Spherical harmonics (normalized)

$$X_\ell^m(\Delta) \simeq \frac{1}{\pi\sqrt{\sin\Delta}} \cos\left(k\Delta - \frac{\pi}{4} + \frac{m\pi}{2} + \frac{1}{8k}\cot\Delta\right)$$

$$k = \ell + 1/2$$

Zeroth order:

$$X_\ell^m(\Delta) \simeq \frac{1}{\pi\sqrt{\sin\Delta}} \cos\left(\ell\Delta - \frac{\pi}{4} + \frac{m\pi}{2}\right)$$

of Legendre polynomials in the asymptotic expression for the displacement due to a seismic source for an epicentral distance close to 90°.

Figures 7 and 8 illustrate such operations on the vertical and longitudinal component records of the Costa Rica earthquake observed at SSB. The original data are represented on Figures 7a and 8a with the successive surface wave trains of Rayleigh waves and the successive X phases. Figures 7b and 8b show the extracted fundamental modes of Rayleigh waves.

The second and third overtones (Figure 7, c and d) have been extracted from the

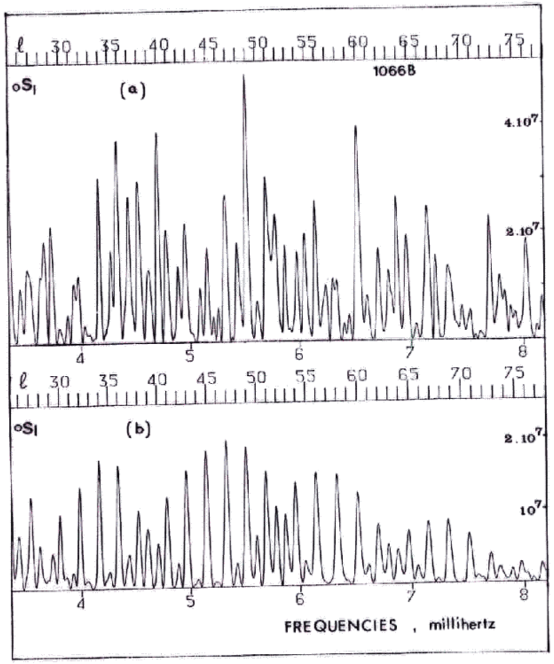


FIG. 6. Energy spectra versus frequency in milliHertz, for the longitudinal component recorded at SSB (France) for the Costa-Rica earthquake (3 April 1983). (a) Before variable filtering; (b) after variable filtering.

residual obtained by subtracting the pure fundamental mode from the original data. The amplitudes of the extracted higher modes compared to those of the respective original data show an important excitation of the overtones on the longitudinal components (Figure 8, c and d), about four times greater than on the vertical one.

EXAMPLES OF ENERGY SPECTRA

The records are tapered by the Connes window function (see "Methods") and

(3)

HETEROGENEOUS EARTH – PERTURBATION THEORY

$$\rho\, \partial_t^2 \underline{u} = H \underline{u} + \underline{f}$$

$$\underline{u} = u_0 + \delta u \qquad \rho = \rho_0 + \delta\rho \qquad H = H_0 + \delta H$$

To First order:

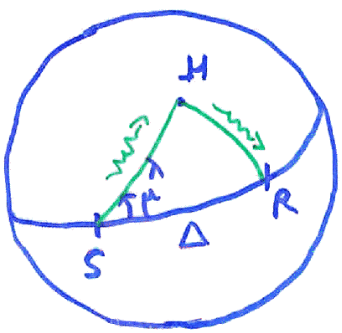
$$\underline{v}\cdot\underline{s}(t) = \mathrm{Re}\Big\{ \sum_k \Big[\sum_m R_k^m S_k^m$$ ← spherical - zeroth order

