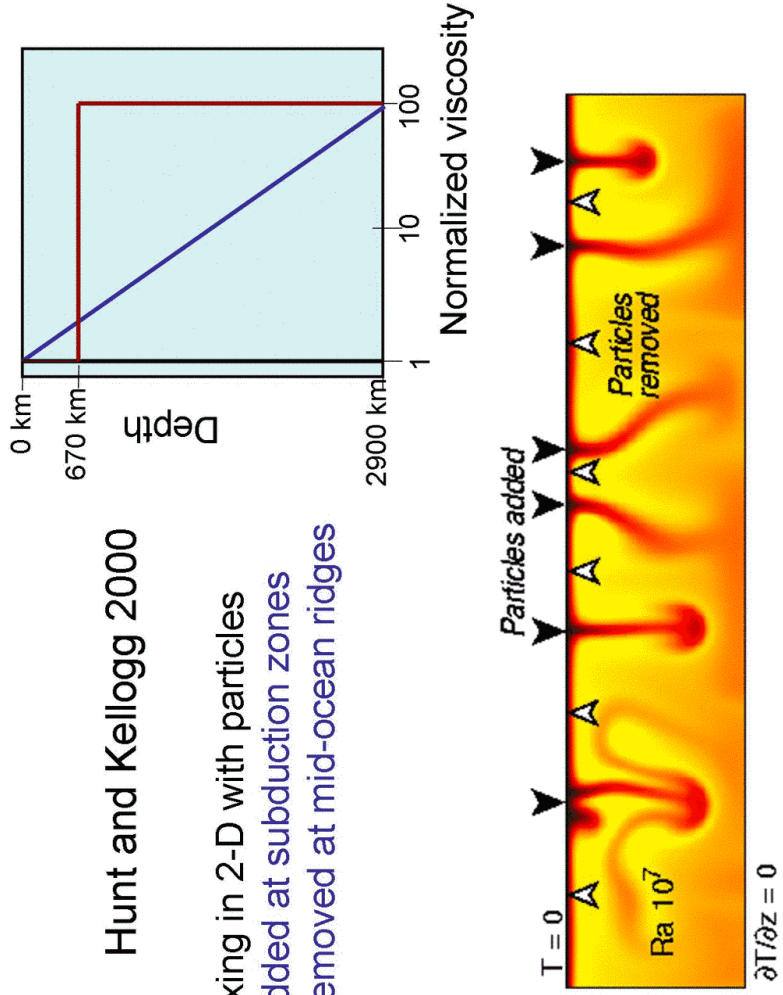
Numerical method:  
 the finite element  
 method



### Hunt and Kellogg 2000

Mixing in 2-D with particles

- Added at subduction zones
- Removed at mid-ocean ridges



Hunt & Kellogg - effect of viscosity on mixing

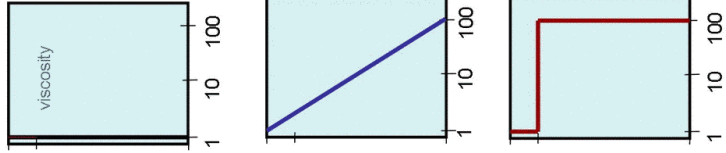
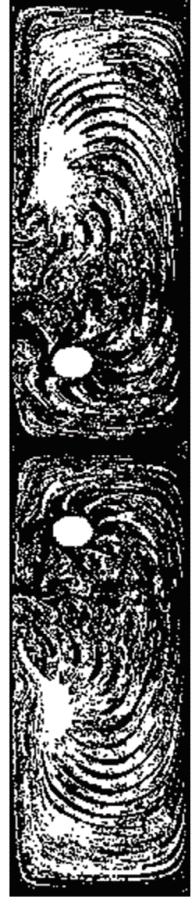
### Constant viscosity