$$+\, it \sum_{mm'} R_k^m H_{kk}^{mm'} S_k^{m'}$$ ← Isolated multiplet

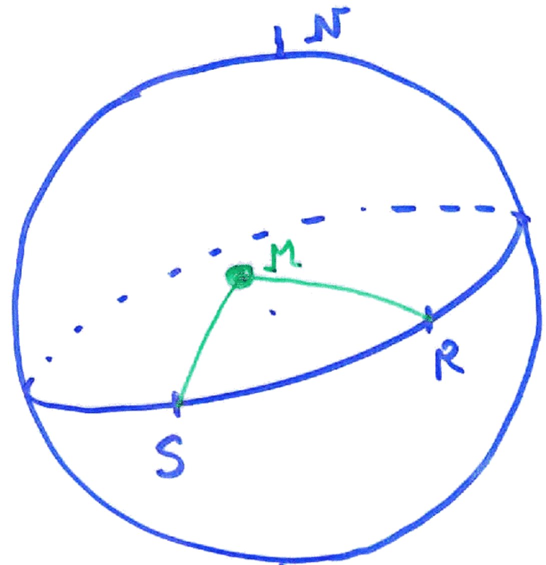
$$+ \sum_{\omega_{k'} \neq \omega_k} \frac{1}{\omega_k^2 - \omega_{k'}^2} \Big(\sum_{mm'} R_k^m H_{kk'}^{mm'} S_{k'}^{m'} + R_{k'}^{m} H_{k'k}^{m'm} S_k^{m} \Big) \Big\}$$

$$\Big] \exp i\omega_k t$$

coupling between different multiplets

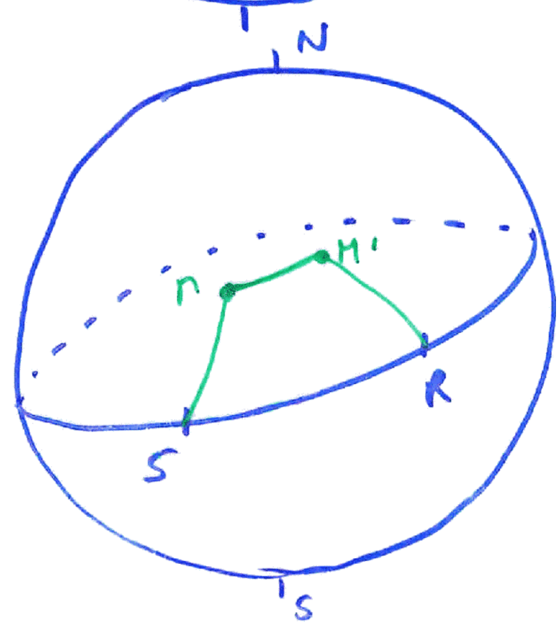


Single scattering (Born)



"Multiple scattering"

2nd order →



4

- Isolated multiplet case

$$\underline{v}\cdot\underline{s}(t) = \mathrm{Re}\sum_k \left(\sum_m R_k^m S_k^m + it\sum_{mm'} R_k^m H_{kk}^{mm'} S_{k}^{m'}\right) \exp i\omega_k t$$

$$= \mathrm{Re}\sum_k \left(\sum_m R_k^m S_k^m \left(1 + it\Lambda_k\right) e^{i\omega_k t}\right)$$

Λ_k = "location parameter" Jordan (1978)

frequency shift }

$1 + it\Lambda_k \simeq e^{it\Lambda_k}$

To zeroth order in l :