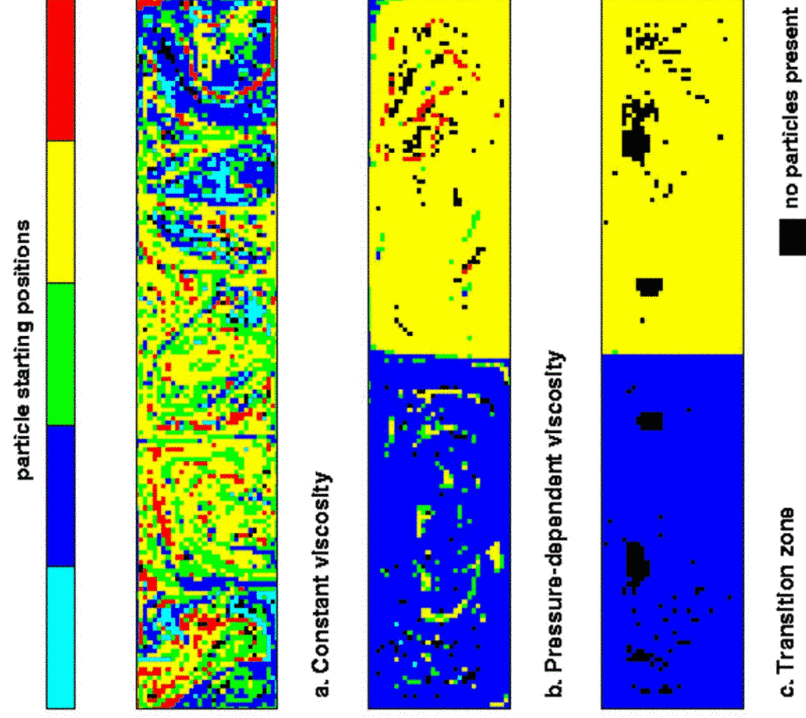
### Pressure-dependent viscosity: smooth increase



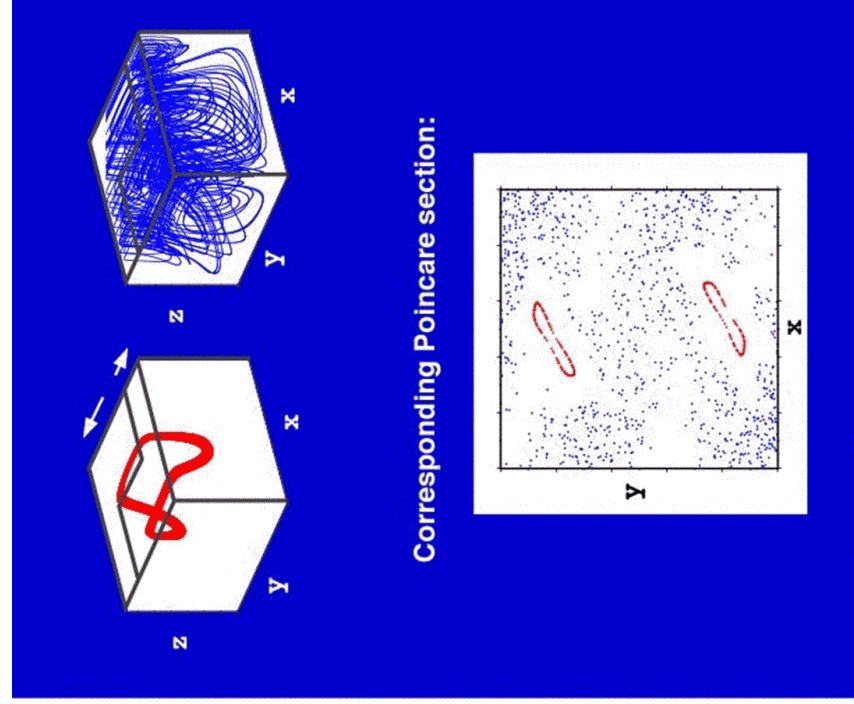
### Transition zone viscosity: Jump at 670 km





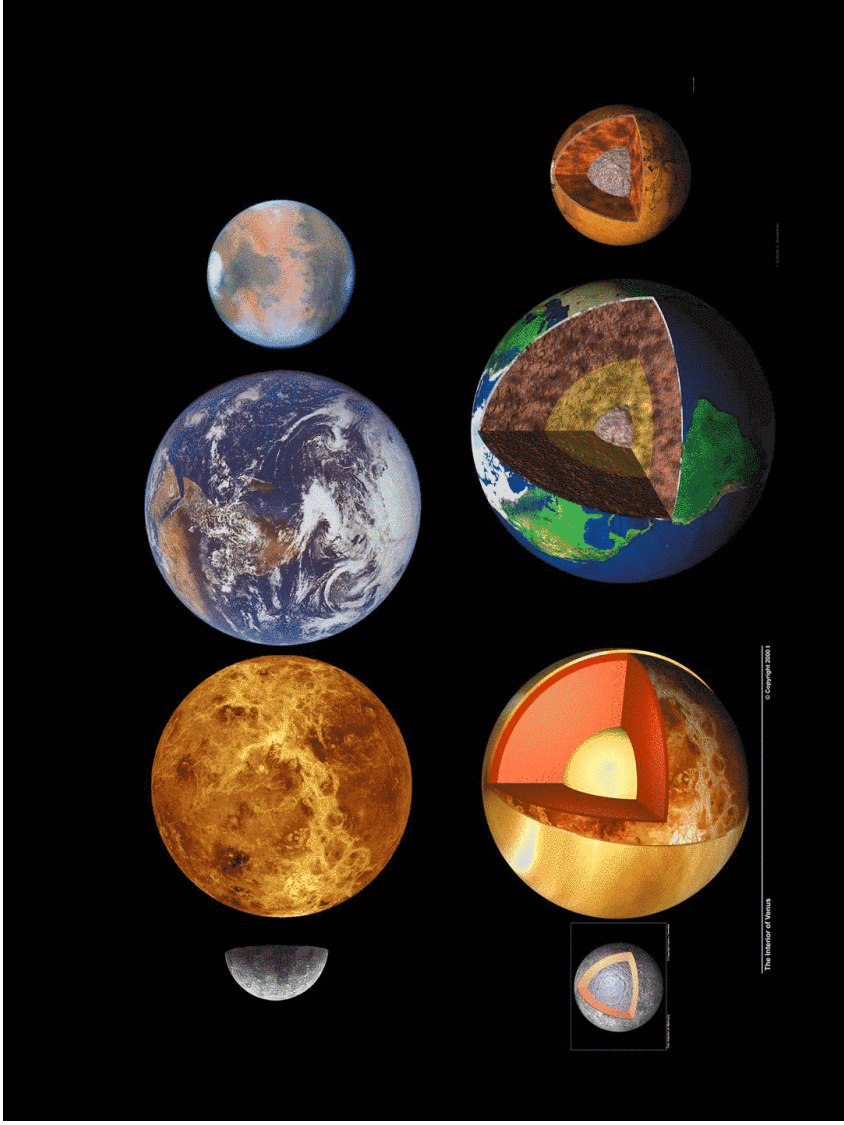


Initial location of particles (Hunt and Kellogg model)

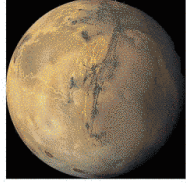
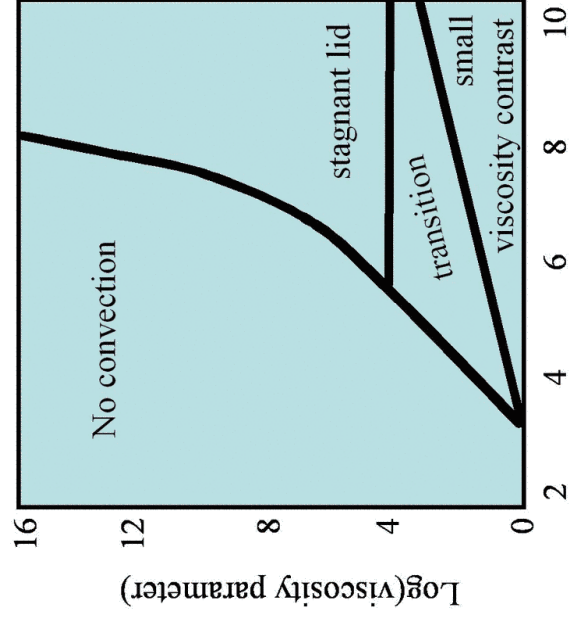


Ferrachat & Ricard, Mixing in 3-D plate driven flows

Chaotic trajectories occur even in steady-state flows



## A planetary perspective on mantle flow



See, for example, work by Solomatov, Moresi, Lenardic, Stevenson, others