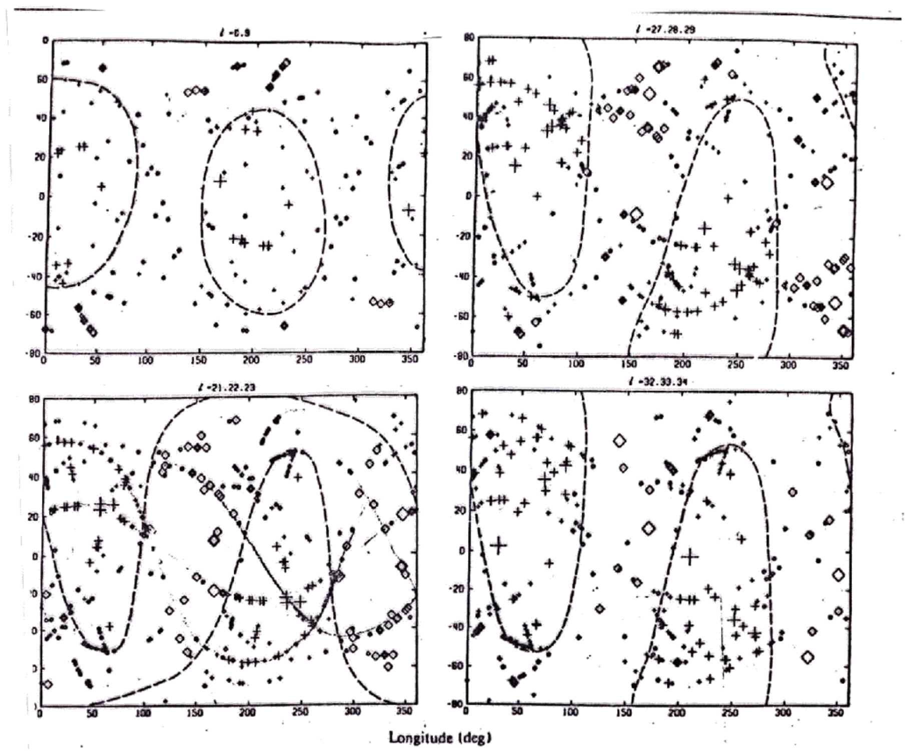
$$\Lambda_k = \delta\hat{\omega}_k = \frac{1}{2\pi}\int_0^{2\pi} \delta\omega_k(\lambda, 0)\, d\lambda$$

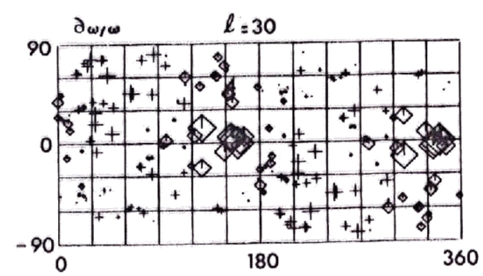
$$\delta\omega_k = \int_0^a \left[M_k^s(r)\frac{\delta v_s}{v_s} + M_k^p(r)\frac{\delta v_p}{v_p} + M_k^\rho(r)\frac{\delta\rho}{\rho}\right] r^2 dr$$

"local frequency"

Frequency shift Λ_k depends only on the average structure along the great circle path connecting source and receiver

→ even order structure





5

- COUPLING INCLUDED AMONG DIFFERENT MULTIPLETS

(A) "Along branch" coupling only
To zeroth order
"PATH AVERAGE APPROXIMATION"

Additional term from coupling :

$$-k\,\delta\Delta_k \,*\, \frac{\sin(k\Delta - \pi/4)}{\pi\sqrt{\sin\Delta}} \exp i\omega_k t$$

$$\delta\Delta_k = \frac{a\Delta}{kU}\left(\delta\hat{\omega}_k - \delta\tilde{\omega}_k\right) \qquad k = l + 1/2$$

$$\delta\tilde{\omega}_k = \frac{1}{\Delta}\int_0^{\Delta} \delta\omega_k(\lambda, 0)\, d\lambda$$

"Minor arc average"

$$\underline{v}\cdot\underline{s}(t) = \mathrm{Re} \sum_k a_k \, \Psi_l^0(\Delta + \delta\Delta_k) \exp i(\omega_k + \delta\hat{\omega}_k)t$$

($\delta\Delta_k$: distance shift; $\delta\hat{\omega}_k$: freq shift)

more generally

$$\underline{v}\cdot\underline{s}(t) = \mathrm{Re} \sum_k A_0^k(\Delta + \delta\Delta_k) \exp i(\omega_k + \delta\hat{\omega}_k)t$$

A_0^k = amplitude of mode k in SNREI model

(6)

(A) "Along branch coupling" PAVA
zeroth order

1D kernels
"Surface wave approx

(B) Across branch coupling
zeroth order
→ ray character
→ 2D Kernels
→ "NACT" Li and Romanowicz 1995

A, B : sensitivity in the vertical plane containing source and receiver

(C) Add higher order terms in 1/l to A and B
→ out of plane effects
"focusing"
depend on $\nabla_2^T \delta\hat{\omega}$, $\nabla_2^T \delta\tilde{\omega}$
second transverse derivatives along source station path
→ 2.5D Kernels